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Small Strain Soil Constitutive Models HS-SMALL & RU+MCC – Calibration and Verification

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Abstract. The paper presents two reasonably advanced constitutive laws for soil. First one is a hybrid of the Modified Cam Clay and a new RU (reload, unload) development, and it is author's conception. The second one is a HS-small model by Benz. In the RU+MCC model, the Modified Cam Clay model, which is an isotropic elastoplastic hardening, describes normal consolidation behaviour - plastic model originated by Burland in 1967 within the critical state soil mechanics. This model describes realistically mechanical soil behaviour in normal consolidation states. The RU part is designed to ensure more adequate soil responses to reloading paths, particularly in the range of small strains. The HS-small model is an improvement of HS model originated form Schanz and Vermeer. This concept base on cap yield surface with incorporation of two hardening mechanisms, stiffness variation at small strains, densification mechanism, Rowe's dilatancy and some others. The author in the FEM computer code Z_SOIL.pc has implemented the RU + MCC model. The HS-small model has been implemented into the same software by Truty. To test the influence of the small strain nonlinearity on soil - structure interaction as well as to exhibit the ability of the proposed model to simulate realistically this effect, a comparative study based on the FEM solution has been carried out. As a benchmark, a trial loading test of strip footing was used. The calibration process has based on advanced laboratory and field soil tests like resonant columns, triaxial test, dilatometer test and many others.

1. Introduction

Nonlinearity of the stress-strain characteristics of soil is a fact well known for a long time. It is revealed even in the simplest of research such as oedometric tests, triaxial tests, direct shear tests, or trial load tests. It is difficult to find a straight part of any characteristic. A classic description (like Coulomb-Mohr or Modified Cam-Clay etc.) does not account for strong physical nonlinearity in the range of small strains. This phenomenon has been observed in many high quality triaxial tests including local strain and wave velocity measurements, carried out for last thirty years. The essence of the above phenomenon is an abrupt (more than tenfold) drop in soil stiffness at increasing deformations in the range from 0.001% to 0.1%. In the light of results of a number of studies [1, 2] accounting for that experimental fact appears to be crucial for realistic prediction of subsoil responses to working loads transmitted from structures.

Therefore, in the range of small strains, the models under consideration should be particularly adequate (for small strains, the range of 10^{-6} - 10^{-3} was assumed after Atkinson and Sallfors [3]. Nonlinearity in this range, compared to the strain called large or moderate, is much more significant, due to a very narrow range. Hooke's law, in turn, is true only in the very small range, with 10^{-6} as the upper limit.

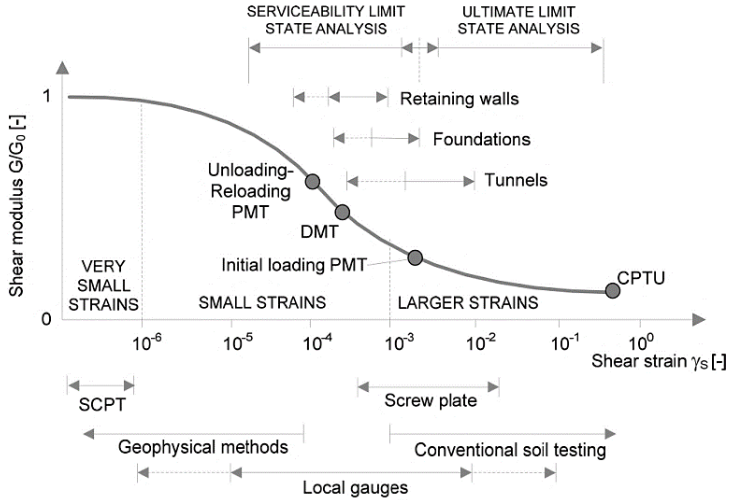

Figure 1. Typical representation of stiffness variation in as a function of the shear strain amplitudes [3]

The problem of constitutive models calibration is inseparable from their specificity. Each time, the estimation of model parameters must be preceded by an analysis of the mechanisms and theories comprised in the calibrated models. In the case of selected models RU + MCC and HS-Small, it will be on one hand the nonlinear elastic set of equations describing the area of small deformations. On the other hand, it will be normal consolidation with elasto-plastic description, where the deformations exhibit much higher values. Unfortunately, calibration process of advanced models, due to their greater complexity, is always more difficult to carry out. Not only the procedure is more prolonged and demanding, but also the amount of the input data usually has to be higher. In addition, this leads to increase number or complexity of soil tests necessary to carry out.

