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Property Modelling and Durability of Composite Materials

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Property Modelling and Durability of Composite Materials

Irina Garkina [1], Alexander Danilov [1]

[1] Penza State University of Architecture and Construction, Herman Titov Street, 28, Penza, Russia, 440028

i.a.naum@mail.ru

Abstract. Composite materials are considered as complex systems with corresponding system attributes. The necessity of studying the properties and structure of composite materials as a whole as systems connected by relationships that generate integrative qualities is pointed out. When synthesizing a composite, it is proposed to use the principle of simulability: a complex system is represented by a finite set of models reflecting a certain facet of its essence. This makes it possible to investigate a certain property or group of properties of a building material using one or several simplified (narrowly-oriented) models. Identification of new properties and entities is done on the basis of building up a set of simplified models (the reflection of a complex system as a whole is provided by the interaction of simplified models). Using the principle of purposefulness makes it possible to describe the quality of the material by some functionality for an integrated system. Internal causal relationships, the existence and function of the composite material (system) are based on the principle described physicality (any system regardless of the inherent physical laws of nature, possibly unique); no other laws are required to describe the operation of the system. The study of material properties is made on the basis of parametric identification of kinetic processes of formation of physical and mechanical characteristics of composite materials. For special purpose building materials, a number of particular criteria (properties) are determined, and their description is formalized using the principal component method (reducing the dimensionality of problems in assessing the quality of a material with simultaneous determination of a set of independent partial criteria). The results of the practical application of the method for evaluating the quality of composite materials with special properties are presented. Among the priority criteria were: strength, density and porosity of the material. Their dependences on the coded volume fractions of aggregate and filler were obtained by methods of mathematical experiment planning. It is shown that for most properties one can confine oneself to second-order differential models; each of the properties of the composite is considered as one of the particular criteria. Appendices are given to the development of composite materials for various purposes. Using the methods of control theory, an analytical description of the durability of the composite material is given; parametric identification is reduced to the determination of time constants in a second-order differential model, taking into account their dependencies on the prescription and technological parameters. Assessment of durability of a radiation protective composite taking into account dependences of the main properties on the prescription and technological parameters received on the basis of the generalized model is made.

1. Introduction

The necessity to solve the problems of engineering protection of personnel, population, equipment, buildings and structures in a number of industries, including the storage of highly toxic and radioactive waste and materials, significantly increased the relevance of creating composite materials with special

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properties and the possibility of regulating their structure. The complexity of the problem makes it necessary to develop fundamental foundations and new mathematical methods, algorithms for interpreting the full-scale experiment on the basis of its mathematical model. An important task is not only the creation of a theoretical basis for obtaining various materials with a given set of operational properties, but carrying out in-depth analysis using the system approach and control theory. Investigation of the properties and structure of composite materials is carried out as integral systems with elements connected by relationships that generate integrative qualities. Based on this, the methods, stages of studying and developing the material are determined. The role of random factors is determined from the standpoint of the synergetic approach, and the possibilities of analyzing the impact of these factors on the properties of systems are indicated.

From the standpoint of the homeostatic approach to composite materials, mechanisms and permissible limits for controlling integrative parameters of the structure and properties of the material are determined. Systemic, general homeostasis ensures the preservation of integrative quality, and the particular - the specific component. When the integrative parameters of the system approach the maximum permissible, a systemic crisis occurs - the system enters the zone of bifurcation. Proceeding from this, the durability is determined.

2. Parametric identification of kinetic processes

Parametric identification of the kinetic processes of the formation of physico-mechanical characteristics of composite materials [1 ... 3] reduces to the study of the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

(in most cases - the second order). In fact, the problem reduces to solving the operator equation

$$\left(p^2 - \sigma p + \Delta\right)X = 0 \tag{1}$$

or

$$\left(T^2 p^2 + 2\xi T p + 1\right)X = 0\,,$$
$$T^2 = \frac{1}{\Delta},\quad 2\xi T = -\frac{\sigma}{\Delta}, T = \frac{1}{\sqrt{\Delta}}, \xi = -\frac{\sigma}{2\sqrt{\Delta}}\,.$$

