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A Study on the Laterally Loaded Pile Behaviour in Liquefied Soil Using P-Y Method

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Abstract. Under repeated seismic loads during an earthquake, increase of pore water pressure in cohesionless, saturated and loose soils leads to decreased shear strength and therefore liquefaction occurs. Literature reports of numerous cases involving damage due to liquefied soil which makes the pile behaviour and performance in liquefied soil even more important. It is possible to investigate the behaviour of piles located in the liquefied soil under inertial loads using linear spring model, p-y method or numerical analyses and a suitable liquefaction constitutive model in time. This study investigates the behaviour of two piles, located in the foundation of the "Niigata Family Courthouse" which was damaged during the Niigata earthquake of 1964, under inertial loads in liquefied soil. The soil profile identified for this case and the piles are analyzed using p-y method and linear spring approach. Pile deformations and bending moments calculated with these two methods were compared.

1. Introduction

Piles are structural members commonly made of wood, concrete, steel or composite materials in order to convey the load arising from upper structure to the load bearing lower layers which are located deep in the soil profile [1]. General purpose of piles is to convey the vertical loads of the foundation acting on soil layers with poor load bearing properties to soil layer with better loadbearing capacity [2]. Moreover, piles are also designed in order to bear lateral loads in some cases. It is possible to conduct simple calculations with empirical approaches for piles performing under vertical loads, however, it is rather hard to talk about a widely accepted design principle for piles bearing lateral loads. The behaviour of laterally loaded piles is a problem of three dimensional and nonlinear soil-structure interaction which is significantly affected by the stress-deformation behaviour of the soil (shear strength, elasticity modulus and volume change characteristics of soil) and the soil-pile interface behaviour [3].

Laterally loaded piles are significantly distinguished from vertically loaded piles in terms of their behaviour under load and design principles. The design of laterally loaded piles requires the estimation of bending moment and the lateral displacement under the load with respect to desired project criteria, soil conditions and pile geometry [4].

It is not uncommon for the pile foundations to be damaged with varying deformations due to liquefaction under repeated loads during earthquakes. Damages in piles due to liquefied soil are a result of the lack of definition of the additional dynamic loads acting on the piles as soil liquefies during an earthquake in the design process [5].

Literature offers many studies on the damages on laterally loaded piles during an earthquake which also included small scale lab and centrifugal experiments. Many of these studies identified the pile length, its diameter and soil profile as the factors effecting pile behaviour.

Cubronovski and Bowen (2008) reported a case analysis for liquefied soil profile. This study compared the effective stress analysis and pseudo-static analysis. Authors investigated the performance of pile foundations in liquefied soil, under seismic loads using time-history analysis [6]. Takahashi and Takemura (2005) analyzed the pile-supported port structure which was damaged in the Hyogo-ken Nambu earthquake in Takahama, 1995 using a parametric design and centrifugal experiments [7]. Finn and Fujita (2002) emphasized the importance of pile design for soil liquefied under earthquake loads. Authors investigated a pile sample of 1.5 m diameter designed for a 14-floors building and its foundation. Their conclusion was that the most critical criteria for pile design is secured pile tips, the ability to find the soil with liquefying and non-liquefying layered structure and inclusion of earthquake loads [8]. Bhattacharya and Bolton (2004) noted that bending effects must be considered when designing a pile under earthquake loads in liquefied soil. Bhattacharya (2003) found in a centrifugal test that piles placed in liquefied soil are not supported sufficiently as the soil loses its strength which in return leads to the collapse of piles due to the bending effect. Authors suggested that load carrying capacity tests along with bending investigations must be conducted when designing piles [10]. Finn (2015) investigated in a structural model the changes in rigidity due to the rotation and displacement of piles under earthquake loads in liquefied soil. As a conclusion, author emphasized the importance of considering the structure-pile-soil interaction and the environmental loading in the pile design [11]. Hamada (1992) reported the damages the Niigata Family Courthouse suffered during the Niigata Earthquake of 1964 [12]. In their publication, Madabhushi, Knappett and Haigh (2010) used the data from Hamada's report and explored the damages in the piles [13]. Yao et al., (2004) conducted a vibrating table experiment in order to explore the pile-soil-upper structure behaviour in liquefied soil using a laminar box of 4 m in length and 2 m in width and height. Authors emphasized the need to consider the non-steady state of soil before liquefaction as bending moment and the pressure of the soil acting on the pile may reach the maximum level at the liquefaction process [14].

