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Dynamic Response of Overhead Transmission Line in Turbulent Wind Flow with Application of the Spectral Element Method

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Abstract. In the paper, the response of overhead transmission line in turbulent wind flow with use of the spectral method is investigated. The analysis is performed for wind flow model that reflects the real conditions. Numerical analysis investigates the vibrations of the conductor due to different parameters of turbulence. For comparison, the excitation of the sine function is investigated. Spectra of longitudinal wind velocity for the numerical case, as well as the spectra of Karman, FSU and the proposal of author's models are analysed. Counihan and ESDU integral length scale are performed.

1. Introduction

Overhead transmission lines are subjected to many environmental factors, among which, the wind is the most important. Numbers of researchers have been made by CIGRE [1] due to wind is the cause of approx. 20% of all failures [2]. Depends on the magnitude of the wind, turbulence (Figure 1), vibrations of small amplitudes and significant frequencies are generated, as well as high-amplitude and low frequency [3-7]. The Reynold's formula can be used to describe the effect of the wind acting on the conductor:

$$Re = \frac{VD}{\nu} \quad (1)$$

where: V is the wind speed, D is the diameter of the conductor, ν is the kinematic viscosity of the fluid. The Reynolds number is a dimensionless quantity characterizing the ratio of inertia forces to viscous forces occurring during fluid flow [3]. The value of the Reynolds number corresponding to the change of flow from laminar to turbulent is called critical [8-9].

The researchers have shown that average wind speeds, measured for averaging times of 5 minutes, 10 minutes and even 1 hour, do not differ significantly. The pulsations of wind speed during periods lasting over 5 minutes hardly occur [10]. The wind speed is assumed, which can be exceeded on average once in the estimated time of use of the building. This approach is used in [11].



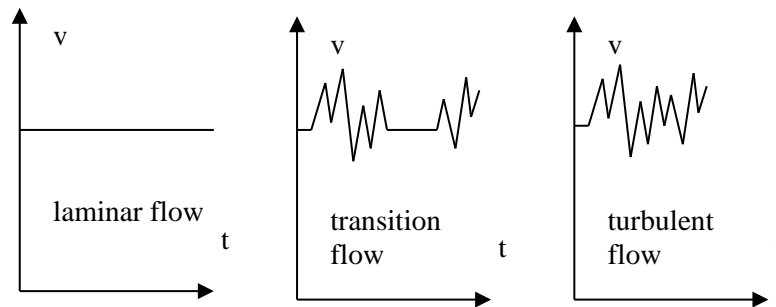


Figure 1. Character of laminar and turbulent flow

1.1. Wind turbulence characteristics

In order to describe the distribution of turbulence with frequency, the spectral density function is used. The standard deviation can be written in the following formula

$$\sigma_u^2 = \int_0^{\infty} S_u(n) dn \quad (2)$$

where n is the frequency, S_u is the spectral density function for u .

There are many mathematical forms that have been used for $S_u(n)$ in wind engineering. The most common is the von Karman form [12].

Another spectral density function model was also presented by Tieleman [13].

Based on [14], the recommendation of the authors of this paper as the fittest model for the data of wind distribution taken into consideration in numerical example analysed is:

$$\frac{n S(z, n)}{\sigma_u^2} = \frac{5.168f}{(1.3 + 10.12f^{1.5})^{5/3}} \quad (3)$$

$$f = \frac{nL_u}{\bar{u}(z)} \quad (4)$$

where L_u is the turbulent length scale [15], $\bar{u}(z)$ is the mean velocity, z is the height above ground.

Spectra of longitudinal wind velocity and turbulent length scale according to [15-16] are presented in Figure 1. Analysis of wind turbulence is presented in paper as well in [15-16].

1.2. Development of spectral method

The Spectral Element Method (SEM) is a meshing method similar to Finite Element Method (FEM), where the approximated element shape functions are substituted by exact dynamic shape functions obtained from the exact solution of governing differential equations. Therefore, a single element is sufficient to model any continuous and uniform part of the structure. This feature reduces significantly the number of elements required in the structure model and improves the accuracy of the dynamic system solution. At the same time, there are some drawbacks like the unavailability of exact wave solutions for most complex and 2D and 3D structures. In these cases, approximated spectral element modelling can be used and may still provide very accurate solutions. Although SEM ensures exact frequency-domain, it is not true for time-domain solutions, because errors due to aliasing or leakage are inevitable in the use of the inverse-DFT process. Thus, special attention in obtaining the inverse-DFT is required. In recent years some researchers were performed with use of SEM. The extensive study of the

fundamentals and a variety of new applications such as composite laminated, periodic lattice, damage detection were presented in [17]. The wave behaviour in composites and inhomogeneous media are studied in [18]. Studies related to structural damage detection have been developed in [19]. Other works using wave propagation and SEM to detect damage under the presence of structural randomness can also be found in references [20-22].