High quality documented experimental field of the University of Texas was chosen for the calibration process. The original purpose of that experiment was to analyse the problem of vertical displacement of various foundation dimensions (1.0 ÷ 3.0 m). The experiment was described by Briaud and Gibbens [4], and the scheme is shown in Figure 2.

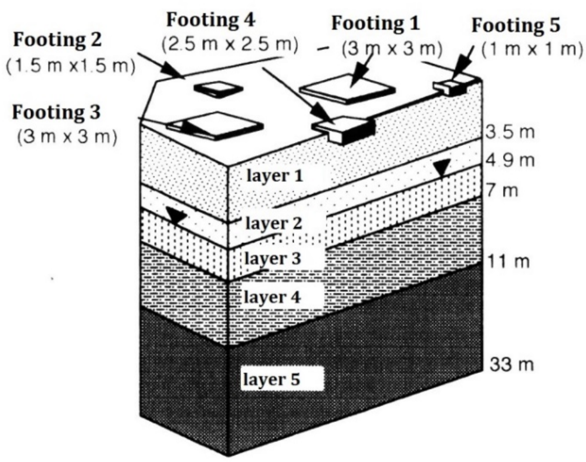

Figure 2. The scheme of investigation field [4, 5]

Within the experimental procedure a trial load tests of several foundations were carried out. Foundations were localised in a small but sufficient distance from each other within the same soil conditions. A large number of laboratory or in situ soil tests have been performed independently.

Soil conditions were recognized by a wide spectrum of different soil tests. Among others, the advanced methods were used, like DMT, CPT, PMT, RC, triaxial and Cross-Hole tests.

2. RU+MCC model

The concept of the RU+MCC model is an extension of the classical theory of critical state. It fulfils all the conditions imposed to models at this class. It is defined by: yield surface, plastic law, isotropic hardening law and an elastic law. The base model of the RU + MCC is a classic Modified Cam-Clay (MCC). It is also an improvement of the FC+MCC small strain model [6], which based on Fahey-Carter nonlinear material function

The novelty of the model RU+MCC is a new material functions, causing answer of the model more realistic and adequate. Proposed material functions governing the process are consistent with state-of-the-art in constitutive modelling of soil, and specify known dependences of shear modulus with stress and strain invariants. Aspect of plasticity is not forgotten. Classic MCC with modification of the shape of yield surface by van Eekelen was used.

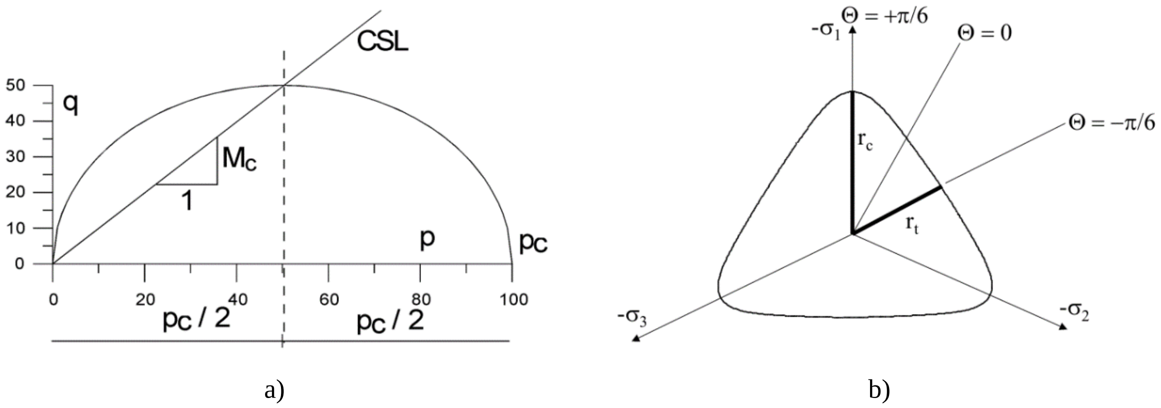

Figure 3. Yield surface of RU+MCC in p'-q space (a) and it's deviatoric cross section (b)

A key element of the model development process was to find a material function of the shape similar to those obtained experimentally. This shape is defined in the literature as "sigmoidal" (Figure 4) and is relatively common in biology, statistics, or in research of neural networks. Finally, selected function was *arctan (x)*, which, after transformation, is presented in eq. 1.

$$G_S = G_0\left(\frac{\left(-arctg\left(t_2\left(\log_{10}\left(\varepsilon_S\right)+6\right)t_1\right)\right)}{\pi}+0.52\right), \tag{1}$$

$$\nu = const. \tag{2}$$

where G_0 is the initial shear modulus [kPa], ε_S is the shear strain, t_1 and t_2 are parameters and υ is the Poisson's ratio.

