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Composite Materials: Identification, Control, Synthesis

Irina Garkina ¹, Alexander Danilov ¹

¹ Penza State University of Architecture and Construction, Herman Titov Street, 28,
Penza, Russia, 440028

i.a.naum@mail.ru

Abstract. A new methodology for research into the development of composites is analysed as a set of methods designed to study the properties and structure of the material as a whole. It is noted that in the traditional approach the composite is considered as an unstructured medium, and in the case of a system approach, it is structured: in essence, a paradigm shift takes place in construction materials science. It is shown that the main feature of the system approach is the presence of the dominant role of the whole over the particular, complex over the simple; the more complex the system, the greater the effect of applying a systemic holistic approach. The properties of a composite material as a system object are given. It is assumed that there is a complete, integrative property of the system. The synthesis of composites takes into account the paradoxes of integrity (evaluation and analysis of composite materials should be based on the consideration of the material as an integral and unified system, on the other hand - the study of the material is impossible without analysing its parts) and hierarchy (description of the composite material as a system is possible only if its description as an element of the super system and vice versa). In accordance with the polystructural theory, a hierarchical structure of the radiation-protective composite is developed. It is shown that with strictly ordered criteria, the synthesis of the material reduces to a lexicographic optimization problem with the possibility of obtaining not only optimal strategies (the choice of prescription and technological parameters), but also the ordering of all strategies according to the degree of their preference. In the lexicographic optimization problem, an arbitrarily small increment of a more important criterion is assumed, due to any losses with respect to the remaining less important criteria. With conflicting criteria, the direct reduction of the optimization problem to lexicographic is not possible or not expedient. Here we use a not so rigorous ranking of criteria, as in the lexicographical case. One of the possible ways of synthesizing the material is given - the method of successive concessions. Realization of a method at synthesis of a composite of a special purpose is given. An approach is proposed for solving multicriteria problems on the basis of constructing Pareto sets (in informal analysis, those solutions that are obviously bad are excluded); is realized at multicriteria synthesis of epoxy composite of increased density for protection from radiation. High mobility of the obtained concrete mixes makes it possible to make piece products, building structures of any configuration and containers for transportation of radioactive materials. The dense structure of superheavy concrete allows it to be used for lining the enclosing structures of underground and ground bunkers, cemeteries and storage facilities for radioactive solid, liquid and gaseous wastes. Low porosity guarantees high values of brands for frost resistance, water and gas tightness.



1. Introduction

Now in construction materials science the new methodology of scientific research on development of composite materials is formed. Its basis is made four fundamental and mutually supplementing each other of approach to scientific knowledge: systematic, synergetic, information and homeostatic.

The system approach (based on a holistic view of a complex object, phenomenon or process) to scientific knowledge gave a powerful impetus to the development in science of the direction known as "system theory." It is a set of methods, tools that allow you to explore the properties, structure and functions of objects, phenomena or processes as a whole, presenting them as a system with all the complex interelement relationships, the interaction of the elements on the system and on the environment, as well as the influence of the system itself on its structural elements. The main feature of the system approach is the presence of the dominant role of the whole over the particular, the complex over the simple. The main properties and results of the activity of a system of any nature, although they depend essentially on the composition and properties of its constituent elements, but only the characteristics of these elements cannot be recognized in principle at the level of study. The system is a set of interrelated elements, united by unity of purpose and functional integrity, and the property of the system itself is not reduced to the sum of the properties of the elements. The properties of a system as a whole are determined not only by the properties of its individual elements, but also by the properties of the structure of the system. In the traditional approach, the composite is considered as an unstructured medium; at system approach - structured (change of paradigms in the theory of composite materials) [1...4].

In the simplest case, the structure of the system is represented as the totality of all the elements, the relationships between these elements and the relationships between them (sometimes they are identified with the organization of the system). The aggregate of interconnected structural elements forms a system only in the case when the relations between elements generate a new special quality of integrity, called a systemic, or integrative quality. Thus, formal consolidation of layers of multilayer panels from various materials with certain functions (heat-shielding, vapor-proof, constructive, etc.) in a given sequence of layers cannot be regarded as a system: its properties are only a sum of the properties of the layers. Conservative characteristics of the system can remain unchanged for a long time without significant changes in the state of the system. Within the system and between systems, there are connections between all the system elements, sub-systems and systems. Elements (subsystems) are considered interrelated, if changes in one of the elements can be used to judge changes in others. In the formation of interelement bonds, some of the properties of the elements can be suppressed, while others can be strengthened (for example, the introduction of fillers leads to a reduction of shrinkage deformations and an increase in the strength of the composite). The more complex the system, the greater the effect of applying a system-wide holistic approach. Here, all private, local goals obey the common ultimate goal.

