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Stochastic Modelling and Prediction of Fatigue Crack Propagation Based on Experimental Research

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Stochastic Modelling and Prediction of Fatigue Crack Propagation Based on Experimental Research

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Abstract. Numerical analysis of the propagation of an edge crack in a plate was performed in this study. The theoretical model of fatigue crack progression is based on linear fracture mechanics. Calibration functions for short edge cracks are applied in stochastic models and the stochastic dependencies between input random variables and the fatigue resistance are described. Attention is focused on the domain of the relative crack length. Results are obtained using the Latin Hypercube Sampling method. Sensitivity analysis is evaluated using methods ranging from the screening method to quantitative techniques based on correlation measurements. Pearson correlation coefficient, Spearman rank-correlation coefficient and Kendall rank correlation coefficient are used for the evaluation of sensitivity analysis. The study demonstrates the application of several numerical simulation procedures covering both qualitative and quantitative sensitivity analysis using a one-sample base. The effects of non-linear stochastic dependencies and outliers on the results of sensitivity analysis are discussed.

1. Introduction

Problems with older steel bridges are mainly related to insufficient care and associated decreasing durability and lifespan. State-of-the-art reviews on the assessment of the fatigue life of steel bridges are published, for e.g. in [1-5]. The lifetime of Czech steel bridges is affected by the type of steel used, the type of structure chosen and the ever-increasing traffic intensity. Experimental research identifies fatigue characteristics of building structures [6]. The degradation of fatigue life can be countered by timely repairs using information from regular inspections, analysis of the resistance and stress states of the load bearing structural parts [7]. The finite element method is the main tool for the analysis of the limit states of load bearing structural systems [8-10] including the related geotechnical aspects [11]. Other significant parameters include the strength material characteristics of steel, which have been monitored over a long period of time [12-14]. An important part of theoretical analysis is the computer modelling of fracture tests [15, 16] and reliability assessment of the structure based on statistical [17, 18], probabilistic [19-22], sensitivity analysis [23-27] and the optimization branch of non-linear and non-convex programming problems [28]. Probabilistic methods [19] along with linear fracture mechanics [29] and experimentally obtained data [6] are used to identify stochastic interactions of failure prediction [30]. Several methodological complications resulting from the identification of higher statistical characteristics of variables in the limit state functions were found [30]. This overview also includes decision-making methods developed for dealing with uncertainties and applied to the solution of civil engineering problems [31-32]. This article deals with the solution of some

methodological problems we face in stochastic applications of results of experimental research [6] in linear fracture mechanics.

2. Linear fracture mechanics

Linear elastic fracture mechanics examines the propagation of crack length a from initial size a_0 to the critical size a_{cr}, then fatigue fracture occurs. The crack grows depending on the number of fatigue cycles N. Fatigue crack growth is generally described using Paris's law expressed by Paris and Erdogan [34]:

$$\frac{da}{dN} = C(\varDelta K)^m \tag{1}$$

where m and C are Paris-Erdogan law constants that depend on the material, environment and stress ratio and ΔK is the range of the stress intensity factor during the fatigue cycle, which can be determined as:

$$\varDelta K = \varDelta\sigma\sqrt{\pi a}F(a/W) \tag{2}$$

where $F(a/W)$ is the calibration function (geometric factor) describing the course of crack propagation with respect to the sample geometry and $\Delta\sigma$ is the quasi–constant stress range. Integration of Paris's law can be written as:

$$\int_{a_0}^{a_{cr}} \frac{da}{\left[F(a/W)\cdot\sqrt{\pi\cdot a}\right]^m} = C\cdot N_F\cdot\Delta\sigma^m \tag{3}$$

where N_F is the total number of cycles at crack growth from a_0 to a_{cr}, $\Delta\sigma$ is the quasi–constant stress range. C, m are material constants according (6)

$$\log(C) = c_1 + c_2 m \tag{4}$$

where c_1, c_2 can be considered as c_1 = -11.141, c_2 = -0.507 for steel grade S235 [16]. $F(a)$ is the calibration function evaluated from experimental research for pure bending in the form published in [6], see also Figure 1. The full line in Figure 1 represents pure bending, the other curves are obtained for three- and four-point bending specimen configuration, see the detailed specification in [6].