In the paper, the response of overhead transmission line in turbulent wind flow with use of spectral method is investigated. The analysis is performed for wind flow model that reflects the real conditions. Numerical analysis investigates the vibrations of the conductor due to different parameters, such as a tension force and the area of the cross section of the conductor.

2. Theoretical background of spectral approach

Considering a simplified cable model, the governing differential equation for the undamped free vibration is given by Clough and Yu [23-24]:

$$EI \frac{\partial^4 v}{\partial x^4} - T \frac{\partial^2 v}{\partial x^2} + \rho A \frac{\partial^2 v(x, t)}{\partial t^2} = 0 \quad (5)$$

The undamped Euler-Bernoulli beam equation of motion subjected to axial force and under bending vibration is governing by equation (5). A structural internal damping is introduced into the beam formulation by adding into Young's modulus weighted by a complex damping factor $i\eta$, $i = \sqrt{-1}$, η is the hysteretic structural loss factor, to obtain $E = E(1 + i\eta)$.

The general solution for the Euler-Bernoulli beam spectral element subjected to axial load of length L , can be expressed in the form

$$v(x, \omega) = a_1 e^{-ikx} + a_2 e^{-kx} + a_3 e^{-ik(L-x)} + a_4 e^{-k(L-x)} = \mathbf{s}(x, \omega) \mathbf{a} \quad (6)$$

where

$$\mathbf{s}(x, \omega) = \{e^{-ikx}, e^{-kx}, e^{-ik(L-x)}, e^{-k(L-x)}\} \quad (7)$$

$$\mathbf{a}(x, \omega) = \{a_1, a_2, a_3, a_4\}^T \quad (8)$$

where k is the wave number.

The spectral nodal displacements and slopes of the beam element are related to the displacement field at node 1 ($x=0$) and node 2 ($x=L$), by

$$\mathbf{d} = \begin{bmatrix} v_1 \\ \phi_2 \\ v_2 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} v(0) \\ v'(0) \\ v(L) \\ v'(L) \end{bmatrix} \quad (9)$$

By substituting equation (6) into the right-hand side of equation (9) and written in a matrix form gives:

$$\mathbf{d} = \begin{bmatrix} s(0, \omega) \\ s'(0, \omega) \\ s(L, \omega) \\ s'(L, \omega) \end{bmatrix} \mathbf{a} = \mathbf{G}(\omega) \mathbf{a} \quad (10)$$

, where

$$\mathbf{G}(\omega) = \mathbf{d} = \begin{bmatrix} 1 & 1 & e^{-ikL} & e - kL \\ -ik & -k & ie^{-ikL} & ke - kLk \\ e - ikL & e - kL & 1 & 1 \\ -ie - ikLk & e - kLk & ik & k \end{bmatrix} \quad (11)$$

The frequency-dependent displacement within an element is interpolated from the nodal displacement vector \mathbf{d} , by eliminating the constant vector \mathbf{a} from equation (9) and using equation (10) it is expressed as

$$v(x, \omega) = \mathbf{g}(x, \omega) \mathbf{d} \quad (12)$$

where the shape function is

$$\mathbf{g}(x, \omega) = \mathbf{s}(x, \omega) \mathbf{G}^{-1}(\omega) = \mathbf{s}(x, \omega) \mathbf{\Gamma}(\omega) \quad (13)$$

The dynamic stiffness matrix for the spectral beam element under axial tension can be determined as:

$$\mathbf{S}^e(\omega) = EI \left[\int_0^L \mathbf{g}''(x)^T \mathbf{g}''(x) dx - k^4 \int_0^L \mathbf{g}(x)^T \mathbf{g}(x) dx \right] \quad (14)$$

where 'express the spatial partial derivative. By solving the integral the dynamic stiffness matrix is

$$\mathbf{S}^e(\omega) = \frac{EI}{\Delta} \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ & S_{22} & S_{23} & S_{24} \\ & & S_{33} & S_{34} \\ sym & & & S_{44} \end{bmatrix} \quad (15)$$

where $\Delta = \cos(kL) \cosh(kL) - 1$ and the components of element matrix (eq.15) are given according to [25]

As far as structure beam is uniform without any sources of discontinuity, it can be represented by a single spectral element with very accurate solutions [26]. However, if there exist sources of discontinuity such as the point loads the beam should be spatially discretized into spectral elements. Analogous to Finite Element Method (FEM) [27], the spectral elements can be assembled to form a global structure matrix system [17].

