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To cite this article: Petr Kral *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **471** 052052

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Four-Point Bending Test on a High Reinforced Concrete Beam: Nonlinear Numerical Analysis Using Material Parameter Identification

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Abstract. The numerical analysis of concrete or reinforced concrete structures using nonlinear mechanics tools is currently the focus of interest of many scientific institutions. Today, numerical modelling of the real behaviour of concrete and concrete reinforcement is mainly performed with the aid of nonlinear constitutive relations (nonlinear material models). Current modern computational systems based on the implicit or explicit finite element method include a relatively large amount of nonlinear material models intended for modelling the real behaviour of concrete and concrete reinforcement. However, the accuracy of the simulated behaviour of real concrete and reinforced concrete structures depends on the correct definition of the input parameter values of these material models. This makes the nonlinear numerical analysis process quite difficult because the correct definition of the input parameter values of (in particular) the material models for the modelling of concrete is often not a trivial task. However, a combination of nonlinear numerical analysis with the identification of the input parameter values of the used material models based on relevant experimental data can be currently employed to address this task. The aim of this paper is to perform a nonlinear numerical analysis of a high reinforced concrete beam stressed by four-point bending so that the numerical response of the beam corresponds to the real response of the beam as closely as possible. For this purpose, an optimisation-based parameter identification process is used in this paper. Within this process, the input parameter values of a nonlinear material model of concrete which is known as the modified CSCM model and implemented in LS-DYNA finite element software are identified (optimised). Specifically, the parameter identification is based on the combination of optimisation methods with nonlinear numerical simulations and experimental data. The optimised input parameter values of the material model are an important result of the performed parameter identification process because they make it possible to achieve the primary aim. It can be concluded that the applied procedures and obtained results can be advantageously used in further analyses of concrete or reinforced concrete structures.

1. Introduction

Concrete is currently one of the most used materials in the construction industry. Its extensive use for the construction of new building structures leads to the refinement of the design of these structures through advanced numerical analysis [1-3]. Today, the advanced numerical analysis of concrete and reinforced concrete structures using nonlinear mechanics tools is the focus of interest of many scientific institutions. The use of nonlinear mechanics tools mainly means the use of physical



nonlinearity in numerical calculations because concrete and reinforced concrete structures belong between structures which exhibit small deformations [4, 5]. Therefore, numerical modelling of the response of concrete and concrete reinforcement is mainly performed with the aid of nonlinear constitutive relations (nonlinear material models) in the present days. Current modern computational systems based on the implicit, explicit or combined finite element method [6-8] include a variety of nonlinear material models intended for modelling of concrete and concrete reinforcement. However, the accuracy of the simulated response of real concrete or reinforced concrete structures depends on the correct definition of the input parameter values of these material models. Unfortunately, the correct definition of the input parameter values of (in particular) the material models intended for the modelling of concrete is very often not a trivial task. This makes the whole nonlinear numerical analysis process quite difficult. At present, however, there is an approach that allows the mentioned nontrivial task to be addressed. This is a combination of nonlinear numerical analysis with the complex process of identification of the input parameter values of the used material models which is based on relevant experimental data [9, 10].

Generally, the process of identification of the input parameter values of the nonlinear material models is based on the combination of computer simulations and experimental data with identification methods. The basic method that can be used for the purposes of identification of the material parameter values is the classic Trial and error method [11]. However, this method is very noneffective with a large number of identified parameter values. Very popular identification methods are currently methods based on the exercise of artificial neural networks [12] which are very effective for the mentioned purposes. Last but not least, optimisation methods can be used for identification purposes too. These methods can advantageously be combined with sensitivity analysis [13-16] and (like methods based on the exercise of artificial neural networks) they are very effective for identification purposes.

