IOP Conference Series: Materials Science and Engineering

A Modeling Method for Abrupt Change Shafts in Gearboxes and Experiment Verification

IOP Conf. Series: Materials Science and Engineering 470 (2019) 012034 doi:10.1088/1757-899X/470/1/012034

A Modeling Method for Abrupt Change Shafts in Gearboxes and Experiment Verification

Z C Liu, W Song and Q Lu

ABB Corporate Research Center, Kangxin highway 4528, Pudong, Shanghai, China

Zhanchi.liu@cn.abb.com

Abstract. The modeling method of the shaft with abrupt change of section is essential in gearbox design. This study reviewed the proposed piece-wise beam model for abrupt change shaft, which is dedicated to estimate shaft deflection. The proposed model is an equivalent model, which is established by comparing and analysing the distribution of the flexural rigidity of the shaft model using solid element. Based on the proposed model a design tool is developed for creating the piece-wise beam model with high efficiency and convenience. As verification of the piece-wise beam model, a test bench with abrupt change shaft is designed and constructed. The shaft deflection with static load has been measured. Experiment analysis has been made by comparing the results between experiments and simulation, which is obtained using solid model, the traditional beam model and the proposed piece-wise beam model. Result shows that the piece-wise beam model is more accurate than the traditional beam model in shaft deflection estimation. The experiment result reveals the correctness and effectiveness of the piece-wise beam model and the design tool.

1. Introduction

To realize high transmission ratio in limited space, it is quite common to mount a large gear onto a shaft. Shrink fitted gear is normally used to realize a stable matching condition. In design of the gearbox it is important to estimate the deflection of the shaft. For shafts with shrink-fitted disk, an equivalent diameter is well defined using the pitch and bore diameter of the gear according to ISO-6336[1]. The resultant shaft is often a shaft with abrupt change section which means a sudden change of diameter at the equivalent part.

Modeling of the abrupt change shaft is essential in gear design. Solid element is a good choice when performing the simulation using finite element method. However, 1-D beam element is still preferred by engineers due to its simplicity of pre-process and high efficiency. Moreover, with some commercial software focused on gearbox like KissSoft, beam element is the only choice for users. But it is proved that traditional beam element is questionable in shaft bending analysis when it comes to abrupt change shafts. It is because traditional beam element failed to depict the distribution of the rigidity at the abrupt change section. In gearbox design, precise estimation of shaft bending is of great importance because it affects tooth modification which will further influence the wearing and lifetime of the gear. Thus a relative accurate beam model is desired for abrupt change shaft in gearbox design.

Literatures [2-4] investigated constrains of the traditional beam elements which will increase the errors when dealing with shafts or beams with abrupt change sections. It is also pointed out that the reason of the error is the constant rigidity property of the tradition beam element. Actually the distribution of the shaft rigidity is continuous in the neighbour of the discontinuity of the geometry. [3, 5] investigated the distribution of the flexural rigidity for abrupt change shafts. The equivalent second moment of area $I(x)$ was used, which can be obtained using finite element method with solid element. Some dimensionless design curves is supplied, in which the continuous $I(x)$ is supplied.

$$I(x) = \frac{M(x)}{E\frac{d^2y}{dx^2}} \tag{1}$$

Based on the limitation of the previous study work, [6] investigated the application in practical engineering. A generalized shaft model is proposed, which is applicable to the most case of the shaft with spur or helical gears. Using finite elements analysis, a piece-wise beam model for abrupt change shaft is proposed. This model could be used to estimate the deflection of the shaft. In the supplied design case the error between the beam model and solid model could be highly decreased. Figure 1 shows the general idea of development of the piece-wise beam model from an abrupt change shaft solid model.