3. Numerical example

On the basis of the formulas presented in equations (14-16), it is possible to obtain the Frequency Response Function (FRF) of the overhead transmission conductor. For the numerical tests, it is assumed a pinned-pinned boundary condition. The beam structures are made with aluminium whose mechanical properties are: $E = 74 \text{ GPa}$, $\rho = 2700 \text{ kg/m}^3$, $\eta = 0.01$. The geometry properties are: length of $L = 213 \text{ m}$ and circular section area of $A = 300 \text{ mm}^2$. The substitute four node forces are applied to the conductor. The wind force acting on the overhead transmission line may be determined by the expression:

$$F = \frac{1}{2} \rho V^2 DC \quad (16)$$

where $\rho = 1.25 \text{ kg/m}^3$, V is the wind speed, $D = 0.03 \text{ m}$ is the diameter of the transmission line, C is the aerodynamic coefficient, equal to 2.

The value of tension force of 10 kN is assumed in the tests. In all tests, five spectral elements on the span are used. The analysis is made for two case of turbulence: $Tu = [10.6, 19.3]$ for the assumed distribution of wind speed. In the two cases the maxim amplitude of wind speed and the excitation forces are the same. The results of analysis in Figure 2 and 3 presents that for the higher turbulence the maximum displacement of the cable is higher in the analysed point. For comparison the response of the conductor on the sine function excitation is in Figure 4 and vibrations conductor in Figure 5.