   In this article, a new curve is determined (4), which supplements the experimental results published in [6] and presented here in Figure 1 on the interval of relative crack length $a/W \in \langle 0,\ 0.5 \rangle$.

$$F\left(\frac{a}{W}\right) = 1.114 - 0.8975\left(\frac{a}{W}\right) + 2.752\left(\frac{a}{W}\right)^2 + 1.1323\left(\frac{a}{W}\right)^3 \tag{5}$$

where W is specimen width in the direction of crack propagation. The question, which arises, is for which aspects of the reliability analysis can the extended domain $a/W \in \langle 0,\ 0.5 \rangle$ be beneficial and under what circumstances can the standard interval $a/W \in \langle 0,\ 0.3 \rangle$ be considered.
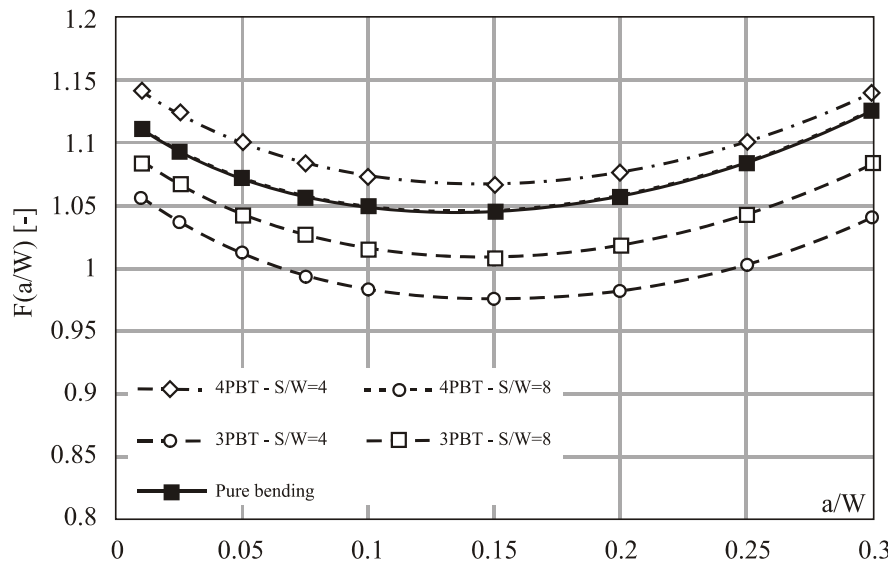
Figure 1. Experimentally determined calibration functions $f(a/W)$ [6]

The propagation of a fatigue crack is a stochastic phenomenon brought about by inherent uncertainties stemming from material properties, environmental conditions and cyclic loads. Stochastic processes thus provide a suitable framework for the modelling and prediction of crack propagation.

3. Statistical and sensitivity analysis

The influence of uncertainties can be considered using stochastic modelling strategies. The modelling of fatigue crack propagation using the framework of stochastic processes has been addressed in several papers. This framework enables the introduction of certain variabilities to the typical deterministic laws to describe fatigue crack growth under constant or variable amplitude fatigue loading, see for e.g. [34].

The main analysed variable in the assessment of limit states is the fatigue resistance N_F. The input random variables of the probabilistic model are listed in Table 1. The statistical characteristics of input variables were taken from [35] with the exception of the second and third variables, which are introduced with approximately the same variation coefficients as in [35]. Initial crack size a_0 has a log-normal probability density function (pdf), the other random variables have Gauss pdf with the exception of $\Delta\sigma$, which is modelled with Hermite pdf, which is available in the software Statrel 3.10.