The aim of this paper is to perform a nonlinear numerical analysis of a high reinforced concrete beam stressed by four-point bending so that the computer numerical response of the beam corresponds to the real (experimental) response of the beam as closely as possible. For this purpose, an optimisation-based parameter identification process is used in this paper. This process is thus based on the combination of nature inspired and direct optimisation methods with nonlinear numerical simulations and experimental data. Within the parameter identification process used, the input parameter values of a nonlinear material model of concrete which is known as the modified CSCM model and implemented in LS-DYNA finite element software [17] are identified (optimised). The result of the numerical simulation using the resulting identified parameter values of the modified CSCM model is finally compared to the experimental data used.

2. Task formulation and experimental data

In this paper, the solved task is a nonlinear numerical simulation of the four-point bending test carried out on a high reinforced concrete beam. This test was performed within the experimental measurement described in [18]. Based on this test, experimental data which have been used for identification purposes in this paper were obtained. A schematic representation of the test is shown in figure 1.

Figure 1 shows the geometry of a tested beam reinforced by a reinforcing bar whose geometry and position are shown in figure 1 too. The beam thickness was 76.2 mm. Further, the geometry and position of the washers located at the loading and support points can also be seen in figure 1. The parameters of the used (28-days) concrete and concrete reinforcement were (acc. to [18]) as follows:

Concrete:

- Modulus of elasticity – 20.68 GPa
- Poisson's ratio – 0.15
- Unconfined uniaxial compressive strength – 24.13 MPa
- Unconfined uniaxial tensile strength – 3.10 MPa

Concrete reinforcement:

- Young's modulus of elasticity – 210.00 GPa
- Yield strength – 344.75 MPa
- Cross-sectional area of reinforcing bar – 0.000071 m²

During the course of loading, the tested beam was loaded (according to figure 1) by forces F whose magnitude linearly and very slowly increased over time up to 80 kN. Therefore, the applied loading had a constant velocity and it was static. The quantity obtained from the performed test was the vertical displacement U which was measured in the middle of the span of the high reinforced concrete beam (point P in figure 1).

Figure 2 shows a graph which typifies the dependence of the force F on the vertical displacement U . This graph overall characterises the experimental data obtained from the test described above. As already mentioned, these data have been used in this paper.

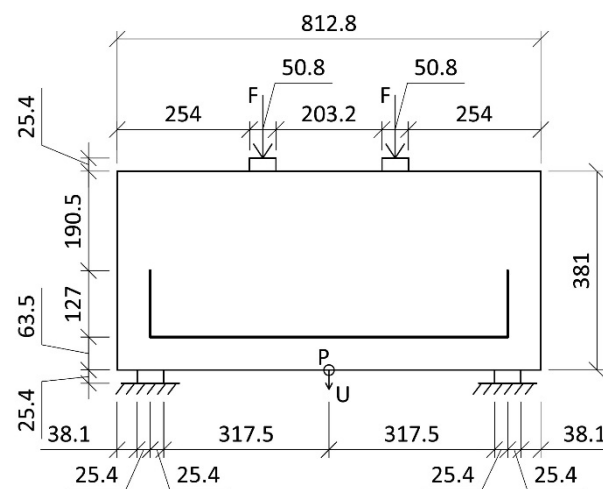


Figure 1. Schematic representation of the four-point bending test (dimensions in millimetres).

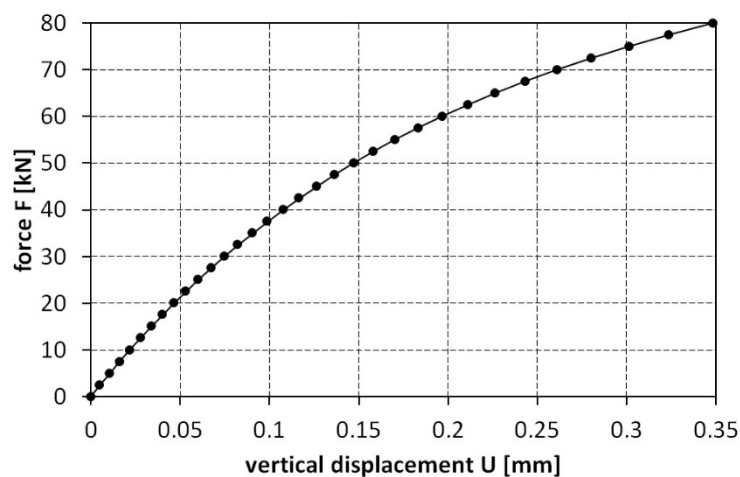


Figure 2. Experimental data.