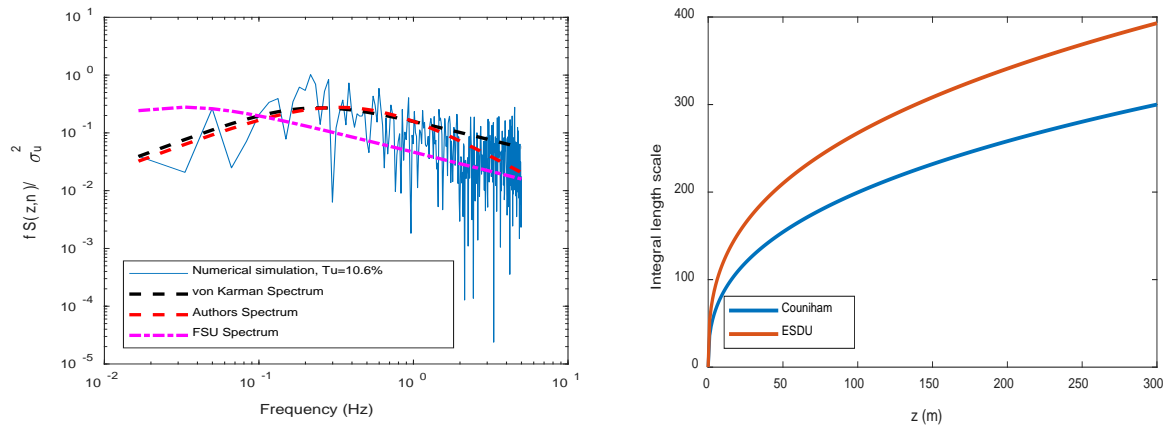
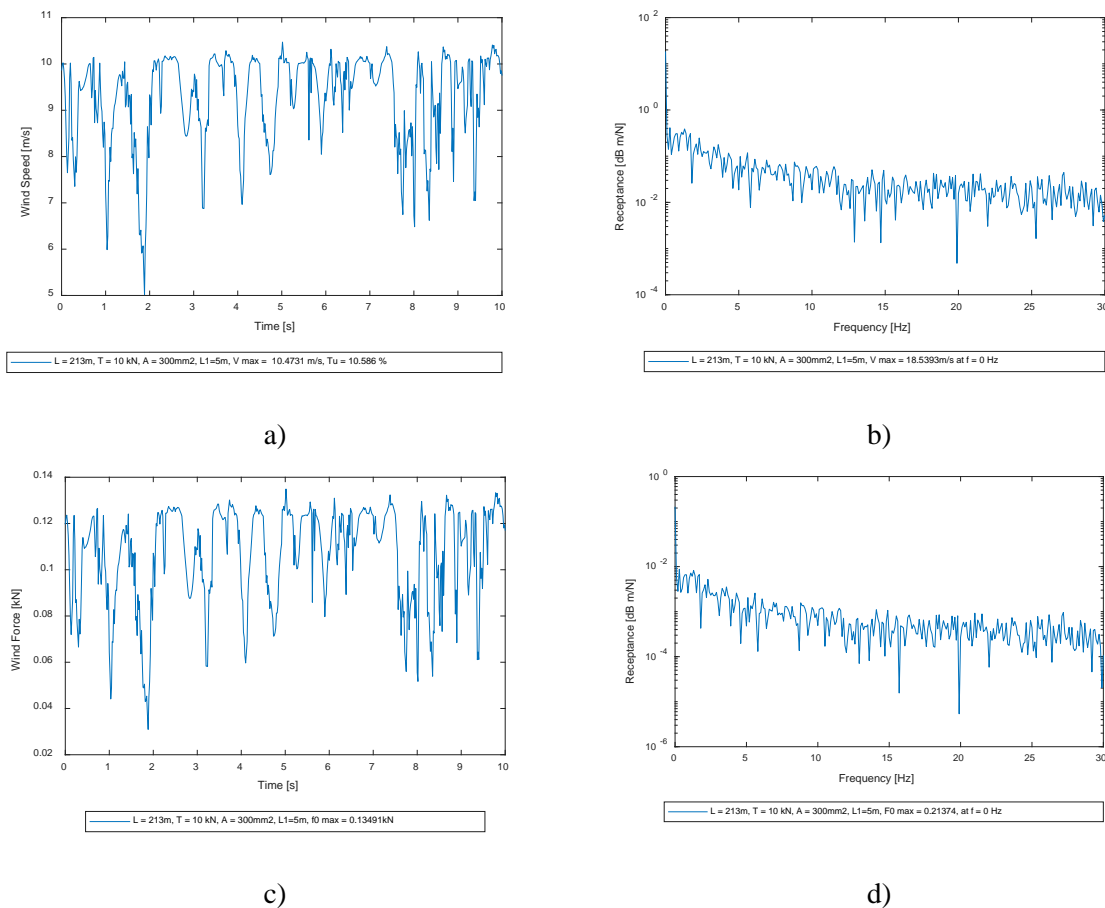
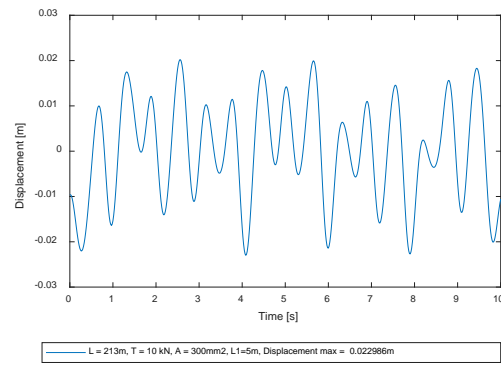
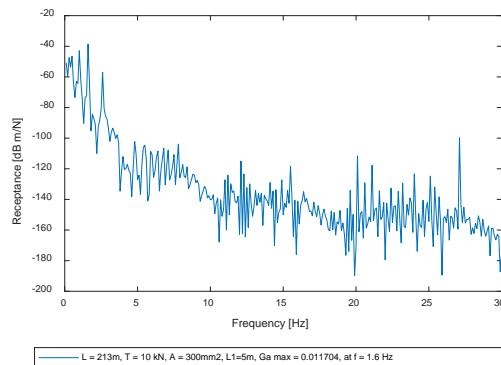
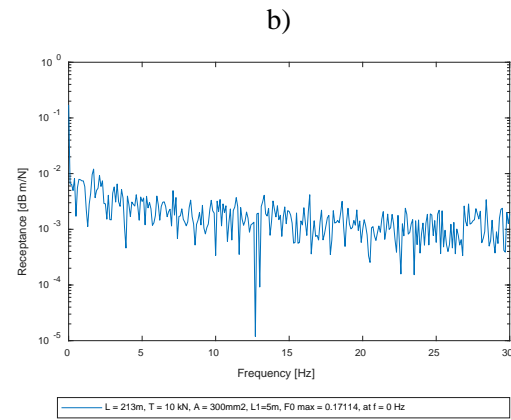
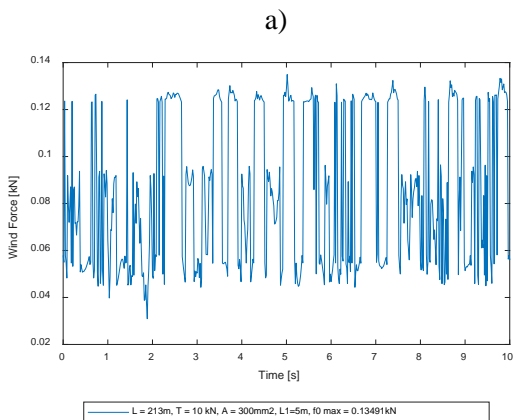
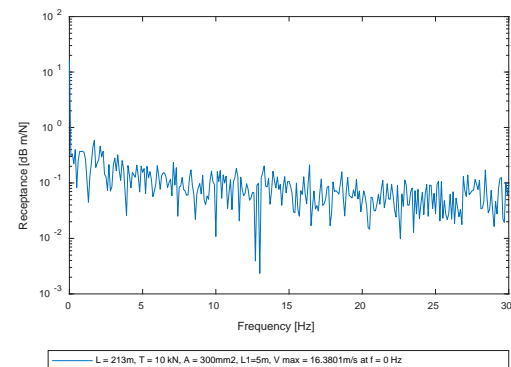
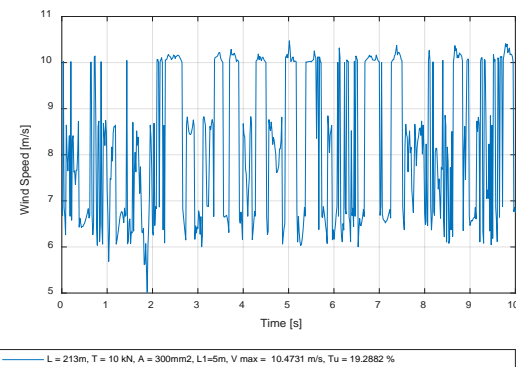