Table 1. Input random quantities

Random variables	Symbol	Distribution	Unit	Mean value	Std. deviation	Skewness	Kurtosis
Initial crack	a_0	Log-norm	[mm]	0.526	0.504	3.73	33.75
Critical crack	a_{cr}	Gauss	[mm]	200	16	0	3
Specimen width	W	Gauss	[mm]	460	23	0	3
Parameter	m	Gauss	[-]	3	0.03	0	3
Stress range	$\Delta\sigma$	Hermite	[MPa]	61.36	10.22	-0.2495	2.9782

Statistical and sensitivity analysis is performed with ten thousand samples using LHS [36, 37]. Statistical characteristics of N_F evaluated for this number of samples are: mean value 526E3 and standard deviation 502E3. Statistical dependencies between N_F and input random variables in Table 1 are depicted in Figure 2 to Figure 7. Figure 2 and Figure 7 show the non-linear dependence between a_0

vs N_F and $\Delta\sigma$ vs N_F. The smaller the size of the initial crack a_0 the greater the value of N_F and the smaller the value of the quasi–constant stress range $\Delta\sigma$ the greater the value of N_F.
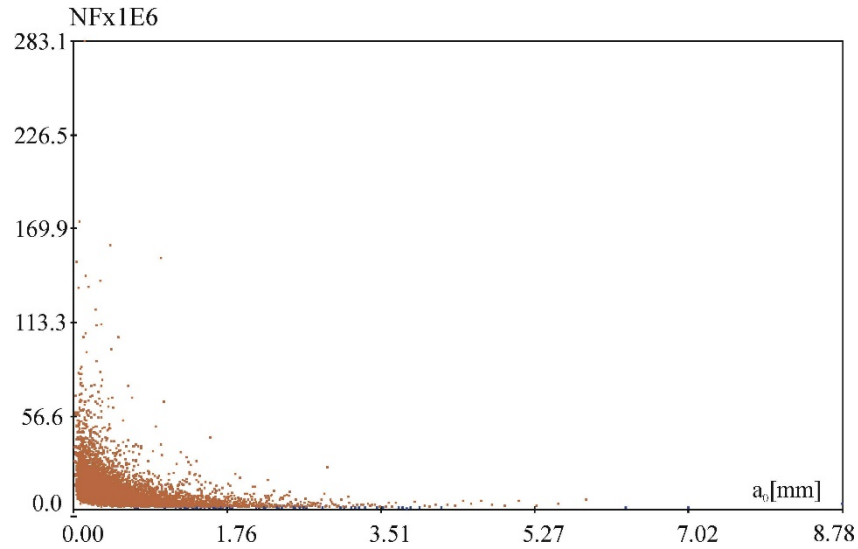

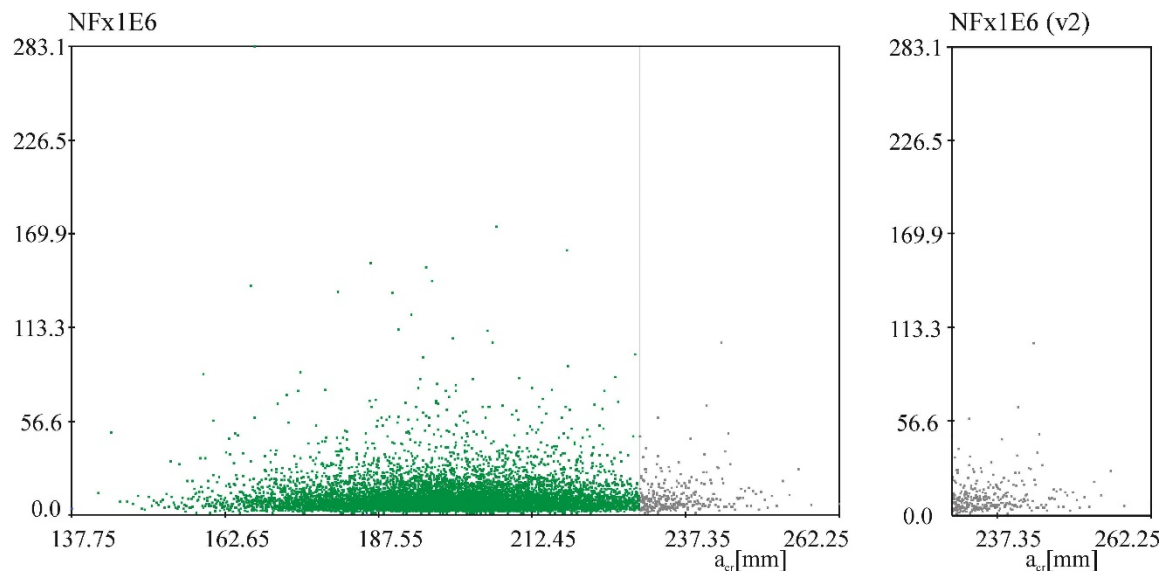**Figure 2.** Sampling-based dependence between a_0 and N_F.