In the current study, displacement and bending moments which may act on the laterally loaded pile in liquefied soil are analyzed using p-y method in LPILE software and linear spring method in SAP2000 software considering only the inertial loads. Deformation profiles and moments were compared with the data obtained from the analyses.

2. Modelling Methods for the Laterally Loaded Pile Behaviour

The issue of laterally loaded piles is a problem of nonlinear structure-soil interaction and the pile behaviour under lateral loads concerns the interaction between the pile and the soil.

Available methods for the analysis of a single laterally loaded pile can be listed as follows: [15]

- Limit state method,
- Foundation soil reaction method,
- P-Y curve method,
- Elasticity analysis method, and
- Finite element method.

Broms method [16] is a limit state method used in order to determine the ultimate lateral load bearing capacity of a single laterally loaded pile. Foundation soil reaction method [16];

1. While the soil strength and pile displacement values are modeled proportionally, studies show that soil strength is not a linear variable.
2. Horizontal coefficient is rather a parameter of the model than an essential soil property.
3. Although soil is continuous, soil strength is modeled using discontinuous springs.
4. Pile geometry is implicitly considered. Soil strength in P-Y method shows properties similar to those in foundation soil method except the fact that it is a nonlinear function of pile displacement.

P-y method is described as spring systems placed with specific intervals along the pile. Where “y” is the displacement of the pile due to lateral forces, “ p_u ” is the force soil acts on the unit length of the pile as a result of the displacement. Figure 1(a) shows the soil-pile interaction, lateral, vertical, and end bearing capacity, lateral springs (p-y spring), vertical springs (t-z springs), end springs (q-z spring) in BNWF model [17]. P-Y curves method is efficient in evaluating the behaviour of laterally loaded piles. Figures 1(b) and 1(c) shows the states of p-y curves before and during liquefaction. Following steps must be observed when creating p-y curves [17];

1. Drill logs must be assessed for soil parameters,
2. The length of the pile foundation must be defined,
3. A stress-unit deformation graph must be produced for liquefying soil,
4. P-Y curves must be produced using the soil stress-unit deformation graph.

Foundation soil reaction method has found itself an extensive area of use as the p-y curve method and beams on elastic foundation method are easy to use also offering logical results.

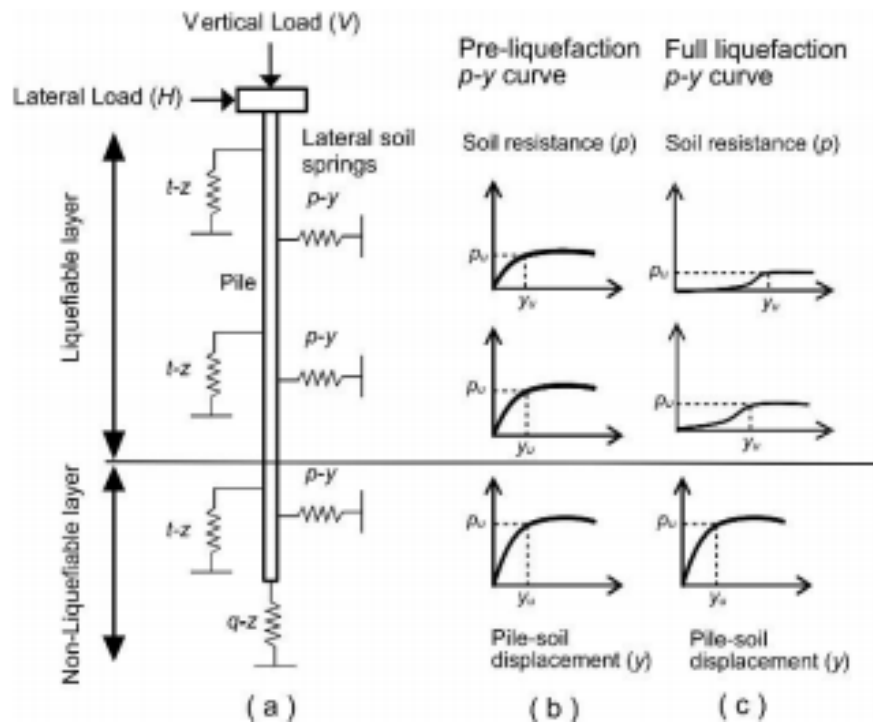


Figure 1. (a) BNWF model of pile-soil interaction, (b) pre-liquefaction, and (c) post-liquefaction p-y curves [17].