2. Composite material as a system object

Composite material as a system object (Figure 1) has the following properties in the most general form:

- is created for a specific purpose and in the process of achieving this goal it functions and develops (changes);
- the system is controlled by information on the state of the object and the external environment on the basis of modeling the behavior of the system under consideration;
- consists of interrelated components performing certain functions in its composition;
- the properties of the system object are not exhausted by the sum of the properties of its components;
- all components, when they work together, provide a new property that each of the components does not possess (the ability to control the properties of the entire system).

When determining the composite material as a system, it is assumed that there is an integral, integrative property of the system. A distinctive feature of composite materials from a mechanical

mixture of components (whose properties are defined as the sum of the properties of the components) is the presence of a phase boundary that determines the intensity of the processes of structure formation and the properties of the material. At the phase boundary, a contact layer is formed that ensures the adhesion of components (adhesion strength is a new integrative property that the elements entering into the system do not possess) and material properties. The combination of components leads to the formation of layers with altered properties at the phase interface that affect the formation of the properties of the system, other than the characteristics of the components (for example, cement hardening processes in large volumes differ from processes in thin layers on limit of the section of phases). On the one hand, the evaluation and analysis of composite materials should be based on the consideration of the material as an integral and unified system; on the other hand, the study of the material is impossible without analyzing its parts (the paradox of wholeness). Therefore, the study of the structure and properties of the material should be carried out and based on the manufacture of prototypes with the study of interelement bonds while maintaining the integrity of the system (this is how the kinetic processes of the formation of physical and mechanical characteristics of the material are studied). The description of a composite material as a system is possible only in the presence of its description as an element of the supersystem (a wider system) and vice versa, the description of the building material as an element of the supersystem is possible only if there is a description of the building material (paradox of hierarchy). The quality of materials is evaluated taking into account their place as an element in the hierarchical structure of an integral supersystem (the criterion for the quality of the subsystem is part of the overall criterion for the quality of the system, determined by its integrative properties).