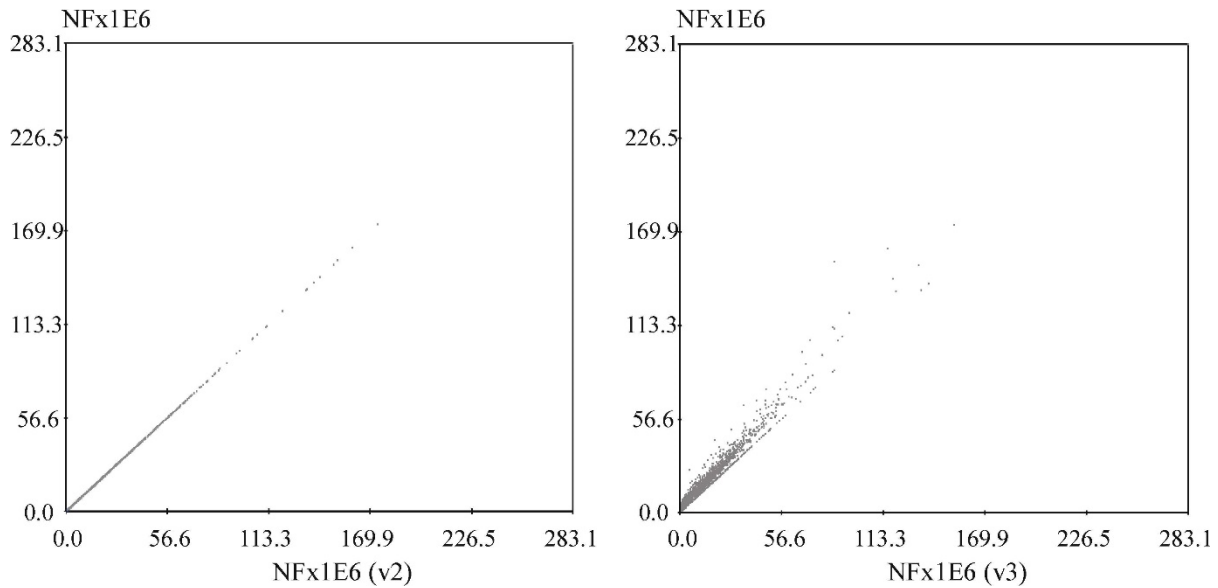**Figure 3.** Sampling-based dependence between a_{cr} and N_F.

Figure 4. Sampling-based dependence between N_F and N_F (v2), N_F and N_F (v3).

Correlation analysis, which is used to quantify the strength of the statistical relationship between two random variables, is among the core research paradigms in nearly all branches of scientific and engineering fields. If the correlation is large and positive, there is a high probability that large (small) values of one variable occur in conjunction with large (small) values of another. If the direction is reversed the correlation should be large and negative.

Figure 3 shows the stochastic dependence between a_{cr} and N_F. The samples shown in Figure 3 are divided into two parts. Samples a_{cr} with size greater than 230 mm are in the grey section. The higher the value of random realization a_{cr} the greater the probability that a/W will be higher than the domain (4). The right side of Figure 3 represents the same realizations a_{cr}, however, N_F is calculated with the difference that brittle fracture can occur only for random realizations of critical crack depths smaller than 230 mm (variant v_2). Practically, random realizations of a_{cr} were fixed at the value of 230 mm in cases where this value was exceeded. It is apparent from Figure 3 that runs N_F calculated in this manner are practically identical with the results on the left side of Figure 3. The left side of Figure 4 shows that the correlation between both variants is 1. On the right side of Figure 4, the study is evaluated using the same methodology, but for the fixed value of 4.6 mm (variant v_3). In this case, the correlation is no longer 100%, because the achieved number of cycles in variant v_3 is significantly lower than in the basic (first) variant of the N_F calculation.

