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Finite Difference Method for Simulating Transverse Forced Vibration of Vertical Rope with Multi-Rope Winding Hoister

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Abstract. During the high speed of the mine hoisting system, external excitation will produce large transverse vibrations on the vertical rope, which may eventually lead to dynamic instability. In this paper, the Hamiltonian principle is used to establish the partial differential equations transverse vibration of the vertical rope. By using the Finite Difference Method, the differential vibration control equation under lateral excitation is discretized and solved, and the transverse vibration characteristics of the vertical rope are improved. Taking the multi-cable winding hoisting system as an example, the effect of external excitation on the transverse vibration of the vertical rope was simulated and analysed. The simulation results show that the solution of vertical rope vibration equations based on the Finite Difference Method can improve the vibration results of the system.

1. Introduction

With the increasing focus on exploitation of deep mineral resources, stable design and operational reliability of hoisters for long distance transport have received increasing attention. The hoisters are used to transport mineral resources in deep mines to the surface platform. During the transport process, the vertical rope of the mine hoisting system will produce transverse vibrations, and these vibrations have a very significant impact on the performance of the hoisting system. With the gradual development of the coal mine in deeper areas, it will have more increased transverse vibration problems. Therefore, it is necessary to analyse the transverse vibration of the vertical rope with the mine hoisting system.

Many scholars have extensively studied the characteristics of mine hoisting systems. It involves the modelling, the kinematics and dynamics analysis of the hoist system. [1, 2] based on Galerkin method, study the transverse vibrations characteristics and the longitudinal-transverse coupling vibration analysis of rope for friction hoisting system. By considering the head sheave eccentricity and the defect of the rigid guide to the vibration influence of the rope, and analysing the vibration characteristic of the rope in different operational phases. Take the multi-cable winding hoisting system as the example, [3] analyses the longitudinal vibration characteristics of steel wire ropes in ultra-deep mine hoisting test bench. [4] analyse the modelling of rope transverse vibration for flexible hoisting systems with time varying length.

However, most scholars use the Galerkin method to solve discretion partial differential equations into ordinary differential equations. The Finite Difference Method has the following advantages compared with the Galerkin method: The Finite Difference Method includes all vibration modes of the rope vibration, and is more suitable for the complex constraint conditions. [5] uses the Finite



Difference Method to simulate transverse forced-vibration of elevator suspended system. Discrete differential equations are discretized into ordinary differential equations by using the Finite Difference Method, and then the step differential integration is used to solve the ordinary differential equations. The comparison with the Galerkin method shows that the effectiveness of the Finite Difference Method is verified. [6] uses the Finite Difference Method to simulate transverse forced-vibration of elevator suspended system. [7] uses the direct numerical method of the Finite Difference Method to discretize the partial differential equations into algebraic equations and solve the flexible hosting system.

In this paper, Finite Difference Method is used to study the influence of the transverse vibrations in the main rope during the transport of the mine hoisting system in external excitation. The dynamic model of a multi-cable winding hoister is derived via Hamilton principle. The infinite dimensional distributed-parameter system is discretized by the Finite Difference Method. The effect of external excitation on the transverse vibration of the vertical rope was simulated and analysed. It provides ways and methods for research on the safety of deep mine hoisting systems.

2. Mathematical model

The separation between the vertical rope and the head sheave is used as the fixed point of the vertical rope. The dynamic model of multi-cable winding hoisting system is shown in Figure 1.