For an aperiodic system ($\sigma^2 - 4\Delta > 0$; $\sigma = a_{11} + a_{22},\quad \Delta = a_{11}a_{22} - a_{12}a_{21}$) it is true:

$$T^2 p^2 + 2\xi T p + 1 = (T_1 p + 1)(T_2 p + 1) = T_1 T_2 p^2 + (T_1 + T_2)p + 1\,;$$
$$T_1 T_2 = T^2,\quad T_1 + T_2 = 2\xi T\,;\ T_1 = \frac{T}{\xi + \sqrt{\xi^2 - 1}} = T\left(\xi - \sqrt{\xi^2 - 1}\right),\ T_2 = \frac{T}{\xi - \sqrt{\xi^2 - 1}} = T\left(\xi + \sqrt{\xi^2 - 1}\right).$$

In this case, the roots of the characteristic equation will be equal to

$$\lambda_1 = -\frac{1}{T_2}, \lambda_2 = -\frac{1}{T_1}\,;\ 0 < T_1 < T_2\,.$$

The solution of equation $\left(T^2 p^2 + 2\xi T p + 1\right)X = 0$ has the form

$$x(t) = Ae^{-\frac{1}{T_2}t} + Be^{-\frac{1}{T_1}t}\,;$$

under the initial conditions (trial effects) $x(0) = x_0, \dot{x}(0) = 0$ we have:

$$A = \frac{T_2}{T_2 - T_1}x_0 > 0,\quad B = \frac{T_1}{T_2 - T_1}x_0 < 0\,.$$

Without loss of generality we can put $x_0 = 1$.
Correct:

$$x(t) = \frac{T_2}{T_2 - T_1} e^{-\frac{t}{T_2}} - \frac{T_1}{T_2 - T_1} e^{-\frac{t}{T_1}} = y_1(t) - y_2(t);$$

$$x(t) = \frac{m}{(m-1)} e^{-\frac{t}{T_2}} - \frac{1}{(m-1)} e^{-\frac{t}{T_1}} = y_1(t) - y_2(t);\ m = \frac{T_2}{T_1}.$$

The function $x(t), t > 0$ is monotonically decreasing ($\dot{x}(t) = \frac{1}{(m-1)T_1}\left(e^{-\frac{t}{T_1}} - e^{-\frac{t}{T_2}} \right) < 0$ by $e^{-\frac{t}{T_1}} < e^{-\frac{t}{T_2}}$).

We have

$$\ddot{x}(t) = \frac{1}{(m-1)T_1}\left(\frac{1}{T_2} e^{-\frac{t}{T_2}} - \frac{1}{T_1} e^{-\frac{t}{T_1}} \right).$$

At the inflection point (t_n - parameter optimization) $\ddot{x}(t_n) = 0$. We have

$$t_n = T_2 \frac{\ln m}{m-1} = T_2 \ln(m)^{\frac{1}{m-1}}.$$

The value t_n, determined from the experimental plot, can be used for monitoring or for approximate calculation m:

$$\ln(m)^{\frac{1}{m-1}} = \frac{t_n}{T_2},\ (m)^{\frac{1}{m-1}} = e^{\frac{t_n}{T_2}}.$$

We also note that $\forall t > 0$

$$y_1(t) > y_2(t)$$

($t > 0$; $\frac{m}{m-1} > \frac{1}{m-1}$; $m > 1$: $e^{-\frac{t_n}{T_2}} > e^{-\frac{t_n}{T_1}}$; $T_2 > T_1$).

We find the value $t = t_k$, for which $\frac{y_1(0)}{y_1(t_k)} = k_1, \frac{y_2(0)}{y_2(t_k)} = k_2, \frac{x(0)}{x(t_k)} = k$.

We have:

$$y_1(t_k) = \frac{m}{m-1} e^{-\frac{t_k}{T_2}}, \quad y(0) = \frac{m}{m-1}.$$

From here

$$k_1 = e^{\frac{t_k}{T_2}}, \frac{t_k}{T_2} = \ln k_1, t_k = T_2 \ln k_1.$$

From the relation k_2 we obtain

$$k_2 = \frac{1}{m-1} : \frac{1}{m-1} e^{-\frac{T_2 \ln k_1}{T_1}} = e^{-m \ln k_1} = e^{\ln k_1^{-m}} = k_1^{-m} = \frac{1}{k_1^m}.$$

In this way, $k_2 = \frac{1}{k_1^m}$. We define k by k_1.