Elastic analysis method is an easy-to-use method for pile displacement calculations taking into account the soil at all times. Moreover, soil modulus is taken elastic and considered to be changing with the changes in the stress.

The concept of modulus of subgrade reaction is based on the assumption that the soil is elastic and that it consists of an infinite number of springs with minimal distance between them. Where modulus of subgrade reaction is k_s , stress at any point of the soil is q and the placement at that point is calculated as a ratio of ΔH in Figure 2 [18].

$$k_s = \frac{q}{\Delta H}$$

Figure 2. Definition of modulus of subgrade reaction [18]

In the aforementioned methods, the displacement of the foundation system due to lateral load will not reach to a point to affect the upper structure, a range of load will be defined to not exceed the soil collapse threshold taking into consideration the shear strength of the soil around the pile under load, and the pile dimensions will be defined accordingly. The lateral load acting on the pile will be sufficient against bending and collapse and a security coefficient will be utilized. Among these methods, the modulus of subgrade reaction method and p-y method are the most commonly preferred in practice. In the modulus of subgrade reaction method, it is assumed that soil acts as a vertical beam having modeled it with individual springs. p-y method, on the other hand, takes the plasticity properties into account and defines the changes in the form as a function of these properties.

Literature offers abundant research on the analysis of laterally loaded piles in liquefying soil. These studies commonly used p-y method and modulus of subgrade reaction method. Among the techniques used in p-y curves are as follows [19]:

- **p multiplier (α):** This technique uses strength reduction factor SPT $N_1(60)$ number of impacts [18], This technique is the most popular one for the liquefying soils in the BNWF (Beam on Nonlinear Winkler Foundation) model [19]. Figure 3(a) shows that when lateral displacement relatively small, soil-pile interaction depends on the onset rigidity of the curve. However, when the displacement is significant, then the ultimate soil strength is affected by the foundation rigidity. If p-y curves are concave upward, then the pile will have a nonlinear behavior with small displacement and zero strength (Figure 3(b)) [20]. In liquefying soil, a pile will behave like a free moving column and will bend under significant axial loads.

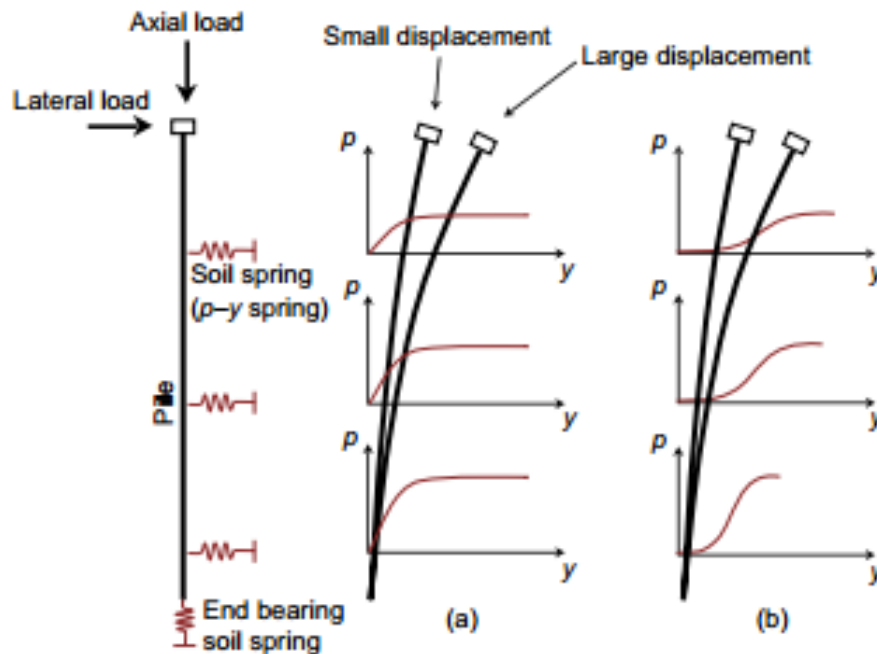


Figure 3. Effect of the shape of p-y curve on the mobilised strength and stiffness of a pile foundation in a liquefied layer (solid lines schematically indicate the deflected shape of the pile at small and large displacement): (a) p-y curves from p-multiplier approach; (b) proposed strain-hardening p-y curves [20].