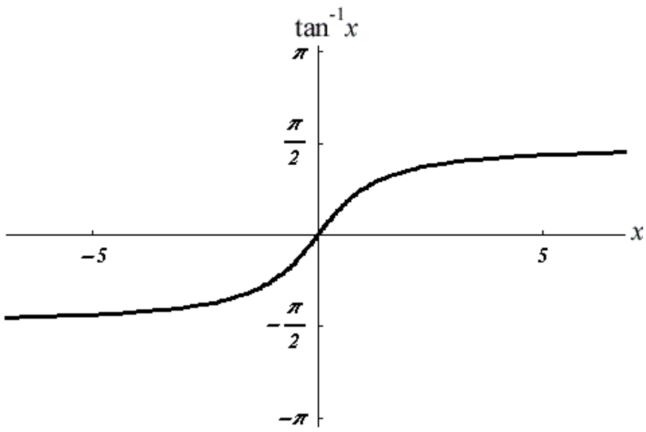

Figure 4. Sigmoidal function arctan(x)

In eq. 1 very important is an initial shear modulus, known before small strain phenomena discovery as dynamic shear modulus. The most common specification of that modulus were incorporated by Hardin [7]. Simplified version of G_0 with only two parameters was used in the model:

$$G_0 = G^* \ p_a \left(\frac{p'}{p_a} \right)^n , \tag{3}$$

where *p’* is the mean effective stress, *G** and n are parameters and *pa* is the reference pressure of 1 kPa.

In eq. 1, 2 and 3 only four parameters can be found. Even together with classical MCC model parameters, there are just nine of them. This number is acceptable for practical use of the RU+MCC model.

3. HS SMALL model

The Hardening Soil model (HS-Standard) was designed by Schanz [8] in order to reproduce basic macroscopic phenomena exhibited by soils such as:

- densification (a decrease of voids volume in soil due to plastic deformations),
- stress dependent stiffness (phenomena of increasing stiffness modules with increasing mean stress),
- soil stress history (for overconsolidation effects),
- plastic yielding (development of irreversible strains with reaching a yield criterion),
- dilatancy (an occurrence of negative volumetric strains during shearing).

The modification by Benz [9] includes additionally:

- strong stiffness variation with increasing shear strain amplitudes in the domain of small strains
- hysteretic, nonlinear elastic stress-strain relationship which is applicable in the range of small strains

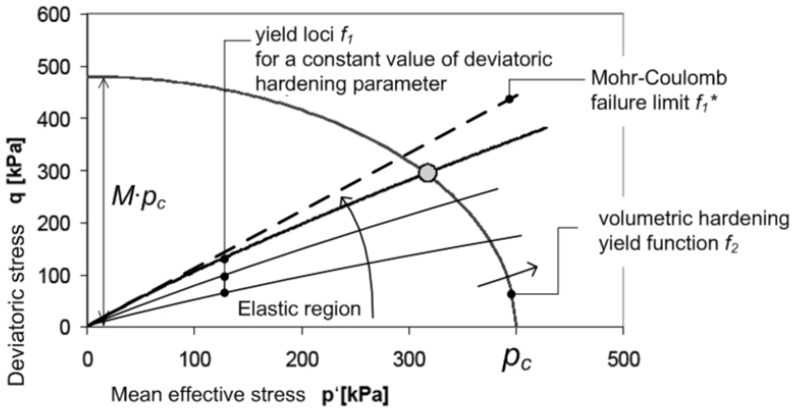

Figure 5. HS-small model scheme [10]

In the model shear, strain hardening loci is described by the function:

$$f_1 = \frac{q_a}{E_{50}} \frac{q}{q_a - q} - 2\frac{q}{E_{ur}} - \gamma^{PS} = 0, \tag{4}$$

where:

γ^{PS} is the plastic strain hardening parameter,

q_a is the asymptotic deviatoric stress.

