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To cite this article: Marcin Bujko 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **471** 042013

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Identification of Elasto-Plastic Phenomena in Soils in the Range of Small Strains

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Abstract. Accurate modelling of the phenomena occurring in loaded soil materials is the primary source of development of the current normative regulations as well as smarter and more efficient engineering practice. The aim of this research is to develop a more accurate description of the complex nature of the soil material and thus a more accurate evaluation of the subsoil-construction interaction in the whole range of stress, not only in chosen limit states. The proper description of the behaviour of soil in the range of small strain is an extremely important element to forecast displacements of construction interacting with subsoil. Due to this fact, it has a huge influence on the quality of mapping the actual internal forces in the whole structure - including the foundation. Stiffness moduli for small strains are actually recognized as one of the most important properties of the soil. RC/TS is the reference apparatus in which they are determined. A technique based on the analytical solutions was used to calculate the dynamic shear modulus, dynamic modulus of elasticity and the damping coefficient of soils and rocks. In theory, the movement of the tested soil particles resulting from the propagation of elastic waves is non-destructive. Despite the apparent differences in the behaviour of soil material under dynamic loads with a significant frequency (approx. 60-160Hz - RC test) and quasi-static loads, characterized by very low frequency amplitude (below 10Hz, in practice 0.1Hz below - TS test), small strain conditions are modelled numerically using a combination of linear and non-linear laws of elasticity. The main drawback of standard RC/TS research methodology is the assumption that soil material is subject only to visco-elastic strain. This assumption is manifested, among others in the interpretation of the damping coefficient during TS test, which is defined as the proportion of the energy dissipated by the material during the cyclic torsional shearing to the potential energy accumulated in the material during elastic deformations. However, observations confirm the hypothesis that the behaviour of the soil, even in the range of small strains, should be considered as elasto-plastic - including the generation of a plastic, irreversible deformation. The research method involves executing multiple (repeated) torsional shear tests on natural and artificial (repeatable) structure samples of soil. Samples were subjected to varying conditions of the amplitude and frequency of cyclic load. Using modified procedure of registration test results allows the observation of changes in viscoelastic strain of material and measurement of permanent deformation after the completion of the torque impact. Multiple repetitions of tests on samples of the natural and artificial structure allows to identify the cause of any differences that may arise in the individual reactions of the material (phase elastic, viscous, plastic).



1. Introduction

The aim of the article is to identify phenomena in soil materials during the process of torsional shearing executed in RC / TS apparatus, in the range of small strains ($\gamma < 10^{-3}$).

Due to analyses of existing studies, it was found that in certain load conditions, soil materials do not behave in accordance with the theoretical model, Maxwell, Kelvin-Voigt or Burger. In spite of deformation in the range of small strains, soil materials exhibit properties of the visco-elastic-plastic body.

The null research hypothesis was that soil materials subjected to even light loads causing small strains fall within the permanent, plastic deformation, which can be observed in research conducted in torsional shear apparatus RC / TS. The alternative research hypothesis was the assumption that the viscosity is the phenomenon of significant importance to the process of deformation of soil material - even at very low frequencies of cycling load i.e. lower or equal to 0.01Hz, while permanent deformations are unnoticeable.

Experiments verify the hypotheses and create the basis of a numerical model of the material, which were used to carry out the computational analyses.

2. Significance of the subject

Dynamic testing of mechanical properties of materials has been an important part of many aspects of maritime engineering, seismic engineering and designing foundations of machines or structures subjected to different quick-variable loads for a long time.

Within the last two decades, there has been great progress in the development of laboratory and field dynamic test methods, which have become routine research and analytical techniques of engineering practitioners. This has created the possibility of solving complex problems of the dynamic nature in the issue of construction-subsoil interaction.

However, these methods are still not applied enough by engineers solving problems of statics. One of the reasons of the existing gap between soil dynamics and traditional geotechnical engineering used to be (or still is) the assumption that the dynamic properties of soil cannot be applied in analyses of static geotechnical issues. It turns out that in the range of small strains, soil parameters describing the stiffness are comparable in statics and dynamics issues [1-2].