- **C_u factor:** This method presumes that strength reduction factor changes depending on the level of liquefaction in a totally liquefied soil in accordance with the p multiplier method [19].

- **Residual strength method:** Many researchers claimed that sandy soil acts like soft clay at the moment of total liquefaction. According to researchers, p-y curves are similar to the curves of soft clay under repeated loading [19]. However, maximum lateral strength can be replaced with undrained residual shear strength of sand [19].
- **Zero strength method:** Residual shear strength in liquefying soil depends on several parameters which in return makes it rather challenging to assess [19]. It was speculated that effective stress must be zero at the total liquefaction moment and that the shear strength of the soil must also be zero [19].

Indeed, p-y curve will show increased slope with increased deformation in liquefying soil, but this behaviour can be explained as hardening with larger displacement and it occurs under specific conditions. Nevertheless, the common practice attempts to decrease the slope of the p-y curve for liquefied soils. A comparison of the results obtained from analyses and experiments showed that p-y method is more reliable in the estimation of pile behaviour than modulus of subgrade reaction method. Experiments conducted on sand samples with varying density showed that soil density affects the pile behavior. It was reported that pile displacement decreases as the level of soil density increases [21].

Dynamic soil-pile interaction is a complicated phenomenon. Lateral soil rigidity is generally explained as a p-y curve for earthquake load and it is accepted as a parameter used to estimate the pile reaction. The shape of p-y curve and rigidity parameters play an important role in the pile reaction to lateral seismic loading [13]. In pile groups, the slope of p-y curves decreases for continued deformation in non-liquefying sand soil, while the slope increases for liquefying soils [22]. This tendency is explained by the load-induced expansion and the decreasing pore water pressure around the piles [22]. Rigidity of p-y curves and excessive pore water pressure decreased with increasing depth. Structure-soil interaction is a result of the inertial and kinematic interactions. The assumption that inertial effect is not significant when compared to kinematic interaction for high-rise, long-period structure is generally valid, however, it is a simple idea [23].

In this study, only the inertial effects and not the kinematic effects were considered in the analyses. It is necessary to obtain design extents having combined the effects act on the soil and piles during kinematic and inertial interaction phases, taking both interactions into account [24]. The effect of inertial interaction (for the range of foundations and structure parameters considered) is, in general, to increase the pile head response but to significantly decrease the response of the structure [25]. It was found that the LPILE software is recommendable for soil-pile interaction analyses when SAP2000, Abaqus/Cae and LPILE software, were compared. Increased number of springs improves the results obtained from the analyses conducted on SAP2000 [26]. Numerical analyses were run on LPILE and SAP2000 software. Rollins' liquefaction model is used with LPILE software, while analyses were conducted using modulus of subgrade reaction with SAP2000. Empirical formulas were used in the definition of coefficients in SAP2000 model and the modulus of subgrade reactions were reduced by one tenth at the liquefying soil layer as the p multiplier of $N_{1(60)}$ corrected SPT value 7-8 corresponds to 0.1 as suggested by Brandenburg et al., (2005) [27]. Deformation profiles and moments were compared with the data obtained from the analyses.

3. A case sample and parametric study

For the parametric study, the case of Niigata Family Courthouse (NFCH) building which was damaged in the Niigata earthquake of 1964 was taken. The pile system available for the NFCH building was damaged due to displacement which in return led to 1° vertical slope of the building. After being used for another 25 years, the building was rebuilt. During the construction work, 2 piles were extracted and investigated. 1st pile was 6 m long and 35 cm in diameter and the 2nd pile was 9 m long and 35 cm in diameter [13]. Pile samples were then analyzed comparing the displacement and moment values for liquefying and non-liquefying soil profiles at 4 m, 5 m, and 8 m using LPILE and SAP2000 software. Table 1 shows the properties of the piles with 35 cm diameter [28].