Figure 2 a) spectra of longitudinal wind velocity, b) Counihan and ESDU integral length scale



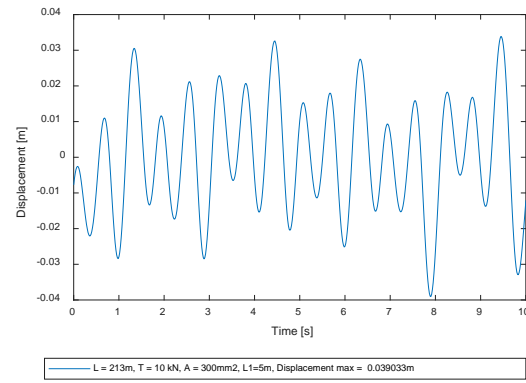
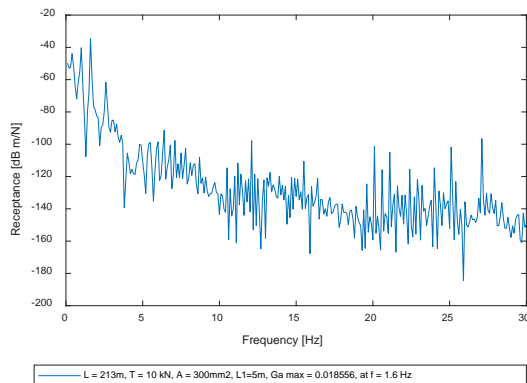


e) f)
Figure 3 a-f. Vibrations of the conductor in the wind flow of turbulence 10.6%

