IOP Conference Series: Materials Science and Engineering

PAPER • **OPEN ACCESS**

Novel Topology Optimization Techniques Adapted to Strengthening of Civil Structures Suffering from the Effects of Material Degradation

To cite this article: Katarzyna Tajs-Zielinska and Bogdan Bochenek 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **471** 052010

View the article online for updates and enhancements.

This content was downloaded from IP address 117.93.176.10 on 14/10/2019 at 22:09

IOP Conf. Series: Materials Science and Engineering 471 (2019) 052010 doi:10.1088/1757-899X/471/5/052010

Novel Topology Optimization Techniques Adapted to Strengthening of Civil Structures Suffering from the Effects of Material Degradation

Katarzyna Tajs-Zielinska [1], Bogdan Bochenek [1]

[1] Institute of Applied Mechanics, Cracow University of Technology
Warszawska 24, 31-155 Cracow, Poland

Katarzyna.Tajs-Zielinska@pk.edu.pl

Abstract. Over years, cultural heritage monuments and historical buildings, as well as old civil structures, have been exposed to slow aging processes, material damage and degradation due to climate and environmental changes or problems resulting from poor maintenance. The task of repairing and strengthening of historical structures can be extremely challenging, because of architectural value of buildings, from one side, and selection of appropriate engineering techniques, from the other. The estimation of technical condition of the old structures, from the engineering point of view, is usually difficult or even impossible to carry out in a precise way. Because of a limited knowledge about stiffness of the considered structure, the designing of strengthening elements for historical structures becomes a motivation for developing new concepts and engineering techniques. In what follows some of the well-known, conventional reinforcing methods, like implementation of wood and steel frames or tie rods, have been gradually replaced by innovative approaches, which allow the resulting structures to become lighter, stronger and also attractive from the aesthetic point of view. This paper presents the novel design technique for retrofit strengthening of existing civil structures suffering from the effects of material degradation. The unknown stiffness distribution within weakened structure is modelled by randomly distributed material data, including cracks and reinforcements of original structure. The idea is to adopt topology optimization techniques to find the optimal layout of strengthening elements for loaded, initially and randomly weakened structure. Classical approach to topology optimization is to find within considered design domain the distribution of the material, which is optimal in some sense. The material data for design-active and design-passive regions of the structure are defined first. Then, optimization process, understood as minimization of the structure compliance, is performed and material is removed from the parts of the structure where it is not necessary to the parts, where it is essential. Finally, the considered structure takes on a new shape, while the amount of the material is reduced to assumed volume fraction. The new idea discussed in this paper is to implement into optimization process a random information about the stiffness of the original structure by modelling design-passive regions with randomly distributed values of material data. As a result, the new layout of strengthened structure for which the assumed volume fraction is preserved can be obtained. This approach allows to reduce a mass of strengthening elements, but what is essential, the complete information about the material degradation level of the original structure is not necessary to perform the optimization process.

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

1. Introduction

Maintaining architectural heritage should be a concern not only for historians but also for architects and engineers. It is important to invent and implement the best, the most effective techniques to restore cultural heritage monuments and historical buildings. Among frequently applied strategies, one can find many well-known ideas like re-pointing, injection grouting and the use of steel ties, jackets or tie-beams, but also innovative usage of technologies like shape memory alloys, self-compacting concrete, thin lead layers. Nowadays, designing of a restoration process has ability to enjoy the benefits of the best innovations in engineering and science. The efficient conceptual designing has to be based on numerical methods, and among them optimization ones. This paper is focused on one of the new concepts, namely implementation of topology optimization to retrofitting of civil structures suffering from the effects of material degradation. Topology optimization is a numerical method, which allows obtaining stiffer, lighter and cheaper constructions keeping their aesthetic value by developing new, original shape of strengthening. The idea of topology optimization is well known among researchers and can by successfully adapt to many industry applications, including civil engineering. This paper expands the area of those considerations and presents a new idea of adaptation of a model of material degradation described by random stiffness information of the original structure. That can be achieved by modelling design-passive regions with randomly distributed values of material data. This distribution is varied at each iteration step to avoid the effect of result dependence on once defined condition for whole structure. This approach allows to reduce a mass of strengthening elements, but what is essential, the complete information about the material degradation level of the original structure is not necessary to perform the optimization process. This concept is illustrated by numerical example and followed by the discussion of obtained results.