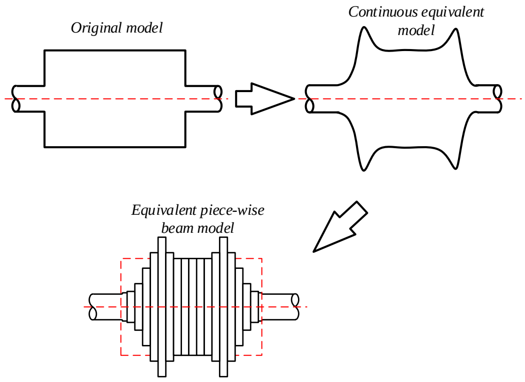

Figure 1. From original model to equivalent piece-wise beam model.

However, there are still limitations of the application in design practice. Obviously, the analysis using solid element will decrease the efficiency of the application of the piece-wise beam model. Furthermore, trying shafts with different geometry parameters might be necessary for designer. The current solution is not convenient to embed the proposed model in the fast design loop. Finally the piece-wise beam model is not verified in real design case so far, which add questionable factors to the propose beam model. Therefore, there is still a gap between the study outcomes and applications. Experiment verification is also necessary for the proposed piece-wise beam model.

The present work is focused on the application method of the piece-wise beam model and experimental verification. Firstly, the establishment of the piece-wise beam model has been reviewed. The application method of the piece-wise beam model is proposed. Then a design tool for abrupt change shaft is developed, in which design data of thousands of shafts is stored. Using this tool the piece-wise beam model for abrupt change shafts in a certain geometry range can be obtained with high efficiency, which make it possible to merge the proposed beam model well into the design loop. A shaft test rig has been designed and constructed. The deflection of a shaft with abrupt change section has been measured under static loading condition. Comparison of test and simulation results has been made, which verifies the effectiveness of the piece-wise beam model in increasing the accuracy of deflection estimation for abrupt change shafts.

2. Equivalent piece-wise beam model

2.1. Shaft model preparation

The generalized shaft model[6] contains three parts: basic shaft 1, stepped shaft and basic shaft 2 as is shown in figure 2. It can be well defined using the following three dimensionless parameters:

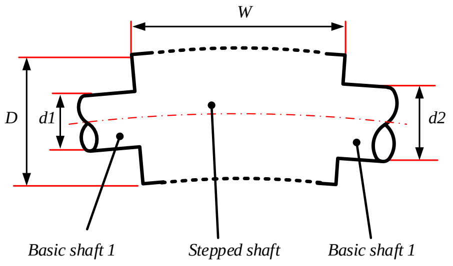

Figure 2. Generalised shaft model with different basic shafts.

$$\begin{cases} W_{dim} = \frac{W}{d_1}, & W_{dim} \in R \\ D_{dim} = \frac{D}{d_1}, & D_{dim} > 1 \\ d_{2dim} = \frac{d_2}{d_1}, & 1 < d_{2dim} < D_{dim} \end{cases} \tag{2}$$

where W and D are the length and diameter of the stepped shaft; d_1 is the diameter of the basic shaft 1; d_2 is the diameter of the basic shaft 2; subscript *dim* means dimensionless. In similarity condition (2), it is assumed that $d_2 \geq d_1$. This condition can be easily met by switching them if $d_2 < d_1$ in design practice.

2.2. Continuous equivalent model

For the abrupt change shaft with specific geometry dimension, a finite element shaft model can be employed with the same geometry parameters at the abrupt change section, which means the W_{dim}, D_{dim} and d_{2dim} are the same with the objective shaft model. Figure 3 shows an example of the shaft model with simple supports and pure bending moments. Then the static analysis can be performed to obtain the shaft deflection. An easy treatment is using the displacement of the shaft centre line as the shaft deflection. Figure 4 shows the deformation of the shaft using solid element. Figure 5 shows the deflection of the shaft which is extracted from the shaft center line of solid model.

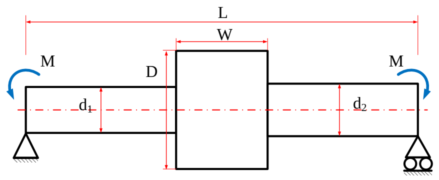

Figure 3. Set-up of the shaft model.