Ultimate deviatoric stress q_f is described by the Mohr-Coulomb criterion:

$$q_f = \frac{2\sin\phi}{1-\sin\phi}\left(\sigma_3 + c\cot\phi\right), \tag{5}$$

where:

c is a cohesion,
ϕ is an internal friction angle.

Volumetric hardening yield function f_2 is:

$$f_2 = \frac{q^2}{M^2 r^2(\Theta)} + p'^2 + p_c^2 = 0, \tag{6}$$

where:

M is the model parameter which defines the shape of the cap surface and is related to K_0^{NC} ,
r(Θ) is a van Eekelen's function,
p_c denotes the preconsolidation pressure.

The small strain behaviour is described by Hardin & Drnevich [11] formula:

$$G = \frac{G_0}{1 + a\dfrac{\gamma}{\gamma_{0.7}}}, \tag{7}$$

where:
a, $\gamma_{0.7}$ are parameters,
G_0 is an initial shear modulus, assumed by formula:

$$G_0 = G_0^{ref} \left(\frac{p'}{p'^{ref}} \right)^m \tag{8}$$

where:
p' is a mean effective stress,
G_0^{ref} and m are parameters.

The G_0 initial shear modulus is defined by the same formula in both models. Different is S-shape function, which governs the modulus degradation.

4. Models calibration

Soil conditions were recognized by the wide spectrum of simple and advanced soil tests. Among others a SPT, DMT, CPT, PMT, resonant columns, triaxial and Cross-Hole tests were carried out. Mentioned below results of that tests were used for soil parameters estimation. There was a general assumption of a direct calibration if possible.

In the subsoil five soil layers were separated of thickness within 1.4 and 22 m (Figure 6), but layer 2 and 3 were the same layer with different groundwater condition. Layers 1-4 were medium dense sands, when the layer 5 was a dark grey marine clay. The consistency of the clay was stiff.

Depth	dt	Shear wave velocity	Initial shear modulus
[m]	[ms]	[m/s]	[kPa]
2	10	240	104
4	8	300	162
6	8.5	281	142
8	12	199	71
10	10	238	102

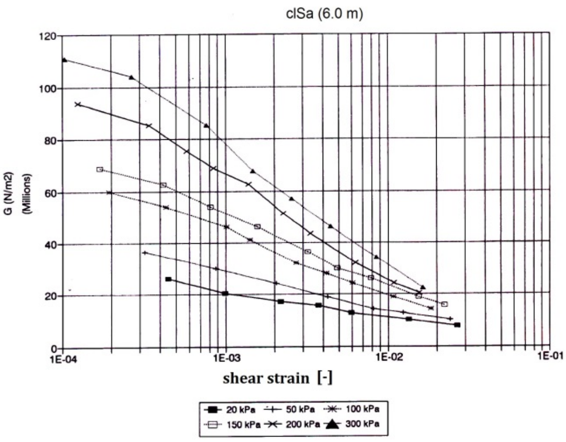

Figure 6. Calibration data – results of cross hole (left) and resonant columns (right) tests [4]

Within the performer tests were resonant columns test and Cross-Hole test, which were totally satisfied for calibration process of small strain material functions. Resonant columns test is rather rare in engineering practice, but triaxial test with local strain measurement is also sufficient. For initial stiffness calibration a SCPTu or SDMT are useful. In case of lack of that data, the correlation dependences are available and helpful, but less accurate.
More problematic was moderate strain zone calibration. In this, a crucial is an estimation of preconsolidation pressure. This parameter defines the overconsolidation stress zone range and

separates elastic and elasto-plastic behaviour in both models. Finally, a global calibration of pc0' were done.

Back analysis of one of the footings (size of 3x3m) was used to estimate the values of missing parameters pc_0 and λ. The results of this are shown in Table 1 and Table 2. A good correlation was found for both models. Final presentation of estimated parameters in both models is shown in Table 1 and Table 2. The layer of dark clay, which influence in a boundary problem is marginal, was finally described by linear elastic model.

