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Use of Probabilistic Analysis in Design of Shallow and Deep Foundations

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Abstract. The paper deals with some aspects of reliability analysis, with special attention drawn to Point Estimation Method, in foundation design. The aim of calculations is to establish reliability index β and probability of failure P_f for two main types of foundations – spread footing and pile system. Most certainly foundations are the most important part of any structure. The correctness of its design determines the durability and safety of the facility at every stage of construction and service life. The resistance of the selected type of foundation depends on its geometry and to a large extent on the soil parameters, which have a strongly stochastic character. In most publications, authors rarely or almost never deal with the problem of deep foundations. For this reason, in the paper equal attention is paid to both types of foundations – shallow and deep (focusing on piles in the second case). The probabilistic analysis is conducted in accordance with the recommendations included in the EN-ISO 2394 standard titled *General principles on reliability for structures*. Subsoil bearing capacity of the spread footing is determining in relation to both Eurocode 7 and Polish standard. It should be noted that in the reliability of the foundations the applied safety measures take into account the randomness of both: geotechnical parameters and loads (i.e. statistical analysis), such approach is not fully possible according to the standards used in the design of foundations. On the other hand, probabilistic analysis is much more sophisticated and requires the understanding not only of engineering but also mathematics. For example the difficulty of defining correlation coefficient ($\rho_{R,S}$) between resistance R and loads S forces most authors to assume that this variates are not correlated, which is correct in case of pile foundation, but can be very disputable in case of footing. Thus in the paper the influence of correlation coefficient on the reliability index is also shown.

1. Introduction

Most certainly foundations are the most important part of any structure. The correctness of its design determines the durability and safety of the facility at every stage of construction and service life. The resistance of the selected type of foundation depends on its geometry and to a large extent on the soil parameters, which have a strongly stochastic character. Therefore, there was a need to introduce also here elements of reliability theory, well known and used in other fields of civil and off-shore engineering.

In the design of foundations there are three types of approaches: deterministic, semi-probabilistic and probabilistic. Nowadays the first one is only used (in Polish standards) in Serviceability Limit State expressed by an inequality of the form: $s \leq s_{dop}$, where s is a generalized displacement caused by actions on the foundation, and s_{dop} allowable value of serviceability criteria. The characteristic values



of loads and soil strength parameters are considering in this case and the permissible values S_{dop} are imposed in advance and result from the professional experience of the geotechnical community.

The semi-probabilistic approach is the most commonly used in engineering, it takes into account the design values of effects of actions E_d and resistances R_d , so the Limit State can be written down as $E_d \leq R_d$. The partial factors used here are calculated based on statistical analysis of soil sample tests, for example:

$$\gamma_m = 1 + \frac{1}{x^{(n)}} \left[\frac{1}{N} \sum_{i=1}^N (x_i - x^{(n)})^2 \right]^{\frac{1}{2}} \quad (1)$$

where:

γ_m - safety factor for material parameters (here geotechnical)

N - number of soil samples tested,

x_i - set of results of geotechnical parameter investigated during test,

$x^{(n)}$ - arithmetic average values of x_i .

Fully probabilistic approach employs statistical analysis to take into account the distribution of random variables (geotechnical parameters along with loads), leading to limit-state function which separates safe from failure region (see figure 1). The paper deals with selected aspects of foundation design in terms of reliability theory. Studies presented in the monographs and books listed in the bibliography rarely or almost never deal with the problem of deep foundations. For this reason, in the paper equal attention is paid to both types of foundations – shallow and deep (focusing on piles in second case). The aim of probabilistic analysis here is to establish a reliability index β and failure probability P_f , based on the Point Estimation Method (PEM), in accordance with the recommendations included in the EN-ISO 2394 standard titled *General principles on reliability for structures* [9]. Subsoil bearing capacity of the spread footing are determining in relation to both Eurocode 7 [12] and Polish standard [10].