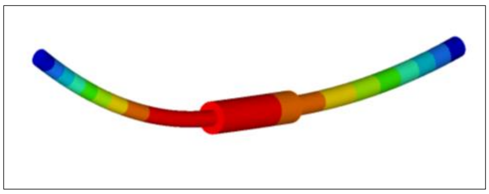

Figure 4. Deformation of the FE shaft model.

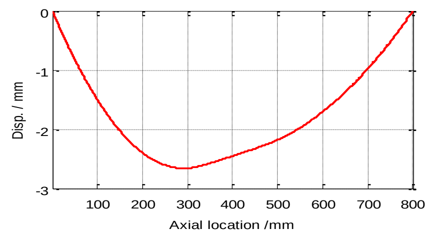

Figure 5. Deflection of shaft center line.

With shaft deflection, the local curvature can be calculated at each node axial place. Then the local second moment of area $I(x)$ can be obtained using formula (1) as shown in figure 6, in which all the parameters has been made dimensionless with r_1. Because $I(x)$ is also a property of the cross section $I = \frac{\pi r^4}{4}$, the equivalent radius of the solid model can be calculated as is shown in figure 7. Using the continuous radius an equivalent solid model is obtained.

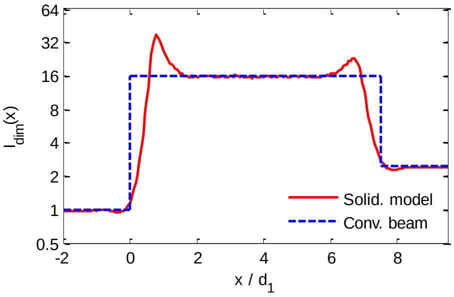

Figure 6. Equivalent second moment of area. Dash line: $I(x)$ of conventional beam, solid line: $I(x)$ of the solid model.

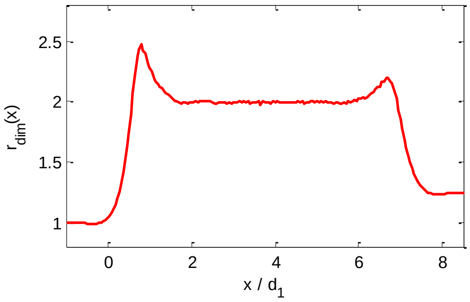

Figure 7. Distribution of dimensionless radius $r_{dim}(x)$.

2.3. Piece-wise beam model

The piece-wise beam model can be obtained by cutting the equivalent continuous model into pieces. Figure 8 demonstrate an example of piece-wise beam model for the continuous shaft, which is shown in figure 7. For further operation, it is easy to make it in to a model with dimension using d_1.

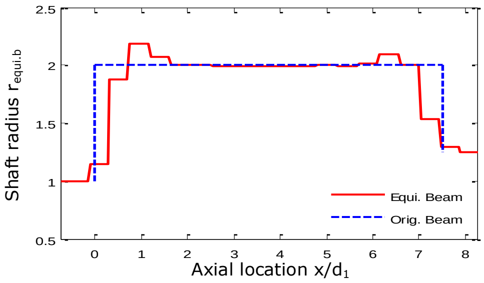

Figure 8. Dimensionless equivalent piece-wise beam model.

3. Development of the design tool for abrupt change shafts

In this section the development of a design tool will be demonstrated, which contains the stored data of simulation for thousands of shafts and a searching algorithm for creating the right piece-wise beam model for later use. Figure 9 shows the basic concept of the design tool. With this tool the piece-wise beam model can be generated instantly based on the in-put shaft dimension, which bridge the gap between the principle of the piece-wise beam model and its application.