Table 1. Material properties for piles

Pile Properties	Pile No 1	Pile No 2
Length (m)	6	9
EI (kN.m ²)	5625	7500
Length (m)	6	9

Analyses with LPILE software were proved to be impossible as the pile was too fragile and the cross-section was insufficient which leads to yielding to the bending effects. Table 2 shows the maximum pile displacement and maximum moment values for Piles No. 1 and 2 obtained from the analysis conducted with SAP2000 software.

Table 2. Results of SAP2000 for piles no 1 and piles no 2.

SAP2000 Results		
Liquefaction Depth	Max. displacement (mm)	Max. moment (kN.m)
	Pile No 1	
4	54	37.26
5	54	37.24
No Liquefaction	9	20.08
	Pile No 2	
4	50	40.49
5	51	40.40
8	51	40.06
No Liquefaction	4	15.89

In this case sample, nonlinear analyses conducted with LPILE software were overwhelmed due to the mechanical properties of the piles and insufficient diameter, however, analyses conducted with SAP2000 software gave results as it runs elastic analyses. The analyses with SAP2000 were not the correct approach as it assumes that the soil is linear and as it models using springs. In the SAP2000 analysis, moment values and displacements were not significantly different as the liquefaction depth increased. The differences found are in relation with the socketing depth of the pile and the pile behaved more rigidly as the depth decreased receiving more moment. As the cross-section and mechanical properties of the pile were insufficient, a parametric design was used increasing the cross-section and mechanical properties of the pile. In this study, two piles, 9 m long with 70 cm in diameter and 6 m long with 50 cm in diameter were analyzed. The effects of pile diameter and length were investigated for the pile behavior in liquefying soil. Analyses were run on SAP2000 and LPILE software for pile models with 50 cm and 70 cm diameter. Liquefied soil profile was altered in the analyses and the analyses were repeated with 50 kN lateral load per pile for liquefaction at 4m, 5m and 8m. Soil properties are shown in Table 3 and the properties of the concrete used for the pile are shown in Table 4.

Table 3. Soil properties

Precedent	Soil Profile (Sand)	Effective Unit Weight (kN/m ³)	p-y curve Soil model name
Liquefaction at 8 m	0-8 Loose	7	Liquefied Sand (Rollins)
	8-9 Medium Dense	10	Sand (Reese)
Liquefaction at 5 m	0-5 Loose	7	Liquefied Sand (Rollins)
	5-9 Medium Dense	10	Sand(Reese)
Liquefaction at 4 m	0-4 Loose	7	Liquefied Sand (Rollins)
	4-9 Medium Dense	10	Sand(Reese)

Table 4. Concrete properties of piles

Concrete Type	Characteristic Compressive Strength (MPa)	Unit Weight (kN/m ³)	Modulus of Elasticity (MPa)
C25	25	24	30000

Table 5 shows the maximum moment and maximum displacement values obtained from the analyses performed on LPILE and SAP2000 software for the sample of 9 m length and 70 cm diameter.

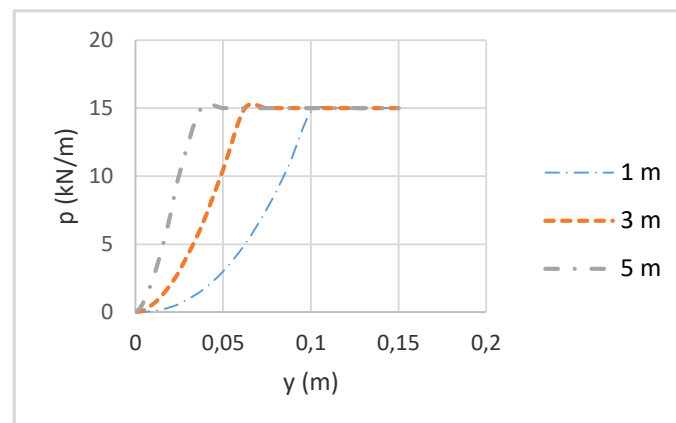
Table 5. Maximum moment and maximum displacement values obtained for the sample of 70 cm diameter

LPILE Results		
Liquefaction Depth	Max. displacement (mm)	Max. moment (kN.m)
4	200	225
5	140	263
8	110	128
No Liquefaction	77	108
SAP2000 Results		
Liquefaction Depth	Max. Displacement (mm)	Max. Moment (kN.m)
4	13.5	149.91
5	17.3	130.6
8	21.3	95.96
No Liquefaction	1.6	44.53

