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Preventive Services of Residential Buildings According to the Pareto Principle

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Preventive Services of Residential Buildings According to the Pareto Principle

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Abstract. The problem of ensuring an adequate level of the technical condition of a building occurs over the entire period it is in service. In solving problems connected with developing a prediction of changes in performance characteristics of a residential building, it is suggested that algorithms of determining changes in the reliability of technical devices be used. The process of changes in the technical condition of technical equipment can be managed by making decisions regarding repairs or replacing components. The prognosis of unfavourable processes will make it possible to determine the time frame in which the technical condition of a building will be unsatisfactory in the future, and thus necessitate repair works. Applying optimal prophylactic replacements requires a knowledge of the time span that the components of the building can be expected to work properly. To model situations in survival analysis, the Rayleigh distribution for the random variable of time was accepted. In the article, the model of the life span curve for a residential building has been presented, where the Pareto principle was applied as the strategy for undertaking renovation works. Modelling various scenarios of use helps to choose the optimal planning of renovation works on a building. The characteristics of various strategies influence the shape of the life cycle curve of the building. Applying the Pareto principle is an example of a strategy of renovation works on residential buildings. Applying the Raleigh distribution to predict reliability is possible thanks to the analysis of a set of data including the values of the degrees of wear of actual residential buildings

1. Introduction

The process of changes in the technical condition of technical equipment can be managed by making decisions regarding repairs or replacing components. In the case of mechanical devices, so-called strategies of prophylactic replacements, also referred to as preventive renewals, are applied. As a result of its replacement, a component characterized by a low level of reliability, but still effective, is replaced by a new element of the same kind [1, 2]. Structures composed of many components are an organized group of such, but replacements do not include all components and instead, rather the gradual replacement of different groups. The optimization of component replacements depends on choosing the times of carrying out prophylactic replacements in such a way that the selected goal function, dependent on these times, achieves an extreme value.

The best-known and most commonly applied strategy in the use of equipment is the so-called simple periodical strategy of prophylactic replacements in an unrestricted stretch of time. It refers to elements working in an unrestricted period of time, subjected to ongoing observation, and having an increasing function of damage intensity. The timeframe x - the period of prophylactic replacements, is constant for a given type of equipment [1-3].

In the case of complex systems composed of many sets of elements, the structure of the equipment, interrelationship of the times that the elements function properly, different types of economic relationships, etc. are taken into consideration when constructing an effective service strategy. The times of prophylactic servicing are indicated on the basis of reliability characteristics of the equipment using technical and economic criteria. A model describing the process in which failures and damage occurred is essential.

2. Mathematical model of changes in the technical condition

The reliability of technical equipment is defined as its ability to carry out a task resulting from its intended use under specified conditions over the course of its operation [4-7]. This means expecting the equipment to fulfil a given function for a given period of time t, and under given operating conditions. It is assumed that the measure of the reliability of equipment in regards to a given task is the probability of carrying out this task. The measure of reliability expressed as such is a function of the time that the equipment functions properly and is referred to as the reliability function.

To model a situation in survival analysis, where the probability of a failure changes over time, the Weibull distribution is applied as the distribution of the random variable of the time the equipment is operational [8-11]. The density of the probability of a failure is determined by the relationship:

$$f(t) = \alpha\beta^{\alpha}t^{\alpha-1}\exp(-(\beta t)^{\alpha}) \text{ dla } t \geq 0 \tag{1}$$

where: t - the time of using the equipment,
α - scale parameter (real number), $\alpha > 0$,
β - shape parameter (real number), $\beta > 0$.

The α parameter of the distribution determines the behaviour of the probability of a failure over time:
- for $\alpha < 1$ the probability of a failure decreases over time; when modelling the failure of equipment, this suggests that individual specimens can have manufacturing defects and slowly fall out of the population;
- for $\alpha = 1$ (exponential distribution) the probability is constant, this indicates the fact that the failures have the character of external random events;
- for $\alpha > 1$ probability increases over time, which suggests the wear of elements with the passing of time as the main reason behind failure;
- for $\alpha = 2$ (Rayleigh distribution) probability increases linearly with the passing of time.

The β parameter is a coefficient characterizing the speed at which reliability is lost.