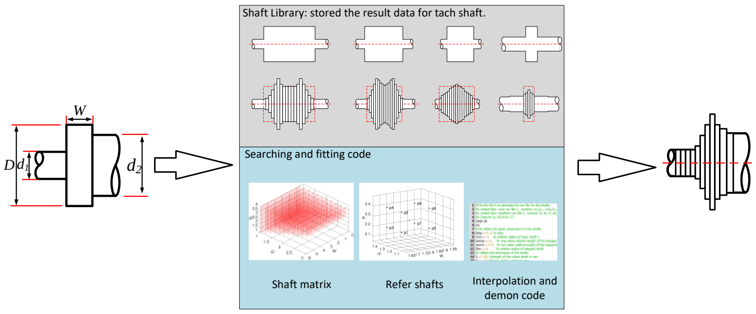

Figure 9. Design tool of abrupt change shaft.

3.1. Model dimension and finite element analysis

The shaft geometry dimension range could be defined with the current product data. Then the range of the abrupt change shaft parameters: W_{dim}, D_{dim}, and d_{2dim} can be defined. With an appropriate step, a shaft matrix containing different combination of shaft parameters can be established. In this demon case the shaft dimension range is defined as:

$$\begin{cases} W_{dim} = \frac{W}{d_1}, & W_{dim} \in [0.5,\ 3] \\ D_{dim} = \frac{D}{d_1}, & D_{dim} \in [1,\ 4] \\ d_{2dim} = \frac{d_2}{d_1}, & d_{2dim} \in [1, 3] \end{cases} \tag{3}$$

With the step 0.2, the shaft matrix can be obtained, which contains 3146 shafts of different geometry. Figure 10 shows the shaft matrix, in which every dot stands for a shaft model.

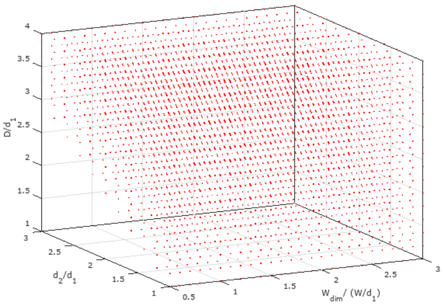

Figure 10. Abrupt change shaft matrix.

For each shaft, static finite element analysis, which contains the same procedure as shown section 2, has been made. In this example static analysis is performed in HyperWorks which supplies the script interface for repetitive work. Figure 11 demonstrates some of the finite element shaft models creating using script. After analysis the deflection of shaft center line is extracted and stored for later usage.

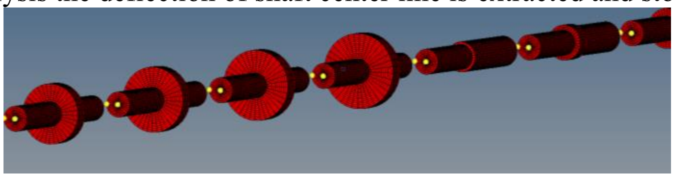

Figure 11. Part of shafts of shaft matrix.

3.2. Design tool development

For any combination of input parameters, which are W_{dim}, D_{dim}, and d_{2dim} of objective shaft, the design tool will supply shaft dimension parameters of the equivalent piece-wise beam model as output. It contains the shaft radius of the each axial sections, which can be defined according to user's requirements.

To achieve this goal, a searching algorithm is developed to obtain the deflection of the objective shaft. When the matrix contains the objective shaft, which means exist a shaft with the same dimensionless parameters as the input shaft, the deflection result can be used directly. When the matrix doesn't contain the shaft with the same dimension with objective shaft. Samples of shaft will be found as envelop shafts. The IDW (Inverse Distance Weighted) methods will be used to generate the deflection of the input shaft. Figure 12 shows the relationship of the input shaft (objective point) and envelop shafts (sample points).

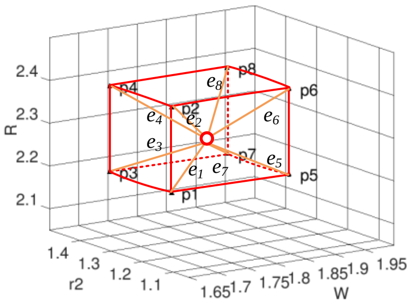

Figure 12. Envelop shafts (sample points) and input shaft (objective point). (○: input shaft; p1-p8: envelop shafts).