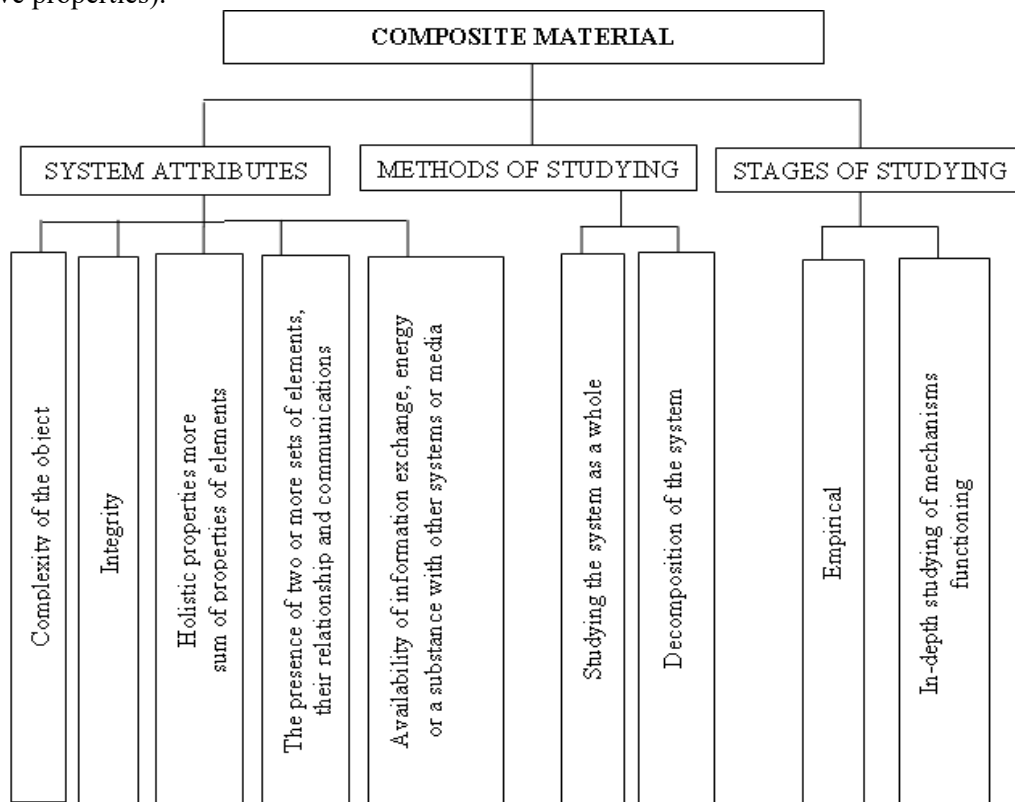


Figure 1. Characteristic features of composite material

Such an approach to the synthesis of composites was used in the development of special-purpose materials [5,6]. First, a cognitive map was constructed on the basis of interdisciplinary studies, then a hierarchical structure of the quality criteria was developed with the prioritization of particular criteria.

In accordance with this structure, proceeding from the polystructural theory [3], a hierarchical structure of the radiation-protective composite was constructed (the fragment in Figure 2). Further, the solution of one-criterion optimization problems for the selection of prescription parameters was carried out.

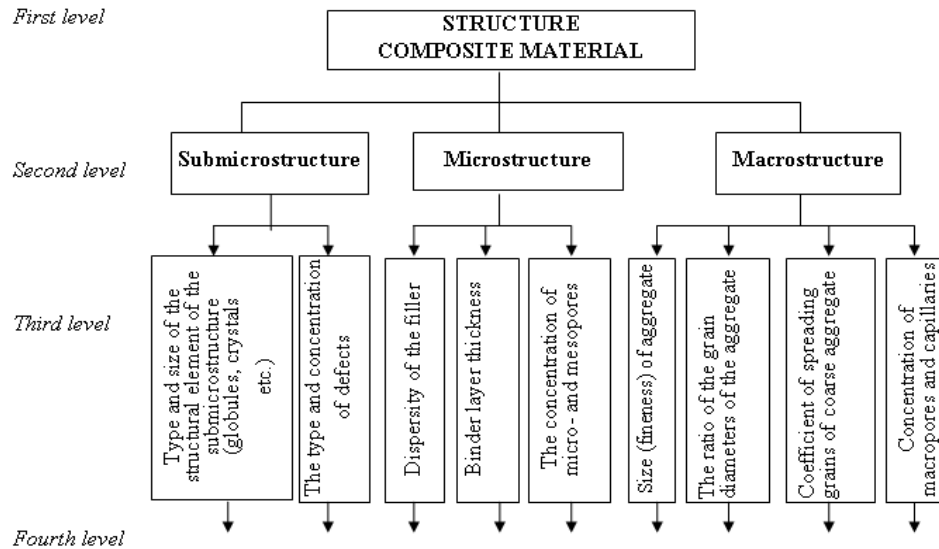


Figure 2. Structure of composite material

3. Lexicographic a task of optimization

With strictly ordered criteria of importance q_1, q_2, \dots, q_m the synthesis of the material is reduced to the so-called lexicographic a task of optimization [3]. As a result of its solution, it is possible to obtain not only optimal strategies (choice of prescription and technological parameters), but also order all strategies according to their degree of preference (this is how the words in the dictionary are arranged). In the deterministic (in the absence of random and uncertain factors) the lexicographic problem of each strategy corresponds to certain numerical values of particular criteria. Optimization of the structure and properties of materials as complex systems consisting of interconnected subsystems belonging to different hierarchical levels is also easy to present in lexicographical form. When comparing the two strategies, the first criterion is first used: the best strategy is that for which the value of this criterion is greater. If the values of the first criterion for both strategies are equal, then the second criterion should be used; preference is given to the strategy for which its value is greater. If the second criterion does not allow to allocate the best strategy, the third one is used, etc. Formally, the lexicographic relation of preference is defined as follows:

- strategy u is preferable to strategy v (it is denoted by $u \overset{lex}{<} v$) if one of the following conditions is fulfilled:

1. $q_1(u) > q_1(v)$;

2. $q_1(u) = q_1(v), q_2(u) > q_2(v)$;

- ...
- r. $q_1(u) = q_1(v), q_2(u) = q_2(v), \dots, q_{r-1}(u) = q_{r-1}(v), q_r(u) > q_r(v)$;

