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FEEDBACK AND FORWARD-LOOKING MODELS OF THE DEMAND FOR MONEY:

THE CASE OF THE UNITED KINGDOM 1963-86.

VITO ANTONIO MUSCATELLI

A dissertation submitted February 1989

at the University of Glasgow,

Department of Political Economy,

for the degree of

Doctor of Philosophy

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## SUMMARY

In the last two decades there have been numerous attempts to model the demand for money in a single-equation context. This study critically examines some of the approaches which have been suggested by applied economists. One bewildering aspect of the current literature on modelling monetary aggregates is the wild diversity of the approaches which have been advocated. Using recent developments in applied the econometrics literature, we categorise and examine the various ways in which one may construct a model which appropriately characterises the long-run and the short-run demand for money.

In the first part of the study (Chapters 2 and 3) we assess the various methods by which one may construct what have often been called 'feedback-only' single-equation models of the demand for money. We show that different approaches can sometimes lead the applied economist to surprisingly different conclusions about the properties of the demand for money.

In the second part of the study (Chapters 4 and 5) we focus on a different approach to modelling the demand for money, which treats the decision by economic agents to hold money as a forward-looking one. This approach is based on a view of the money stock as a 'buffer asset' in economic agents' portfolios. We demonstrate how existing single-equation models may be extended in order to render them applicable in the context of a multi-asset model with saving flows. We also consider which

interpretation of the demand for money (i.e. 'feedback' or 'forward-looking') is likely to be valid in the case of the M1 aggregate in the United Kingdom.

Finally, in Chapter 6 we examine the way in which the presence of forward-looking behaviour in the money market may affect the role and usefulness of monetary aggregates as information variables or indicators of monetary policy.

## CHAPTER 1

### CHAPTER 1: INTRODUCTION

#### SECTION ONE: INTRODUCTION AND THEORETICAL FOUNDATIONS OF THE DEMAND FOR MONEY

##### 1.1.1 Introduction

The main aim of this thesis is to examine some of the recent developments in the theoretical and empirical modelling of the demand for money. This area has generated a vast literature, and inevitably any attempt to focus closely on a small number of aspects of recent research work is bound to raise a number of objections. There are a number of ways in which the subject of modelling the demand for money may be approached, and the purpose of this chapter is to provide the reader with a guide to some of the main issues in the literature, and to set the scene for the rest of the thesis.

We begin, in this section, with an introduction to the theoretical foundations of the demand for money. This will serve as the basis for our empirical modelling exercises in the following chapters. In section two we examine the reasons why the demand for money matters for economic policy-making, and survey some of the recent empirical evidence on the demand for money in the UK. In section three we set the scene for the rest of the thesis with a discussion of the issues which will be confronted.

##### 1.1.2 Theoretical Foundations of the Demand for Money

Ever since the publication of Keynes' (1936) General Theory

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there has been a tendency to distinguish between the different motives for holding money. Although most theorists regard this distinction as unhelpful, in the sense that the three motives for holding money (transactions, precautionary, and speculative) identified by Keynes do not lead to three independent decisions on how much money to hold on the part of economic agents, there has nevertheless been a tendency by post-war economists to develop separate theories of the demand for money each of which tended to focus on one motive for holding money<sup>1</sup>.

In general, most empirical demand for money models are based on theoretical models which recognise the role of money in transactions, and as a financial asset (and hence a substitute for alternative financial assets). These theories suggest to us that the demand for money will be related to a scale variable (usually real income or wealth) and the yield on alternative assets. We now turn to a brief exposition of these post-war theories, beginning with those focusing on the transactions motive.

The link between the number of transactions and the demand for money was already present in pre-Keynesian economics, as is evident both the Fisher and Cambridge views of the Quantity Theory (see Fisher, 1911, Desai, 1981). Although the Fisher approach is not generally considered as being formulated in terms of 'desired money holdings' in the sense that the money stock  $M$  is generally considered to be exogenous, whilst the Cambridge

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approach is seen as providing a link between desired money stock holdings and income (where the latter is a proxy for the number of transactions), both of these theories provide the original basis for post-war theories of the transactions demand for money<sup>2</sup> by linking  $M$  to  $Y$ <sup>3</sup>.

This link between money and real income provided by the quantity theorists was founded mainly on utility theory as applied to the choice between money and goods. No detailed analysis was provided of the costs of holding money for transactions purposes, or of the role of uncertainty in the time profile of disbursements until the post-Keynesian period. In the 1950s there were a few notable studies which sought to build an inventory-theoretic model of the demand for money. The best known of these are the studies by Baumol (1952) and Tobin (1956). These works are well known and basically involve a trade-off on the part of the economic agent between costs of the brokerage and time-inconvenience type in holding all wealth in alternative assets (usually bonds) and only obtaining money as the need arises, and the higher yield obtained by holding wealth in bonds. On the basis of these considerations, the individual's desired demand for money may be shown to be positively dependent on the volume of transactions (proxied by the level of real income), and negatively dependent upon the yield on the alternative asset(s).