The shaft deflection can be calculated with interpolated value w_i. A general form of finding the interpolated values at given distance between objective point (input shaft) and sample points (envelop shafts) as be expressed as:

$$w_i = \frac{e_i^{-p}}{\sum_{j=1}^{n} e_j^{-p}} \tag{4}$$

where i is the index of sample; p is a positive real value, usually use 2; n is the total number of the samples; e_i is the distance of between the objective point and the corresponding sample point. With shaft deflection the piece-wise beam model can be calculated same process demonstrated in section 2. Example of output is shown in the next section.

4. Experiments verification

4.1. Set-up of test rig and experiments

To verify the proposed equivalent piece-wise beam model an abrupt change shaft test rig is designed and constructed. The deflection of shaft can be measured using linear variable deformation transformers (LVDT) under static loading condition. Figure13 and 14 shows 3D model and the test rig, which contains an abrupt change shaft simply supported between two ball bearings. Table 1 lists the shaft parameters of the test rig.

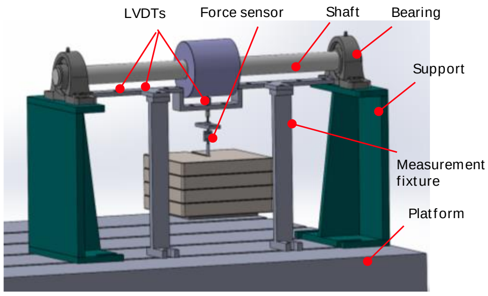

Figure 13. 3D model of the test rig.

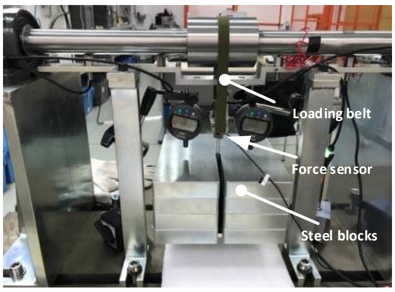

Figure 14. Abrupt change shaft test rig.

Table 1. Shaft parameters of test rig.

Shaft diameter /mm	Diameter of stepped section /mm	Length of the stepped section /mm	Distance of supports L/mm
36	72	100	600

In experiments seven displacement measure points are set along the shaft. The axial position coordinates of the measure points are listed in Table 2. Because the bearing radius stiffness is found to have obvious influence to the test results. Two measuring points near supports are used as the reference points (calibration points). Under this circumstance, the deflection between the reference points can be used to verify the shaft model, without influence of the bearing stiffness. In the experiment result analysis the displacement value relative to the reference points is calculated.

Table 2. Axial location of measuring points

Measure point	1 l_1/mm	2 l_2/mm	3 l_3/mm	4 l_4/mm	5 l_5/mm	6 l_6/mm	7 l_7/mm
Distance	45	100	210	300	390	500	555

Static loading condition is achieved by hanging weights in the middle of the shaft with force sensor on the loading mechanism as is shown in figure 14. The maximum static loading force is 313.6 N (32kg force). The results of the experiment will be analyzed in 4.3 results analysis.

4.2. Simulation with different shaft models

To verify the effectiveness of the proposed piece-wise beam model, the deflection is estimated using the solid model, traditional and piece-wise beam model. The simulation result is compared with the experiment result.

Firstly the piece-wise beam model is achieved using the design tool. The stepped part of the shaft is cut into 8 pieces. Figure 15 shows the geometry of the piece-wise beam model. Table 3 lists the value of the original and piece-wise beam model in detail.