- ...
- m. $q_1(u) = q_1(v), q_2(u) = q_2(v), \dots, q_{m-1}(u) = q_{m-1}(v), q_m(u) > q_m(v)$;

- strategies u and v are equivalent ($u \sim v$), if

$$q_1(u) = q_1(v), q_2(u) = q_2(v), \dots, q_m(u) = q_m(v);$$

- strategy u is lexicographically no worse (no less preferable) than strategy v (denoted by $u^* \underset{\sim}{\overset{lex}{>}} v$) if one of the above $(m+1)$ conditions is fulfilled. Note that any two strategies u and v are comparable in the preference relation under consideration, that is, one of the conditions

$$u \underset{\sim}{\overset{lex}{>}} v; v \underset{\sim}{\overset{lex}{>}} u; u \sim v.$$

In general, the effectiveness of the strategy u within the framework of the model used is characterized by a set of numbers q_1, q_2, \dots, q_m , reflecting the degree of achievement of the objective of the operation when using this strategy.

In the lexicographic optimization problem, an arbitrarily small increment of a more important criterion is achieved by any loss for the remaining less important criteria.

Lexicographically optimal is strategy u^* , which is no worse than any other strategy v , if $u^* \underset{\sim}{\overset{lex}{>}} v$.

In the presence of only one criterion of efficiency, the optimal strategy u^* is determined from condition

$$q_1(u^*) = \max_{u \in U} q_1(u), \quad (1)$$

where U is the set of all strategies.

Optimum strategies are determined on the basis of the solution of the lexicographic optimization problem. Since all such strategies are equivalent, we can confine ourselves to finding not only the set U^* , but only one optimal strategy. Each subsequent partial criterion narrows the set of strategies obtained with the help of all the previous partial criteria:

$$U \supseteq U_1^* \supseteq U_2^* \supseteq \dots \supseteq U_m^*; U^* = U_m^*. \quad (2)$$

If there are several solutions in the initial optimization problem with one scalar criterion, and for further selection consistently apply additional criteria, the obtained strategies will be optimal for the corresponding lexicographic problem with a vector criterion consisting of all the alternately used criteria.

4. The method of successive concessions

Usually, in the development of composite materials, partial criteria tend to be contradictory. Direct reduction of the optimization problem to lexicographic becomes inexpedient. In this case, the ranking criteria should not be as stringent as in the lexicographic case. Here one of the possible ways of synthesizing the material is to use the method of successive concessions. It includes:

- the location and numbering of all partial criteria in order of relative importance;
- maximization of the first, most important criterion;
- assignment of the allowable reduction (assignment) value of the value of this criterion;
- maximization of the second most important private criterion without decreasing the value of the first criterion relative to the maximum by more than the amount of assignment;
- choosing the size of the assignment according to the second criterion;
- maximization of the third criterion provided that the values of the first two criteria do not differ from the previously found maximum values more than the values of the corresponding concessions, etc.

Thus, the multicriteria problem is reduced to the alternate maximization of particular criteria and the choice of the sizes of concessions (the concessions are smaller, the priority is tougher). Any strategy that provides a conditional maximum of the last criterion of importance is optimal.

Let us also consider another approach to the synthesis of materials, based on its representation as optimization problems with constraints (in the general case, they may turn out to be contradictory). In

this case the problem reduces to determining the maximum of the functional $f_0(u)$ on the set U under the condition that the inequalities

$$f_j(u) \geq 0, j = \overline{1, r}. \quad (3)$$

Under consistent constraints, the set of desired strategies (points of) is not empty. In the absence of information on compatibility of inequalities, the set may turn out to be empty. In this case, the original formulation of the problem can be modified: if the constraints are compatible, make it equivalent to the usual maximization problem, and in the case of an empty set make it meaningful. To do this, we use the idea of lexicographic optimization. Let us estimate the degree of non-fulfillment of constraints with the aid of the residual functional

$$\psi(u) = \begin{cases} 0 & \text{at not consistency of constraints,} \\ < 0 & \text{in the case of contradictory constraints.} \end{cases}$$