Further refinements have been carried out to the theory of



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the demand for money for transactions purposes following this seminal work. Sprenkle (1969, 1972) considers the use of multiple holdings by large firms with multiple accounts. Akerlof and Milbourne (1980) consider the use of a target-threshold model, where an upper limit is set by economic agents to money holdings, with adjustment to portfolios carried out when a limit is hit<sup>4</sup>. In the absence of uncertainty in the agent's transactions, this model leads to different results regarding the effect of  $Y$  on  $M$  compared to models of the Baumol (1952)-type. In fact, Akerlof and Milbourne (1980) show that the short-run income-elasticity of the demand for money may even be negative.

Uncertainty may be introduced in transactions-demand models, although some observers may argue that this makes them more akin to Keynes' description of the precautionary demand for money. Once again, the link between money holdings and the frequency of transactions and the opportunity cost of holding money (the interest rate on bonds) may be established (see Miller and Orr, 1966, 1968). Overall, the main thrust of these models is to confirm the link stressed by both versions of the quantity theory, but by providing 'sounder microfoundations'. Other variables enter the demand for money in some models. For instance the expected rate of inflation in Santomero, 1974, (analogously with the work of Cagan (1956) and Friedman (1956) which we examine below), the brokerage cost (see Baumol, 1952), and the variance of transactions (see Miller and Orr, 1966, 1968).

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However, as we shall see below, of these only the inflation rate enters most empirical studies because of measurement problems in the case of variables which are specific to the economic agent and for which aggregate measures are difficult to define<sup>5</sup>.

Turning next to the asset motive for holding money, this once again has its origins in the pre-Keynesian era. There is plenty of evidence that the Cambridge version of the quantity theory recognised the links between  $k$  and the interest rate, although these were rather underemphasised, in the sense that  $k$  was considered to be stable in most 'normal economic conditions'. The link between the money stock and the interest rate was also established in Wicksell's 'indirect mechanism' of money supply effects on the real economy (see Wicksell 1898).

However, once again it fell upon Keynesian writers, in the light of Keynes' account of the theory of liquidity preference in the 'General Theory' to develop the microfoundations of the asset motive for holding money. This involved the application of Von Neumann-Morgenstern utility theory (see Von Neumann and Morgenstern, 1947). The most cited contributions in this regard are those of Tobin (1958), Markowitz (1959), and Sharpe (1964). These models assume that the investor has the choice of investing his wealth either in a number of assets with an uncertain return over the holding period, or in an asset with a safe (usually but not necessarily zero) return, money. The motive for holding diversified portfolios in an uncertain environment is easy to see

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if we consider a risk-averse investor. The risk-averse investor will always tend to hold part of his wealth in a riskless asset, even though it bears no yield, as a type of insurance on his total portfolio. It can be shown (see for instance Bhattacharyya, 1979, Courakis, 1988, Stevenson et al., 1988) that if the investor has an exponential utility function of the type:

$$U(R) = a_0 - a_1 \exp(-a_2 R) \quad (1.1)$$

where  $R$  is the return on the portfolio, and he may choose between a safe asset with zero return (money,  $M$ ), and some alternative asset (say bonds,  $B$ ) whose return,  $r_b$ , is random and normally distributed:

$$r_b \sim N(\mu_b, \sigma_b^2) \quad (1.2)$$

then the following demand functions may be derived for the two assets:

$$M^d/P = -r_b/a_2\sigma_b^2 + W/P \quad (1.3)$$

$$B^d/P = r_b/a_2\sigma_b^2 \quad (1.4)$$

where the usual 'adding-up restrictions' of demand systems hold, and where  $P$  is the price level and  $W$  denotes nominal wealth (i.e.  $W \equiv M + B$ ).

This simple example can clearly be extended to a context where more than two assets are held, in which case the variances and covariances of each risky asset's return enters all the demand functions. It is typically assumed that the variances and covariances of asset returns do not change, and hence most empirical studies assume that asset demands depend on observed

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asset returns and not their variances. This requires us to assume that the distribution of these returns does not change (which may be a rather heroic assumption in an economy where conditions in financial markets are constantly evolving and new financial instruments are constantly being introduced). Furthermore the neat separability of the means and variances is in part due to the choice of an exponential utility function. An alternative choice (say, a quadratic utility function would not enable us to focus solely on the mean returns, see Bhattacharyya, 1979)<sup>6</sup>. Furthermore, it is apparent that this type of exercise need not be restricted to a closed economy, and that we may also use it to illustrate the portfolio choice of economic agents in an open economy context (see Branson and Henderson, 1985). Lastly, it is possible to combine the asset and transactions motives for holding money into a single decision by altering the simple exponential utility function so that money yields some utility per se, because of its usefulness in conducting transactions. Such a model is found, inter alia in Branson and Henedrson, 1985 and Muscatelli et al., 1989).

Before we conclude this section, it is worthwhile to mention the monetarist perspective on the motives for holding money, as exemplified in Friedman's 'Restatement' of the Quantity Theory (see Friedman, 1956). There are some superficial similarities between Friedman's approach, and that of the 'portfolio theorists' described above which led to the development of the

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microfoundations of the 'asset motive' in the Keynesian era. In line with traditional utility theory, the demand for money is partly a function of an individual's total level of resources (i.e. his wealth, which is proxied by permanent income), and the returns on alternative (real and financial) assets. However, it should be stressed that Friedman did not employ Von Neumann-Morgenstern utility theory, as his approach had far more elements in common with the traditional quantity theory. Furthermore, there seems to be more emphasis in Friedman's work on the 'uniqueness' of money, and hence the absence of close substitutes. Furthermore, substitution between a greater number of assets is considered, even to the point of including real assets.