The proper description of the behaviour of soil in the range of small strain is an extremely important element to forecast displacements of the construction interacting with subsoil. Due to this fact, it has a huge influence on the quality of mapping the actual internal forces in the whole structure - including the foundation.

Stiffness moduli for small strains are actually recognized as ones of the most important properties of the soil. Because of this fact, in geotechnical engineering, the results of field and laboratory dynamic tests as well as seismic tests are widely used to solve traditional issues of the subsoil-construction interaction.

To such a group of techniques belong all the geophysical tests based on the elastic wave propagation theory. In majority of cases, the aim of these tests is to define the value of the speed of shearing wave propagation or/and compression wave between two points of subsoil. Stiffness of the soil layer is related to shear modulus G or elastic modulus E . They are calculated according to relations:

$$G = \rho \cdot V_s^2 \quad (1)$$

$$E = \rho \cdot V_L^2 \quad (2)$$

where: ρ - density of the soil; V_s - speed of shearing wave; V_L - speed of compression wave.

In theory, the movement of the tested soil particles resulting from the propagation of elastic waves is non-destructive. The resulting modulus value corresponds to a small strain, thus these moduli are often called initial (G_0 , E_0 or G_{\max} , E_{\max}). Techniques leading to measuring the speed of propagation of the elastic wave are mostly applied in field conditions (CHT - crosshole test, DHT - downhole test,

suspension logging, seismic reflection, seismic refraction and SASW - spectral analysis of surface waves). Moreover, sensors receiving seismic waves are installed in traditional field tests (SCPTU or SDMT).

Comparing the results of seismic and traditional field tests, correlations are made which allow the estimation of the initial strain modulus [3].

In order to confirm the results obtained in the field, direct laboratory measurements of the mentioned above parameters are required. The results of tests carried out in the laboratory are often considered as reference values in relation to the geotechnical characteristics measured in the field tests.

In order to improve the reliability of the results of in-situ tests, a comparison of the results of field tests has been made with traditional laboratory tests. In the laboratory, according to the assumed path of stress, there is a possibility to freely change the state of stress of soil samples. In laboratory dynamic tests it is also possible to analyse the influence of such parameters as: frequency of loading, the number of cycles, void ratio, or OCR ratio on the nature of the response of soil in given conditions. Correlations for some sensitive soils are useless and require direct research in the laboratory.

For 20 years, velocity of the wave propagation in soils has been usually determined in the laboratory by piezoelectric bender elements [4]. However, the methods of interpreting the results, which lead to the determining the shear wave velocity still raise doubts. This issue was presented in [5]. The use of the 'bender' elements spread in the 90s of the twentieth century with the necessity of integration non-linear characteristics of the soil behaviour in the engineering analyses [6]. As the output parameter characterizing the stiffness of soil is G_{max} , a significant attention is paid to the development of techniques for determining its value. As the most authoritative technique to estimate the initial strain modulus is generally considered the test in the resonant column (RC test).

Test in resonant column is the method known in geotechnical engineering since 30s of twentieth century. The first work presenting the method has been published by Japanese engineers Ishimoto and Iida [7, 8]. One of the earlier constructions of the resonant column apparatus was built in the USA, which was used to determine the speed of the shearing wave in the torsional shear rock samples [9]

Next researches took into account the loads of an anisotropic stress [10] and modifications of the apparatus allowing the use of hollow samples [11], which minimised the variability of the amplitude of shear strain along the radius of the cross-section sample under loads causing torsional shearing.

The apparatus has also been modified to allow for testing at large strains [12], and at high pressure in the cell [13]. Further construction works on the device helped to expand its capabilities, leading to the creation of its new version - TS (cyclic torsional shear apparatus) [14]. Present versions of the apparatus apply the amplitudes of large strains in combination of cyclic torsional shearing and resonant column. The device developed by a professor Stokoe at the University of Texas in Austin is commonly known as RC / TS apparatus (resonant column / torsional shear). Currently RC / TS method is considered as one of the most reliable, effective and pragmatic laboratory methods for the determination of shear modulus and damping ratio of soil and other materials [15].