As $\psi(u)$ one can use the functional

$$\min\{0, f_1(u), f_2(u), \dots, f_r(u)\}, \\ (\max \psi(u) = 0).$$

In a lexicographic problem with two criteria $q_1 = \psi(u), q_2 = f_0(u)$, if the constraints are compatible ($\psi(u) = 0$), q_1 reaches its maximum on U . The set of maximum points coincides exactly with \tilde{U} (the solution of the lexicographic problem under consideration coincides with the solutions of the maximization problem $f_0(u)$ under constraints $f_j(u) \geq 0, j = \overline{1, r}$). If the constraints are inconsistent, then maximizing the criterion q_1 maximizes the degree of fulfillment of constraints, and then criterion q_2 is maximized on the set of points that ensure the highest degree of fulfillment (the smallest degree of non-fulfillment) of these constraints. The resulting two-criteria lexicographic problem is in fact a generalization of the maximization of $f_0(u)$ under given constraints and the lack of information on their compatibility.

5. Multi-criteria synthesis of superheavy concrete for protection from radiation

5.1. Overcoming uncertainty of goals

The approach discussed was used in the development of superheavy concrete to protect against radiation [7 ... 10]. With strictly ordered criteria of importance q_1, q_2, \dots, q_m , it was reduced to the lexicographic optimization problem. When determining the search area when solving the optimization problem, taking into account the inconsistency of the quality criteria, the method of successive concessions was used [3]. In the future, in this area, optimization was carried out with four ways to overcome the uncertainties of goals. It was assumed that the hierarchical structure of the quality criteria corresponds to the target vector function $\mathbf{q}(\mathbf{x}) = (q_1(\mathbf{x}), q_2(\mathbf{x}), \dots, q_m(\mathbf{x}))$ of the multidimensional variable $\mathbf{x} = (x_1, x_2, \dots, x_n)$; $x_i, i = \overline{1, n}$ - controlled factors; $q_j, j = \overline{1, m}$ - partial quality criteria.

In the simplest way to overcome the uncertainties of goals, the ordering of particular quality criteria is made, taking into account their priorities. With the single (main) quality criteria singled out, the optimization of the structure and properties of the material in the general case is reduced to the solution of the nonlinear programming problem: to find the value of the multidimensional variable $\mathbf{x} = (x_1, x_2, \dots, x_n), x_i \geq 0, i = \overline{1, n}$, which delivers the extremum of the objective function $q_1(\mathbf{x})$ under the conditions

$$-q_j(x_1, x_2, \dots, x_n) \leq q'_j, \quad q_j(x_1, x_2, \dots, x_n) \leq q''_j, \quad j = \overline{1, m}$$

(single-objective problem $q_1(\mathbf{x}) \rightarrow \max$ under the indicated restrictions).

In the case of linearity of the functions $q_j(\mathbf{x})$, $j = \overline{1, m}$ the required solution is defined as a solution of the linear programming problem. In the development of superheavy concrete for protection against radiation, the volume fractions x_1, x_2 (structure: technical sulfur; barite, $S = 250 \text{ m}^2/\text{kg}$; the modifying additive – mix of asbestos fibers, paraffin and soot in the ratio 12,5:1:2,5; filler – lead shot, diameter of 4-5 mm). Methods of mathematical planning of an experiment have received dependences of porosity $q_1, \%$, durabilities on compression $q_2, \text{ MPa}$ from volume fractions x_1, x_2 fillers:

$$\begin{aligned} q_1(x_1, x_2) &= 196,9 - 1217x_1 + 623,6x_2 - 1064x_1x_2 + 1532x_1^2, \\ q_2(x_1, x_2) &= -305,3 + 1188x_1 + 57,20x_2 - 1148x_1^2. \end{aligned}$$

The minimum value of porosity is reached in a point $M_1(0,519; 0,35)$ for which $q_1(0,519; 0,35) \approx 2,73\%$. Durability maximum in a point $M_2(0,518; 0,4)$, for which $q_2(0,518; 0,4) \approx 25,1 \text{ MPa}$. Further at synthesis of material performance of conditions $q_1 \leq 4\%$, $q_2 \geq 22 \text{ MPa}$ (area D_a , Figure 3) is supposed.