2. Elements of reliability theory

As previously mentioned, in a probabilistic approach to the safety of structures, the reliability index β plays the main part. It determines target safety (reliability levels) for the structure, which takes into account consequences of failure and costs of safety measures. For the purposes of these study, the reliability index β_c is define as in equation (2) proposed by C.A. Cornell [5].

$$\beta_c = \frac{E\{g(\mathbf{X})\}}{\sqrt{\text{Var}\{g(\mathbf{X})\}}} \quad (2)$$

where:

$\text{Var}\{g(\mathbf{X})\}$ - the symbol of the variance operator,

$E\{g(\mathbf{X})\}$ - expected value (EV) of variate in $g(\mathbf{X})$,

$g(\mathbf{X})$ - limit-state function.

The role of the limit-state function is to separate the safe (reliability) region from the failure (unreliability) region, as depicted in figure 1. Reliability is defined by the ability of the system (structure or its elements) to meet specific requirements in its service life. System failure is reached when it cannot offer the service that it was designed to provide or insufficient bearing capacity occurs [1, 5]. Limit-state function returns a negative value under system failure and a positive value when the system is stable. Therefore, it can be viewed as the difference between resistance R , and load S , both of which are understand as a resultant forces maintaining the structure in a equilibrium state and leading to its loss, respectively.

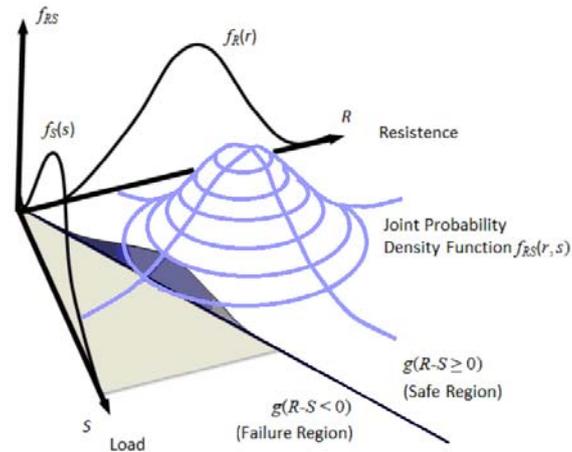


Figure 1. Limit-state function [13].

The resistance of soil R and loads apply to a structure S both depend on random variables. Consequently, they each have a probability distribution $f_S(S)$ and $f_R(R)$ – a normal distribution is assumed here – which in turn combine to generate a joint probability density function (see figure 1). Reliability index according to Cornell formula can be rewritten now as:

$$\beta_C = \frac{m_R - m_S}{\sqrt{\sigma_R^2 + \sigma_S^2 - 2\rho_{R,S}\sigma_R\sigma_S}} \tag{3}$$

where:

- σ_R, σ_S - standard deviations (SD) of resistance and load distributions,
- m_R, m_S - EV of variate R and S respectively,
- $\rho_{R,S}$ - coefficient of correlation between variate R and S

Expected value and the standard deviation can be established from the following formulas:

$$m_R = \sum_j^{2^k} P_j R_j, \quad m_S = \sum_j^{2^k} P_j S_j \tag{4}$$

$$\sigma_R^2 = \sum_j^{2^k} P_j R_j^2 - m_R^2, \quad \sigma_S^2 = \sum_j^{2^k} P_j S_j^2 - m_S^2 \tag{5}$$

where:

- $P_j = \frac{1}{2^n}$ - for 2^n number of combinations for all random data,
- $k = 2$ - is a number of random variable,

3. Reliability index for spread foundation

A shallow foundation of a production hall of overall dimensions of 64×40 m, with spread and continuous footing is considered. In order to identify the subsoil lithology, a series of field tests and boreholes were carried out. The first two layers are Made Ground with organic content and a thickness of 1m, whose average bulk density equals: $\rho = 2.15 \text{ Mg/m}^3$. They are followed by deposit of fine sand

(FSa) in a medium state of compaction ($I_D = 0.4 \div 0.5$) with a low water content and the floor at 2.5 m (see figure 2). It is the first layer which parameters could be determined and are following:

- friction angle: $\varphi = 35.6^\circ$,
- bulk density of the bearing soil layer: $\rho = 1.80 \text{ Mg/m}^3$

The last layer comprises of medium sands (MSa) in a medium state of compaction ($I_D = 0.6$), partially saturated (water table located at 3.0 m) and with slightly higher mechanical and physical parameters (see figure 2). Such a geotechnical conditions of subsoil allow for shallow foundation which after considering the local frost depth are placed at 1.2 m below ground level.