The cumulative distribution function that is the function of the reliability of equipment for the Weibull distribution:

$$F(t) = 1 - \exp(-(\beta t)^{\alpha}) \tag{2}$$

The reliability function - change in the probability of no damage over time:

$$R(t) = 1 - F(t) = \exp(-(\beta t)^{\alpha}) \tag{3}$$

The intensity of damage $\lambda(t)$ is an indicator characterizing reliability, also defined as the intensity of the probability of damage, or speed at which unreliability increases in relation to reliability:

$$\lambda(t) = \frac{dF(t)}{dt}\frac{1}{R(t)} \tag{4}$$

The exponential distribution is a specific case of the Weibull distribution where shape parameter α=1. The exponential function is applied very often when assessing the distribution of the time of proper functioning [5, 9-11]. A characteristic feature of the exponential distribution is the constant intensity of damage over the entire time the building is in use λ(t) = const. The relationship defining the reliability function (3) for the i-th component of a building according to the exponential distribution:

$$R(t) = \exp\left(-\left(\frac{t}{T_{Ri}}\right)\right) \tag{5}$$

Another specific example of the Weibull distribution, when the shape parameter is α=2, is the Raleigh distribution. This distribution is a single-parameter distribution, occurring only when the wear of the object with the passing of time is the main reason behind failure [9, 10, 12]. Choosing to apply the Rayleigh distribution for building structures seems to be the most appropriate. All buildings and their components over the course of use are subject to wear, and Rayleigh's distribution is applied when the wear of the object increases with the passing of time that it is in service. The reliability function (3), in this case, takes the form of:

$$R(t) = \exp\left(-\left(\frac{t}{T_{Ri}}\right)^2\right) \tag{6}$$

Intensity of damages according to the Rayleigh distribution

$$\lambda(t) = 2\frac{t^2}{T^2}, \tag{7}$$

In Figures 1 and 2, the results of the changes in the reliability of one of the components of a building - masonry walls made of brick over the course of a 100-year service period of the building have been presented. The reliability functions (5) and (6) were indicated for three cases: the minimal, average and maximal lifespan (period of durability) provided in literature.

3. Mathematical model of changes in the technical condition

When considering the reliability of technical equipment, the intensity of damage is dependent on the wear [3]:

$$S_z = \int \lambda(t)dt \tag{8}$$

where: S_z - degree of wear of the manufactured products.

The degree of wear according to the exponential distribution, where the intensity of damage is constant (7), is a linear function:

$$S_z = \frac{t}{T_R} \tag{9}$$

where: S_z -   the degree of technical wear of the technical equipment expressed in percentages,
             t   -   age of the equipment
             T_R -   expected lifespan of the equipment expressed in years.

The obtained relationship is one of the time methods, applied in practice, used to determine the degree of the technical wear of buildings poorly maintained at any given moment of use.

For the Rayleigh distribution, where α = 2, β = 1/T_R, the degree of wear is equal to:

$$S_z = \frac{t^2}{T_R^2} \tag{10}$$

For each element of the building, made from specified construction materials, a prediction of the degree of wear over the entire course of its use can be indicated. The periods of durability of building elements with selected material-construction solutions are provided in literature (e.g. [9]), and thanks to

being applied in dependencies (9) and (10), a prediction of the degree of wear can be obtained according to the exponential and Rayleigh distribution.
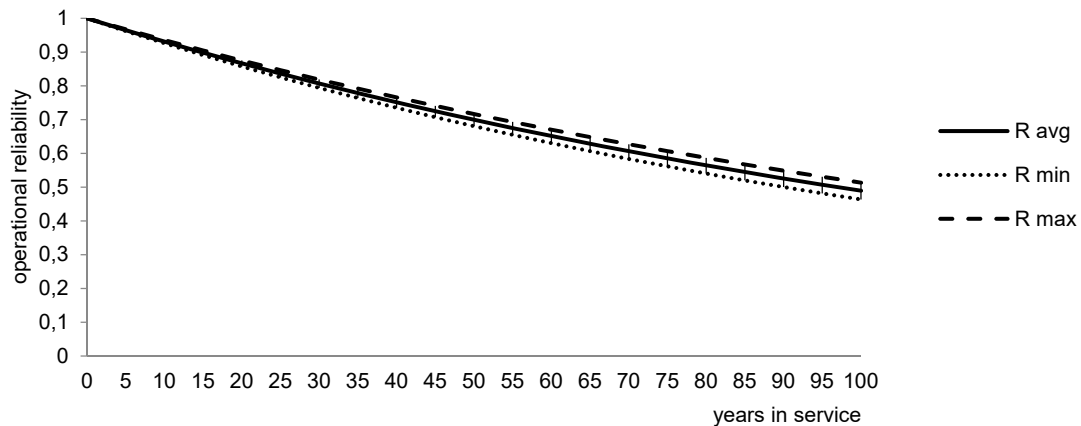