We have

$$\frac{y_2(t)}{y_1(t)}=\frac{\frac{1}{m-1}e^{-\frac{t}{T_1}}}{\frac{m}{m-1}e^{-\frac{t}{T_2}}}=\frac{1}{m}e^{\left(\frac{1}{T_2}-\frac{1}{T_1}\right)t}<1.$$

$\forall t$, as $T_2>T_1,\frac{1}{m}<1$.

We define the value $t=t_{21}$ for which $\frac{y_2}{y_1}=\frac{\beta}{100}$. Since $\frac{y_2(t)}{y_1(t)}$ - is a monotonically decreasing function, since

$$\left(\frac{y_2(t)}{y_1(t)}\right)'=\frac{1}{m}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)e^{-\left(\frac{1}{T_2}-\frac{1}{T_1}\right)t}<0,$$

then for $t\geq t_{21}$

$$\frac{y_2}{y_1}\leq\frac{\beta}{100}.$$

Then for $t\geq t_{21}$ with an accuracy of $\beta\,\%$

$$x_1(t)=y_1(t).$$

Define t_{21}. For $t\geq t_{21}$

$$\frac{y_2}{y_1}=\frac{1}{m}e^{\left(-\frac{1}{T_2}-\frac{1}{T_1}\right)t_{21}}\leq\frac{\beta}{100},$$

$$\left(\frac{1}{T_2}-\frac{1}{T_1}\right)t_{21}\leq\ln\frac{m\beta}{100},\ t_{21}\leq\frac{T_2}{1-m}\ln\frac{m\beta}{100}.$$

For $t=t_{21}$

$$\frac{T_2}{1-m}\ln\frac{m\beta}{100}=T_2\ln k_1$$

$$\ln\left(\frac{m\beta}{100}\right)^{\frac{1}{1-m}}=\ln k_1,\ k_1=k_1(t_{21})=\left(\frac{m\beta}{100}\right)^{\frac{1}{1-m}}.$$

At a value of $t_{21}=\frac{T_2}{1-m}\ln\frac{m\beta}{100}, k_1=k_1(t_{21})$ and after $t=t_{21}$ can be $y_2(t)$ neglected with an accuracy of $\beta\,\%$.

By virtue of $x_1(t)\approx y_1(t)$ for $t\geq t_{21}$

$$k=\frac{x_1(0)}{y_1(t_{21})}=\frac{m-1}{m}e^{\ln\left(\frac{m\beta}{100}\right)^{\frac{1}{1-m}}}=\frac{m-1}{m}\left(\frac{m\beta}{100}\right)^{\frac{1}{1-m}}.$$

In this way,

$$k(t_{21})\approx\frac{m-1}{m}\left(\frac{m\beta}{100}\right)^{\frac{1}{1-m}}.$$

The true value $k(t_{21})$ is determined by the formula

$$k(t_{21})=\frac{m-1}{m\left(\frac{100}{m\beta}\right)^{\frac{1}{1-m}}-\left(\frac{100}{m\beta}\right)^{\frac{1}{1-m}}}.$$

The value T_2 is determined at the end of the transient process $x(t)$ ($x(t)=ce^{-\frac{t}{T_2}}$ -solution of the equation $(T_2p+1)x=0$; c is determined from the initial condition $x(0)=x(t_k)$, $c=x(0)$).
We have

$$\frac{x(0)}{x(t_k+\mathrm{T})}=k_{\mathrm{T}}=\frac{x(0)}{x(0)e^{-\frac{1}{T_2}\mathrm{T}}}=e^{\frac{\mathrm{T}}{T_2}}.$$

Then

$$T_2=\frac{\mathrm{T}}{\ln k_{\mathrm{T}}}.$$

If a transient (kinetic) process is known, then for the chosen t_k we have: $k=\frac{x(0)}{x(t_k)}$; $\ln k_1=\frac{t_k}{T_2}$, $T_2=\frac{\ln k_{\mathrm{T}}}{\mathrm{T}}$. Knowing k_1, from $k=\frac{m-1}{\frac{m}{k_1}-\frac{1}{k_1^m}}$ we define m, and then the second required optimization parameter $T_1=\frac{T_2}{m}$.