3. Nonlinear numerical analysis

The process of identification of the input material parameter values used in this paper required the repeated performing of nonlinear numerical simulations of the solved task. For this purpose, material models were selected (for description of simulated materials), and a computational model was created in LS-DYNA finite element software.

3.1. Material models

For modelling of concrete, a nonlinear material model known as the modified CSCM model (Continuous Surface Cap Model) [19] was selected. This is an elasto-plastic constitutive model of concrete which is based on the yield function [20] that can be mathematically written as:

$$Y(I_1, J_2, J_3) = J_2 - \Re(J_3)^2 F_f^2(I_1) F_c(I_1, \kappa) \quad (1)$$

where I_1 is the first invariant of the stress tensor, J_2 and J_3 are second and third invariants of the deviatoric stress tensor, $\Re(J_3)$ is the Rubin strength reduction factor and κ is the cap hardening parameter. The yield function is composed of two parts, these being the hardening compaction function $F_c(I_1, \kappa)$ and the shear failure function $F_f(I_1)$. These two functions are combined using a multiplicative formulation. The expression of the hardening compaction function is given by equations:

$$F_c(I_1, \kappa) = 1 - \frac{(I_1 - L(\kappa))^2}{(X(\kappa) - L(\kappa))^2} \text{ for } I_1 > L(\kappa) \quad (2)$$

$$F_c(I_1, \kappa) = 1 \text{ for } I_1 \leq L(\kappa) \quad (3)$$

together with equations:

$$L(\kappa) = \kappa \text{ for } \kappa > \kappa_0 \quad (4)$$

$$L(\kappa) = \kappa_0 \text{ for } \kappa \leq \kappa_0 \quad (5)$$

$$X(\kappa) = L(\kappa) + R F_f(I_1) \quad (6)$$

where R is the cap aspect ratio. The shear failure function is defined by the equation:

$$F_f(I_1) = \alpha - \lambda \exp^{-\beta I_1} + \theta I_1 \quad (7)$$

where α , β , λ , and θ are material constants which are usually determined on the basis of the triaxial compression tests.

Within its formulation, the modified CSCM model allows the effect of strain rate on the resulting response of material to be taken into account. However, this capability of the model can be neglected within the calculations through the model parameter *IRATE* (*IRATE* = 1: Rate effects turned on; *IRATE* = 0: Rate effects turned off). If the *IRATE* parameter is equal to zero, the response of the model is static, independent of loading rate. It follows that the material model can be used not only for dynamic calculations, but also for static calculations. The parameter *IRATE* equal to zero has been used within the calculations performed for this paper, so the response of the model was always static. It is also good to mention that the model includes an algorithm for limiting the dependence of results on the size of the finite element mesh.

The modified CSCM model contains a total of 3 input material parameters which must be defined for calculations. On the basis of these 3 parameters, other parameters are automatically generated. These 3 parameters of the modified CSCM model were optimised within the context of parameter identification process performed. Descriptions of the parameters are given in table 1 [17] along with the units used.

For modelling of concrete reinforcement, the Plastic Kinematic material model was selected. This is a bilinear material model that is suitable for steel modelling. This model allows modelling of the

material without or with hardening while the hardening can be isotropic or kinematic. Further, the model also allows the effect of strain rate on the resulting response of material to be taken into account. In this paper, the material was modelled without hardening, and the effect of strain rate on the resulting response of material was neglected by defining the relevant parameters with zero values. The basic input material parameters of the Plastic Kinematic model are presented in table 1 [17] along with their descriptions and the units used.

