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Large Displacement Analysis of Elastic Pyramidal Trusses

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Abstract. The aim of the present study is the analysis of the loss of stability of von Mises planar trusses with initial imperfections. The loading process is controlled by the size of the vertical static load in the top joint. Load-deflection curves are determined using the elastic geometric nonlinear solution based on beam finite elements. The influence of the random variability of initial imperfections on the load-deflection curves is the subject of research. The results showed the influence of asymmetric random imperfections on the load-deflection curves.

1. Introduction

Shallow structures, for e.g. shells, arches and reticulated structures, are commonly used to span large spaces. These structures generally have a highly nonlinear response, characterised by loss of effective stiffness in the presence of compressive stresses. The elastic von Mises truss model is a prototype structure with highly nonlinear response in the presence of static and dynamic loads [1, 2]. The von Mises truss is associated with the name of von Mises [3], and von Mises and Ratzersdorfer [4]. Loss of effective stiffness leads to bifurcation of equilibrium and loss of stability in the case of elastic and perfectly straight bars. If the bars have very small initial geometric imperfections, the initial curvatures increase from the onset of loading until the effective stiffness of the structure is zero, see e.g. [5].

The complex analysis of the von Mises truss is based on elastic geometric nonlinear modelling, which does not limit the structural behaviour by creating permanent deformations. Geometric nonlinear computational models must also provide load-deflection analysis for those loading states in which the determinant of the tangential stiffness matrix is equal to zero or is negative [6]. Thus, the computational models differ from conventional approaches of geometric [7-9] or geometric and material [10-16] nonlinear modelling. Load-deflection elastic analysis of a von Mises truss can be efficiently performed using the beam finite element method [17] or rigid segments connected by springs [18]. Difficulties associated with the modelling of von Mises trusses using the beam finite element method can be effectively overcome by designing a system to introduce a proportional fraction of the load in its top joint so that the determinant of the tangential stiffness matrix is always positive [17]. This approach is applied in this article.

2. Von Mises planar truss without imperfections

The von Mises planar truss with cross-section area $A = 0.00538 \text{ m}^2$, second moment of area $I = 13.36\text{E}6 \text{ mm}^4$, Young's modulus $E = 210 \text{ GPa}$ and initial angle $\alpha = 15^\circ$ was analysed, see Figure 1. In the case of a perfectly straight bar, the load-deflection elastic analysis can be performed using analytical equations published, for e.g. in [19-20].



When the axial force in the bars is equal to Euler's critical force $N_{cr} = 1538.34$ kN then buckling occurs in the plane of the truss due to bending about the minor axis. Buckling occurs when $\varphi = \varphi_{cr} = \arccos(\cos \alpha / (1 - \pi^2 / \lambda^2))$, where $\lambda = l/i$ is the slenderness ratio of the bar, $l = \sqrt{18}$ m is the length of the bar and $i = 49.8$ mm is the radius of gyration [17]. Buckling occurs if $\lambda > \pi^2 / \sqrt{1 - \cos \alpha}$.

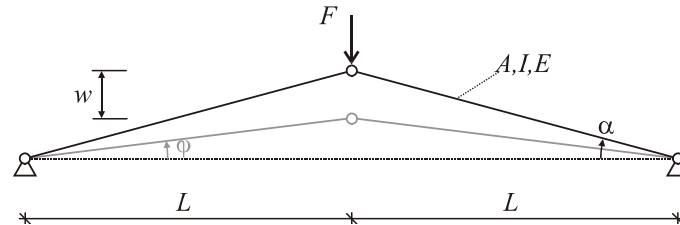


Figure 1. Von-Mises planar truss with perfectly straight bars.

The red point “E” depicted in Figure 2 represents the bifurcation point at which buckling occurs. At this point, a bar can be in equilibrium either in the straight or in a slightly deflected (buckled) configuration. After buckling, either there is a slight increase in force F resulting in snap “E” $\rightarrow$ “F” (dotted red line in Figure 2), or there is a decrease in force F due to a slight increase in the vertical deformation of the top joint (full red curve in Figure 2). The full black curve depicted in Figure 2 corresponds to a truss in which only axial compression (without buckling) of bars, described by the relation $F = 2EA(\sin \varphi - \cos \alpha \tan \varphi)$, occurs, see [17]. Point “A” in Figure 2 is the peak of the load-deflection curve reached upon increasing the load F if the bars are fixed against buckling. A slight increase in the load F at point “A” results in snap “A” $\rightarrow$ “B”.