The identification method was used when setting up a particular composite model with known parameters $T_1=0{,}5, T_2=1; m=2$. Here the exact solution has the form:

$$x(t)=2e^{-t}-e^{-2t}.$$

The numerical values $x(t_k)$ of are given in the table:

t	0	0,1	0,25	0,5	1	2	2,5	3	4	5	3,3
$y_1=2e^{-t}$	2	1,8	1,55	1,2	0,74	0,27	0,164	0,1	0,037	0,013	0,074
$y_2=e^{-2t}$	1	0,82	0,61	0,37	0,14	0,018	0,0007	0,0025	0,00034	0,00004	0,0014
x_k	1	0,98	0,94	0,83	0,6	0,25	0,16	0,098	0,037	0,013	0,063

We give the corresponding estimates of the values T_1 and T_2, determined from the numerical values of x_k. So when $t_k=2$ we have: $\mathrm{T}=3-2=1$, $k_{\mathrm{T}}=\frac{0{,}25}{0{,}1}=2{,}5$, $T_2=\frac{\mathrm{T}}{\ln k_{\mathrm{T}}}=\frac{1}{\ln 2{,}5}=1{,}09$ (the time constant was determined with the relative error $\frac{1{,}09-1}{1{,}09}\cdot 100\%=8\%$).

We have $\beta\le\frac{100\%}{2{,}73}e^{\frac{1-2{,}73}{1{,}09}\cdot 2}=1{,}5\%$; true value $\beta=\frac{y_2(2)}{y_1(2)}100\%=6{,}7\%$.

True:

$$T=\sqrt{T_1T_2}=\sqrt{1{,}09\cdot 0{,}4}=0{,}66;\ \xi=\frac{T_1+T_2}{2T}=1{,}13;\ \omega_0=\frac{1}{T}=\frac{1}{0{,}66}=1{,}52;$$

$$\Delta=\frac{1}{T^2}=\frac{1}{0{,}66}=2{,}29;\quad \sigma=-2\xi T\Delta=-3{,}42.$$

From where

$$a_{11}a_{22}-a_{12}a_{21}=2{,}29;$$

$$a_{11} + a_{22} = -3{,}42 .$$

From the last equations, two of the coefficients a_{ij} can be determined for two given coefficients. If the inflection point t_n is determined by the form of the kinetic process, then for the definition of m it is better to use formula

$$(m)^{\frac{1}{m-1}} = e^{\frac{t_n}{T_2}} .$$

Now let $x(t)$ be registered at a time with $\dot{x}(t)$. The necessary condition for an extremum $\dot{x}(t)$ is $\ddot{x}(t) = 0$, that is, an extremum is possible for $t = t_n = T_2 \ln(m)^{\frac{1}{m-1}}$.
As

$$\ddot{x}(t) = \frac{1}{(m-1)T_1 T_2^2}\left(m^2 e^{-\frac{t}{T_1}} - e^{-\frac{t}{T_2}} \right) < 0$$

by $m > 1$, $e^{-\frac{t}{T_1}} < e^{-\frac{t}{T_2}}$, then $\dot{x}(t)$ it reaches a maximum at $t = t_n$.

Since $\dot{x}(0) = 0, \dot{x}(\infty) = 0$, then the definition of t_n under the schedule $\dot{x}(t)$ is not difficult. Therefore, in the presence of realizations $x(t)$ and the estimate $\dot{x}(t)$ is better defined by the relation $(m)^{\frac{1}{m-1}} = e^{\frac{t_n}{T_2}}$.

3. Ranking of specific quality criteria

Traditionally, material quality management is performed on the basis of a set of experimentally determined basic properties [5 ... 7]. The number of elements of this set is established proceeding from a differential threshold at allocation of classes of quality (with maintenance of a necessary level of a signal-to-noise ratio). As a rule, partial criteria (assessment of the completeness of their set is subjective) are contradictory and dependent. Recently, a chemometric approach has been used to reduce the dimensionality of problems in assessing the quality of a material and to determine the set of independent criteria. Here we use projective mathematical methods that allow us to extract latent variables in large data sets and analyse the connections in the system under study. Unfortunately, despite the simplicity and effectiveness of this (often visual) approach to the analysis of experimental data, it is practically not used in construction materials science. The method of the principal components of K. Pearson is also effective; consists in finding a multidimensional ellipsoid of dispersion of empirical data in the factor space, which is determined by the location and lengths of the semi axes (main directions and standard deviations in the space of the principal directions).