Figure 1. Changes in the reliability of brick masonry walls according to exponential distribution
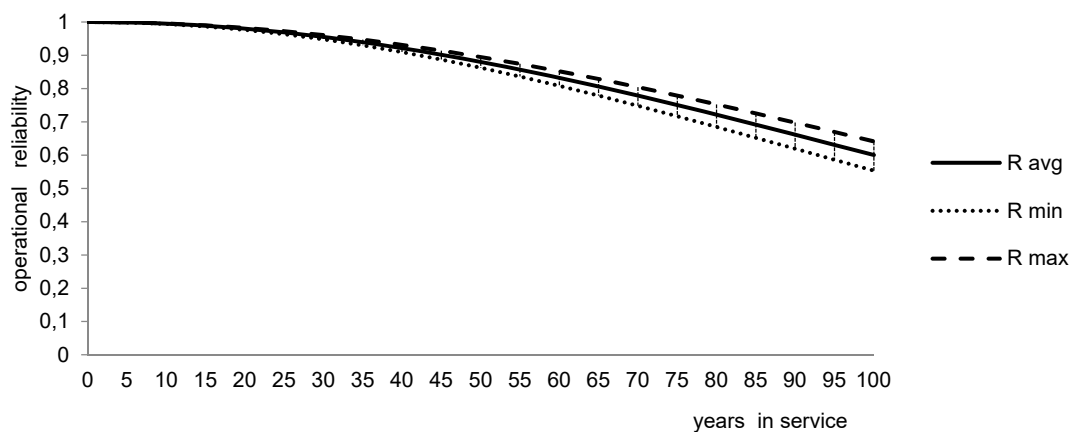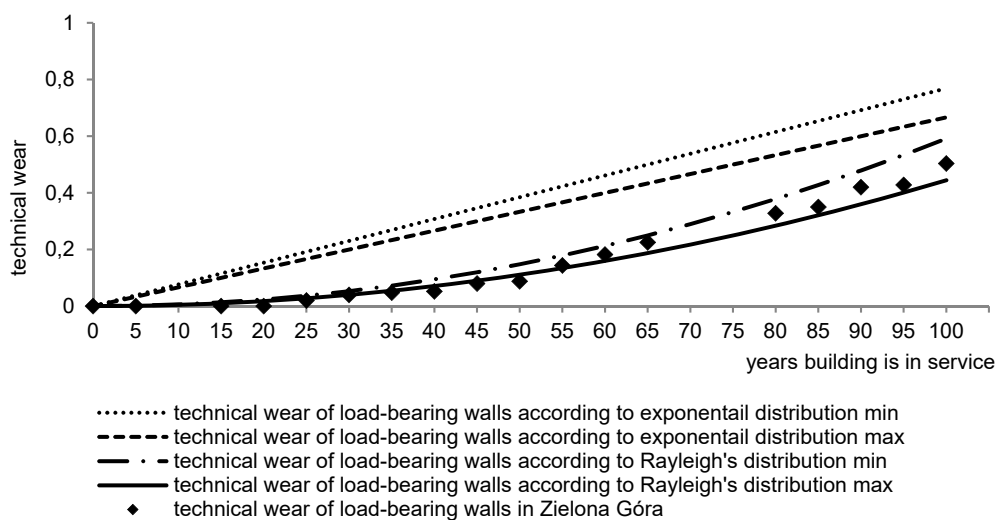

Figure 2. Changes in the reliability of brick masonry walls according to Rayleigh distribution

In the case of masonry walls made of brick, the period of durability is determined to be in the range of 130 to 150 years. For the minimum (130 years) and maximum (150 years) value, the degrees of wear have been expressed according to the exponential (9) and Rayleigh (10) distribution. The obtained results have been presented in Table 1 and in Figure 3. In order to verify the proposed methods, the average values of the degree of wear for the load-bearing walls of buildings in Zielona Góra [13] have also been indicated [12].

The values of the degree of wear of walls according to the Rayleigh distribution were subjected to verification with the Student's t-test. Assuming a 5% probability of estimation error and 19 degrees of freedom, the critical value of the test is 2.0930. The result of the test in the study was 3.05515, which means that the results are statistically significant at a level of $p=0.05$ [12].