Nevertheless, the original 'Restatement' should not be strictly interpreted as a guide for empirical work. In later work, Friedman's views on the empirical modelling of the demand for money become more apparent (see Friedman, 1959). In his 1959 study it becomes clear that, though Friedman regards the substitutability of money with a number of alternative assets as important, he 'drops' the interest rate effect seeing it as empirically insignificant, concentrating instead on the effect of permanent income on the demand for money. Thus, although Friedman sees the demand for money as a stable function of a number of variables, the choice as to which variables are ultimately retained as empirically significant is regarded as an empirical

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matter, making it difficult to distinguish between the 'modern quantity theory' and alternative approaches, such as the portfolio approach. Ultimately, the variables contained in the 'Restatement' are mostly those suggested by theoretical models of the transactions, precautionary, and asset motives for holding money.

We have seen so far that there are a number of variables which are seen as important in affecting the demand for money. The main ones are the return on alternative assets, the own return on money (if this is relevant to the definition of money under scrutiny), the variances of asset yields, the expected exchange rate (in cases where we are considering foreign assets in the portfolio choice of economic agents), the volume of transactions (usually proxied by a current real income variable), the variance of transactions, the costs of switching between money and alternative assets, and wealth. Given the limitations of economic time series data, in that only a limited number of observations are available, and there are no direct measures available for some of the variables which are likely to influence the demand for money, most empirical analysts have had to limit the number of variables which enter their models. We have already cited Friedman (1959) as a classic example of simplification (although some would argue that he was guilty of oversimplification in this case), and in section two we shall see how other studies have also been limited in their scope.

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Ultimately, it is arguable that empirical verification should provide economists with a clue as to whether their 'synthesis' and simplification of theoretical studies leads to acceptable results.

Before turning to an outline of the scope of this thesis in section three, we should complete our brief survey of the existing literature on the demand for money by providing an account of the importance of the stability of the demand for money for monetary policy, and of some empirical studies which will have some bearing on later chapters.

### SECTION TWO: THE STABILITY OF THE DEMAND FOR MONEY AND EXISTING EMPIRICAL EVIDENCE

#### 1.2.1. The Stability of Money Demand and Monetary Policy

The importance of the stability of the demand for money lies in the implications which it has for the transmission of monetary policy. It should be apparent that if the demand for money is unstable, then it becomes difficult for the monetary authorities to actively use monetary policy in order to achieve their final policy objectives. This point was recognised from the very outset of the Keynesian-monetarist debate, with Friedman's (1959) assertion that the transmission mechanism of monetary policy may be more stable than the Keynesian multiplier. This became the basis for the famous Friedman-Meiselman (1963) 'test' of the money and autonomous expenditure multipliers. Although their attempt was subsequently discredited (see Desai. 1981 for an

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outline of this debate), this episode illustrates the importance placed by monetarists on the stability of the demand for money, in the same way as the quantity theorists placed great weight on the stability of 'velocity' or the 'Cambridge k'.

The importance of a stable demand for money also raised its head once monetary policy was used more actively, and economists examined the advantages of different variables as intermediate targets of monetary policy. Poole's famous (1970) analysis illustrates how the relative stability of the IS and LM curves may lead one to choose either the interest rate or the money stock as an intermediate objective. Although Poole's analysis is extremely simple in that it ignores the possibility of instrumental uncertainty, and that it is particular to the IS-LM model, it also exemplifies the importance of the stability of the demand for money as a pre-requisite for the adoption of monetary targets (the basic plank of any monetarist strategy). Even once we move to more complex models to analyse the advantages of different intermediate targets and indicators (see B.Friedman, 1975, and Chapter 6 below), Poole's basic result can still be shown to hold: the stability of the demand for money is an important pre-requisite for any strategy advocating monetary targets.

### 1.2.2. Empirical Evidence on the Demand for Money

The main difficulty encountered in comparing empirical studies on the demand for money is the different definitions of



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the money stock which have come under scrutiny. This makes comparisons across countries also difficult, because of the differences in definitions of, say, M1 and M3 in different countries. Furthermore, the empirical economist also has a varied choice when it comes to deciding on variables such as the returns on alternative assets, real income, and the price level. In the case of the 'own return' on a particular money stock definition, this is made difficult by the fact that such definitions typically encompass 'narrower' definitions of the money stock, and thus there cannot be a uniform 'rate of return' on any given money stock definition. In the case of alternative assets, the close correlation between different interest rates makes it impossible to enter the returns of all the possible alternatives to money. Typically the researcher focuses on one, or at most two alternative rates of return. In the case of real income and the price level there are also various definitions available such as total final expenditure (and its corresponding implicit deflator) and real personal disposable income (and its implicit deflator).