For modelling of washers, the Linear Elastic material model was selected because more detailed model was not necessary for describing their behaviour. The input material parameters of the Linear Elastic model are given in table 1 [17] along with their descriptions and the units used.

Table 1. The input material parameters of the used material models.

Parameter	Parameter description	Unit
Modelling of concrete (the modified CSCM model)		
<i>RO</i>	Mass density	Mg/mm ³
<i>FPC</i>	Unconfined uniaxial compressive strength	MPa
<i>DAGG</i>	Maximum aggregate size	mm
Modelling of concrete reinforcement (the Plastic Kinematic model)		
<i>RO</i>	Mass density	Mg/mm ³
<i>E</i>	Young's modulus of elasticity	MPa
<i>PR</i>	Poisson's ratio	-
<i>SIGY</i>	Yield strength	MPa
<i>ETAN</i>	Tangent modulus (<i>ETAN</i> = 0: No hardening)	MPa
Modelling of washers (the Linear Elastic model)		
<i>RO</i>	Mass density	Mg/mm ³
<i>E</i>	Young's modulus of elasticity	MPa
<i>PR</i>	Poisson's ratio	-

3.2. The computational model

In order to create the computational model of the solved task, finite element models for high beam, reinforcing bar and washers had to be created, and boundary conditions (supports, loading) had to be applied. The finite element model of the high beam was created by discretisation of its geometric model by a regular mesh of explicit 3D structural finite elements (bricks). The size of the finite elements has been chosen so that the length of the time step in explicit algorithm was not too small. This (together with the use of the symmetry of the task) has resulted in an acceptable calculation time. The finite element model of the reinforcing bar was created from explicit 3D beam finite elements. The lengths of the individual beam finite elements were adapted to the lengths of the brick edges in order to ensure the continuity of both finite element models (both finite element types had common nodes). The dimensions of the rectangular cross-section of the beam finite elements were chosen so that the resulting area corresponded to the cross-sectional area of the reinforcing bar from the experiment (0.000071 m²). The geometric models of the washers were discretised by a regular mesh of bricks. The size of the finite elements in the washer models corresponded to the finite element size in the model of the high beam. The complete finite element model of the four-point bending test of the high reinforced concrete beam is shown in figure 3 (only half of the model → the task symmetry was used).

In terms of boundary conditions, the supports were applied to the models of the washers and to the model of the high beam in the symmetry axis. The washer at the support point of the beam was prevented from moving horizontally and vertically, and the washer at the loading point of the beam was prevented from moving horizontally only. The symmetrical boundary conditions were applied to the model of the high beam. Loading was applied in the form of a linearly increasing pressure over

time acted to the washer (the final pressure value corresponded to a force of 80 kN), and in the form of taking into account the own weight of the structure.

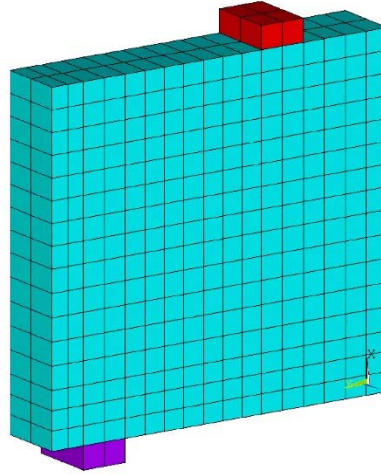


Figure 3. The finite element model of the four-point bending test.

4. Optimisation-based parameter identification process

Within this paper, the used parameter identification process was performed using the optiSLang optimisation software [21]. The whole process consisted of performing the global and local optimisation.