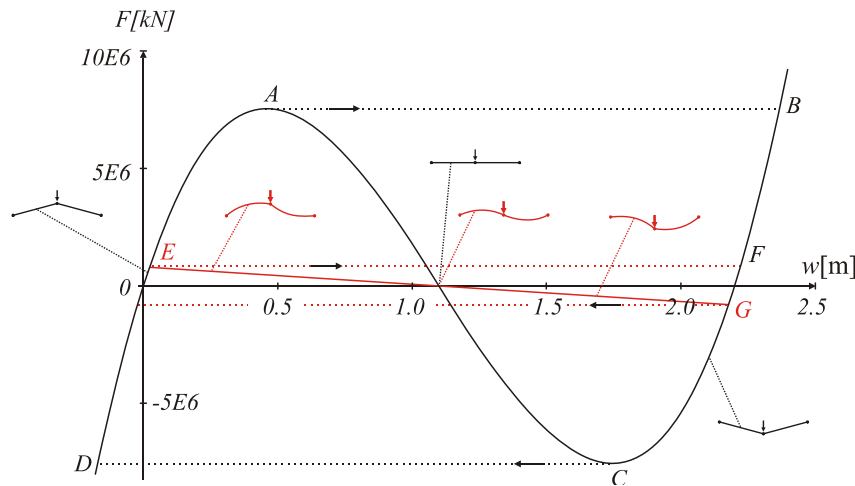


Figure 2. Snap-through effects of the von-Mises truss with perfect straight bars.

3. Von Mises planar truss with initial imperfections

Figure 1 shows the von Mises planar truss from the previous chapter, which has initial imperfections. The initial top joint horizontal imperfection e_0 , amplitudes of initial sinusoidal curvature of bars e_{01} , e_{02} , and stiffness K are considered as initial imperfections, see Figure 3. The vertical deformation was introduced in the von Mises planar truss using the geometric nonlinear computational model with a long draw rod, which is described in detail in [17]. The author of the presented paper [17] developed the computer algorithm. The software based on nonlinear beam theory allows the evaluation of the increasing and decreasing branch of the load-deflection curves.

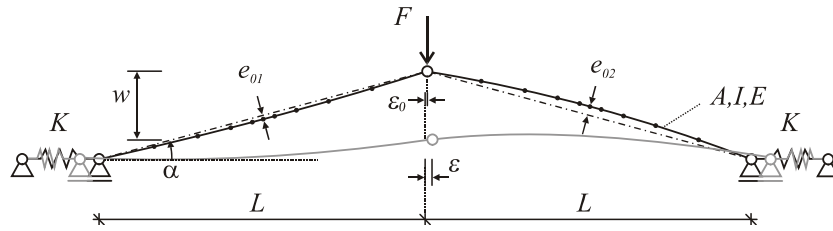


Figure 3. Von-Mises truss with initial imperfections ε_0 , e_{01} , e_{02} and stiffness K .

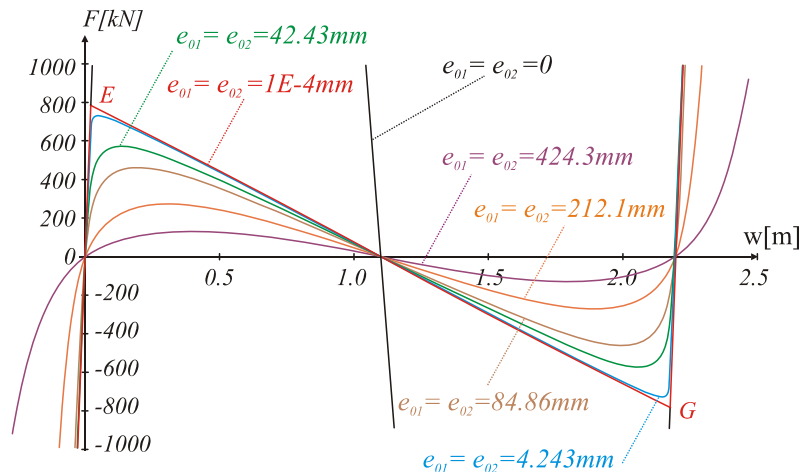


Figure 4. Load-deflection curves for $K = \infty$, $\varepsilon_0 = 0$ and $e_{01} = e_{02}$.