Table 1. Estimated parameters of RU+MCC model

layer		model	p_{c0} [kPa]	λ	M_c	υ	e_0	G*	n	t_1	t_2
I	0-3.5m	RU+MCC	270	0.1	1.41	0.25	0.75	18 000	0.5	6	2
II	3.5-7.0m	RU+MCC	350	0.1	1.41	0.25	0.78	30 000	0.5	5	4
III	7.0-11.0m	RU+MCC	450	0.1	1.41	0.25	0.75	18 000	0.5	5	1.67
IV	11.0-33.0m	Elastic	E=200000 [kPa]			0.3					

Table 2. Estimated parameters of HS-Small model

layer		model	p_{c0} [kPa]	E_0 [kPa]	υ	m	c [kPa]	ϕ [°]	M	H [kN/m²]	$\gamma_{0.7}$	E_{50} [kPa]
I	0-3.5m	HS-Small	270	260 000	0.25	0.5	1	35	0.67	3000	0.002	40 000
II	3.5-7.0m	HS-Small	350	400 000	0.25	0.5	1	37	0.67	3000	0.002	50 000
III	7.0-11.0m	HS-Small	450	350 000	0.25	0.5	1	35	0.67	3000	0.0009	45 000
IV	11.0-33.0m	Elastic	E=200000 [kPa]		0.3							

5. Verification analysis

To obtain a results of interaction of foundation with subsoil, a three-dimensional FEM model of a quarter part of the tasks in the plan dimensions of 12x12 m and a depth of 32.5 m was created. The model view, indicating areas of different material is shown in the Figure 7, where marked three finite elements with resultants shear characteristics are presented.

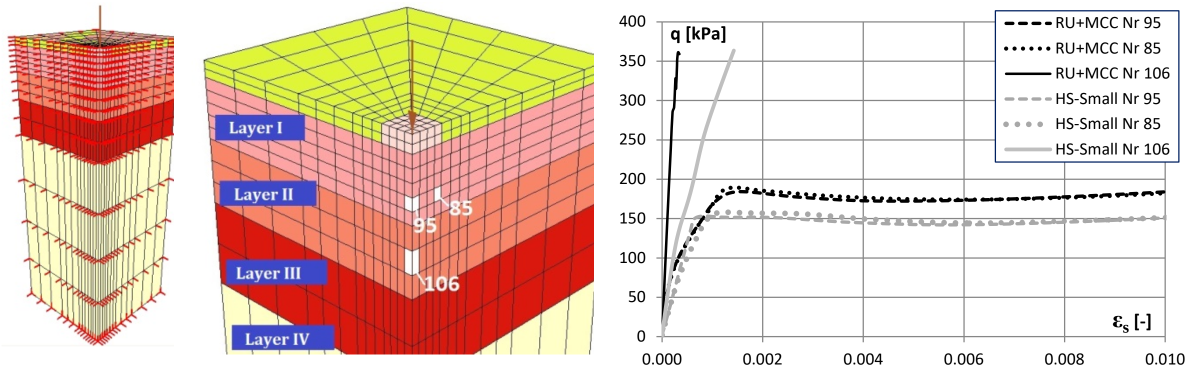

Figure 7. FEM model of analysed pad foundation (Z_Soil.pc) and shear characteristics in elements 85, 95 and 106

As shown in Figure 8 two different constitutive models influenced the resultants shear characteristics even in case of similar calibration process.