2. Generation of optimal topologies

Structural optimization has been present in engineering practice since Galileo considered a problem of designing of bending beams. Since the optimization as a mathematical problem was born, the optimum-structural-design began to be a daily practice of designers and constructors. Among various concepts of optimal design, one can find the relatively new one called topology optimization, which was started by Dorn et. all [1] and then formulated by Bendsoe and Kikuchi [2] and Bendsoe [3] in late eighties of the last century. The principle of the topology optimization is to find within a design domain the distribution of material that is optimal in some sense. During optimization process, material is redistributed and parts that are not necessary from objective point of view are selectively removed. Topology optimization ends up in finding material distribution that is visualized by black and white regions over the design domain (see figure 1).



Figure 1. Generating optimal topology of a plane elastic structure.
Initial structure (left) and final topology (right)

The application of topology optimization requires a decomposition of considered design domain into lattice of finite elements and design variables are selected for these elements. The distribution of the design variables creates the resulting topology. The most common approach to topology optimization called the power-law approach known also as SIMP (Solid Isotropic Material with Penalization [4], [5]) assumes, that design variables are defined as relative densities of a material d_i. The elastic modulus of each element is modelled as a function of relative density using power law, as it is presented in equation (1), where the power p has been introduced in order to penalize intermediate densities:

$$E_i = d_i^p E_0, \qquad d_{min} \le d_i \le 1 \tag{1}$$

Elastic modulus E_0 stands for the density of a solid material.

2.1. Formulation of topology optimization problem

From the mathematical point of view, optimization is formulated as a minimization of the objective function subject to imposed constraints. The objective function in topology optimization is typically defined as a compliance of the structure (see equation (2)).

$$U(\boldsymbol{d}) = \sum_{i=1}^{n} d_i^p \boldsymbol{u}_i^T \boldsymbol{k}_i \boldsymbol{u}_i \tag{2}$$

In equation (2) $\boldsymbol{u}_i$ and $\boldsymbol{k}_i$ are the element displacement vector and stiffness matrix respectively and n is a number of elements.

The constraints must be imposed on the smallest values of design variables to avoid numerical instabilities (see equation (1)), but additional constraints can be defined by, for example, given volume fraction of material for final, optimized structure. That can be written as it is shown in equation (3):

$$V = \kappa V_0 \tag{3}$$

V_0 stands for design domain volume and κ is a prescribed volume fraction.

In the problem formulated in this paper, two possible interpretations of objective function are taken into account. Firstly, it can be interpreted as a minimization of a compliance of the construction as a whole, where the design domain is only a part of the structure. Second way is to interpret the objective function as a compliance calculated only for the domain where the optimization process is performed. In this case, the sum in equation (2) has to be limited to the number of design variables. This approach may be called subdomain oriented topology optimization.

2.2. Local rules for generating optimal topologies

The idea of the updating rules for the design variables utilized in this paper is called Cellular Automata and it was firstly applied to optimization of structure in mid-nineties of 20th century. The basics were described by Inou et al. [6] and later on by Kita and Toyoda [7] or Abdalla and Gurdal [8]. The essence of the idea is to define for each cell (which is equivalent to finite element) a neighbourhood. The neighbourhood is formatted by a particular cell together with cells to which it is connected. The modification, at each iteration t, of the design variable δd_i of each cell is governed by local rules constructed based on information gathered from cells forming the neighbourhood, as it is presented in equation (4):

$$d_i^{(t+1)} = d_i^{(t)} + \delta d_i^{(t)}, \qquad \delta d_i^{(t)} = (\alpha_0 + \sum_{k=1}^{N} \alpha_k) m \tag{4}$$

where m is a move limit. Coefficients α_0 (for central cell) and α_k (for N neighbouring cells) are calculated based on a comparison of the compliance of the considered cell U_0 or U_k with the threshold value U^* as it is shown in equation (5):

$$\alpha_0 = \begin{cases} -C_{\alpha 0} & \text{if } U_i^{(t)} < U^* \\ C_{\alpha 0} & \text{if } U_i^{(t)} \ge U^* \end{cases} \qquad \alpha_k = \begin{cases} -C_{\alpha} & \text{if } U_{ik}^{(t)} < U^* \\ C_{\alpha} & \text{if } U_{ik}^{(t)} \ge U^* \end{cases} \quad k = 1 \dots N. \tag{5}$$

The values of both $C_{\alpha 0}$, which is assigned to central cell and C_α which refers to neighbouring ones are selected so as to keep their sum within neighbourhood equals one. The concept of the above local

rules was discussed and illustrated by many numerical implementations in [9] whereas some extensions have been proposed in [10] and [11].