Figure 4 shows the p-y curve graph obtained for the sample of 9 m length and 70 cm diameter which is produced in accordance with the Rollins' p-y curves method [22] applied at 1 m, 3 m and 5 m of a soil profile where liquefaction occurs at the first 5 m. Where, p is the lateral pressure along the pile (kN/m), y= horizontal displacement of the pile (mm), $A=3 \times 10^{-7} \times (z+1)^{6.05}$, $B=2.80 \times (z+1)^{0.11}$, $C=2.85 \times (z+1)^{-0.41}$, $P_d=3.81 \times \ln(d)+5.6$ z= depth (Eq.1) [22].

$$p = P_d \cdot A \cdot (By)^C \quad (Dr \approx 50\%) \quad (1)$$

The same analysis was also run for liquefaction at 4 m and 8 m.

**Figure 4.** p-y curve graph for the soil profile with liquefaction at the first 5 m

Figures 5 and 6 show the bending moment and depth values obtained from the pile sample of 9 m length and 70 cm diameter.

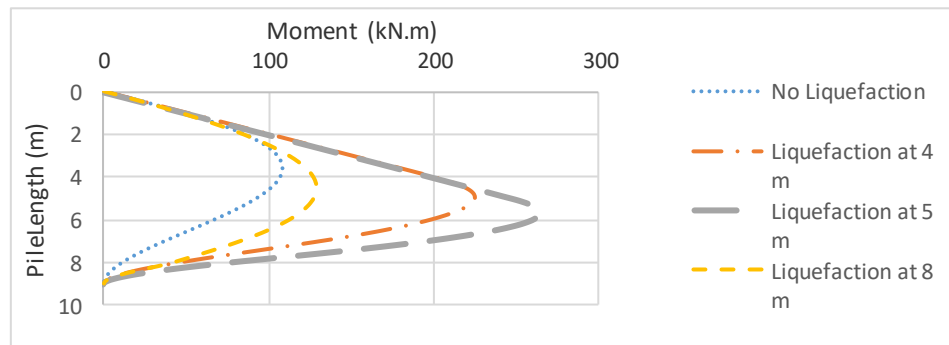


Figure 5. Bending moment-depth graph of the liquefying soil using LPILE software

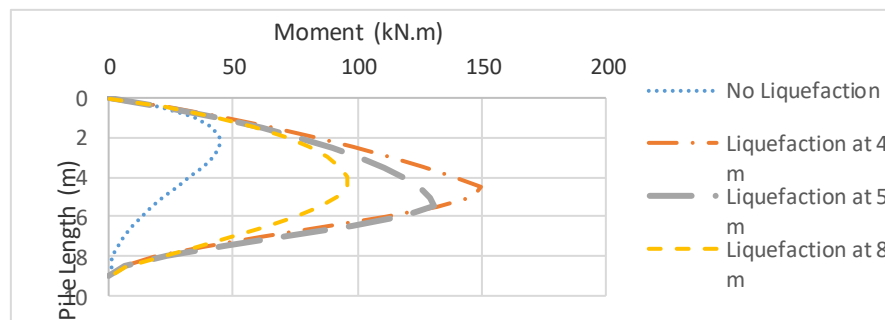


Figure 6. Bending moment-depth graph of the liquefying soil using SAP2000 software

Figures 7 and 8 show the displacement graphs for the pile sample of 9 m length and 70 cm diameter as obtained from the analyses run on SAP2000 and LPILE software.