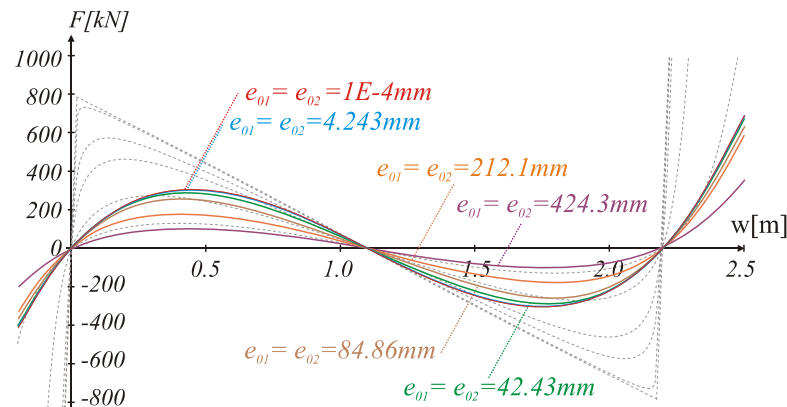
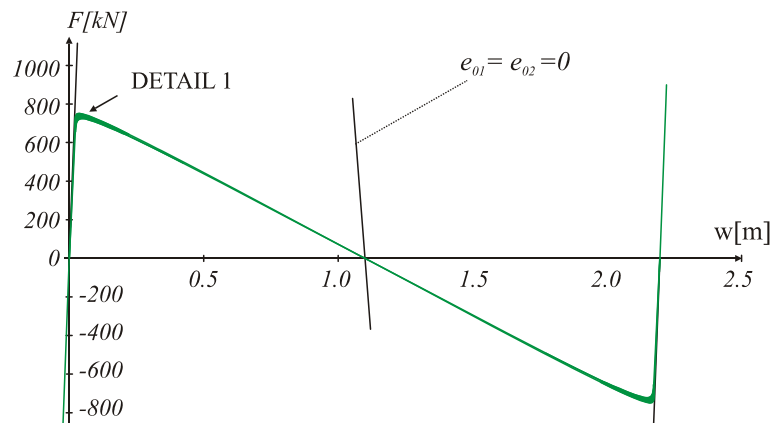
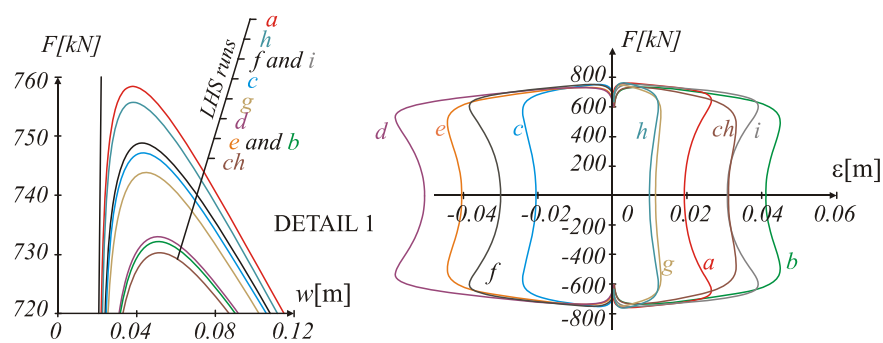


Figure 5. Load-deflection curves for $K = 11.1 \text{ MNm}^{-1}$, $\varepsilon_0 = 0$ and $e_{01} = e_{02}$

Load-deflection curves in Figure 4 and Figure 5 are calculated considering $\varepsilon_0 = 0$, seven values of imperfections e_{01} , e_{02} and two values of stiffness K . Figure 6 shows load-deflection curves that were obtained with consideration to initial random imperfections ε_0 , e_{01} , e_{02} , see Table 1. Random variables ε_0 , e_{01} , e_{02} have Gauss probability density function with mean value of zero. The magnitude of the beam bow imperfection can be considered as $l/1000$, see e.g. [21–23]. The standard deviation $l/1960$ is derived on the assumption that 95 observations of amplitude e_0 lie within the tolerance limits $\pm l/1000$, see e.g. [14]. The runs were simulated using the Latin Hypercube Sampling (LHS) method [24, 25].