4.1. Global optimisation

Within the global optimisation, optimal values of the identified material parameters were sought that would enable the computer simulation result to approximate the used experimental data very well. In other words, optimal values of the identified material parameters were sought that would minimise the value of the selected objective function. The global optimisation was thus based on searching the global minimum of the objective function [22]. The objective function used within this paper had the form of Root-Mean-Square Deviation (*RMSD*), so its mathematical expression was as follows:

$$RMSD = \sqrt{\frac{\sum_{i=1}^n (U_{sim,i} - U_{exp,i})^2}{n}} \rightarrow \min \quad (8)$$

where $U_{sim,i}$ were the vertical displacement values obtained from the appropriate numerically-simulated data for the given forces, and $U_{exp,i}$ were the vertical displacement values obtained from the used experimental data for the same forces. The value of parameter n corresponded to the number of data (the number of dots) on the loading curve in figure 2.

Based on the test calculations, it was found that the shape of the numerically-simulated loading curve was exclusively influenced by the material parameter values of the concrete material model. Therefore, only these parameters have been optimised. Within the global optimisation, the optimised parameters formed the design vector that had the following form:

$$\mathbf{X}_{design} = \{RO, FPC, DAGG\}^T \quad (9)$$

The design vector parameters were defined as continuous random variables with a probability distribution at the interval given by the boundary values. These boundary values corresponded to the

parameter limit values given by LS-DYNA software [17] (see table 2). The initial parameter values necessary for the first optimisation design corresponded to the experiment values for which the numerical simulation result is shown in figure 4 for comparison purposes. The material parameters of other material models were defined deterministically by values from the experiment (see table 2).

For global optimisation, an optimisation algorithm known as an Evolutionary Algorithm (EA) [21] was used. An EA is one of the optimisation algorithms which exploit processes inspired by biological evolution, including (for example) reproduction, mutation and recombination. A total of 400 generations of EA were needed to find the global minimum of the selected objective function. This minimum corresponded to the best generation of EA from which the optimal values of the identified material parameters were obtained.

Table 2. Deterministic and optimal material parameter values of the used material models.

Parameter	Unit	Determinist. values	Initial values	Boundary values (MIN)	Boundary values (MAX)	Optimisation (best generation/iteration)	
						Global (EA)	Local (Simplex)
Modelling of concrete (the modified CSCM model)							
<i>RO</i>	Mg/mm ³	-	2.400 x 10 ⁻⁹	2.100 x 10 ⁻⁹	2.450 x 10 ⁻⁹	2.430 x 10 ⁻⁹	2.433 x 10 ⁻⁹
<i>FPC</i>	MPa	-	24.130	20.000	58.000	20.000	20.001
<i>DAGG</i>	mm	-	16.000	8.000	32.000	8.000	8.002
<i>RMSD</i>	mm	-	-	-	-	0.00745623	0.00745617
Modelling of concrete reinforcement (the Plastic Kinematic model)							
<i>RO</i>	Mg/mm ³	7.850 x 10 ⁻⁹	-	-	-	-	-
<i>E</i>	MPa	210000	-	-	-	-	-
<i>PR</i>	-	0.3	-	-	-	-	-
<i>SIGY</i>	MPa	344.750	-	-	-	-	-
<i>ETAN</i>	MPa	0	-	-	-	-	-
Modelling of washers (the Linear Elastic model)							
<i>RO</i>	Mg/mm ³	7.850 x 10 ⁻⁹	-	-	-	-	-
<i>E</i>	MPa	210000	-	-	-	-	-
<i>PR</i>	-	0.3	-	-	-	-	-

4.2. Local optimisation

Within the local optimisation, just as in the case of the global optimisation, optimal values of the identified material parameters were sought that would minimise the value of the selected objective function given by equation (8). The goal of the local optimisation was to search the vicinity of the global minimum for the purpose of its refinement.