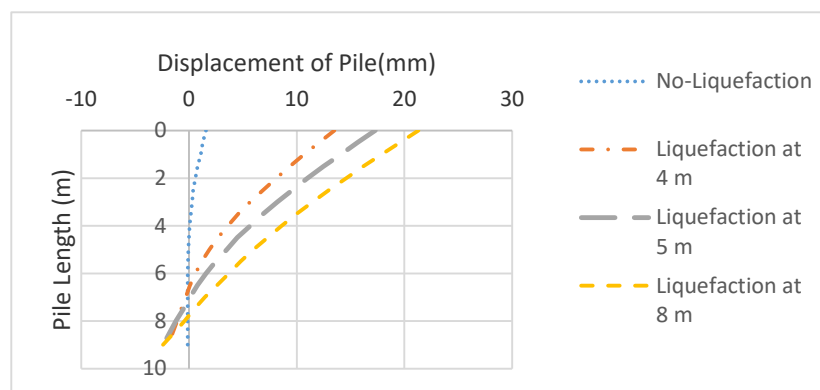


Figure 7. Pile length-horizontal displacement in SAP2000 analysis

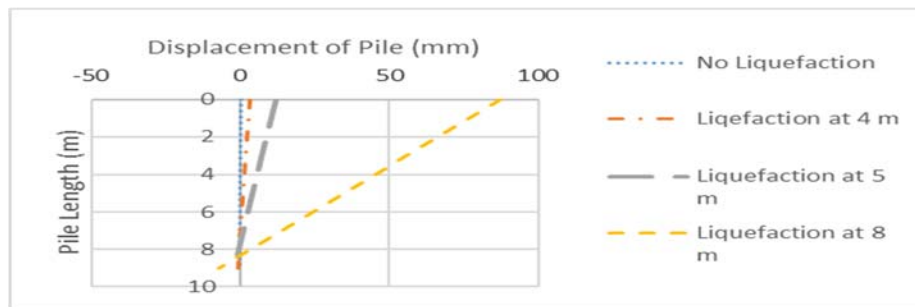


Figure 8. Pile length-horizontal displacement in LPILE analysis

Figures 9 and 10 show the bending moment and depth values obtained from the pile sample of 50 cm length and 6 m diameter.

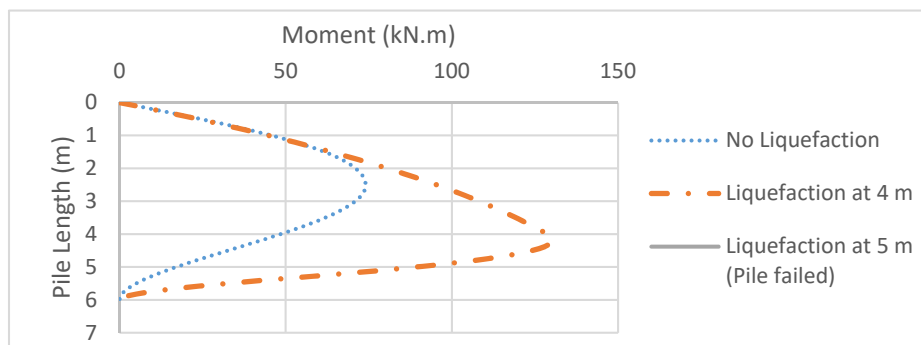


Figure 9. Bending moment-depth graph of the liquefying soil using LPILE software

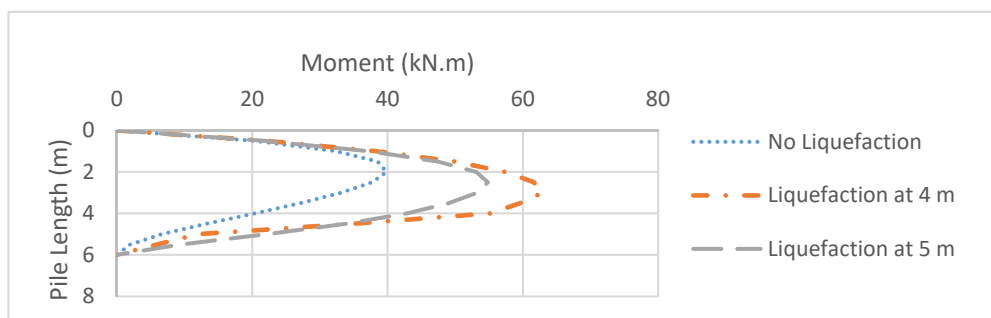


Figure 10. Bending moment-depth graph of the liquefying soil using SAP2000 software

Figures 11 and 12 show the displacement graphs for the pile sample of 6 m length and 50 cm diameter as obtained from the analyses run on SAP2000 and LPILE software.