Using the method of the main components of the PCA (Principal Component Analysis), the quality criteria were ranked according to the obtained values for the experimental samples. The first main component was defined as the direction of the largest change (spread along some central axis-a new variable) of data $\mathbf{q} = \left\| q_{ij} \right\|, i = \overline{1, p}, j = \overline{1, n}$ in the **Descartes** coordinate system $Oq_1 q_2 ... q_p$ (approximately purely geometrically, refinement - based on the best linear approximation of all the points q_{ij} by the least squares method). The second main component was taken (by definition!) orthogonal to the direction of the first (the next largest change in values occurs along it), and the third component - perpendicular to both the first and the second (lies in the direction in which the third largest change in the data occurs). The following main directions were determined in a similar way. The resulting system of principal components gives a set of orthogonal axes, each of which lies in the direction of the maximum data change in order of decreasing these quantities. Due to the orthogonality of the principal components in the resulting new set, the variables - linear combinations of the original variables no longer correlate with each other. The transition from the original **Descartes** coordinate system to a new set of orthogonal axes allows one to get rid of the relationship between the criteria. The upper limit of the number of principal components does not exceed $\max\{n-1, p\}$. The effective dimension of the space of principal components is determined by the rank of the matrix $\mathbf{q} = \left\| q_{ij} \right\|$. The last major component lies in the direction in which the difference between the samples will be minimal (in fact, the distinction of the samples is impossible here, since all these differences are only random noise). The main components with large numbers were considered as directions in which the main component is noise. Thus, the PCA method allowed the decomposition of the original data matrix into a structural part (several main first components lying in the directions of maximum changes) and to noise (directions in which the difference between the position of the points is small and can be neglected).

The computing method admits a compact representation. Here, for sampling $\{x_{iu}\}$, $i = \overline{1,k}$, $u = \overline{1,N}$, the values of the primary characteristics (*k* - the number of features, *N* - the number of measurements), the following procedures are performed successively.

1. Centering of characteristics (partial criteria):

$$\xi_{ui} = x_{ui} - \bar{x}_i,\ i = \overline{1,k},\ u = \overline{1,N},$$

where $\bar{x}_i = \frac{1}{N}\sum_{u=1}^{N} x_{ui}$ selective average of *i-th* attribute.

2. Definition of the covariance matrix:

$$C = (c_{ij}) = \Xi^T \Xi,$$

where $\Xi = (\xi_{ui})$ is the matrix of centered features.

3. Determination of eigenvalues λ_i and eigenvectors of the covariance matrix (always has *k* real non-negative eigenvalues, including multiple ones).

4. Sorting of eigenvectors in order of decreasing eigenvalues. The unit eigenvectors defining the principal directions make the rows of the matrix *L* of the *k*-th order. A linear homogeneous operator with matrix *L* transforms the original centered data into uncorrelated and with decreasing variances.

Unlike the method of least squares, the assumption of the normal distribution of empirical information is not used in the method of principal components (applicable for arbitrary data).

Lowering the dimension (separation of the input data into the content part and noise) within the framework of the principal component method is achieved by discarding directions corresponding to small eigenvalues. Apparently, there are no general rules for choosing the number of significant principal components (it is determined by the values of the eigenvalues of the covariance matrix, the research problems (visualization on the plane or in space), the intuition of the researcher, etc.).