Let as a result of the solution of the single-objective problem

$$q_j(\mathbf{x}) \rightarrow \max, \quad j = \overline{1, m} \quad (4)$$

in each problem, a vector $\mathbf{x} = \mathbf{x}_j$ is defined that delivers the maximum value to criterion $q_j(\mathbf{x})$: $q_j(\mathbf{x}_j) = \hat{q}_j$. The set of scalar quantities \hat{q}_j in the criterion space determines the point of "absolute maximum" $(\hat{q}_1, \hat{q}_2, \dots, \hat{q}_m)$. For different \mathbf{x}_j , $j = \overline{1, m}$ there is no choice that allows to reach this point.

$$h(\mathbf{x}) = \sqrt{\sum_{jk} r_{jk} (q_j(\mathbf{x}) - \hat{q}_j)(q_k(\mathbf{x}) - \hat{q}_k)}, \quad (5)$$

where $\mathbf{R} = \|r_{ij}\|$ is a positive definite matrix. For $\mathbf{R} = \mathbf{E}$ we have $h(\mathbf{x}) = \sqrt{\sum_j (q_j(\mathbf{x}) - \hat{q}_j)^2}$ - Euclidean distance from point $(q_1(\mathbf{x}), q_2(\mathbf{x}), \dots, q_m(\mathbf{x}))$ to point $(\hat{q}_1, \hat{q}_2, \dots, \hat{q}_m)$ in the space of criteria.

In the case under consideration $\hat{q}_1 = 2,73$ at point $M_0(0,519; 0,35)$, $\hat{q}_2 = 25,1$ at point $M_1(0,518; 0,4)$.

The solution of problem $q_1(x_1, x_2) \rightarrow \min, q_2(x_1, x_2) \rightarrow \max$ for $\mathbf{R} = \mathbf{E}$ reduces to the definition in region $0,5 \leq x_1 \leq 0,6; 0,35 \leq x_2 \leq 0,4$; the smallest value

$$h_1(x_1, x_2) = \sqrt{(q_1(x_1, x_2) - 2,73)^2 + (q_2(x_1, x_2) - 25,1)^2}$$

(the non-linear programming problem $h(x_1, x_2) \rightarrow \min$ under the constraints $0,5 \leq x_1 \leq 0,6; 0,35 \leq x_2 \leq 0,4$).

The solution of the problem gives:

$$h_{1, \min} = h_1(0,522; 0,370) = 2,194;$$

$$q_1(0,522; 0,370) = 4,14\%; \quad q_2(0,522; 0,370) = 23,4 \text{ MPa}.$$

As you can see, the minimum point is not included in the search area; $q_1 = 4,14\% > 4\%$. The above restrictions can be explicitly taken into account (discarding points not belonging to the D_a region, Figure 3) by introducing a penalty function or by changing the metric in the space of the

quality criteria. The introduction of the penalty function shifts the point (conditional) of the minimum to position $M_h(0,522;0,368)$ (for this point $q_1 = 4,0\%$, $q_2 = 23,3$ MPa, $h_{\min} = 2,201$). The change in the metric is reduced to replacing the unit matrix by the diagonal matrix ($\mathbf{R} = \begin{bmatrix} 6/5 & 0 \\ 0 & 5/6 \end{bmatrix}$ is assumed).

The problem reduces to minimizing

$$h_2(x_1, x_2) = \sqrt{\frac{6}{5}(q_1(x_1, x_2) - 2,73)^2 + \frac{5}{6}(q_2(x_1, x_2) - 25,1)^2};$$

is attained at point $M_{h,2}(0,521;0,366)$, $q_1 = 3,84\%$, $q_2 = 23,2$ MPa, $h_{2,\min} = 2,14$.

It is possible to build a global objective function based on benchmarks. Here, the choice of parameters x_1, x_2, \dots, x_n is made from the conditions for maximizing the functions $q_j(\mathbf{x})$ under constraints $q_j(\mathbf{x}) \geq q_j^*$, $j = \overline{1, m}$. The objective function is represented in the form

$$q(\mathbf{x}) = \min_j \left\{ \frac{q_j(\mathbf{x})}{q_j^*} \right\} \text{ and a vector } \mathbf{x} \text{ is sought, which provides the maximum value of } q(\mathbf{x}), \text{ that is,}$$

the point M_q , in which $\max \left(\min_j \left\{ \frac{q_j(x_1, x_2)}{q_j^*} \right\} \right)$ is provided. With this value of vector \mathbf{x} , the value

$q(\mathbf{x})$ gives the worst value of the exponents $q_j(\mathbf{x})$, $j = \overline{1, m}$. Thus, condition $q(\mathbf{x}) \rightarrow \max$ means the choice of such a system of parameters (x_1, x_2, \dots, x_n) , which maximizes the ratio of the j -th actually reached value of the criterion to its control value q_j^* . As control values are assumed $q_1^* = 4\%$, $q_2^* = 22$ MPa. Given that the first of the quality indicators (porosity) is minimized, while the second (strength) is maximized, we will have

$$q(x_1, x_2) = \min \left\{ \frac{4}{q_1(x_1, x_2)}, \frac{q_2(x_1, x_2)}{22} \right\}.$$

The maximum $q(x_1, x_2)$ is reached at point $M_q(0,521;0,365)$, for which $q_1(0,521;0,365) = 3,8\%$, $q_2(0,521;0,365) = 23,1$ MPa, $q(0,521;0,365) = 1,052$.