Most empirical studies prior to the 1970s tended to concentrate on a small number of explanatory variables. When such studies were carried out on annual data, the typical form of the models estimated was:

$$m_t = a_0 + a_1p + a_2y - a_3R \quad (1.5)$$

where  $m$  denotes the money stock,  $p$  the price level,  $y$  denotes

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real income, and  $R$  the interest rate on alternative assets, and where lower cases indicate that the variable is in logs. When quarterly data was used, some lags in the regressors were required to improve the fit of the model (see Feige, 1967 for the US, and Laidler and Parkin, 1970 for the UK). Typically, the following simple partial adjustment mechanism was adopted, where (1.5) is now seen as representing desired money holdings,  $m^*$ :

$$m_t^* = \alpha_0 + \alpha_1 p + \alpha_2 y - \alpha_3 R \quad (1.6)$$

and actual money holdings adjust towards  $m^*$  according to the following simple adjustment mechanism:

$$m_t - m_{t-1} = \lambda(m_t^* - m_{t-1}) \quad 0 < \lambda < 1 \quad (1.7)$$

Generally, most studies both in the UK and the US found 'well determined' and stable demand for money functions (see for instance Meltzer 1963, Brunner and Meltzer 1964, Laidler 1966 for the US, and Barrat and Walters 1966, Laidler 1971 for the UK). It was recognised that the use of the money stock to proxy the demand for money might not have been totally correct, but the assumption of instantaneous equilibrium in the money market did not seem invalid at the time. Furthermore, attempts to estimate the supply and the demand for money simultaneously (see Teigen, 1964) did not significantly affect the results obtained.

Unfortunately this early optimism regarding the stability of the demand for money did not last through the 1970s. There seemed to be a 'breakdown' in the stability of the demand for money in a number of OECD countries during the 1970s (see OECD, 1979). In

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the UK, Artis and Lewis (1976) illustrated this 'breakdown' for different specifications of the demand for money, showing that the period 1971-73 seemed particularly problematic. In the US, Goldfeld (1976) also showed that a demand for money function estimated over the 1952-73 period did not provide accurate forecasts for the 1970s.

To some extent we can account for this 'breakdown' on the grounds that the earlier specifications of the demand for money had a dynamic structure which was inadequate. It should be apparent that applying the simple partial adjustment process (Equation 1.7) to a static demand for money function (Equation 1.6), produces an estimating equation with a very simple dynamic structure (i.e. with no lagged explanatory variables, and with a single lagged dependent variable):

$$m_t = \alpha_0\lambda + \alpha_1\lambda p_t + \alpha_2\lambda y_t - \alpha_3\lambda r_t + (1 - \lambda)m_{t-1} \quad (1.8)$$

There is no reason to believe that, if there are some costs in adjusting money balances, the partial adjustment process provides an optimal response on the part of economic agents. As we shall see in Chapters 4 and 5, this is a rather simple mechanism, and was merely adopted on an ad hoc basis because of its simplicity. The dynamic adjustment of the demand for money in the 1970s seemed to be characterised by more complex dynamics, and to some extent this provides an 'econometric' response to the apparent 'breakdown' of the demand for money.

Several alternative models were proposed (see for instance

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Coghlan, 1978). One in particular by Hacche (1974) received a lot of attention, because it set out to model the demand for M3 in the UK by defining the variables in terms of first differences. Furthermore, this study found significant serial correlation in the resulting model, which was corrected for using conventional GLS-type methods.

The problems with this methodology are many. For one thing, estimating a model in first differences implies that there is no long-run relationship between the variables in the model (see Chapter 2). Furthermore, one is replacing an ad hoc dynamic structure (given by the partial adjustment mechanism) with another. Hendry and Mizon (1978) pointed out that in the absence of strong theoretical priors regarding the dynamic structure of a model, the data should play a larger part in constructing the short-run dynamic properties of an empirical model. In fact, in this context, the serial correlation found by Hacche might have been caused by dynamic misspecification. In such a case, 'correcting' for serial correlation is not the optimal strategy: one is required to respecify the model.

The alternative strategy proposed by Hendry and Mizon (1978) and Hendry (1979), which has become known as the 'general-to-specific' models selection procedure (see Chapters 2 and 3 for a more in-depth outline), is to begin with a sufficiently general dynamic specification where a number of lags of the dependent and explanatory variables are introduced in the model. Typically such

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a general model will suffer from 'overfitting', and many of the regressors will be insignificant. The dynamic structure of the model is then simplified by imposing statistically acceptable restrictions on the general model to obtain a more parsimonious model which adequately describes the data, and passes a number of data coherency criteria (see Hendry, 1983) or 'diagnostic tests'.

Using this technique, Hendry and Mizon find the following 'preferred' specification for the demand for M3 in the UK over the data period used in their study:

$$\begin{aligned} \Delta(m - p)_t = & 1.60 + 0.21\Delta y_t + 0.81\Delta R_t + 0.26\Delta(m - p)_{t-1} \\ & (0.65) \quad (0.09) \quad (0.31) \quad (0.12) \\ & -0.40\Delta p_t - 0.23(m - p - y)_{t-1} - 0.61R_{t-4} + 0.14y_{t-4} \\ & (0.15) \quad (0.05) \quad (0.21) \quad (0.04) \end{aligned}$$

$$R^2 = 0.69 \quad \hat{\sigma} = 0.91\% \quad \chi^2(12) = 6.4 \quad (1.9)$$

where the numbers in brackets denote estimated standard errors,  $\hat{\sigma}$  denotes the standard error of the equation, and the  $\chi^2(12)$  statistic is the Box-Pierce portmanteau statistic for 12 lags.