3. Proposal of a new concept of topology optimization oriented at strengthening of civil structures suffering from the effects of material degradation

Some important aspects of retrofitting and strengthening of heritage structures have been reported in literature and the books [12], [13] may serve here as examples. This paper considers a new point of view, namely adaptation of topology optimization techniques to searching for the optimal layout of strengthening elements to be implemented into damaged and/or weakened structures.

The conventional approach to topology optimization, where material is fully defined according to specified distribution of the Young modulus within a design space has to be reconsidered taking into account the new concept. The idea is to implement a random stiffness information of the original structure into the optimization process. It can be done, by modelling design-passive regions with randomly distributed values of material data. The unknown stiffness distribution within weakened structure is modelled in order to apply effects of material degradation, including cracks and reinforcements of original structure. As a result, the new layout of strengthened structure for which the assumed volume fraction is preserved can be obtained. This approach allows reducing a mass of strengthening elements, but what is essential; the complete information about the material degradation level of the original structure is not required to perform the optimization process. To achieve a reasonable numerical model without precise information about stiffness of the considered damage structure the random distribution of material is generated at each iteration.

It is important to underline, that this new concept covers also a multi-material topology optimization approach discussed previously in [14].

4. Numerical example

As a numerical example illustrating the above discussed approach, the bridge structure shown in figure 2 has been chosen. It has been assumed that the bridge has been weakened by both structural damage and by the effect of material degradation. In order to strengthen the structure, the application of the topology optimization technique is proposed. The upper part plotted in black colour is representing a road; this part is excluded from optimization. The Young modulus of the road equals $E_1 = 27\ 10^9$ Pa.

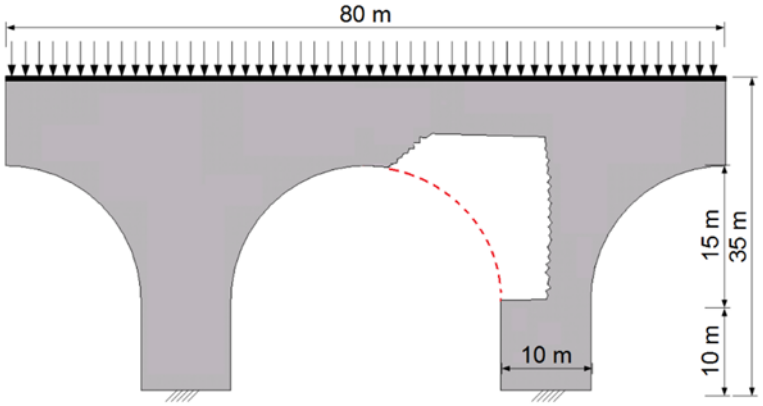

Figure 2. The damaged bridge structure with applied loads and supports

4.1. Weakened material representation

Two models of weakened material are considered. The first one it is uniformly distributed material of decreased, as compared with the original structure, value of the Young modulus. The constant value of $0.8\ E_1$ has been selected for the structure presented in the figure 3.



Figure 3. Uniformly distributed weakened material

In the second model of weakened material, the Young modulus values are randomly selected from the interval: 0.8 E_1– E_1. The material degradation is therefore illustrated in the figure 4 by randomly distributed green/grey regions. In addition, some more weak light grey regions with the Young modulus equals 0.001 E_1 and the randomly distributed void elements for which the Young modulus values are ranging from 0.0001 E_1 to 0.001 E_1 have been included. These isolated elements are represented by white dots in figure 4. It is important to underline, that generation of random data is performed at each iteration.