Let us present the results of the practical application of the method of principal components in evaluating the quality of composite materials with special properties. Among the priority criteria were: strength, density and porosity of the material. The dependence of the porosity $q_1(x_1,x_2)$,%, compressive strength $q_2(x_1,x_2)$, MPa and density $q_3(x_1,x_2)$, kg/m^3 from the coded volume fractions of the aggregate (lead shot with a diameter of 4-5 mm) $x_1 \in [0{,}5;0{,}6]$ and the filler (barite, $S_{ss} = 250$ m^2/kg) $x_2 \in [0{,}35;0{,}4]$, obtained by methods of mathematical planning of the experiment:

$$q_1(x_1,x_2) = 5{,}18 + 3{,}44x_1 + 0{,}96x_2 - 1{,}33x_1x_2 + 3{,}83x_1^2;\ q_2(x_1,x_2) = 22{,}5 - 3{,}72x_1 + 1{,}43x_2 - 2{,}87x_1^2;$$

$$q_3(x_1,x_2) = 7143 - 147x_1 - 181{,}7x_1^2.$$

The covariance matrix obtained on the experimental values ξ_{ui} of the listed indicators has the form:

$$C = \frac{1}{N-1}(\xi_{ui})^T(\xi_{ui}) = \begin{pmatrix} 0{,}169 & 0{,}023 & -1{,}35 \\ 0{,}023 & 0{,}220 & 0{,}149 \\ -1{,}35 & 0{,}149 & 21{,}5 \end{pmatrix};$$

$\lambda_1 = 0{,}226$, $\lambda_2 = 0{,}077$, $\lambda_3 = 21{,}6$ - a Eigen values, $\mathbf{v}_1 = (0{,}221;0{,}975;0)$, $\mathbf{v}_2 = (0{,}973;-0{,}221;0{,}063)$, $\mathbf{v}_3 = (-0{,}063;0;0{,}998)$ - a Eigen vectors of the covariance matrix:

$$\lambda_1 = 0{,}226\,,\ \mathbf{v}_1 = (0{,}221; 0{,}975; 0);$$
$$\lambda_2 = 0{,}077\,,\ \mathbf{v}_2 = (0{,}973; -0{,}221; 0{,}063);$$
$$\lambda_3 = 21{,}6\,,\ \mathbf{v}_3 = (-0{,}063; 0; 0{,}998).$$

The transition matrix to the principal components $\Gamma_1, \Gamma_2, \Gamma_3$ has the form:

$$L = \begin{pmatrix} -0{,}063 & 0 & 0{,}998 \\ 0{,}221 & 0{,}975 & 0 \\ 0{,}973 & -0{,}021 & 0{,}063 \end{pmatrix};$$

the main components are related to the initial exponents q_1, q_2, q_3 linearly:

$$\Gamma_1 = -0{,}063 q_1 + 0{,}990 q_3\,,$$
$$\Gamma_2 = 0{,}221 q_1 - 0{,}975 q_2\,,$$
$$\Gamma_3 = 0{,}973 q_1 - 0{,}021 q_2 + 0{,}063 q_3\,.$$

By virtue of $\lambda_3 >> \lambda_1$ and $\lambda_3 >> \lambda_2$, the significant principal component is unique and corresponds to the principal direction $\mathbf{v}_3$; the vector of the first principal direction forms a small angle with the axis of the third original variable. The dominant is the average density (third indicator).

Simulation of longevity

In system theory, the destruction of a system is seen as a catastrophe associated with a disruption of homeostasis. It is believed that the systems always work as damaged: the system continues to function, since it contains many additional means of ensuring stability; its work can be considered as a permanently changing combination of failures and component recovery. When noticeable global failures occur, and several small, individually harmless failures, unite, a global systemic failure is created. Each of the failures provokes an accident (the result of a joint impact of failures); the possibilities for the occurrence of systemic accidents are much greater than the manifest incidents. With an aperiodic reduction in the performance characteristics of the material, the evaluation of durability is reduced to parametric identification of the function $x(t)$ (Fig. 1, without loss of generality of reasoning for linear systems, the performance value can be assumed $x(0) = 1$).

Formally the function $x(t)$ can be considered as a solution of the operator equation (1). We have:

$$T^2 p^2 + 2\xi T p + 1 = (T_1 p + 1)(T_2 p + 1) = T_1 T_2 p^2 + (T_1 + T_2) p + 1\,;$$
$$T_1 T_2 = T^2, \quad T_1 + T_2 = 2\xi T\,;$$
$$T_1 = \frac{T}{\xi + \sqrt{\xi^2 - 1}} = T\left(\xi - \sqrt{\xi^2 - 1}\right),\ T_2 = \frac{T}{\xi - \sqrt{\xi^2 - 1}} = T\left(\xi + \sqrt{\xi^2 - 1}\right);$$

the roots of the characteristic equation are equal to $\lambda_1 = -\dfrac{1}{T_2}, \lambda_2 = -\dfrac{1}{T_1}$; $0 < T_1 < T_2$.