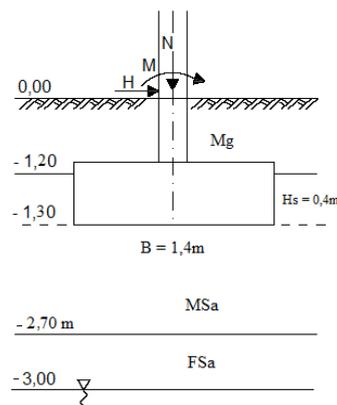


Figure 2. Foot geometry, applied loads and subsoil lithology.

The probabilistic analysis is conducted, based on the point estimation method (PEM), as mention previously. The following parameters were subjected to random variability:

- friction angle: $\varphi = 35.6^\circ$,
- overburden pressure (at the level of the foundation base): $q = 19.1 \text{ kPa}$
- unit weight of the bearing soil layer: $\gamma = 18.0 \text{ kN/m}^3$

The values of geotechnical parameters are changing in the range determined by means of coefficients of variation (CV), summarized in table 1, which can be in fact interpreted as the uncertainty coefficients, increasing (or decreasing) the mean value by the estimated measurement error.

Table 1. Coefficients of variation used for geotechnical parameters and loads.

Geotechnical parameter or load	Coefficients of variation	Range of data	Units
Friction angle	$v_\varphi = 0.1$	32.04 \div 39.16	degrees
Unit weight	$v_\gamma = 0.08$	16.56 \div 19.44	kN/m^3
Overburden pressure	$v_q = 0.08$	17.57 \div 20.63	kPa
Life load	$v_Q = 0.3$	184.6 \div 342.8	kN
Dead load	$v_G = 0.05$	341.4 \div 377.3	kN

3.1. Preliminary analysis

Firstly, the generated values of soil parameters are used to examine how the resistance calculated from Eurocode 7 (EC7) and Polish standard PN-B-03020 (PN-B) differs. In case of EC7 the Drained Condition along with second Design Approach (DA2* to be precise) are considered. In DA2* the Ultimate Limit State is define by means of characteristic values and despite several differences in

formulas between PN-B [10] and EC7 [12] it gives quite similar values of characteristic resistance (see figure 3). Discrepancy increases closing to maximum value of friction angle within data range (figure 3b) or is almost constant in case of unit weight (figure 3a). But the distinction will be much larger after applying partial factor of $\gamma_R = 1.4$ which reduces characteristic value of resistance by almost 30% (while in Polish code maximum diminution is of about 20%).

An important remark should be made here – in the Polish approach the subsoil resistance is calculated from design values of geotechnical parameters established by means of material safety factor from equation (1). As a consequence, PN-B always gives lower values of resistances in comparison to EC7, which is proved in this preliminary analysis. But the situation will be reversed in reliability analysis which does not allow for design values.