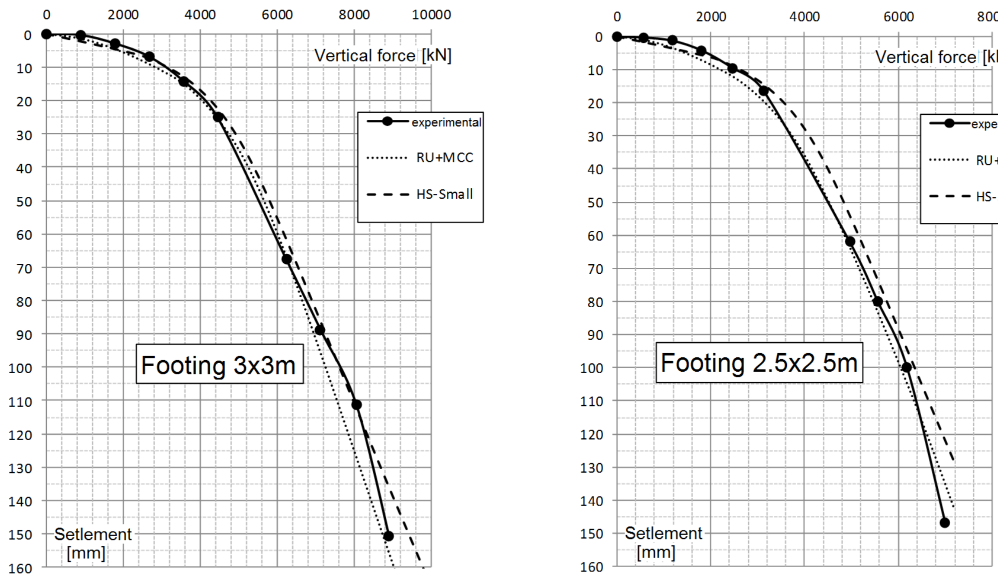

Figure 8. Vertical displacement characteristics of 3x3 m and 2.5x2.5 m footings

In part of elasto-plastic behaviour characteristics are very similar, visible different is nonlinear elastic behaviour governed by S-shape material function, even with the same initial modulus condition. Nonlinear soil behaviour is obvious and visible in Figure 8 and 9. While analysing the local characteristic, the first stage of loading is inclined higher in case of RU+MCC model then in HS-small.

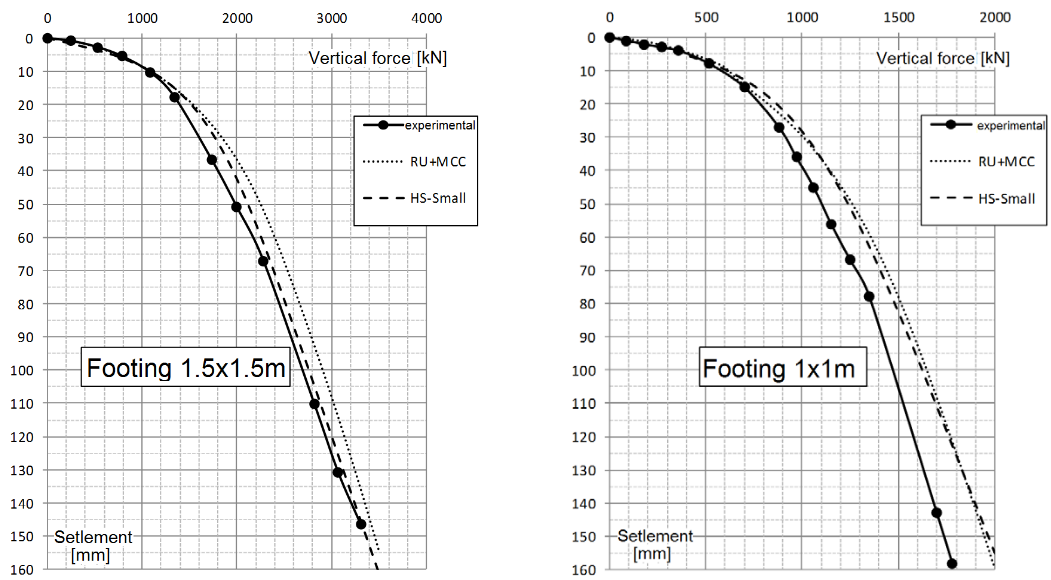

Figure 9. Vertical displacement characteristics of 1x1m and 1.5x1.5m footings

In global characteristics all three curves, an experimental and two simulations are very close to each other. However, in global characteristics of larger footings (2.5x2.5 m and 3.0x3.0 m) small differences at first stage of loading are visible, while in two others are not. Generally, both constitutive models gave adequate and accurate results of settlement.

6. Conclusion

Presented RU+MCC and HS-small models realistically describe the behaviour of soil under monotonic loading. The precision of the prediction of settlement is satisfying. Proper calibration of small strain model like RU+MCC or HS-small with modern soil testing methods like CPTU, triaxial test, Cross-Hole and resonant columns can give adequate answer of the model qualitative and even quantitative.

Numerical FEM simulations of trial loads compared with obtain from laboratory and in situ tests calibrated small strain models show good and correlation with real behaviour of the foundations. In first stage of loading, the results are very close to each other. This relation is observer for RU+MCC model as well as for the second one HS-Small. Stage of plasticity domination is also well simulated. In this case, the differences are more visible on the graphs, but presented models have more limitations in this part and for preserving relative simplicity of them, this imperfection is intentional.

It is necessary to mention, that the boundary problem of simple spread footing could be different in behaviour from other geotechnical problems. To confirm the quality of small strain models it is necessary to analyse more cases, which are different.

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