Thus, one possible response to the demand for money 'breakdown' is that of re-estimating these equations allowing for more general dynamic specifications to check if an improved model can be found. Another possible response is to recognise that simple models such as Equations (1.5), (1.8) and (1.9) only contain a small number of variables which may affect the demand for money. Variables which have been unimportant in the past may suddenly become relevant in the money demand decision if there is

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a change in the economic environment (e.g. if there is a change from a regime of fixed to one of flexible exchange rates). Furthermore, innovation in the financial system may lead to important changes in the behaviour of the supply-side of the money market, and there is no reason to believe that models such as (1.8) or (1.9) should remain invariant to such changes.

There have been a number of studies which have sought to assess the importance of 'additional' variables in the demand for money. For instance there have been attempts to allow for an 'own rate' variable as it has become common practice to pay interest on components of the money stock which previously yielded no return (see Goodhart, 1984 for UK studies, and Baba et al., 1987 for a recent US study). Alternatively, Judd and Scadding (1982) have tried to proxy the effect of financial innovation for the US. Another 'conspicuous absentee' from many demand for money studies is wealth. Usually this is due to the lack of reliable data on wealth, but one cannot deny the importance of this variable if one is to believe theoretical asset demand models of the demand for money. Some researchers have tried to construct their own data sets, and there is some evidence that the introduction of a wealth variable in demand for money studies may be a fruitful avenue of research (see Grice and Bennett, 1984). The role of the foreign sector on the demand for money is difficult to assess, as the effect of foreign interest rates on different money stock definitions will clearly be different. In

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the case of M1 and M3 we may not see such variables as relevant; in the former because the demand for M1 may be primarily for transactions purposes, and in the latter because a large component of M3 is denominated in foreign currency anyway. In the case of fM3 McKenzie and Thomas (1984) have experimented (with some success) with a foreign interest rate variable. Lastly, when discussing portfolio models we noted that the variances of asset returns were usually assumed constant in simple demand for money models. In a recent study Baba et al (1987) have shown how a stable demand function for M1 may be found for the US if one explicitly allows for the variance of asset returns.

The above brief survey of recent developments in the literature shows that there are a number of promising avenues of research in modelling the demand for money. In the next section we set the scene for this thesis by discussing the aggregates and the types of model which we shall consider in the following chapters.

### SECTION THREE: SETTING THE SCENE: THE AIMS OF THIS STUDY

As we have seen from the previous sections, there are a number of possible routes which an empirical economist may take in constructing a model of the demand for money. However, in this thesis we shall consider and develop only a small subset of the theoretical and empirical issues raised by the recent literature on the demand for money.

There are two broad types of models which we shall consider

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in the chapters which follow. First, we shall examine in more depth the problem of econometric specification in feedback-only models (i.e. standard, backward-looking dynamic models). The recent econometric literature has suggested a number of competing methodologies which may be used in constructing a satisfactory empirical time-series model. In this thesis we apply all of these techniques to the same UK demand for money data set, thereby providing the first direct comparison of these techniques in the existing literature. The purpose of Chapter 2 is that of providing a coherent taxonomy of the different methodologies which have appeared independently in the econometric theory literature. The purpose here is to highlight the differences and similarities between the different model selection procedures which have been advocated. In addition in Chapter 2 we provide some exhaustive tests of the cointegratedness of the variables used in our demand for money models. Again, this is probably the first comprehensive exercise of its kind on UK demand for money data. In Chapter 3 we then apply these various model selection procedures to the same UK demand for money data set to provide a comparison of these different methods.

The second type of model considered here is the so-called 'forward-looking' or rational expectations model of the demand for money, which was developed as part of the recent literature on money as a 'buffer asset'. Although different versions of the forward-looking model have already been widely tested in the



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existing literature, our aim here is to extend the multiperiod quadratic costs-of-adjustment approach to include saving behaviour. Existing models seem to be couched exclusively in a constant-wealth two-asset framework, which is rather restrictive. The purpose of Chapter 4 is to provide a guide to the existing literature, and to extend existing models to incorporate saving behaviour. We estimate a demand for money model which includes saving, and we also provide some pointers to the extension of the simple model to a multi-asset case, which can be shown to be a 'forward-looking' version of the famous Brainard-Tobin (1968) interdependent asset adjustment model.

These two different approaches to the demand for money, namely the rational expectations/'forward-looking' approach and the dynamic feedback-only adjustment approach can be shown to be observationally equivalent, and in Chapter 5 we discuss this well-known proposition. In this chapter we also propose and implement a new way of comparing the two approaches to gauge whether a forward-looking or a feedback-only interpretation is more appropriate. We also place our methods of assessing the competing models in the context of the recent Hendry (1988) critique of forward-looking models. Chapter 5 therefore provides a link between the models estimated in Chapters 2 and 3, and the models estimated in Chapter 4.

Our last innovation is that of illustrating the consequences of introducing a forward-looking model of the demand for money

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into a simple macroeconomic model in order to evaluate the suitability of monetary targets in these circumstances. This is done in Chapter 6. Despite the numerous attempts to empirically assess the validity of the 'forward-looking' model of the demand for money, this appears to be the only attempt to date to incorporate such 'forward-looking' behaviour into the literature on monetary targets. We also suggest possible extensions to our simple model to treat the dynamic adjustment of money markets as the result of a dynamic game interaction between the private sector and the monetary authorities.

Finally, in Chapter 7 we provide some concluding remarks on the work presented in this thesis, and provide some pointers to possible future work on the modelling of the demand for money.