Sensitivity analysis is performed using Pearson correlation coefficients between input random variables and N_F. It is apparent form Table 2 that the variability of $\Delta\sigma$ and a_0 has a dominant effect on N_F. The influence of the other variables is relatively small. Pearson correlation coefficient is the basic method for evaluating statistical dependencies that reflect linear dependence. Pearson correlation coefficient is greatly influenced by outliers found in this presented study.
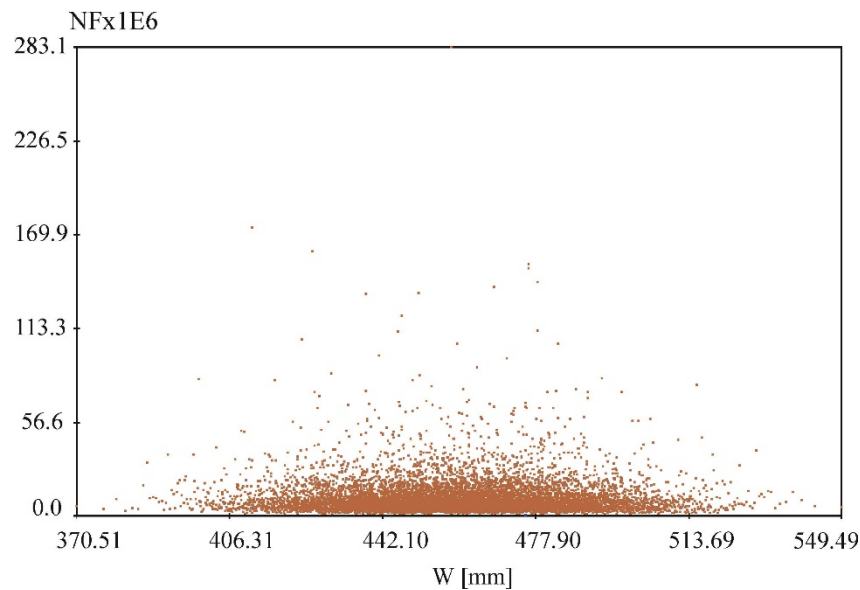
Figure 5. Sampling-based dependence between W and N_F.

An extension for the evaluation of certain types of non-linear dependencies is presented by Spearman rank-correlation coefficient. It is a nonparametric correlation coefficient, which is robust with regard to outliers and generally to deviations from normality because, like many other nonparametric methods, it only works with the rankings of the observed values. Unlike Pearson correlation coefficient, which describes the linear relationship between input and output variables, Spearman rank-correlation coefficient describes how well the relationship between two random variables corresponds to a monotonic function, which may be non-linear. This Spearman rank-correlation coefficient property is often used in non-linear structural mechanics, see e.g. [38]. The comparison of both approaches in Table 2 shows that the results are almost identical. This could be due to the small sensitivity of Spearman rank-correlation coefficient to outliers. However, outliers may be an important part of reliability assessment, and therefore should be components of sensitivity measurements in advanced building industry [39].

The third approach to evaluating sensitivity is using Kendall rank correlation coefficient (Kendall's τ). Kendall's τ is a measure of rank correlation: the similarity of the orderings of the data when ranked by each of the quantities. Kendall's τ has greater sensitivity to certain non-linear relationships.

Table 2. Correlation coefficients

Random variables	Symbol	Pearson correlation	Spearman correlation	Kendall's tau correlation
Initial crack	a_0	-0.2999	-0.2999	-0.4216
Critical crack	a_{cr}	0.0025	0.0025	0.0088
Specimen width	W	-0.0123	-0.01233	-0.0052
Parameter	M	-0.1004	-0.1004	-0.1078
Stress range	$\Delta\sigma$	-0.6566	-0.6566	-0.5488