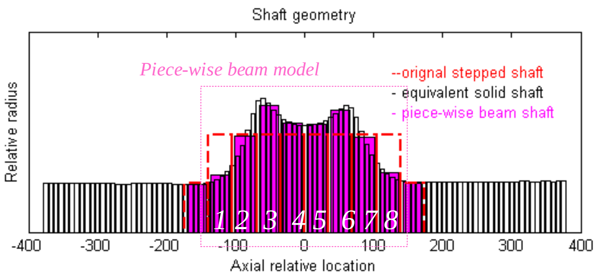

Figure 15. The obtained piece-wise beam model. (Red dash line: original stepped shaft; black: equivalent solid model; pink: piece-wise beam model.)

Table 3. Radius value of the piece-wise beam model

Section	1	2	3	4	5	6	7	8
Length/mm	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5
Radius (traditional)/mm	18	18	18	18	18	18	18	18
Radius (piece-wise)/mm	21.33	35.58	46.97	40.40	40.16	45.12	35.40	22.08

Figure 16 and 17 show the comparison of the shafts using traditional beam model and piece-wise beam model. These are also the model using for shaft deflection estimation. A 1-D beam based finite elements code is developed to finish the shaft deflection estimation [7]. Then the displacement of the testing points (5 points) is extracted, which would be compared with the experiment result in the following subsection. For the simulation using solid element, there is no special treatment. Thus the detailed information will not be supplied.

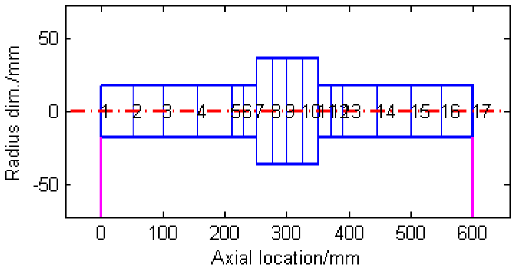

Figure 16. Shaft with traditional beam model.

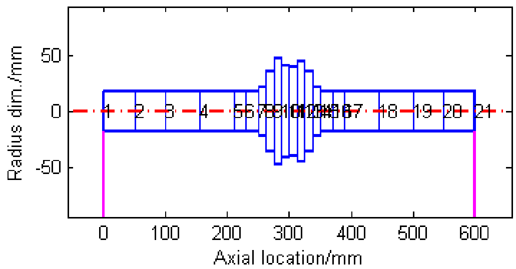

Figure 17. Shaft with piece-wise beam model.

4.3. Results analysis

Figure 18 is the deflection estimation of the shaft between the two calibration points with solid shaft model, traditional beam model, piece-wise beam model and the test result. The maximum displacement, which would be the mesh point for gear in this case, is used to evaluate the accuracy of the shaft with different models. Test result is used as reference for simulation results. The difference of the results is listed in table 4.

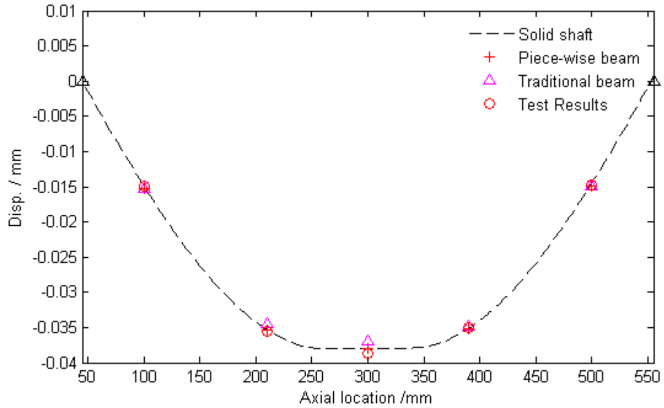

Figure 18. Comparison of shaft deflection using solid model, traditional beam model, piece-wise beam model and the test result.