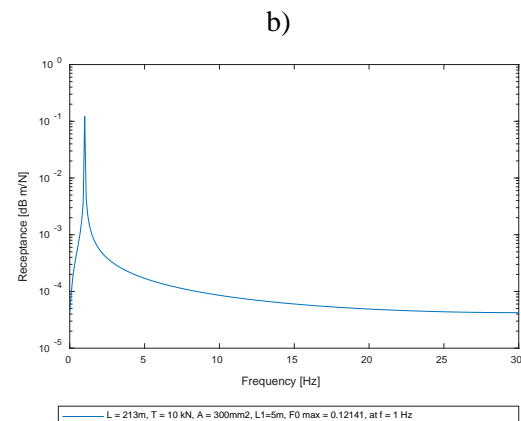
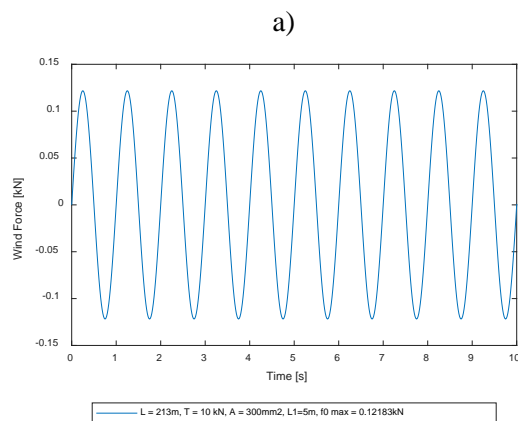
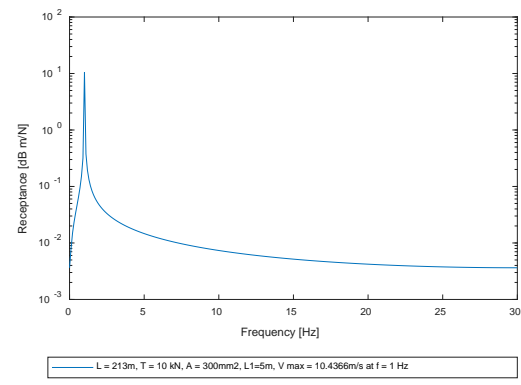
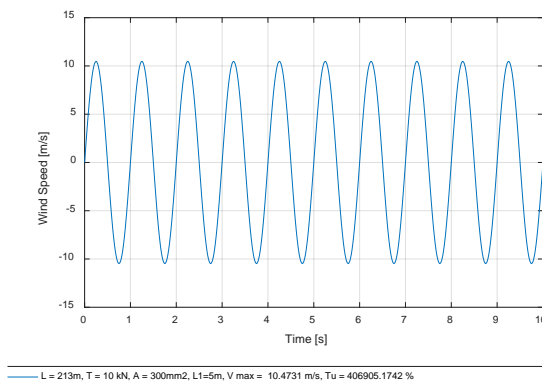


c)

d)



e) f)
Figure 4 a-f Vibrations of the conductor in the wind flow of turbulence 19.3%



c)

d)

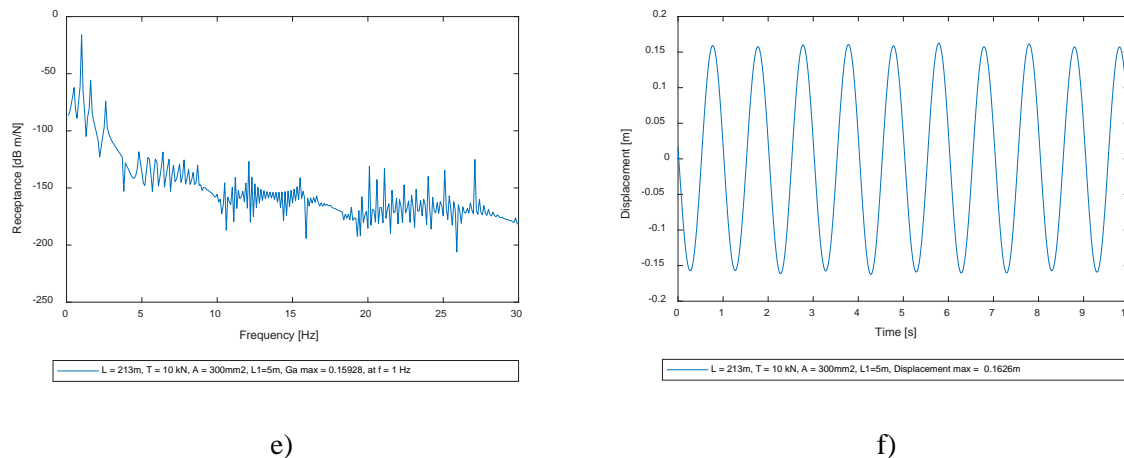


Figure 5 a-f Vibrations of the conductor in the wind flow under sine function excitation

4. Conclusions

In the paper the turbulence of the wind flow is the subject of the research. Among many spectra of longitudinal wind velocity, the Karman, FSU and author proposal is presented. The response of overhead transmission line in turbulent wind flow with use of spectral method is investigated. The analysis is performed for wind flow model that reflects the real conditions. Numerical analysis investigates the vibrations of the conductor due to two parameters of turbulence. For comparison the excitation of sine function is performed.

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