At this stage we should also point out the issues regarding the modelling of the demand for money which we do not explore in this thesis. The main restriction on the range of our empirical research is provided by the limited set of explanatory variables considered in the demand for money models which we estimate. In general, we limit ourselves to the use of a price variable, a real income variable, and of a single return on an alternative asset to capture the substitutability of money with other assets in the portfolio. There are some good practical reasons for the strict limit imposed on the number of explanatory variables used. The main reason, of course is the usual one of data limitations. Many of the empirical exercises carried out in this thesis seek

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to compare different approaches to empirical modelling. As a result, the use of a large number of explanatory variables would have rendered the individual models rather cumbersome. At the end of the day, the main aim was not that of constructing the 'best' model of the demand for money in absolute terms, but to compare different models which had been constructed using the same data set.

There are other good reasons for restricting the range of explanatory variables to  $P$ ,  $Y$ , and  $R$  alone which become apparent once we consider the aggregates used in the estimations presented in this thesis. In Chapters 2 and 3 we shall use UK data for the M3 definition of money to estimate our demand for money function. This is because this has provided a more troublesome aggregate to model in terms of 'stability' than narrower measures such as M1, where the problems of 'breakdown' have been less acute in the UK (see for instance Hendry, 1985). Thus, M3 provides a more demanding testbed for our comparison of different model selection procedures in Chapters 2 and 3. On the other hand, in Chapters 4 and 5 we use M1 data to construct our models because existing empirical tests of the 'forward-looking' approach have been carried out in this context, and thus by using M1 we can ensure the comparability of the results presented here with the results obtained in earlier studies.

The exclusive use of these two aggregates does tend to help our aim of reducing the number of explanatory variables in

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the demand for money equations which we estimate. Turning first to M1, it appears from existing studies (see for example Hendry 1979, 1985) that the exclusive use of P, Y, and R has been sufficient to construct models which successfully characterise the behaviour of the M1 series over the 1963-1984 period, which is the sample period used in our study. Whilst in recent US studies additional variables such as a measure of the 'own rate' on M1 and the variability of returns have been needed to achieve stability (see Baba *et al.*, 1987), such measures have as yet not been required for the UK. Furthermore, one could successfully argue that a wealth variable is less important in the case of M1 as this aggregate may mainly reflect the transactions motive for holding money. Also, the substitutability between M1 and foreign assets may also be rather limited. This is not to say that other variables may not become important at some future date. In fact, there is some evidence (see Cuthbertson and Taylor, 1988) that additional variables such as wealth may help explain the behaviour of M1 in the UK in the post-1984 period.

In the case of M3, on the other hand, one could argue that some additional variables, such as wealth or an 'own rate' may have a certain role to play. Again, the main justification for the use of a simple model structure is the need to retain a simple framework for our comparison of different model selection procedures. In this sense, our models in Chapters 2 and 3 are very much in the spirit of the original study by Hendry and Mizon

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(1978), which based a case study of a modelling strategy on an extremely simple model of M3 in the UK. It is also in the spirit of the strategy followed by Friedman (1959) where a complex demand for money function is reduced to one which 'adequately captures' the behaviour of the money stock.

In the case of M3 it is arguable that the inclusion of an 'own rate' on money balances would be appropriate. In Chapter 2 we do experiment with such a variable, but it turns out that our results are more satisfactory than when it is excluded. This is probably due to the high degree of collinearity between different interest rates. For the same reason, we decided not to experiment with returns on foreign assets. Though M3 contains balances denominated in foreign assets, it is arguable that the yield on foreign bonds may have a significant influence on this aggregate. On the other hand, given the difficulty in incorporating a role for exchange rate expectations in the demand for an aggregate which to some extent is made up of a foreign-currency element, we decided to eschew the open-economy aspects of the demand for M3. Similarly, on the grounds of simplicity we thought it best not to include variables such as the variance of asset returns. In the case of wealth, the arguments for its exclusion are twofold: first, it is difficult to obtain reliable data for 'wealth', and most aggregates constructed usually only incorporate some components of financial wealth. It is arguable that the construction of a wealth series for the purpose of modelling a

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particular economic relationship is in itself a major achievement (see for example Grice and Bennett, 1984, and Molana, 1987). Secondly, it is arguable that some measure of permanent income to a large extent overcomes the need for a wealth variable, again in the spirit of Friedman (1959). In Chapter 2 we shall see that most of the demand for money models which we estimate for M3 amongst other things examine the long-run relationship between real income and the demand for money. By concentrating on the low-frequency component of real income (and indeed on the expected value of future income in Chapters 4 and 5) we may be seen as allowing a role for the 'permanent' component of real income as opposed to its short-run fluctuations, thus partly circumventing the need for a wealth variable.

Ultimately, therefore, we have preferred to use demand for money models with simple structures as the basis for our empirical experiments. To a large extent this was dictated by the main aim of our study, which is that of comparing different methodologies in modelling the demand for money, not that of testing the importance of particular variables proposed by individual demand for money theories, or that of proposing a 'new theory' of the demand for money. Clearly, if a researcher is interested in testing the importance of, say, the open economy aspects of the demand for money, he would sacrifice other aspects of the model building procedure in order to test the importance of the role of different foreign interest rates and/or the

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expected exchange rate in the demand for money. Such matters are of secondary importance in our study.