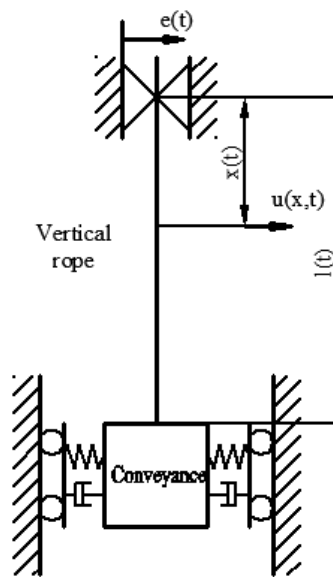


Figure1. Model of multi-cable winding hoisting system

Where $l(t)$ is the length of the vertical rope, $e(t)$ is the transverse interference excitation of the vertical rope. ρ is the rope linear density, A is the rope effective steel area, E is the rope effective Young's Modulus, m is the mass of conveyance. The vertical rope is seen as an axially moves string. The conveyance is seen as a rigid body suspended from the lower end of the vertical rope, and the transverse direction is constrained by the springs and dampers of the rigid guides. Due to the axial movement of the head sheave, a transverse interference excitation is generated on the upper end of the vertical rope. During the transport of vertical rope, the transverse vibration at $x(t)$ on the vertical rope is recorded as $u(x,t)$, and the vertical downward direction is the positive direction.

In this paper, the assumption of the establishment and solution model as follows:

- The slight elastic deformation of the vertical rope due to transverse vibration is far less than the length of the entire string.
- The material of the vertical rope obeys Hooke's law.

- Not consider the effect of longitudinal and torsional deformation of the vertical rope.
- Multi-rope is considered to have the same physical parameters as a single rope and the physical parameters remain unchanged.
- Ignore the effects of bending stiffness, air flow in shaft, damping and friction of rope.
- The conveyance as a rigid body.

The total kinetic energy of the vertical rope in transverse vibration can be represented as

$$E_k = \frac{1}{2} \rho \int_0^{l(t)} \left(v \frac{\partial u(x,t)}{\partial x} + \frac{\partial u(x,t)}{\partial t} \right)^2 dx + \frac{1}{2} m \left(v \frac{\partial u(x,t)}{\partial x} + \frac{\partial u(x,t)}{\partial t} \right)^2 \Big|_{x=l(t)} \quad (1)$$

The elastic potential energy of the vertical rope in transverse vibration can be represented as

$$E_p = \frac{1}{2} \int_0^{l(t)} \left[\left(\rho(l-x) + m \right) g + EA \left(\frac{\partial u(x,t)}{\partial x} \right)^2 / 4 \right] \left(\frac{\partial u(x,t)}{\partial x} \right)^2 dx \quad (2)$$

The gravity potential energy of the vertical rope in transverse vibration can be represented as

$$E_g = -mgu(l,t) - \int_0^{l(t)} \rho gu(x,t) dx \quad (3)$$

According to the generalized Hamilton's principle, the true motion of the system between $t = t1$ and $t = t2$ satisfy:

$$\int_{t1}^{t2} (\delta E_k - \delta E_p - \delta E_g) dt = 0 \quad (4)$$

Using the variation principle and the sub-integration method, the transverse vibration governs equation of the vertical rope can be obtained. For clarity, subscripts x, t represent partial derivatives are used throughout this paper.

$$\rho u_{tt} + \rho 2v u_{xt} + \rho(1+a)u_x + (\rho v^2 - T)u_{xx} - 3EAu_x^2 u_{xx} / 2 = 0 \quad (5)$$

The vertical rope transverse vibration boundary condition in $x = l(t)$ is obtained as Eq.(6)

$$mu_{tt} + 2mv u_{xt} + (ma + T)u_x + mv^2 u_{xx} + EAu_x^3 / 2 = 0 \quad (6)$$

The vertical rope transverse vibration boundary condition in $x = 0$ is obtained as Eq. (7)

$$u(x,t) = e(t) \quad (7)$$

Hypothesize the system is motionless state under initial conditions.

$$u(x,0) = 0, u_t(x,0) = 0 \quad (8)$$

3. Discrete mathematical model

The mathematical model of the multi-cable winding hoister is an infinite distributed-parameter system. There are no accurate analytic solutions. Using the Finite Difference Method, govern equations obtain the approximate solutions.