Table 4. Difference of shaft deflection estimation using different model and experiment result

Testing Point		1	2	3	4	5
Experiments /um		-14.93	-35.61	-38.62	-35.01	-14.78
Solid shaft model	disp/um	-14.90	-35.35	-38.05	-35.15	-14.45
	Diff/%	0.19	0.74	1.46	0.15	2.19
Piece-wise beam model	P.W./um	-15.31	-35.41	-37.93	-35.15	-14.95
	Diff/%	2.5	0.56	1.79	0.16	1.16
Traditional beam model	Tradi./um	-15.31	-34.56	-36.89	-34.80	-14.95
	Diff/%	2.50	2.96	4.51	0.83	1.16

It can be seen that compared with test results, simulation with solid shaft model is with the highest accuracy. The traditional beam model is with the lowest accuracy. The result of the piece-wise beam model is in the between of the two other models, and much closer to the test results than the traditional beam model. The different at the middle point, which could be meshing point for gear, between experiment and estimation using traditional beam and proposed beam model is decreased from 4.51% to 1.79%. It can be seen that with traditional beam model, the stiffness of the shaft can be overestimated. It will result in the estimation of the shaft deflection to be smaller than the real value, which is influenced the tooth modification for gear design.

It can be seen that with the proposed piece-wise beam model, the shaft deflection can be estimated with higher accuracy compared with the traditional beam model. In this case the initial difference between the simulation and the experiment results is in a low level. Under some circumstances the initial difference could be larger than 10%, then the accuracy could be increased much higher [6].

The experiment results and the analysis reveal the correctness and effectiveness of the piece-wise beam model.

5. Future work and conclusion

5.1. Future work

The correctness and the effectiveness of the piece-wise beam in static analysis has been verified. However the dynamic characteristic of the shaft is critical important in the design for rotating machinery. Thus the work of next step will be focused on the dynamic characteristic of the piece-wise beam element. Obviously if the geometry shape of the shaft is varied, the mass and moment of inertia, which are key factors for dynamic characteristic analysis, would all be changed. In this condition some special treatment would be needed to ensure a higher accuracy in simulation. This would be the focus of the work in the future.

5.2. Conclusion

The main focus of present work is on the establishment and verification of the piece-wise beam model for shaft with abrupt change sections. The establishments of the piece-wise beam model is reviewed and constrains of its application is analysis. Based on the proposed beam model a design tool for abrupt change shaft is developed, which can be used to creating the piece-wise with high efficiency and convenience for engineers.

To verify the proposed beam model, a test-bench with abrupt change shaft has been constructed. Shaft deflection has been measured under static loading conditions. Experiment result shows that using the proposed beam model, the deflection of the abrupt change shaft can be estimated with higher accuracy compared with the traditional beam model. At the same time it reveals the correctness and effectiveness of the piece-wise beam model and the design tool.

Acknowledgement

The authors would like to thank Honghua Huang, MinYuan, and Ming Yan, ABB Power Transmission China. Their inputs, comments and discussions on this work are greatly appreciated.

References

[1] Calculation of load capacity of spur and helical gears –Part 1: Basic principles, introduction and general influence factors [S]. Geneva: ISO 6336-1:2006-09
[2] Sato H. Free vibration of beams with abrupt changes of cross-section[J]. Journal of Sound & Vibration, 1983, 89(1), pp. 59-64.
[3] Kitching R, Sanderson N, Hinduja S. Flexibility of rectangular beams with abrupt changes of section[J]. International Journal of Mechanical Sciences, 1975, 17(6), pp. 403-410.
[4] Sanderson N, Reid S R. Photoelastic approach to the determination of the flexural rigidity of rectangular beams with abrupt changes of section[J]. International Journal of Mechanical Sciences, 1976, 18(7), pp. 341-346.
[5] Sanderson N, Kitching R. Flexibility of shafts with abrupt changes of section[J]. International Journal of Mechanical Sciences, 1978, 20(3), pp. 189-199.
[6] Liu ZC, Song W and Lu Q, A Piece-wise Beam Model for Shafts with Abrupt Changes of Section in Gearbox Products[C]. ICAMechS 2017, Xiamen
[7] Yamamoto T, Ishida Y. Linear and Nonlinear Rotordynamics[M]. John Wiley & Sons, Inc., 2013