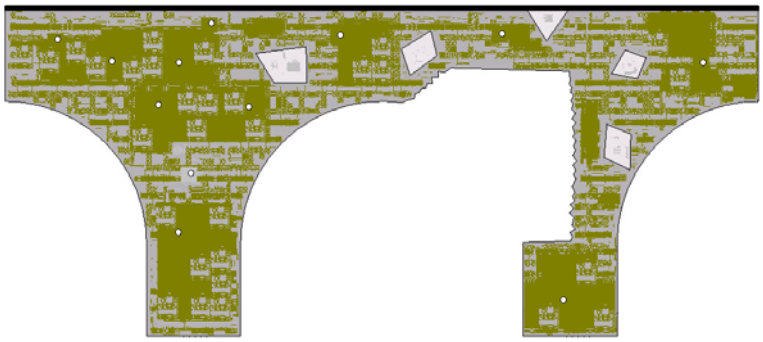

Figure 4. Randomly distributed, damaged material – grey/green domain, weakened material – light grey sub-domain, randomly distributed voids – white sub-domain

4.2. Generation of optimal topologies

The region, in which the strengthening should be implemented is marked by red colour in figure 5. Since optimization is performed within this area the finer mesh has been applied there.

Performing optimization process within the selected design domain results in creating a topology of strengthening which minimizes the total structure compliance. In the figure 6, a visualization of expected optimization result is presented.

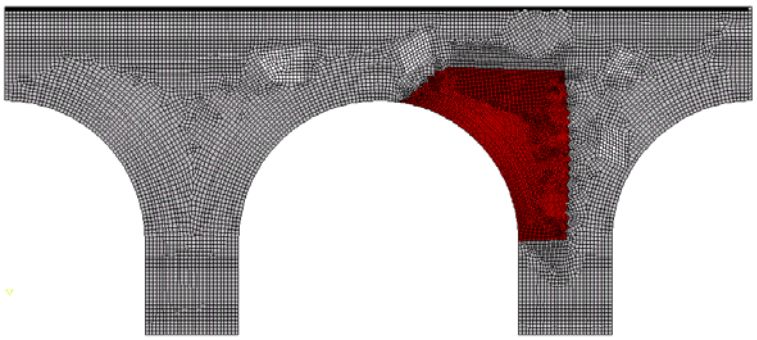

Figure 5. The bridge structure– finite element mesh. Domain subject to optimization – marked red

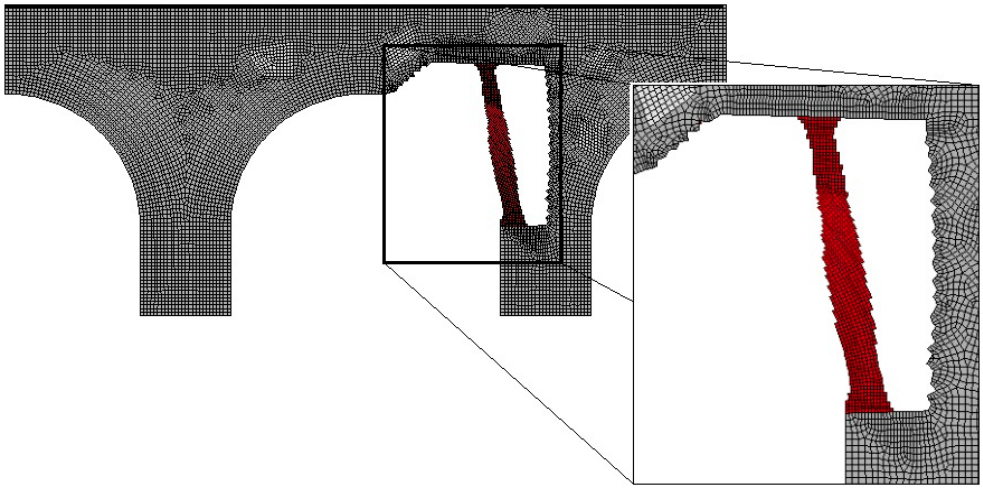

Figure 6. Final topology of strengthening of bridge structure

As the first task, the topology optimization has been performed for the damaged structure with uniformly weakened material shown in figure 3. The Young modulus of a material in the domain selected for optimization equals E_1 =27 10^9 Pa. The value of distributed load is 2500 N/m whereas assumed volume fraction of material for resultant, optimized topology equals 0.2. It is worth noting that structural damage together with material degradation caused significant increase of compliance from 20441 Nm calculated for original perfect structure to 70574 Nm.