The lengths of vertical rope $l(t)$ are time-varying. By introducing a new independent variable $\xi = x/l(t)$, relative to x of the time-varying spatial domain $[0, l]$ is translate into relative to ξ of the fixed domain $[0, 1]$. The relational expression can be obtained as

$$\hat{u}(\xi, t) = u(x, t) \quad (9)$$

The govern equation and boundary condition of the vertical rope can changed as

$$\begin{aligned} \hat{u}_{tt} + (2v(1-\xi)/l)\hat{u}_{\xi t} + \left(\left(v^2(\xi-1)^2 - l(1-\xi)g - mg/\rho \right) / l^2 \right) \hat{u}_{\xi\xi} \\ + \left((al - 2v^2)(1-\xi) + l \right) \hat{u}_{\xi} - 3EA(\hat{u}_{\xi})^2 (\hat{u}_{\xi\xi}) / 2\rho l^4 = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} & \hat{u}_{tt} + (2v(1-\xi)/l)\hat{u}_{\xi t} + (v^2(\xi-1)^2/l^2)\hat{u}_{\xi\xi} \\ & + \left((m(al-2v^2)(1-\xi) + \rho gl^2(1-\xi) + mgl) / ml^2 \right) \hat{u}_{\xi} + (EA/2ml^3)\hat{u}_{\xi}^3 = 0 \end{aligned} \quad (11)$$

$$\hat{u}(0, t) = e(t), \hat{u}(\xi, 0) = 0, \hat{u}_t(\xi, 0) = 0 \quad (12)$$

Application of difference approximations to the time, space, and mixed partial derivatives leads to

$$\begin{aligned} \hat{u}_{tt}(\xi, t) &= (\hat{u}(\xi, t+1) - 2\hat{u}(\xi, t) + \hat{u}(\xi, t-1)) / \tau^2 \\ \hat{u}_{\xi t}(\xi, t) &= (\hat{u}(\xi, t) - \hat{u}(\xi-1, t) - \hat{u}(\xi, t-1) + \hat{u}(\xi-1, t-1)) / h\tau \\ \hat{u}_{\xi}(\xi, t) &= (\hat{u}(\xi+1, t) - \hat{u}(\xi, t)) / h \\ \hat{u}_{\xi\xi}(\xi, t) &= (\hat{u}(\xi+1, t) - 2\hat{u}(\xi, t) + \hat{u}(\xi-1, t)) / h^2 \end{aligned} \quad (13)$$

Where h, τ are spatial and time intervals.

Substituting Eqs. (12) into the Eqs. (10)-(11),

$$\begin{aligned} \hat{u}(\xi, t+1) &= -2\tau v(1-\xi)(\hat{u}(\xi, t) - \hat{u}(\xi-1, t) - \hat{u}(\xi, t-1) + \hat{u}(\xi-1, t-1)) / lh \\ & - \tau^2 (v^2(\xi-1)^2 - l(1-\xi)g - mg/\rho)(\hat{u}(\xi+1, t) - 2\hat{u}(\xi, t) + \hat{u}(\xi-1, t)) / l^2 h^2 \\ & - \tau^2 ((al-2v^2)(1-\xi) + l)(\hat{u}(\xi+1, t) - \hat{u}(\xi, t)) / l^2 h + 2\hat{u}(\xi, t) - \hat{u}(\xi, t-1) \\ & + 3\tau^2 EA(\hat{u}(\xi+1, t) - \hat{u}(\xi, t))^2 (\hat{u}(\xi+1, t) - 2\hat{u}(\xi, t) + \hat{u}(\xi-1, t)) / 2\rho l^4 h^4 \end{aligned} \quad (14)$$

$$\begin{aligned} \hat{u}(\xi, t+1) &= -2\tau v(1-\xi)(\hat{u}(\xi, t) - \hat{u}(\xi-1, t) - \hat{u}(\xi, t-1) + \hat{u}(\xi-1, t-1)) / lh \\ & - \tau^2 v^2(\xi-1)^2 (\hat{u}(\xi+1, t) - 2\hat{u}(\xi, t) + \hat{u}(\xi-1, t)) / l^2 h^2 \\ & - \tau^2 (m(al-2v^2)(1-\xi) + \rho gl^2(1-\xi) + mgl)(\hat{u}(\xi+1, t) - \hat{u}(\xi, t)) / l^2 mh \\ & - \tau^2 EA(\hat{u}(\xi+1, t) - \hat{u}(\xi, t))^3 / 2ml^3 h^3 + 2\hat{u}(\xi, t) - \hat{u}(\xi, t-1) \end{aligned} \quad (15)$$