The design vector for local optimisation naturally corresponded to the design vector from global optimisation. The local optimisation was performed using the direct optimisation method known as the Simplex method [21]. The Simplex method belongs to those iterative methods that are carried out systematically to determine the optimal solution from a set of feasible solutions. For the calculations performed using the Simplex method, the best generation of EA was used as the starting point. A total of 70 iterations of the Simplex method were needed to refine the global minimum of the selected objective function. This refinement corresponded to the best Simplex method iteration from which the optimal values of the identified material parameters were obtained.

4.3. Results

The optimal values of the identified material parameters obtained from the best generation of EA and the best Simplex method iteration are presented in table 2, including the appropriate minimal values of the objective function. It can be seen from table 2 that the Simplex method really provided better result

than the EA because the objective function value from the local optimisation is lower than its value from the global optimisation. However, both *RMSD* values are very similar.

Figure 4 shows a comparison of experimental data with numerically-simulated data obtained from the numerical simulation in which the resulting optimal parameter values of the modified CSCM model from local optimisation (Simplex method) were applied. It can be concluded from figure 4 that the material parameter values of the modified CSCM model were identified very accurately because the numerical simulation result ensures very good approximation of the experimental data. As already mentioned, for comparison, the numerically-simulated data obtained from the numerical simulation, in which the initial (experimental) material parameter values were applied, are illustrated in figure 4, too. These data clearly show that the parameter identification process performed in this paper was needed.

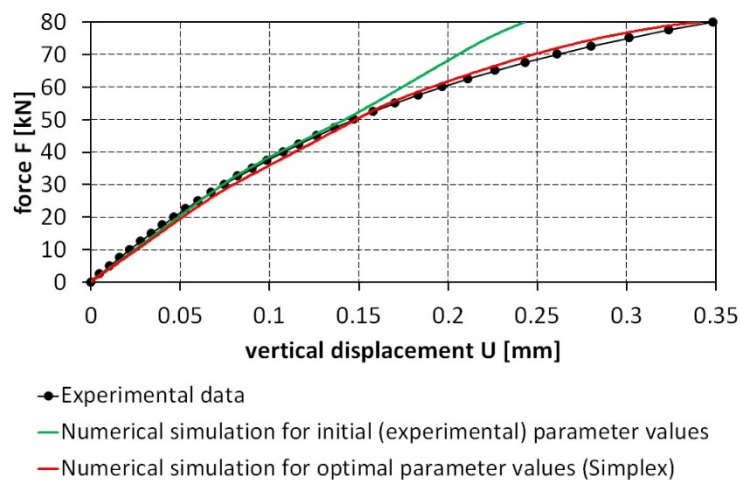


Figure 4. Graphical comparison of experimental data with numerically-simulated data.

5. Conclusions

This paper was focused on performing the nonlinear numerical analysis of the high reinforced concrete beam stressed by four-point bending with the use of the optimisation-based parameter identification process. For this purpose, experimental data obtained from the real four-point bending test, and optimisation methods, such as nature inspired Evolutionary Algorithm and direct Simplex optimisation method, were used. Within the parameter identification process, the input material parameter values of the modified CSCM model were optimised (identified). The results of the parameter identification process showed that the modified CSCM model is a suitable tool to describe the nonlinear behaviour of real concrete. They also indicated that, in cases where the parameter values of the used material models are conveniently selected and entered, we can obtain a very good agreement between the experimental data and numerical simulation of the four-point bending test. Importantly, this claim was proved by the outcome of the numerical simulation performed using the resulting optimal material parameter values of the modified CSCM model from the Simplex method. The discussed result approximated the used experimental data very well.

This paper has also shown that with the help of powerful identification tools it is possible to obtain very good fit for the input parameters of nonlinear material models. The product of the parameter identification process performed will be exploited for further research of authors concerning the nonlinear numerical analysis of concrete structures.

Acknowledgments

This paper has been created with the financial support of the project GACR 17-23578S “Damage assessment identification for reinforced concrete subjected to extreme loading” provided by the Czech Science Foundation, and also with the support of the project FAST-J-18-5604 provided by the Brno University of Technology fund for specific university research.

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