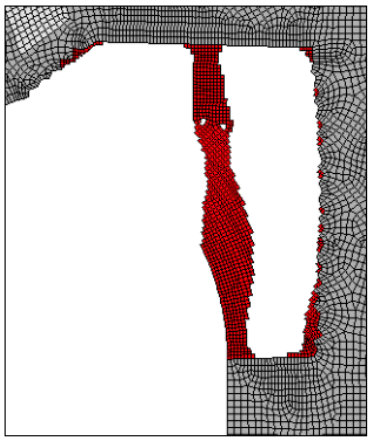

Figure 7. Final topology of strengthening for uniformly distributed weakened material

In the figure 7, the obtained topology optimization result is presented. The total compliance has been reduced to 34314 Nm what confirms the efficiency of the presented approach.

The topology optimization procedure has been applied now to three different cases of random material data distributions for weakened material to check, what extents randomized values of the Young modulus influence final topologies to. Figure 8 presents final topologies generated for all three cases.

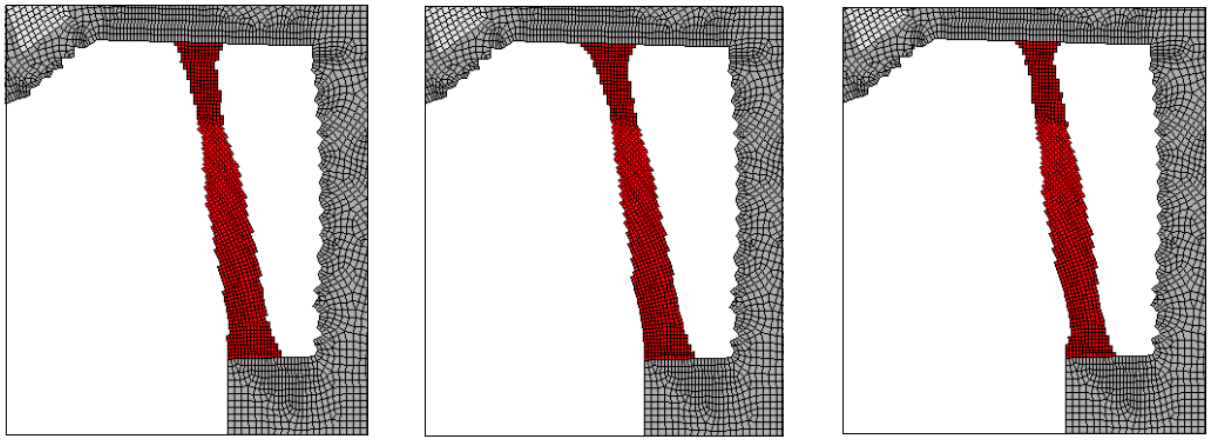

Figure 8. Final topology of the strengthening for three different random distributions of the Young modulus of weakened material.

The final compliances for optimized structures were calculated. Because the values of material data are randomized at each iteration, the values from the last iteration are treated as the resulting ones. Then to make a comparison, the strengthening element obtained under assumption of uniformly distributed weakened material (figure 7) has been implemented instead of the ones from figure 8 and compliances of those have been calculated. The question is whether the resulting topology obtained under assumption of uniform distribution of weakened material can be applied to more realistic random distributions of weakening. The comparison of results is presented in the table 1.

Table 1. Final compliances for optimized structures.