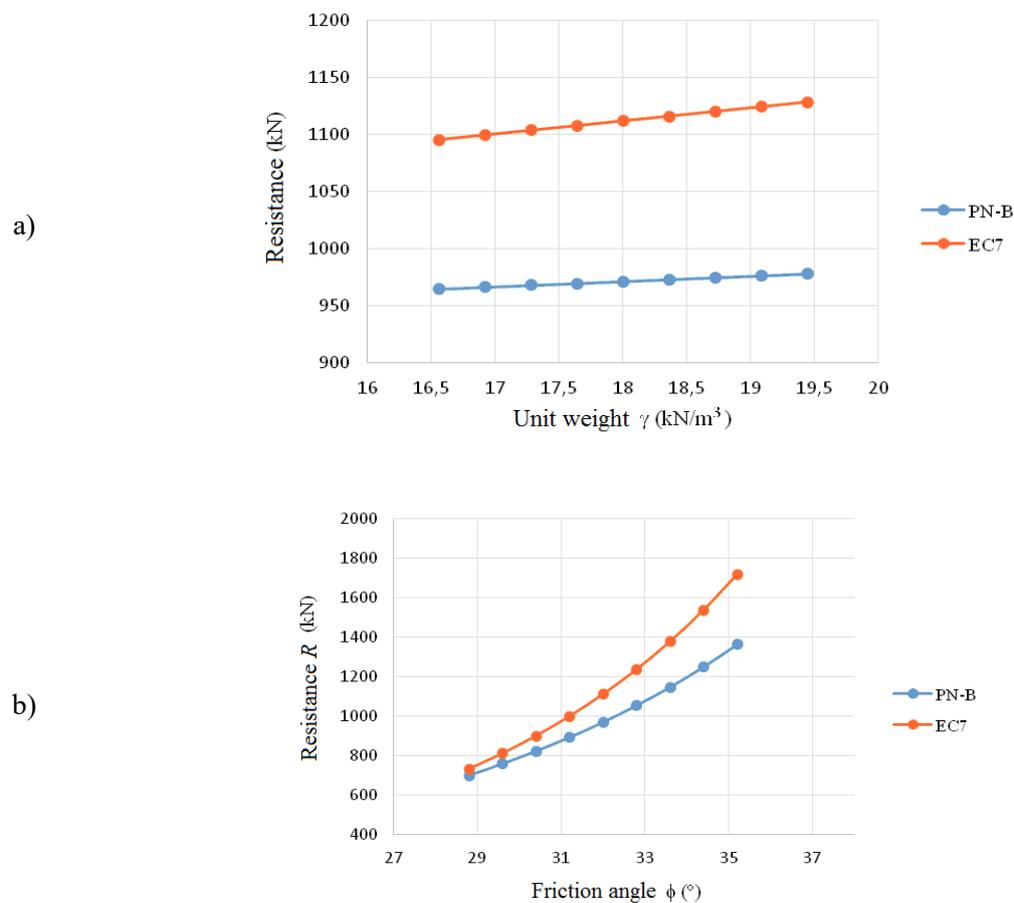


Figure 3. Characteristic resistance in both standards according to: a) unit weight, b) friction angle.

Having the random set of data is very encouraging to study different interesting relationships, like for example influence of safety margin to the value of reliability index. Safety margin is defined as the difference between resistance R and loads S expressed in percentage of R . The increase in the safety margin results in increase of reliability index (see figure 4), which in turn is associated with decrease in the probability of failure. This relationship is linear and has steeper inclination in case of Polish standard, what confirms that EC7 presents more conservative approach.

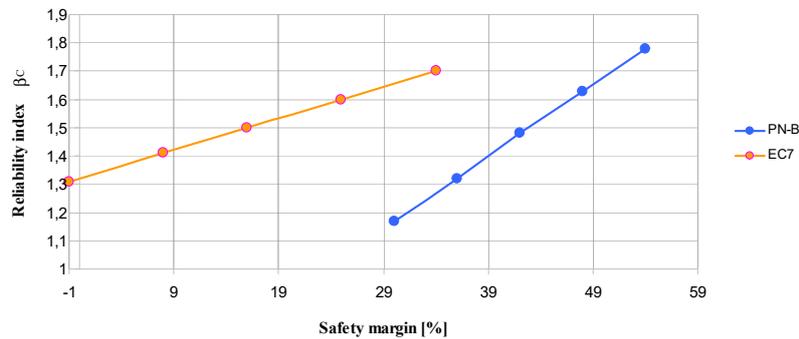


Figure 4. Reliability index vs. safety margin.

3.2. Reliability index based on calculations according to PN-B

The limit-state function base on Polish standard can be written as $g(\mathbf{X}) = Q_n - Q_{fNB}$, where Q_{fNB} is soil resistance defined by the well-known Terzaghi’s formula for Drained Conditions. As a cohesionless soil is considered the bearing capacity of soil depends mostly on friction angle and unit weights, thus they are treated as random (compare table 1) according to normal distribution. Statistical analysis comes down to determining the expected value m , standard deviation σ and variation σ^2 , for both variates in $g(\mathbf{X})$. Calculation of the above for soil resistance is shown in table 2.