Combine Eqs. (12)-(15), the transverse vibrations numerical solution of the vertical rope is obtained.

4. Numerical simulation

In mine hoisting system, transport distance is larger and continuously being increased. Now in China, most of new mines with shaft depths over 1000 m have recently been considered. [8] Typical parameters of the multi-cable winding hoister used in simulation are showed in Table 1.

Table 1. Simulation parameters of the vertical rope values.

Simulation	Values
Vertical rope length $l(t)$ (m)	1100
Hosting time t (s)	90
Rope effective steel area A (m ²)	3×10^{-3}
Conveyance mass M (kg)	5×10^4
Rope effective Young's Modulus E (N/m ²)	1.53×10^{11}
Rope linear density ρ (kg/m)	8.02
Maximum hosting velocity v (m/s)	15
Maximum hosting acceleration a (m/s ²)	0.75

The head sheave eccentricity is used as the upper external disturbance excitation, and it can be represented as a trigonometric function $e(t) = 0.001 \times \sin(\pi t)$. The upward movement profiles of the multi-rope winding hoister are shown in Figure 2- 4. The transverse vibration displacement at the midpoint of the vertical rope is shown in Figure 5.

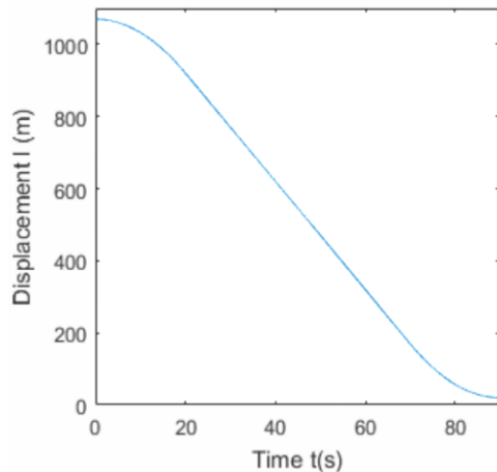


Figure 2. The movement displacement of multi-rope winding hoister.

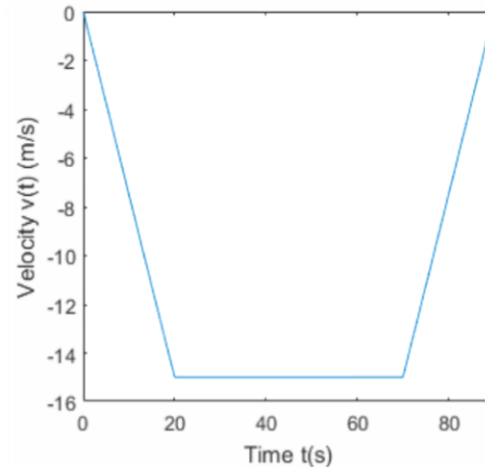


Figure 3. The movement velocity of multi-rope winding hoister.

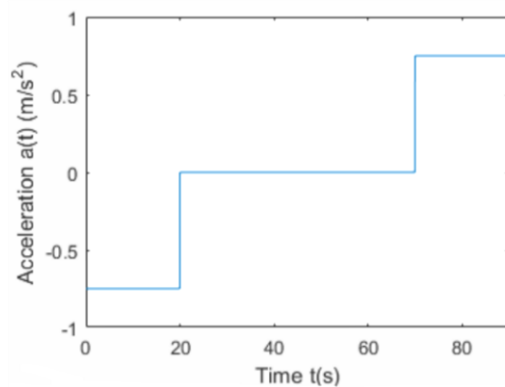


Figure 4. The movement acceleration of multi-rope winding hoister.