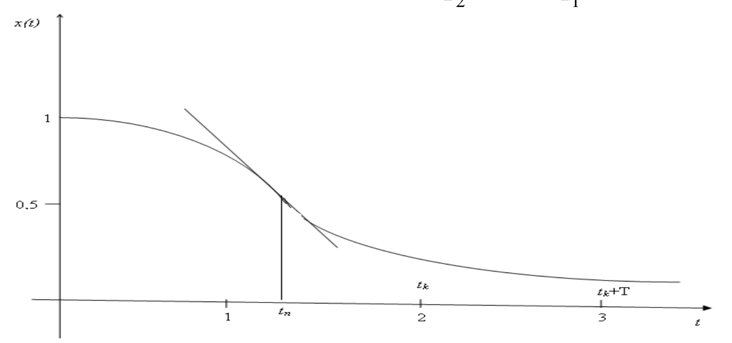

Figure 1. Parametrical identification of performance characteristics

Fair:

$$x(t) = Ae^{-\frac{1}{T_2}t} + Be^{-\frac{1}{T_1}t}.$$

Optimization parameter t_n ($\ddot{x}(t_n) = 0$, $\frac{1}{T_2}e^{-\frac{t_n}{T_2}} = \frac{1}{T_1}e^{\frac{t_n}{T_1}}$, $\frac{T_2}{T_1} = e^{t_n\left(\frac{1}{T_1} - \frac{t}{T_2}\right)} = e^{t_n\left(\frac{m-1}{T_2}\right)}$, $m = e^{t_n\left(\frac{m-1}{T_2}\right)}$, $t_n = T_2\frac{\ln m}{m-1} = T_2 \ln(m)^{\frac{1}{m-1}}$), defined by the graph can be used for control or for an approximate calculation of the value of the exponent m (establishes the relationship between the time constants T_1 and T_2; $t > 0$; $\frac{m}{m-1} > \frac{1}{m-1}$; $m > 1$; $e^{-\frac{t_n}{T_2}} > e^{-\frac{t_n}{T_1}}$; $T_2 > T_1$):

$$\ln(m)^{\frac{1}{m-1}} = \frac{t_n}{T_2},\ (m)^{\frac{1}{m-1}} = e^{\frac{t_n}{T_2}}.$$

True

$$x(t) = \frac{T_2}{T_2 - T_1}e^{-\frac{t}{T_2}} - \frac{T_1}{T_2 - T_1}e^{-\frac{t}{T_1}} = y_1(t) - y_2(t);\ m = \frac{T_2}{T_1};$$

$\forall t > 0$ we have:

$$y_1(t) > y_2(t).$$

Let us find the value $t = t_k$, for which $\frac{y_1(0)}{y_1(t_k)} = k_1, \frac{y_2(0)}{y_2(t_k)} = k_2, \frac{x(0)}{x(t_k)} = k$.

Here

$$y_1(t_k) = \frac{m}{m-1}e^{-\frac{t_k}{T_2}}\ ,\quad y(0) = \frac{m}{m-1};$$

$$k_1 = e^{\frac{t_k}{T_2}}, \frac{t_k}{T_2} = \ln k_1, t_k = T_2 \ln k_1;$$

$$k_2 = \frac{1}{m-1} : \frac{1}{m-1}e^{-\frac{T_2 \ln k_1}{T_1}} = e^{-m\ln k_1} = e^{\ln k_1^{-m}} = k_1^{-m} = \frac{1}{k_1^m};\ k_2 = \frac{1}{k_1^m};$$

$$k = \frac{1}{\frac{1}{m-1}\left(me^{-\ln k_1} - e^{-m\ln k_1}\right)} = \frac{1}{(m-1)\left(me^{\ln k_1^{-1}} - e^{\ln k_1^{-m}}\right)};\ k = \frac{m-1}{\frac{m}{k_1} - \frac{1}{k_1^m}}.$$

We have:

$$\frac{y_2(t)}{y_1(t)} = \frac{\frac{1}{m-1}e^{-\frac{t}{T_1}}}{\frac{m}{m-1}e^{-\frac{t}{T_2}}} = \frac{1}{m}e^{\left(\frac{1}{T_2} - \frac{1}{T_1}\right)t} < 1$$

$\forall t$, as $T_2 > T_1, \frac{1}{m} < 1$.