Table 2. Statistical analysis for soil resistance (variate $R = Q_{fNB}$) based on PN-B.

	$x_1 = \varphi$	$x_2 = q$	$x_3 = \gamma$	$y = Q_{fNB}$	P_j	yP_j	y^2P_j
i	[deg]	[kPa]	[kN/m ³]	[kN]	[-]	[kN]	[kN ²]
1	39.16	20.63	19.44	3752	0.125	469.00	1759688.00
2	39.16	20.63	16.56	3648	0.125	456.00	1663488.00
3	39.16	17.57	19.44	3288	0.125	411.00	1351368.00
4	39.16	17.57	16.56	3185	0.125	398.13	1268028.13
5	32.04	20.63	19.44	1463	0.125	182.88	267546.13
6	32.04	20.63	16.56	1431	0.125	178.88	255970.13
7	32.04	17.57	19.44	1275	0.125	159.38	203203.13
8	32.04	17.57	16.56	1242	0.125	155.25	192820.50
					Σ	2410.50	1151601.75
						m_r	σ^2_R

All loads acting on a shallow foundation are applied to the subsoil by its base thus $S = V_Q + V_G + G_B + G_F$, where V_Q, V_G are the live and dead loads and are randomize here. The last components G_B and G_F comes from backfill and foot weights respectively. Statistical calculations for loads (variate S) are shown in table 3.

As we consider only characteristic values table 3 is valid for both standards EC7 and PN-B. The reliability index obtained using PEM for the foot according to PN-B equals $\beta_C = 1.61$ while probability of failure acquired from table 4: $P_f = 7.2 \cdot 10^{-2}$.

Table 3. Statistical analysis for loads (variate S).

	$x_1 = V_G$	$x_2 = V_Q$	$y = S^a$	P_j	yP_j	y^2P_j
i	[kN]	[kN]	[kN]	[-]	[kN]	[kN ²]
1	377.28	342.79	720.07	0.25	192.51	148252.80
2	377.28	184.58	561.86	0.25	152.964	93592.88
3	341.35	342.79	684.14	0.25	183.53	134740.70
4	341.35	184.58	525.93	0.25	143.98	82923.24
				Σ	673	6580.60
					m_s	σ_s^2

^a S is denoted as Q_n in PN-B and E_k in EC7.

Table 4. Relationship between probability of failure and reliability index [10].

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.3	2.3	3.1	3.7	4.2	4.7	5.2

Table 5. Statistical analysis for soil resistance (variate $R = R_k$) based on EC7.

	$x_1 = \varphi$	$x_2 = q$	$x_3 = \gamma$	$y = R_k$	P_j	yP_j	y^2P_j
i	[deg]	[kPa]	[kN/m ³]	[kN]	[-]	[kN]	[kN ²]
1	39.16	20.63	19.44	2379	0.125	297.36	707395.28
2	39.16	20.63	16.56	2296	0.125	287.06	659217.37
3	39.16	17.57	19.44	2096	0.125	262.03	549287.13
4	39.16	17.57	16.56	2015	0.125	251.87	507527.33
5	32.04	20.63	19.44	883	0.125	110.41	97517.36
6	32.04	20.63	16.56	857	0.125	107.17	91880.25
7	32.04	17.57	19.44	774	0.125	96.72	74832.48
8	32.04	17.57	16.56	749	0.125	93.63	70125.13
					Σ	1506.24	489012.21
						m_r	σ_R^2

3.3. Reliability index based on calculations according to EC7

According to Eurocode 7, foundation will support the design load with adequate safety against bearing resistance failure, when the following inequality is satisfied $E_d \leq R_d$, where R_d and E_d are design values of resistance and effects of action respectively. In fact after taking into account design approach DA2* they are calculated mostly from characteristic values so the inequality can be written as $E_d\{F_d, X_k\} \leq R_d\{F_k, X_k\}$, where X_k are characteristic values of geotechnical parameters and F are the acting forces. Thus the limit-state function $g(\mathbf{X}) = E_k - R_k$ is established using almost the same values as employed in Ultimate Limit State.