Table 1. The random realizations of initial imperfections

LHS run	ε_0 [mm]	e_{01} [mm]	e_{02} [mm]
a	0.279	0.279	1.498
b	-2.310	-0.855	-3.879
c	0.855	-2.310	-0.855
d	-1.498	3.879	0.279
e	1.498	-3.879	0.855
f	-0.855	2.310	-0.279
g	3.879	1.498	2.310
h	2.310	0.855	-1.498
ch	-3.879	-1.498	3.879
i	-0.279	-0.279	-2.310

**Figure 6.** Load-deflection curves for $\varepsilon_0=0$, $K=\infty$.**Figure 7.** Load-deflection curves for $\varepsilon_0=0$, $K=\infty$ and e_{01} , e_{02} from Table 1.

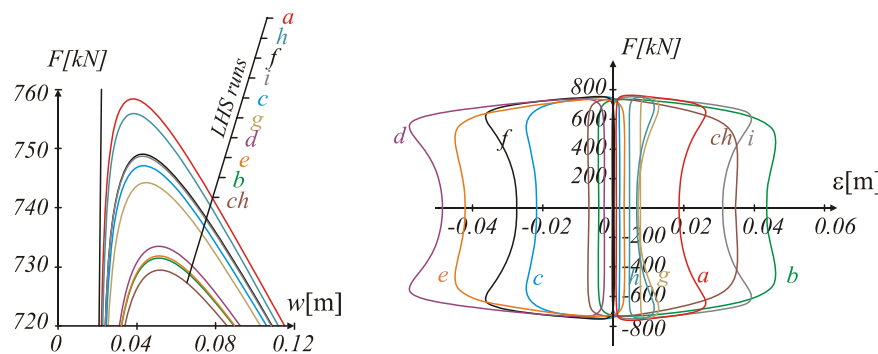


Figure 8. Load-deflection curves for $K=\infty$ and $\varepsilon_0, e_{01}, e_{02}$ from Table 1.

Figure 6 shows that the random variability of initial imperfections e_{01}, e_{02} has relatively small influence on the load-deflection curves. Initial imperfections e_{01}, e_{02} cause that force F brings about changes in deformation ε , which move in loops, see Figure 7. Figure 8 shows the effect of the variability of all three initial imperfections on translations w and ε . The graphs shown in Figure 7 and Figure 8 are very similar, but not identical.

4. Conclusions

The influence of initial imperfections on the load-deflection curve is presented in the article. Random imperfections e_{01}, e_{02} cause asymmetric changes in deformation ε , which move in loops starting from the origin $\varepsilon=0$ if $\varepsilon_0=0$. These asymmetries are highlighted if the behaviour is also influenced by the random variability of e_0 . Stiffness K has a relatively significant influence on the magnitude of the maximum force F . The results may be generally different for bars with different slenderness. The system has the greatest stiffness if the initial imperfections $\varepsilon_0, e_{01}, e_{02}$ are equal to zero. Quantified assessment of the influence of the random variability of initial imperfections on the structural response can be performed using sensitivity analysis [26, 27]. The stiffness of the system decreases if the initial imperfections $\varepsilon_0, e_{01}, e_{02}$ increase or decrease. The stiffness of the system is not monotonously dependent on imperfections $\varepsilon_0, e_{01}, e_{02}$ and thus rank-order correlation coefficients are not suitable indicators of sensitivity of random variables $\varepsilon_0, e_{01}, e_{02}$. Variance-based sensitivity analysis [28] or structural reliability sensitivity analysis [29] may be more appropriate. The list of imperfections can be extended in further studies to include asymmetric stiffness of supports, etc.

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