Other possible approaches to sensitivity analysis include variance-based sensitivity indices [40], SAFD sensitivity assessment [41] or measuring the differences between the unconditional failure probability and conditional failure probability on certain input variables [42], which may be particularly suitable for probabilistic assessment of reliability.
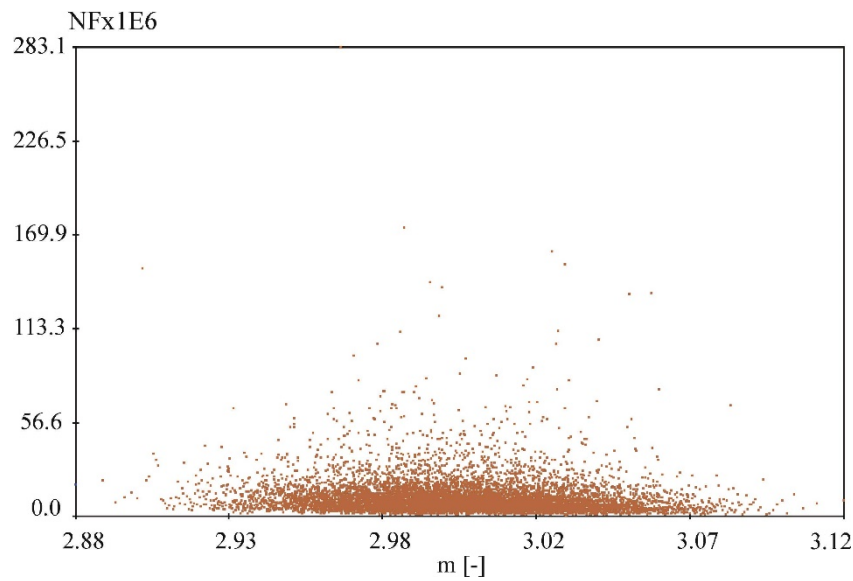
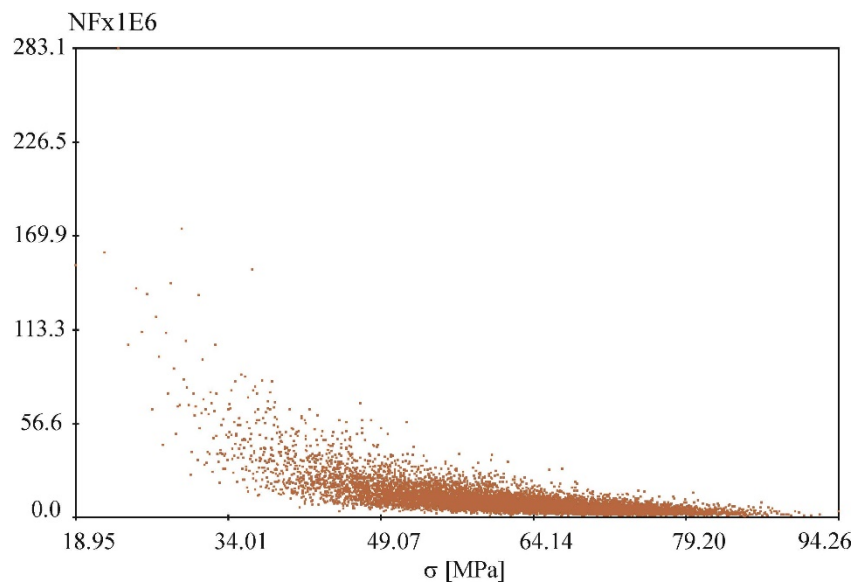**Figure 6.** Sampling-based dependence between *m* and N_F.

Figure 7. Sampling-based dependence between σ and N_F.

4. Conclusions

Stochastic dependencies between input random variables and the fatigue resistance evaluated using linear fracture mechanics as the total number of cycles at crack growth from initial crack size to critical crack size are analysed in this article. The article presents further processing of experimentally obtained results in the course of the calibration function. The calibration functions for short edge cracks are compared for various loads and the basic calibration function for pure bending is presented on an extended domain of relative crack length. It is shown that the calibration function describing the course of crack propagation can be reliably defined on the basic domain of the relative crack length.

Sensitivity analysis showed the dominant effects of initial crack size and quasi-constant stress range on the fatigue resistance. The results of sensitivity analysis obtained using Pearson correlation coefficient were the same as results obtained using Spearman rank-correlation coefficient. Sensitivity

evaluated using Kendall's τ is slightly different. The obtained sensitivity analysis results are in contrast with the expected influence due to the occurrence of outliers. In further research it is necessary to focus on other types of sensitivity analysis aimed at the analysis of the influence of outliers on structural reliability.

Acknowledgment

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