A linear convolution of partial criteria was used. After preliminary normalization of the criteria, the objective function is defined as

$$q(x_1, x_2) = c_1 \frac{q_1(x_1, x_2) - \bar{q}_1}{S_{q_1}} + c_2 \frac{q_2(x_1, x_2) - \bar{q}_2}{S_{q_2}},$$

$$\bar{q}_1 = \frac{1}{S} \iint_S q_1(x_1, x_2) dx_1 dx_2 = 6,457, \quad \bar{q}_2 = \frac{1}{S} \iint_S q_2(x_1, x_2) dx_1 dx_2 = 21,54;$$

$$S_{q_1} = \sqrt{\frac{1}{S} \iint_S (q_1 - \bar{q}_1)^2 dx_1 dx_2} = 2,398, \quad S_{q_2} = \sqrt{\frac{1}{S} \iint_S (q_2 - \bar{q}_2)^2 dx_1 dx_2} = 2,455.$$

The maximum of the objective function is achieved at the point $M_a = \tilde{M}_a$, with this $q(0,518; 0,35) = 0,925$. The locations of the points M_l , M^2 , M_h , M_q and M_a are shown in Figure 4.

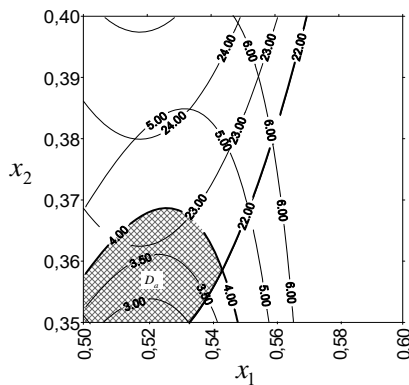


Figure 3. Search scope

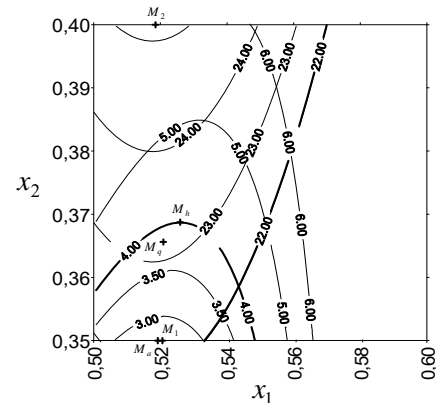


Figure 4. Points of maximum quality

High mobility of the obtained concrete mixes makes it possible to make piece products, building structures of any configuration and containers for transportation of radioactive materials. The dense structure of superheavy concrete allows it to be used for lining the enclosing structures of underground and ground bunkers, cemeteries and storage facilities for radioactive solid, liquid and gaseous wastes. Low porosity guarantees high values of brands for frost resistance, water and gas tightness.

5.2. Construction of Pareto sets

Another approach to the solution of multicriteria problems on the basis of constructing Pareto sets was considered, excluding from the informal analysis those variants of decisions that are knowingly bad. The technique was used for multi-criteria synthesis of epoxy composite of increased density for protection against radiation based on analytical dependencies of medium density ρ , kg/m³ and compressive strength R_s , MPa (obtained by mathematical methods of experiment planning):

$$\rho(X_1, X_2) = 3642,8 - 129,1X_1 + 668,5X_2 + 53,2X_1X_2 - 513,7X_2^2,$$

$$R_s(X_1, X_2) = 118,5 - 19,5X_1 + 20,9X_2 - 3,2X_2^2.$$

Here, X_1, X_2 is the coded values according to the concentration x_1 of the plasticizer (as% of the weight of the resin) and the degree of filling x_2 (P:H by mass). The definition of the Pareto set is made on the basis of a sequential solution of two problems of nonlinear programming:

$$\text{I. } \rho(x_1, x_2) \rightarrow \max, \mathbf{x} = (x_1, x_2) \in G_x, R_s(x_1, x_2) = \text{const}$$

$$\text{II. } R_s(x_1, x_2) \rightarrow \max, \mathbf{x} = (x_1, x_2) \in G_x, \rho(x_1, x_2) = \text{const}$$

($x_1, x_2 \geq 0$ - natural factors) using the method of penalty functions (Arrow-Hurwicz).