	Final compliance [Nm]. The strengthening results from application of topology optimization to randomly distributed weakened material.	Final compliance [Nm]. The strengthening results from application of topology optimization to uniformly distributed weakened material.
Random case 1	75 427	91 716
Random case 2	99 822	105 991
Random case 3	85 095	102 095

5. Discussion of results

The obtained preliminary results show that there is a possibility to model the unknown stiffness distribution of a weakened structure in a way suitable for topology optimization. It is also possible to introduce other than standard uniformly distributed weakened material for non-optimized regions. Figure 8 presents the resulting topologies of strengthening of bridge structure for three different random distributions of the Young modulus of weakened material. As it can be noticed, the results for different randomization are very similar, what can prove, that there may exist a unique solution for the assumed representation of the material degradation. Furthermore, the resulting topologies of strengthening for random and uniformly distributed data show some differences. The proposed approach to topology optimization differs from the standard one based on uniformly weakened material of non-optimized regions. It is very important remark, that when adapt a solution obtained for

uniformly weakened material to the base structure with random weakened material, the value of compliance of received construction is greater, then the value of compliance for obtained topologies in optimization process with proposed model of material degradation. That means, that it is necessary to apply proper model of material while dealing with topology optimization of strengthening in real-life structures, where cracks, damages, holes, etc. are highly irregular and even difficult to identify.

6. Conclusions

The topology optimization adapted to strengthening of civil structures suffering from the effects of material degradation with random model of material weakening is a new concept, which seems to be the issue offering promising results. It is shown in the present paper that this approach opens also new possibilities for novel design techniques for retrofit strengthening of existing civil structures suffering from the effects of material weakening. It has been demonstrated, that assumed model of material degradation allows performing the optimization process and the complete information about the material degradation level of the original structure is not necessary. Additionally, it has been shown, that model of material weakening can influence final topologies of strengthening what means that the standard approach with uniformly distributed material data of non-optimized regions cannot be sufficient to represent the real structure damage while performing topology optimization.

References

[1] W. S. Dorn, R. E. Gomory and H. J. Greenberg, "Automatic design of optimal structures", *Journal de mecanique*, vol. 3(6), pp. 25-52, 1964.
[2] M. P. Bendsoe and N. Kikuchi, "Generating optimal topologies in optimal design using a homogenization method", *Computer Methods in Applied Mechanics and Engineering*, vol. 71, pp. 197-224, 1988.
[3] M.P. Bendsoe, "Optimal shape design as a material distribution problem", *Structural Optimization*, vol. 1, pp. 193-200, 1989.
[4] M. P. Bendsoe, "Optimization of structural topology, shape and material", Springer, 1995.
[5] M. Zhou and G.I.N. Rozvany, "The COC algorithm, part II: Topological, geometry and generalized shape optimization*", Computer Methods in Applied Mechanics and Engineering*, vol. 89, pp. 197-224, 1991.
[6] N. Inou,N. Shimotai and T. Uesugi,"A cellular automaton generating topological structures", *2nd European Conference on Smart Structures and Materials*, 2361, pp. 47-50, 1994.
[7] E. Kita and T. Toyoda, "Structural design using cellular automata", *Structural and Multidisciplinary Optimization*, vol. 19, pp. 64-73, 2000.
[8] M.M. Abdalla and Z. Gurdal, "Structural design using cellular automata for eigenvalue problems'*, Structural and Multidisciplinary Optimization*, 19, 64-73, 2004.
[9] B. Bochenek and K. Tajs-Zielinska, "Novel local rules of Cellular Automata applied to topology and size optimization", *Engineering Optimization*, vol. 44(1), pp. 23-35, 2012.
[10] B. Bochenek and K. Tajs-Zielinska, "Topology optimization with efficient rules of cellular automata", *Engineering Computations*, vol. 30(1), pp.1086-1106, 2013.
[11] B. Bochenek and K. Tajs-Zielinska, "GOTICA - generation of optimal topologies by irregular cellular automata", *Structural and Multidisciplinary Optimization*, vol. 55(6), pp. 1989-2001, 2017.
[12] S. Syngellakis, editor, "Retrofitting of Heritage Structures. Design and evaluation of strengthening technique", *WIT Transactions on State-of-the-art in Science and Engineering*, vol. 62, 2016.
[13] A. Costa, A. Arêde and H. Varum, editors, "Strengthening and Retrofitting of Existing Structures", Springer, 2018.
[14] B. Bochenek and K. Tajs-Zielinska, "Topology Optimization - Engineering Contribution to Architectural Design", *IOP Conference Series: Materials Science and Engineering*, vol. 245(8), pp. 082057, doi:10.1088/1757-899X/245/8/082057, 2017.