The issue of testing the properties of soil material in the range of small strains is still valid and important. In the state-of-art, many publications can be found that deal with laboratory tests with Torsional Shear apparatus - leading to calibration of the numerical models [16-20] or determination of the curve of degradation stress-strain relation of specific soil materials [21-24]. Among the methods of interpretation were also techniques of artificial intelligence [25].

According to the author's research there are proposed the hardware modifications of the equipped RC/TS apparatus to increase the range of forced strain [26], allowing observation of entirely new phenomena in the tested materials [27].

3. Research methodology

All the tests of the mechanical properties of soil were conducted in WF8500 (Figure 5) apparatus, which is equipped in Geotechnical Laboratory of the Institute of Building Engineering of the Faculty

of Geodesy, Geospatial and Civil Engineering of the University of Warmia and Mazury in Olsztyn. The device is the product of British company Wykeham Farrance, purchased in 2011. It is a dual-function apparatus working as a resonant column (RC) or the cyclic torsional shear apparatus (TS). It was designed to test the mechanical properties describing the static and dynamic strain characteristics of the soil samples or weak rock material in the range of small and medium strains (up to 10^{-3}). Due to the type of the load, device is designed for dynamic or static tests of the soil. Cylindrical specimens with a diameter of 50 mm or 70 mm might be used.