In this chapter we have argued that the simple modelling procedures used in the 1960s and early 1970s to estimate demand for money equations (e.g. 'static' and 'partial adjustment' models) yielded empirical models which were not properly designed in a statistical sense. As we pointed out above, in the next chapters we shall try to evaluate, compare and develop some of the alternative modelling strategies which have been proposed to replace these 'simple' models.

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### Footnotes to Chapter 1

(1) To some extent this was due to the desire to develop 'sound microfoundations' for some of the more 'general' theories like the various motives for holding money proposed by Keynes, and the Cambridge approach to the Quantity Theory. One exception to this pattern is provided by Friedman's (1956) 'Restatement' of the Quantity Theory, which is rather more general in nature.

(2) It must be recognised, however, that the 'Cambridge approach' was not purely a transactions model of the demand for money, and that the value of 'Cambridge  $k$ ' was seen as depending on the payments system, interest rates, etc. (see Desai, 1981, Cuthbertson, 1985a). The main point is that ' $k$ ' was seen as reasonably stable and predictable, although Marshall (1925) did point out the dangers of assuming that factors such as price expectations were always stable when exceptional events such as wars, etc. occurred.

(3) The Fisher approach usually linked  $M$  to  $T$ , the quantity of transactions. It is usually assumed that there is a strict proportionality between  $T$  and the level of real income,  $Y$ .

(4) See Chapter 4 for a more detailed outline of a target-threshold model with uncertainty.

(5) See Chapter 4 for some interesting aggregation problems which arise in target-threshold models.

(6) As we shall see below, Baba et al. (1987) have departed from this practice in constructing an empirical model of the demand



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for M1 in the US, as variances of asset returns appear explicitly in their estimations.

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### CHAPTER 2: GENERAL-TO-SPECIFIC AND OTHER TYPES OF FEEDBACK-ONLY MODELS

In this chapter we examine in more detail the general-to-specific approach to modelling the demand for money. In particular, we are interested in the underlying time series properties of the economic data employed in modelling the demand for money. We begin in Section one by surveying the traditional general-to-specific approach developed in the late 1970s. We then turn in Section two to examine its links with the more recent literature on cointegration which has provided a statistical foundation for the type of empirical specification obtained by applying the general-to-specific method. In this context, we present some evidence in Section three on the time series properties of data for the money stock, price level, real income, and the interest rate for the UK. Finally, in section properties of demand for money equations obtained using the general-to-specific method.

#### SECTION ONE: THE GENERAL TO SPECIFIC APPROACH TO EMPIRICAL MODELLING

##### 2.1.1 A General Statistical Framework

A general framework for the analysis and construction of empirical models has been provided by Hendry and Richard (1983), and serves as a useful background for the description of the general-to-specific approach. This framework has already found its way into econometric texts (see, for example Spanos, 1986),

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and hence we shall only provide a brief outline of its salient points.

We may begin by postulating that a sample of data on  $K$  economic variables  $\{x_t\}$  represents a set of realizations (observations) of the  $K$  variables at time  $t$  from the joint density function  $D(X_T | X_0, \theta)$ , where  $X_T = \{x_T, x_{T-1}, \dots, x_1\}$ , and where  $X_0$  represents a matrix of initial conditions, and  $\theta$  an identifiable, finite (but unknown) vector of parameters.

In economics, we are generally concerned with sequential realizations of the elements of  $X_T$ , so that at any point in time,  $t=s$ , we may use the general information set  $X_{s-1}$  to predict this period's realization of the vector  $x_s$ . Provided that the data generation process is not subject to alterations in its structure (due, say, to fundamental changes in the underlying behaviour of economic agents), the density function  $D(\cdot)$  may clearly be factorised as follows:

$$D(X_T | X_0, \theta) = \prod_{t=1}^T D(x_t | X_{t-1}, \theta) \quad (2.1)$$

To narrow down further the approach outlined so far, we should recognize that, given the non-experimental nature of economic data, it typically consists of small size samples on a large number of variables. It therefore makes no sense to focus on the entire data generation process, but on a reduced reparameterization of it which 'adequately' characterizes the data. Thus, typically we consider some reparameterization of  $\theta$ ,

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$(\Phi_1, \Phi_2)$  which enables us to factorise the joint density function further to obtain  $D(w_t | W_{t-1}, \Phi_2)$ , where the vector  $w_t$  contains the economic variables of interest, which are a subset of  $x_t$ . For this reparameterization to characterise the data adequately, this requires that past values of the omitted variables should have no influence on the variables chosen. That is, the omitted variables should not 'Granger-cause'  $w_t$  (see Granger, 1969, 1980). Furthermore, if we are interested in modelling a single series of  $w_t$ , then provided the remaining variables in  $w_t$  are weakly exogenous for the chosen parameterisation one may conduct inference conditionally on the weakly exogenous variables without loss of relevant information (see Engle et al., 1983, Hendry and Richard, 1983).