Table 1. Average values of the technical wear of load-bearing walls obtained
during periodical inspections and theoretically predicted values of the degree of wear

Years of use	Average degree of wall wear determined on the basis of periodical inspections	Predicted degrees of wear according to tested distributions			
		exponential (9)		Rayleigh (10)	
		min	max	min	max
0	0.000	0.000	0.000	0.000	0.000
5	0.000	0.038	0.033	0.001	0.001
10	0.000	0.077	0.067	0.006	0.004
15	0.000	0.115	0.100	0.013	0.010
20	0.020	0.154	0.133	0.024	0.018
25	0.040	0.192	0.167	0.037	0.028
30	0.048	0.231	0.200	0.053	0.040
35	0.052	0.269	0.233	0.072	0.054
40	0.080	0.308	0.267	0.095	0.071
45	0.088	0.346	0.300	0.120	0.090
50	0.144	0.385	0.333	0.148	0.111
55	0.182	0.423	0.367	0.179	0.134
60	0.225	0.462	0.400	0.213	0.160
65	no data	0.500	0.433	0.250	0.188
70	no data	0.538	0.467	0.290	0.218
75	0.328	0.577	0.500	0.333	0.250
80	0.350	0.615	0.533	0.379	0.284
85	0.420	0.654	0.567	0.428	0.321
90	0.428	0.692	0.600	0.479	0.360
95	0.504	0.731	0.633	0.534	0.401
100	0.564	0.769	0.667	0.592	0.444

·········· technical wear of load-bearing walls according to exponentail distribution min
‒ ‒ ‒ ‒ technical wear of load-bearing walls according to exponentail distribution max
‒ · ‒ technical wear of load-bearing walls according to Rayleigh's distribution min
——— technical wear of load-bearing walls according to Rayleigh's distribution max
◆ technical wear of load-bearing walls in Zielona Góra

Figure 3. Comparison of the degree of wear of masonry walls determined according to various
distributions along with the average results of periodical inspections of buildings located in Zielona
Góra

The results of the assessment of the technical state of the buildings in Zielona Góra confirm the effectiveness of the proposed method for indicating the degree of wear applying the Rayleigh distribution. The average values of the degree of wear determined during on-site inspections while assessing the technical condition of the buildings differ only slightly from the proposed function based on the Raleigh distribution. Despite simplifications in the process of mathematical modelling, the obtained results are similar to experimental findings.

4. Modelling the lifespan of repaired objects

Prognostic diagnostics is applied for technical equipment when the elements age over the course of their operation, the intensity of damage increases, and the failure of the element leads to the damage of other elements. Preventive renewals are planned, which are not full renewals of the system (main repairs). Preventive renewals differ from ongoing repairs carried out ad hoc, where minimal renewals are applied upon the equipment becoming damaged. An adaptation of such actions should be applied in residential buildings.

The following assumptions were made to the strategy of preventive services of residential buildings built in traditional technology:

- time periods of inspections and repairs are negligible,
- random variables expressing the time to damage prior to and following repairs have the same distribution,
- the procedure of replacements relies on the simple periodical strategy at a given stretch of time,
- the optimization of the strategy is based on the interrelations of the reliability of individual components accounting for the weight of the element in the structure of the building, while indicating the optimal set of elements and optimal time of replacement results from the length of time that the elements function properly,
- the choice of elements selected for replacement is based on the Pareto principle,
- the remaining renovation works were abandoned.

In the economy and management, the Pareto principle is often applied when making decisions [14-16]. It is used when the aim is preventing negative phenomenon with the highest frequency of occurrence. In accordance with this principle, 20% of the analysed buildings are connected with 80% of certain resources. 100% perfection is treated as unnecessary, while 80% profits regarded as sufficiently high. The principle makes it possible to establish priorities and facilitates time management, thanks to which maximum results are obtained in minimum time.

All kinds of renovation works have a significant effect on the reliability of a building over the course of its later use [17, 18]. Complete characteristics of the reliability of a repaired building must account for initial reliability, as well as changes in reliability following renovation works. In the proposed model, an algorithm for determining the reliability of the entire buildings is used.

Changes in the reliability of a building are described using the R(t) function, where the independent variable *t* is the time variable.

Changes in the reliability of the repaired building over period of its use:

$$R_M(t) = \sum_{i=1}^{s} A_i R_i(t) + \sum_{i=1}^{p} A_i R_i(t - c_j) \tag{11}$$

where:
$R_M(t)$ - the reliability of the renovated building at moment *t*,
$R_i(t-c_j)$ - the reliability of the *i*-th component at moment c_j,
A_i - weight coefficient of the *i*-th component,
c_j - dates of renovation works, under the assumption that $c_1 = 40$ and $c_2 = 80$.
s - number of building components not subjected to renovation,
p - number of renovated building components at date c_j

The weight coefficients A_i of components were determined on the basis of coefficients applied for assessing the quality of residential buildings [12].