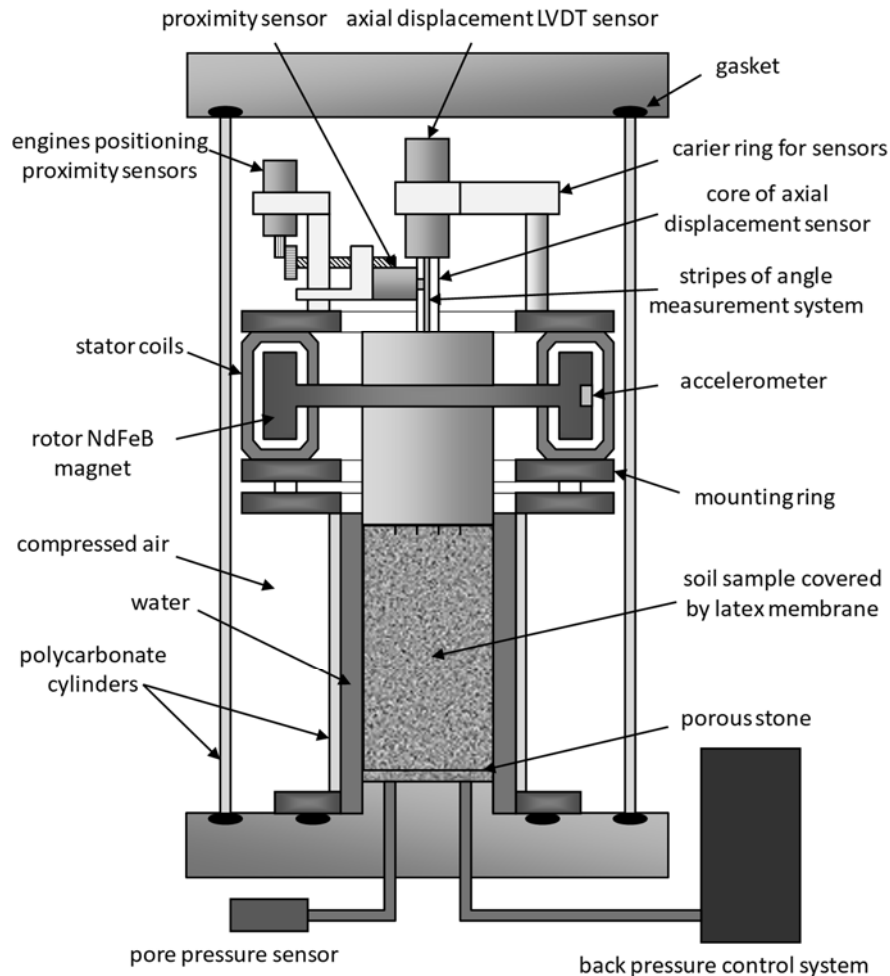


Figure 1. View and scheme of RC / TS apparatus

Testing in WF8500 apparatus in the mode of cyclic torsional shearing (TS) measures the relationship between load and strain of the tested sample of material from which it is possible to derive the value of the shear modulus G and damping factor D . It involves the cyclical loading of samples by torque T of harmonically varying value of the frequency changes less than 10 Hz (in practice less than 1 Hz), causing tangential stress state τ in sample material according to the relationship:

$$\tau(t) = \frac{r \cdot T}{J} = \frac{\kappa \cdot R}{J} \cdot T = \frac{\kappa \cdot R}{J} \cdot T_0 \cdot \sin(\omega \cdot t) = \tau_0 \sin(\omega \cdot t) \quad (3)$$

where: J is a geometrical polar moment of inertia of cylindrical sample, R is the radius of the sample,

r - the distance between the point at which the value τ is determined and the axis of the sample, T_0 - the amplitude of the torque ω - angular frequency, t - time, and κ - a coefficient of 0.8

Second, the effect of torque loading is twisting the free end of the sample, which remains in a fixed relationship with the strain according to the formula:

$$\gamma = \frac{r \cdot \Theta}{H} = \frac{\kappa \cdot R}{H} \cdot \Theta, \quad (4)$$

where: Θ - twist angle, H - the height of the sample.

The course of changes in the value of the angle of torsion, strain and tangential component of stress as a function of time is shown in Figure 7. Eliminating the time variable t , a direct correlation between the tangential component τ and strain γ (detailed output is contained in [31]) can be determined:

$$\tau(\gamma) = \tau_0 \sin\left(\arcsin\left(\frac{\gamma}{\gamma_0}\right) + \varphi\right), \quad (5)$$

where:

$$\varphi = \arctg\left(\frac{C\omega}{K - I_0\omega^2}\right), \quad (6)$$

$$\gamma_0 = \frac{\kappa \cdot R}{H} \Theta_0, \quad (7)$$

where: I_0 - mass moment of inertia of motor, C - constant viscous damping, K - constant elasticity of the torsion

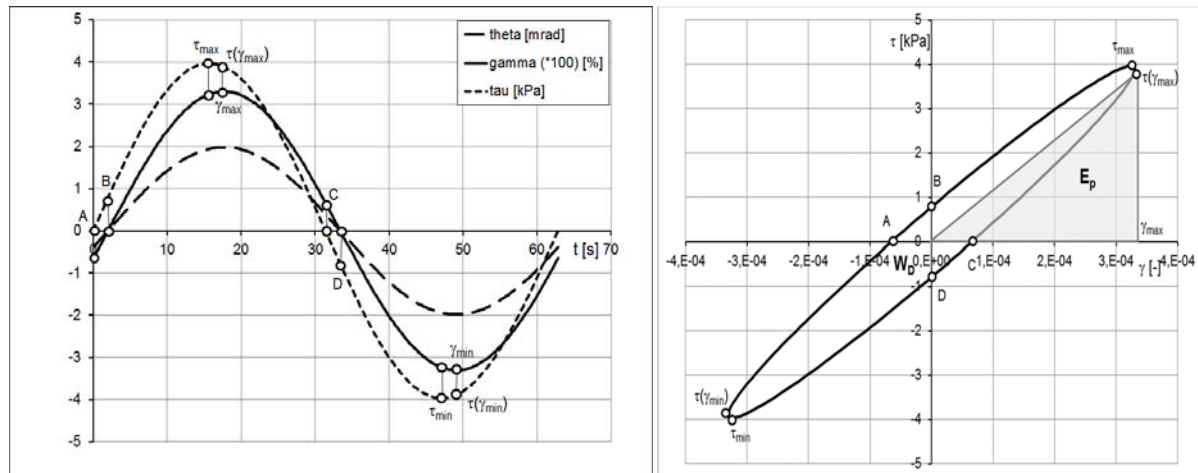


Figure 2. Analytical simulation of TS test [31].