In EC7 the Terzaghi's formula varies from the one applied in PN-B, giving the much lower resistances for the same friction angle. As a consequence, reliability index for Eurocode 7 is also smaller and it equals 1.18, leading to the probability of failure greater than 10^{-1} , which is the highest value occurring in ISO guidelines [9]. Therefore, EC7 once again proves its tendency to overdesign the foundations.

4. Reliability index for deep foundation

Deep foundation composes of 24 driven piles and the capping beam is considered in the example (see figure 5). All piles are 10 m long, have a square cross-section 0,45m by 0,45m and work in compression.

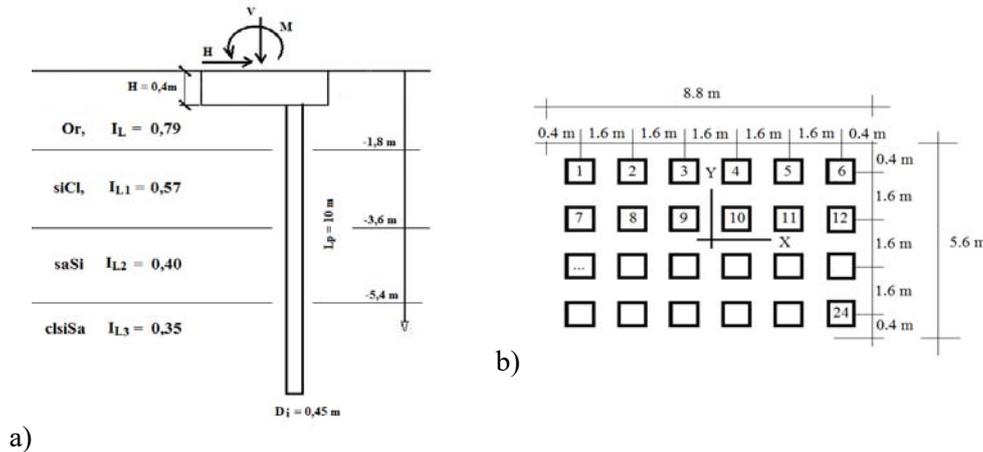


Figure 5. Deep foundation considered in the example: a) geometry of piles and subsoil structure, b) the net of piles

According to Polish code [11] the pile resistance indirectly depends on soil state and type, represented by its name and liquidity index (I_L). Only cohesive soils are considered here as it is more likely for deep foundations. Piles are embedded in 4 layers but the Organic soil is disregarded in calculation as it does not produce any friction nor resistance that follows. Thus only liquidity indexes for soils are recognized as random variables. Coefficient of variation for first soil layer is equal to 0.2 while for other two $v_{IL} = 0.3775$ which gives the ranges of data visible in table 6. The statistical analysis for pile total resistance N_t is also shown in table 6.

Table 6. Statistical analysis for total pile resistance N_t .

i	$x_1 = I_{L1}$ [-]	$x_2 = I_{L2}$ [-]	$x_3 = I_{L3}$ [-]	$y = N_t$ [kN]	P_j [-]	yP_j [kN]	y^2P_j [kN ²]
1	0.68	0.55	0.48	443.1	0.125	55.39	24542.20
2	0.68	0.55	0.22	622.8	0.125	77.85	48484.98
3	0.68	0.25	0.48	457.0	0.125	57.13	26106.13
4	0.68	0.25	0.22	636.6	0.125	79.58	50657.45
5	0.46	0.55	0.48	449.5	0.125	56.19	25256.28
6	0.46	0.55	0.22	629.1	0.125	78.64	49470.85
7	0.46	0.25	0.48	463.3	0.125	57.91	26830.86
8	0.46	0.25	0.22	643.0	0.125	80.38	51681.13
					Σ	543.05	8126.56
						m_r	σ^2_R