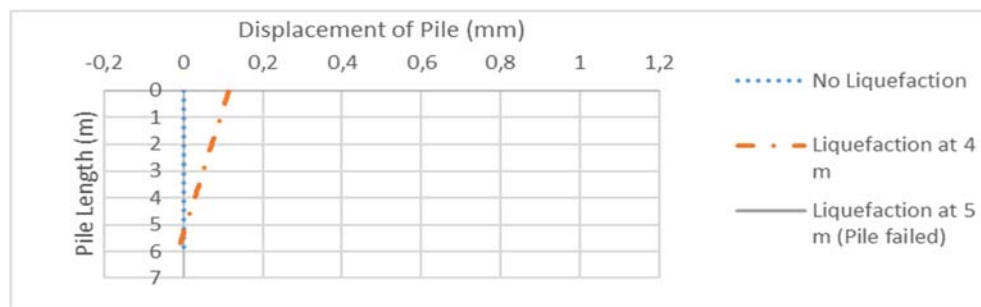


Figure 11. Pile length-horizontal displacement in SAP2000 analysis

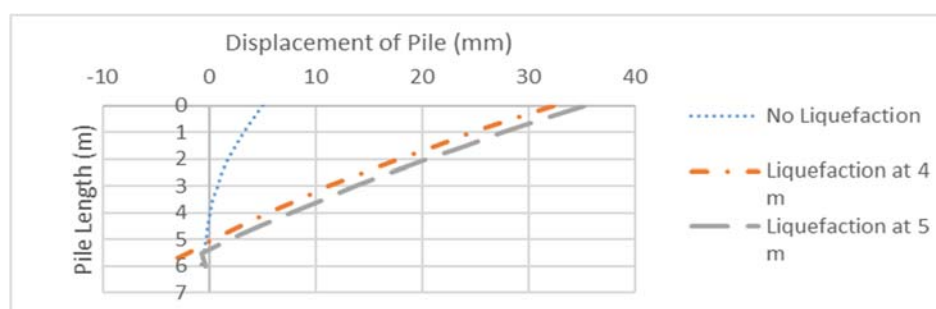


Figure 12. Pile length-horizontal displacement in LPILE analysis

4. Results

This study analyses and compares the laterally loaded pile behaviour in liquefying soil using p-y curve method (LPILE) and linear spring approach (SAP2000). Kinematic interaction was disregarded in the calculations and only inertial interaction was considered. Differences are found in the analysis results obtained using LPILE and SAP2000 software. Especially the pile end deformations calculated using the reduced linear spring approach and maximum cross-section moments are significantly lower than those found with the non-linear spring approach, i.e. reduced linear spring approach offers unreliable results. The main reasons behind these differences are the fact that p-y curves are not linear in liquefying soil and that it hardens with higher deformation; liquefied soil behave as a softer material especially at lower levels. As expected, the lowest moment and the lowest pile displacement was obtained from the soil profile with zero liquefaction. As the liquefying soil layer thickness increases, moment value was increased up to 8 m thickness, however, it was decreased again at 8 m thickness. The reason behind this finding is that socketing depth decreases as liquefying layer thickness increases which leads to lower bending moment and that pile acts relatively rigid causing it to topple. As a result, pile displacement has become more critical and an increase in the displacement was observed. In a soil profile with liquefaction at the first 4 m and 5 m, on the other hand, the pile stands against the displacement due to the non-liquefying soil socketing effect. Nevertheless, moment value was higher as the pile was subject to increased bending. A closer look into the pile length-moment graphs showed that moment value obtained from the analyses performed using LPILE software was the highest for the pile sample of 70 cm diameter in a soil profile with liquefaction at the first 5 m. The highest value from the analyses performed using SAP2000 software, on the other hand, was found for the soil profile with liquefaction at the first 4 m. The pile of 50 cm diameter was yielded in the soil profile with liquefaction at the first 5m in the analysis run on LPILE software. The moment levels obtained from SAP2000 analyses were lower than those of LPILE software and it was observed that the displacement estimations were rather smaller. In the SAP2000 model, linear spring constants were used and the spring coefficient was reduced for the liquefied soil and it was hypothesized that soil does not yield. However, such an assumption is not the correct approach as soil may have a non-linear behaviour under specific loads in the analyses

conducted using inertial loads. In conclusion, it can be said that analyses run on LPILE software offered a better reliability.

As modelling the soil using individual springs is not the best approach, it is necessary to use analysis designs which take elasticity properties along with plasticity properties into account. Therefore, it can be suggested that p-y method in which plasticity properties are considered and deformations are defined as a function of these properties is a better method to use. In the light of other studies available in the literature, it was found that p-y method offers more reliable analyses when compared to the modulus of subgrade reaction method.

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