The relationship $\tau(\gamma)$ is shown (figure 2). Due to the phenomenon of damping, graphs $\Theta(t)$ and $\gamma(t)$ are moved out of phase diagram $\tau(t)$ by an angle ϕ , which results in a hysteresis loop highlighted in the graphical representation according $\tau(\gamma)$. Thus, according to the values of the moment of force T (on the basis of predetermined electrical voltage), the angle of torsion Θ (measured by proximity sensors) and the geometrical dimensions of the sample, the relationship $\tau(\gamma)$ might be calculated and plotted (formulas (1) and (2)). It is the source to get γ_{\max} values and $\tau(\gamma_{\max})$ necessary for determining the modulus G :

$$G = \frac{\tau(\gamma_{\max})}{\gamma_{\max}}, \quad (8)$$

Damping coefficient D_{TS} is determined directly from the TS test results and is defined in

$$D_{TS} = \frac{1}{2\pi} \cdot \frac{W_D}{E_P}, \quad (9)$$

where:

W_D - the energy absorbed by the material during cyclic torsion, the energy dissipated, E_P - potential energy accumulated in the elastic material, fully reversible, strain energy.

Wasted energy W_D is recovered in the system by the external work done by the moment T by the torsion of Θ angle. This energy can be defined by the formula:

$$W_D = \int_Y \tau(\gamma) d\gamma, \quad (10)$$

according to the interpretation of geometric of the damping issue (figure 2), the energy W_D is represented by the area of hysteresis described by functional dependence $\tau(\gamma)$. E_P - recovered potential energy, characterized by a temporary accumulation of energy by the elastic material in the maximum value of strain γ_{\max} . E_P is represented by the marked area under a graph of linear-elastic reaction of the material of the sample (figure 2). This area can be expressed by the following formula:

$$E_P = \frac{1}{2} \tau(\gamma_{\max}) \gamma_{\max} \quad (11)$$

The main drawback of standard RC / TS research methodology is the assumption that soil material is subject only to visco-elastic strain. This assumption is manifested, among others in the interpretation of the damping coefficient during TS test, which is defined as the proportion of the energy dissipated by the material during the cyclic torsional shearing to the potential energy accumulated in the material during elastic deformations. However, observations confirm the hypothesis [28,29] that the behaviour of the soil, even in the range of small strains, should be considered as an elastic-plastic - including the generation of plastic, irreversible deformation.

4. Observations

The experiments are related to the phenomena of the degradation of stiffness of soil material in the range of small strains. During numerous tests, the appearance of permanent deformation of the sample after unloading by torque was noted (figure 3).

In order to clarify the research hypothesis, the initial version of the computer application was developed. It is based on the Finite Element Method (FEM). Non-linear-elastic constitutive law (example of calculation results is shown in Figure 2) and elastic-perfectly plastic law were applied to the criterion of Coulomb-Mohr - an example of the simulation results is shown in Figure 3.