The Pareto set is, as a first approximation, the segment AB , in the second - the broken line $ADCB$. On experimental data, a rectangle corresponding to $-1 \leq X_1 \leq -0,6$; $0,4 \leq X_2 \leq 0,8$ is assumed to be the domain G_x . In this case, in the region G_q (Figure 5: $3900 \leq \rho \leq 3950$; $140 \leq R_s \leq 150$).

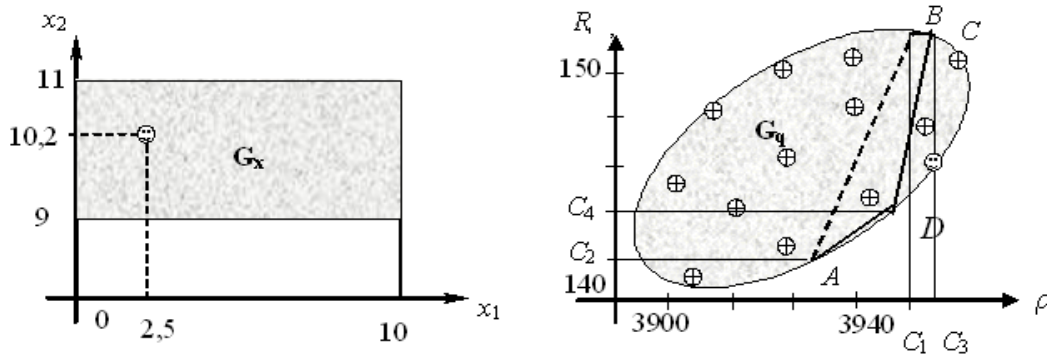


Figure 5. Pareto set

The efficiency of the method is also confirmed by the curves in Figure 6 lines of level $\rho(X_1, X_2) = \text{const}$ (branch of hyperbole), $R_s(X_1, X_2) = \text{const}$ (parabola) of quadratic models of objective functions, which allow us to take as optimal $x_1 = 2,5$; $x_2 = 10,2$. The corresponding values of density and tensile strength: $\rho = 3955 \text{ kg/m}^3$, $R_s = 145 \text{ MPa}$.

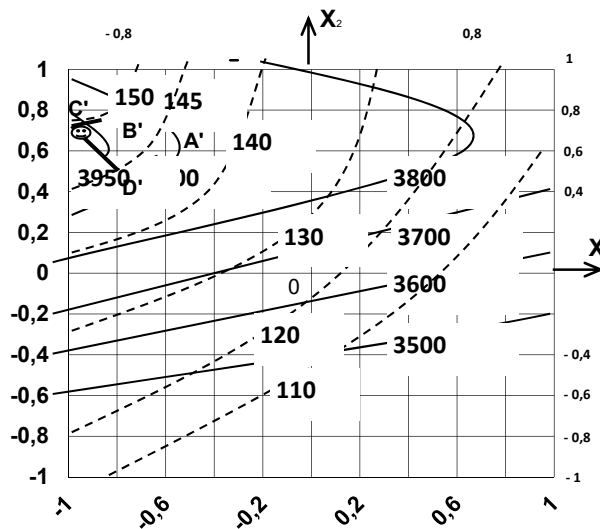


Figure 6. Lines of level $\rho(X_1, X_2) = \text{const}$, $R_s(X_1, X_2) = \text{const}$

6. Conclusions

- The paradigm shift in construction materials science is indicated: in the traditional approach, the composite is considered as an unstructured medium, and at system approach, it is structured.
- The properties of composite material as a system object are given.
- On the basis of the polystructural theory, a hierarchical structure of the radiation-protective composite has been developed.
- With strictly ordered criteria of the material, its synthesis is carried out as a solution to the lexicographic optimization problem.

- The method of successive concessions for the synthesis of a special-purpose composite is given.
- A multicriteria synthesis of epoxy materials based on the construction of Pareto sets was made.

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