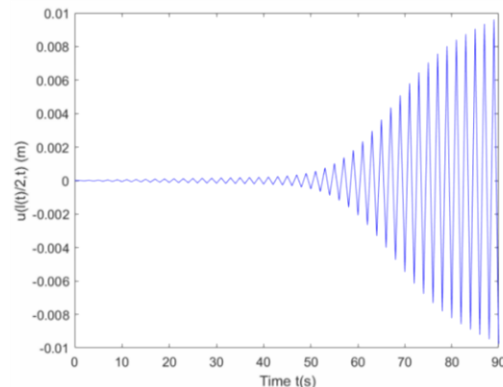


Figure 5. The transverse vibration displacement at the midpoint of the vertical rope

According to Figure 5, the length of the vertical rope gradually becomes shorter, and the transverse vibration displacement gradually increases in the upward movement stage of hoister. In the acceleration phase, the transverse vibration displacement of the vertical rope is small; in the uniform velocity phase, the transverse vibration displacement of the vertical rope sharp increase; in the deceleration phase, the transverse vibration displacement of the vertical rope increases slowly. The sharp increase in transverse vibration displacement of the vertical rope will lead to the dramatic increase in transverse vibration energy, which may ultimately lead to dynamic instability. The simulation results are compared with [4] and the conclusions are consistent, which verifies the validity of the method.

5. Conclusions

In this paper, based on the generalized Hamilton's principle, the partial differential equation for the transverse vibration of the vertical rope was established in the mine hoisting system. By using the Finite Difference Method, the continuous spatial and temporal variables of the partial differential equation are discretized. Through numerical simulation, the transverse vibrations at the upward movement of the multi-rope winding hoister vertical rope are obtained. The analysis of the transverse vibration characteristics of the multi-rope winding hoisting vertical rope studies provides a theoretical basis for the transverse vibration of the vertical rope, and it has certain reference value for the further study about the transverse vibration of vertical rope.

Acknowledgments

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References

- [1] Wu J, Kou Z M, Liang M, and Wu G X. Theoretical model and experimental verification of the coupling longitudinal-transverse vibration of rope for friction hoisting system 2015 *Journal of China University of Mining & Technology*. **44**(5) 1006-1013.
- [2] Wu J, Kou Z M, Liang M, and Wu G X. Analysis and experiment of rope transverse vibration for multi-rope friction hoisting system 2015 *J. Huazhong Univ. of Sci. & Tech. (Natural Science Edition)* **43**(6) 12-21.
- [3] Wang C M, Zhang S L, Wang J S, and Du B. Longitudinal vibration analysis of steel wire rope in ultra-deep lifting test bench 2017 *Machinery Design & Manufacture*. **6** 30-33.
- [4] Bao J H, Zhang P, and Zhu C M. Modeling and analysis of rope transverse vibration for flexible hoisting systems with time-varying length 2012 *Journal of Shanghai Jiaotong University*. **46**(3) 341-345.
- [5] Wang W and Qian J. Finite difference method for simulating transverse forced-vibration of elevator suspended system 2014 *Journal of Vibration Engineering*. **27**(3) 180-185.
- [6] Gao W X, Wu J, and Zhang Q S. Longitudinal vibration characteristics of multi-rope friction hoisting system by the finite difference method 2017 *China Mining Magazine*. **26**(6) 157-160.
- [7] Wang J, Pi Y J, Hu Y M, and Gong X S. Modeling and dynamic behavior analysis of a coupled multi-cable double drum winding hoister with flexible guides 2017 *Mechanism and Machine Theory*. **108** (2017) 191-208.
- [8] Cao G H, Cai X, Wang N, Peng W H, and Li J S. Dynamic response of parallel hoisting system under drive deviation between ropes with time varying length 2017 *Shock and Vibration*. 1-10.