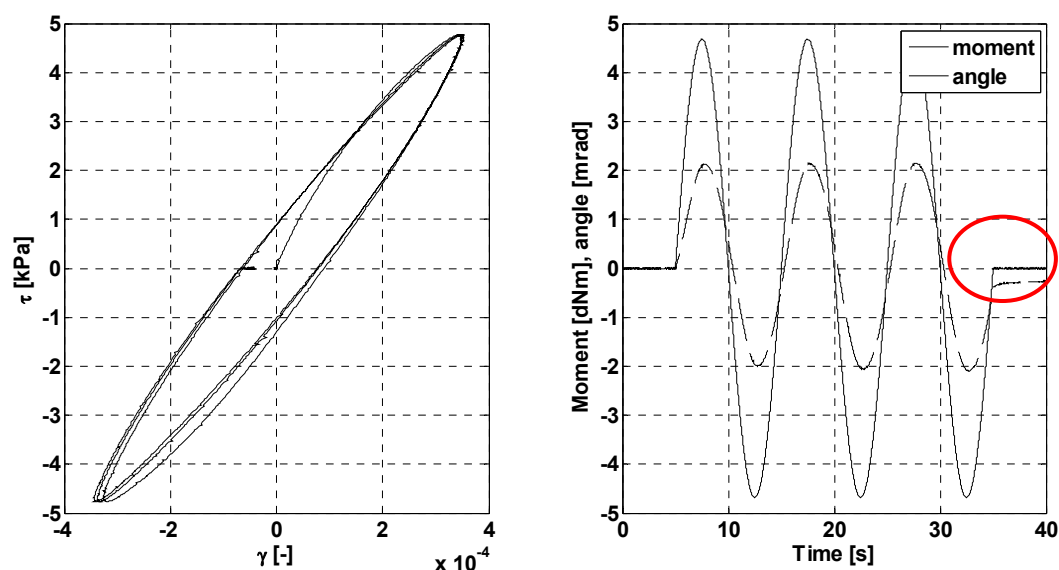


Figure 3. Example of results of cyclic torsional shear test of samples of cohesive soil (marked red: permanent deformation).

According to the latest trends in numerical modelling of granular media [30, 31] the calculation algorithm FEM was created. It includes appearance of plastic deformations during the subjection to relatively small load, so small that the total deformation of the material would be in the range of 10^{-4} - 10^{-3} .

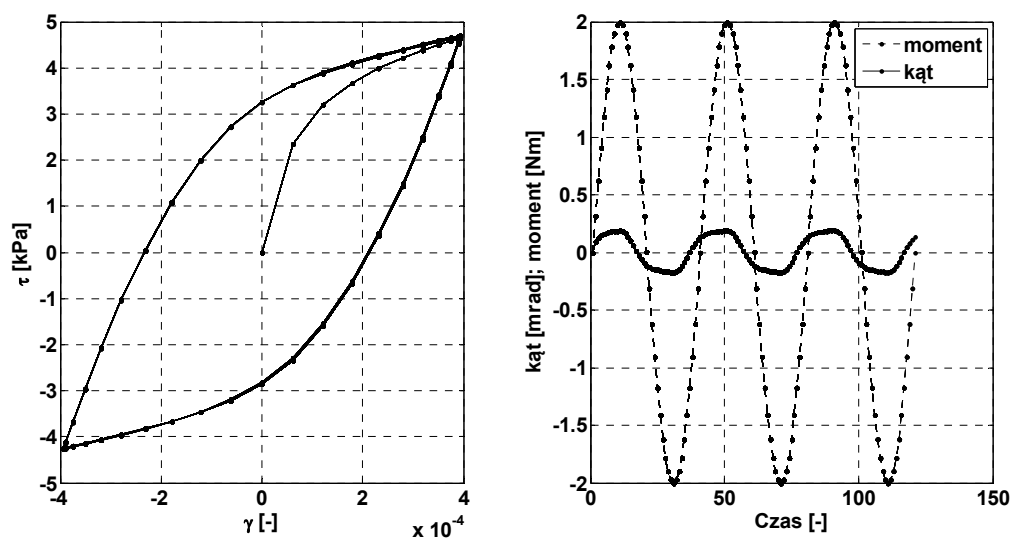


Figure 4. An example of the results of numerical simulation of cyclic torsional shear test on cohesive soil samples using a non-linear-elastic law.