Let us determine the value $t = t_{21}$ of so that for $t \geq t_{21}$ with an accuracy of β % the approximate equality carry out

$$x(t) \approx y_1(t).$$

When $t \geq t_{21}$

$$\frac{y_2}{y_1}=\frac{1}{m}e^{\left(-\frac{1}{T_2}-\frac{1}{T_1}\right)t_{21}}\leq\frac{\beta}{100},\quad \left(\frac{1}{T_2}-\frac{1}{T_1}\right)t_{21}\leq\ln\frac{m\beta}{100},\ t_{21}\leq\frac{T_2}{1-m}\ln\frac{m\beta}{100}.$$

When $t=t_{21}$

$$k_1=k_1(t_{21})=\left(\frac{m\beta}{100}\right)^{\frac{1}{1-m}}.$$

For a value of $t_{21}=\frac{T_2}{1-m}\ln\frac{m\beta}{100}$, $k_1=k_1(t_{21})$ and after $t=t_{21}$, we can $y_2(t)$ neglect with accuracy β %.

By virtue of $x(t)\approx y_1(t)\ \forall t\geq t_{21}$

$$k=\frac{x(0)}{y_1(t_{21})}=\frac{m-1}{m}e^{\ln\left(\frac{m\beta}{100}\right)^{\frac{1}{1-m}}}=\frac{m-1}{m}\left(\frac{m\beta}{100}\right)^{\frac{1}{1-m}}.$$

In this way, $k(t_{21})\approx\frac{m-1}{m}\left(\frac{m\beta}{100}\right)^{\frac{1}{1-m}}$.

The true value $k(t_{21})$ is determined by the formula

$$k(t_{21})=\frac{m-1}{m\left(\frac{100}{m\beta}\right)^{\frac{1}{1-m}}-\left(\frac{100}{m\beta}\right)^{\frac{1}{1-m}}}.$$

The value T_2 is determined at the end of the transition process $x(t)$. Here $x(t)$, in essence, is a solution of equation

$$(T_2p+1)x=0;\ x(t)=ce^{-\frac{t}{T_2}};$$

c is determined from the initial condition $x(0)=x(t_k)$, $c=x(0)$.

We have

$$\frac{x(0)}{x(t_k+\mathrm{T})}=k_{\mathrm{T}}=\frac{x(0)}{x(0)e^{-\frac{1}{T_2}\mathrm{T}}}=e^{\frac{\mathrm{T}}{T_2}}.$$

Then

$$T_2=\frac{\mathrm{T}}{\ln k_{\mathrm{T}}} \tag{2}$$

With known tabular values of the performance characteristics of the material, it is determined:

$$k=\frac{x(0)}{x(t_k)};\ \ln k_1=\frac{t_k}{T_2},T_2=\frac{\ln k_{\mathrm{T}}}{\mathrm{T}}.$$

By k_1 from $k=\frac{m-1}{\frac{m}{k_1}-\frac{1}{k_1^m}}$ is determined m, and then the second optimization parameter

$$T_1=\frac{T_2}{m}. \tag{3}$$

The proposed approach was used to determine the longevity of the radiation-protective composite according to (2) and (3), taking into account the dependences of the basic protective properties on the prescription technological parameters obtained on the basis of the generalized model [7 ... 9].

4. Conclusions

1. Composite materials are presented as complex systems with corresponding system attributes.
2. The technique of objective evaluation of formation of properties and durability of composite materials on the basis of mathematical modeling is given.
3. In modelling the principles of modelling, purposefulness and physicality are fully used.
4. The study of material properties is made on the basis of parametric identification of the kinetic processes of formation of physico-mechanical characteristics of composite materials.
5. Reducing the dimension of tasks to assess the quality of the material with the simultaneous determination of a set of independent partial criteria is made on the basis of the method of principal components.
6. The results of the synthesis of special-purpose materials are presented, based on a set of ranked partial criteria (porosity, compressive strength and density).

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