One must take into account that because of the quite large rotation moment M_n , the lateral load is not distributed equally to all piles, the most strenuous piles are located the farthest from the rotation axis (see figure 5b). The maximum value of axial load (R_{max}) for the whole system of pile is established from the following formula:

$$R_{max} = \frac{Q_n}{n} + \frac{M_n^y x_{max}}{\sum_{j=1}^n x_j^2} \quad (6)$$

where:

n - number of piles,

Q_n - vertical loads at the base of capping beam

x_{max} - coordinate of the most distant pile in relation to rotation axis of the M_n^y .

x_j - coordinates of each pile

Consequently the limit-state function can be define as $g(\mathbf{X}) = N_t - (R_{max} + G_p)$, where G_p is the weight of the pile. Statistical analysis for such loads is performed as shown in table 7. Reliability index for the pile foundation equals $\beta_C = 2.08$ and leads to probability of failure $P_f = 3.0 \cdot 10^{-2}$ (see table 4).

Table 7. Statistical analysis for the maximum loads of a pile $R_{max} + G_p$.

Units i	$x_1 = N_n$ [kN]	$x_2 = M_n$ [kNm]	$y = R_{max} + G_p$ [kN]	P_j [-]	yP_j [kN]	y^2P_j [kN ²]
1	6391.00	1400.00	376.3	0.25	94.08	35400.42
2	6391.00	1100.00	369.6	0.25	92.40	34151.04
3	5229.00	1400.00	327.9	0.25	81.98	26879.60
4	5229.00	1100.00	321.2	0.25	80.30	25792.36
				Σ	348.75	596.86
					m_s	σ_s^2

5. Summary

The two main types of foundation are considered in the paper - spread footing and pile system. Several aspects of their design are mentioned, with special attention drawn to Point Estimation Method, which served to perform the reliability analysis. The results obtain from the reliability analysis for all three foundations are gathered in the table 8. Index β_C appears to be a little low, especially in case of spread footing, hence after some consideration the correlation coefficient $\rho_{R,S} = 0.5$ is taken into account. It increases the values of reliability index but not as significantly as could be expected (see table 8).

Table 8. Results of the reliability analysis.

Foundation type	β_C	β_C^a	P_f	P_f^a
Spread (PN-B)	1.61	1.68	$7.2 \cdot 10^{-2}$	$6.6 \cdot 10^{-2}$
Spread (EC7)	1.18	1.26	above 10^{-1}	above 10^{-1}
Pile system	2.08	n/a	$3.0 \cdot 10^{-2}$	n/a

^a correlation coefficient $\rho_{R,S}$ taking into account in calculations

The probabilistic approach shows that designing of the foundations in accordance with standards often does not provide sufficient information regarding the safety of a given structure. The exemplary life time target values of reliability index (see Table 9) are specified in the ISO code 2394:2012, *General principles on reliability for structures* [9]. For the spread footing (calculated according to PN-B) it can be concluded that in case of a failure, there are some consequences only if the relative costs of safety measures will be high [9].

Table 9. Target values of reliability index - reliability levels [9].

Relative costs of safety measures	Consequences of failure			
	Small	Some	Moderate	Great
High	0.0	1.5	2.3	3.1
Moderate	1.3	2.3	3.1	3.8
Low	2.3	3.1	3.8	4.3

It should be noted that in the reliability of the foundations the applied safety measures take into account the randomness of both: geotechnical parameters and loads, such approach is not fully possible according to the standards used in the design of foundations. On the other hand, probabilistic analysis is much more sophisticated and therefore requires the understanding not only of engineering but also mathematics. For example the difficulty of defining correlation coefficient $\rho_{R,S}$ forces most authors to assume that R and S are not correlated, which is correct in case of pile foundation, but can be very disputable in case of footing. Based on design experience all three foundations should be safe and even in case of failure do not generate great consequences and β_C should be large than recommended 3.1 value. There are several ways of altering this situation. One is to introduce spatial randomization of geotechnical parameters – very efficient method but too sophisticated for everyday design [2, 3]. The other is to implement more advanced probabilistic tools like FORM and SORM although, as the literature shows, they can still produce lower reliability indexes [5]. Finally we can take into consideration narrower ranges of data (data less scattered) along with correlation coefficient explicitly derived for resistance-load relationship (especially when it has a nonlinear characteristics). To find such a simple and practical methods will be a subject of future studies.

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