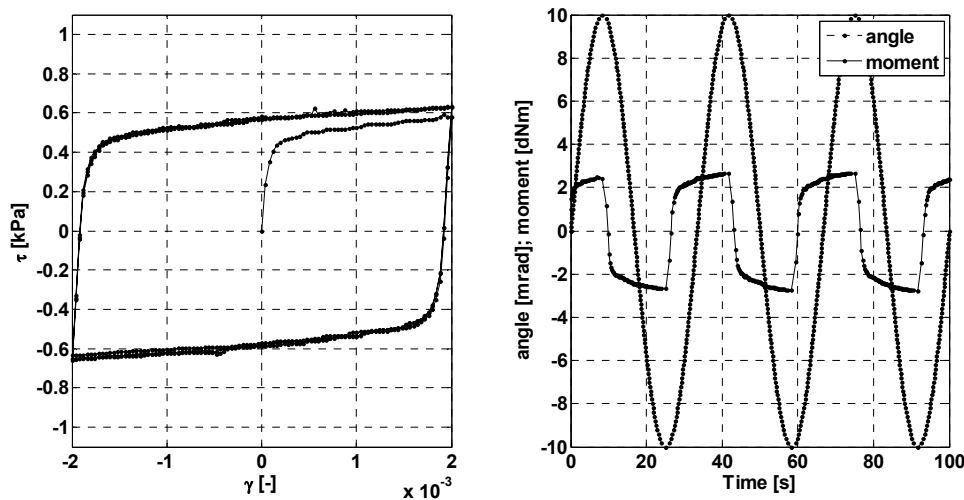


Figure 5. An example of the results of numerical simulation of cyclic torsional shear test of cohesive soil samples using the law of elastic-perfectly plastic.

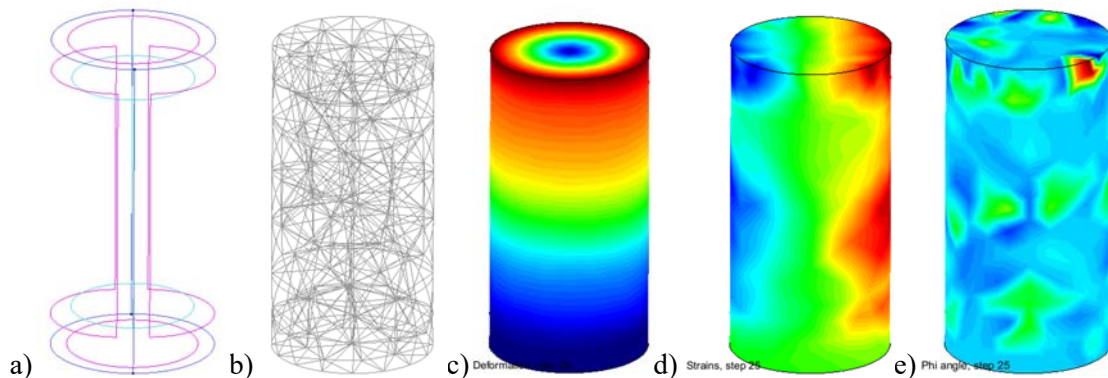


Figure 6. Exemplary results of numerical simulation FEM of clSi sample;
a) the elements of the topology of the sample; b) mesh elements TH10; c) displacement map;
d) map deformation; e) changes in the stiffness map

An example of the results of calculations is illustrated in figure 6. Preliminary analyses indicate the extremely complex nature of the behaviour of soil materials subjected to a load of torque.

5. Modification

Considering the above-mentioned major simplifications in the standard methodology of research and the interpretation of the results, it was decided to carry out the modification of the research technology of torsional shearing samples of soil, implementing upgrades in the construction of RC / TS apparatus - keeping full, original functionality.

An additional, new sensor was used. Microdisplacement transducer was mounted externally to the top cap of sample and measured the angle of the twist of the sample top. The new sensor was a Wheatstone bridge consisting of three precision resistors and one tensometric strain gauge with a combined cantilever plate. The deflection of the cantilever plate causes shortening or lengthening of the strain gauge. The change in resistance is measured by the measuring system: an operational amplifier connected with an analog-to-digital converter which transmits digital data to a computer application via the AVR microcontroller. The measurement is performed in real-time during torsional

shear test and it is parallel to the work of a standard inductive proximity sensor. Monitoring of sensor measurements is possible using an application created by article's author. It might be operated independently from the standard RC / TS software. It allows the reading of the momentary twisting angle of the sample top [mrad] and the shear strain [%]. Extension of the possibilities of interpretation of the results of the torsional shear test were obtained by extending the measurement time. This allows to qualitatively assess whether the sample material returns after unloading to its original state.