The algorithm for determining the elements that take priority is based on the selection of extreme values of weight coefficients, with the simultaneous maximum change in reliability in time $t-c_j$.

Changes in the reliability of a building component $R_i(t-c_i)$ were indicated according to a relationship based on the Rayleigh distribution (6) under the assumption that the renovation works result in an increase in the reliability of a component to its maximum values, i.e. 1.0. Relationship (11) based on the Rayleigh distribution takes the form of:

$$R_{MR}(t) = \sum_{i=1}^{s} A_i \exp\left[-\left(\frac{t}{T_{RiSR}}\right)^2\right] + \sum_{i=1}^{p} A_i \exp\left[-\left(\frac{t-c_j}{T_{RiSR}}\right)^2\right] \quad (12)$$

where:

$R_{MR}(t)$ - the reliability of the renovated building at moment t based on the Rayleigh distribution,

T_{Risr} - the average lifespan of the i-th building component,

s - number of building components not having undergone renovation at moment c_j,

p - number of building components having undergone renovation at moment c_j,

remaining symbols same as above.

The obtained results of the predictions in the changes in the reliability of a renovated building according to rule (12) for the above assumptions have been presented in Figure 4. For purposes of comparison, changes in the reliability during the use of a building which had not undergone renovation works have also been presented.
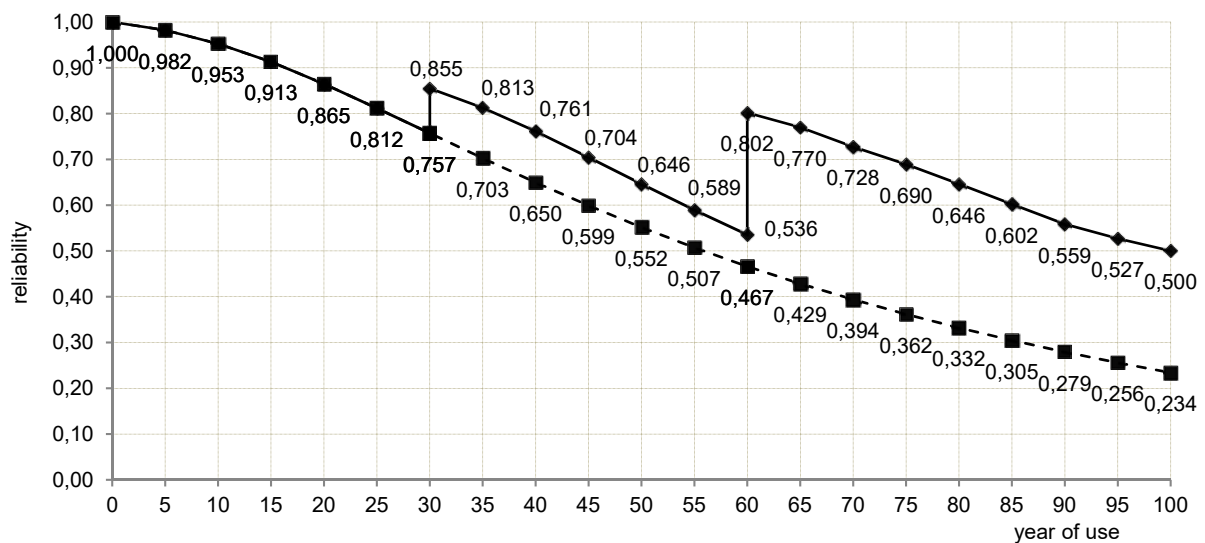

Figure 4. The life cycle curves of a building - model of the renovation strategy acc. to the Pareto principle as well as a model without renovation works

5. Conclusions

Technical diagnostics covers issues connected with assessing the technical condition of an object, as well as the possibilities of its continued use. The diagnosis regards issues connected with the assessment of the current condition, as well as prediction of the development of changes in this condition. The prognostic description of the lifespan of a building in the mathematical approach makes it possible to prepare precise strategies returning an appropriate level of operational performance. The prepared model for predicting the reliability of a building makes it possible to foresee the technical condition as well as model scenarios of renovation works.

Modelling various scenarios of use helps to choose the optimal planning of renovation works on a building. The characteristics of various strategies influence the shape of the life cycle curve of the building. Applying the Pareto principle is an example of a strategy of renovation works on residential buildings. Applying the Raleigh distribution to predict reliability is possible thanks to the analysis of a set of data including the values of the degrees of wear of actual residential buildings.

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