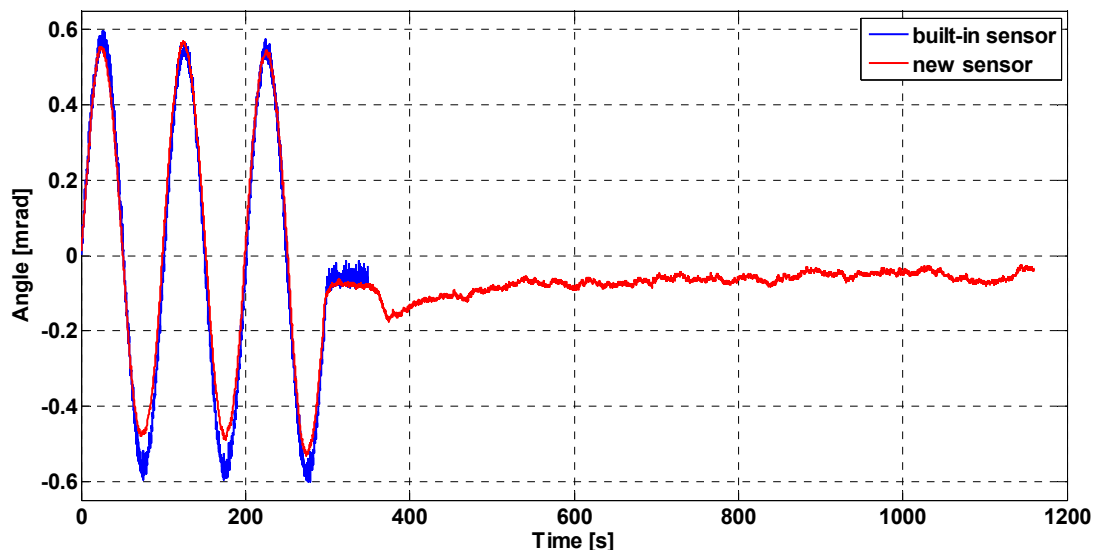


Figure 7. Example of results of cyclic torsional shear test on silty clay with double sensor registration of the twist angle

TS tests with the use of an additional measuring system were performed many times with different settings of torsional shear parameters. The exemplary TS test results conducted on silty clay are shown above. The test was carried out in three torsional shear cycles with motion parameters: amplitude $A = 4V$ and frequency $f = 0.01Hz$. After unloading the sample and completing the standard registration in 400 seconds, the measurement of the new sensor was continued. In the period between 400-550 seconds, a slight increase in the head turning angle was recorded, however, it eventually stabilized at a level below the starting point.

6. Conclusions

The results clearly indicate that after unloading the sample with the torque, twist angle measurement is still possible. The measurement showed that after loading, material does not return to the original state. This allowed to confirm the hypothesis that the soil material reacts elastic-plastically and does not behave in accordance with the theoretical model, Maxwell, Kelvin-Voigt or Burger.

The result of the research will be the concept of visco-elastic and plastic shares in total deformation of the soil materials subjected to cyclic loading. This concept will be implemented in the computer application based on the Finite Element Method and Mohr-Coulomb strength criterion.

With validation processes, the application will accurately simulate the phenomena occurring during the actual testing of the soil samples.

The change of the concept of the soil computational model from visco-elastic to visco-elastic-plastic in the range of a small strain should significantly improve the quality and accuracy of the results of calculations of loaded soil material deformation.

Introducing the concept of elastic-plastic small strain to the practice of design should result in the change of the philosophy of conducting research on soil materials including the modification of the calculation procedures related to verification of the Second Limit State conditions.

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