The framework sketched above does not in itself provide precise guidance to the applied economist on the detail of constructing an econometric model of a behavioural relationship. However, its value lies more in highlighting the individual steps which an applied economist implicitly makes in postulating a single equation model for an economic series of interest. The above steps stress that if in constructing a model the economist does not begin with a reduced reparameterization of the data generation process which is appropriate, it inevitably stores up trouble for the future. Thus, for instance, a postulated model which at the outset does not properly characterise the available data, say because of serial correlation in the residuals (see

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Hendry 1979, 1983, Hendry and Richard 1983), cannot necessarily be improved by 'correcting' for serial correlation using conventional methods (e.g. Cochrane-Orcutt, Durbin 2-stage, see Johnston, 1984). This is because after the transformation the model residuals may still not be white-noise innovations relative to the available information set<sup>1</sup>. It makes far more sense to begin with a sufficiently general model which adequately characterises the data generation process from which the data sample has been obtained.

Thus, the general-to-specific approach takes as its point of departure the need to begin with a model (or, in the terminology of Spanos, 1986, a statistical generating mechanism)<sup>2</sup> which at least provides a crude approximation to the data generation process which gave rise to the data. In general, the type of context in which the above concepts are applied is that of the linear regression model. In the case of a single equation one should clearly isolate the appropriate dependent variable for the model in line with the concept of weak exogeneity mentioned above<sup>3</sup>.

Given that economic theory usually only provides guidance with regard to the relevant variables making up a static (usually long-run) behavioural relationship, and is to a large extent silent on the short-run dynamic structure of the relationship between the variables of interest<sup>4</sup>, it makes sense to design an empirical model which adequately characterises the data without

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being in conflict with the theoretical priors dictated by economic theory. As Pagan (1987) points out, theory and data continually interplay in this methodology.

Thus, in the case of the demand for money one may begin by formulating a general dynamic model of the type:

$$m_t = \text{constant} + \sum_{i=1}^n \alpha_i m_{t-i} + \sum_{i=0}^q \beta_i p_{t-i} + \sum_{i=0}^s \tau_i y_{t-i} + \sum_{i=0}^k \delta_i R_{t-i} \quad (2.2)$$

where  $R$  denotes the interest rate, and  $m$ ,  $p$ , and  $y$  denote respectively the natural logarithms of the money stock, the price level and real income. The choice of maximum lag for each variable is usually set with reference to the type of data available. Thus, with quarterly data it is usually found that the appropriate lag formulation is  $n = q = s = k = 5$  (provided sufficient observations are available). It is usually found that such a dynamic specification is sufficiently general so as to ensure that the model residuals are white noise innovations by construction.

It is important to recognise the main differences and similarities between the type of model illustrated in (2.2) and the type of model which results from the application of a simple partial adjustment or adaptive expectations mechanism to a static model of the demand for money (see Chapter 1).

First, as we pointed out there, it is generally found that static models, or its simple dynamic variants, do not perform adequately in the sense that they do not adequately characterise

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the data generation process. An ad hoc dynamic adjustment mechanism is imposed on the model, and this is generally at odds with the data. If we agree that the demand for money is a theoretical concept which is not directly measurable, it is natural that we should not expect a static demand for money to characterise the data generation process adequately. In the absence of a precise theory of how economic agents adjust their money balances when their portfolios are out of long-run equilibrium, our best move is to use the data to discover a good approximation to the adjustment process. The simple partial adjustment and adaptive expectations models are too restrictive for this purpose.

Second, if we consider a static steady state equilibrium<sup>5</sup>, where all change ceases in all the variables, the model in (2.2) will lead to a conventional static demand for money in logarithms (except for the interest rate)<sup>6</sup>:

$$m_t = k + \pi_1 p + \pi_2 Y + \pi_3 R \quad (2.3)$$

where any restrictions on the signs and sizes of the  $\pi_i$  are testable using the appropriate asymptotic standard errors.

Taking (2.2) as a point of departure, it should be apparent that given the large number of regressors, the 'general' model is overparameterised, and that most of the regressors will appear insignificant. Furthermore, there will be a high degree of multicollinearity amongst the regressors used, making statistixal inference difficult. We now turn to examine the exact procedure

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proposed by the general-to-specific methodology to obtain a more satisfactory empirical model compared to (2.2), and the criteria used to judge the suitability of such a model.

### 2.1.2 From General to Specific: Simplification and Reparameterization.

Having made the case for an initial general model which adequately characterises the data (which will have a very unrestricted dynamic structure), there are four remaining steps necessary to obtain a satisfactory empirical model. (Most of what follows in this subsection reviews the work of Hendry (1979, 1983, 1985, 1986)<sup>7</sup>). First, the model should be reparameterized so as to obtain regressors which are nearly orthogonal, and so as to obtain a model with sensible short- and long-run properties. Second, the dynamic structure of the model should be simplified to the simplest version which appears to be data acceptable. Third, any restrictions imposed on the model should be found to be consistent with the data. Finally, the final (or 'best') version of the model proposed should satisfy a number of criteria of model adequacy.

Taking the last point first, it is usually thought that the resulting model should have an adequate goodness of fit, and white noise residuals. The main point here is that the 'specific' model should also adequately characterise the data at the researcher's disposal. Furthermore, as pointed out above, one would require a model to be consistent with one's prior



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theoretical views of the behavioural relationship under scrutiny. Furthermore, one would hope that a model is also useful for forecasting purposes. In other words, one would hope that the parameterization selected will be constant (i.e. the modeller does not marginalise with respect to important variables whose relationships with retained variables change over time). Lastly, the model should be able to 'encompass' rival models, i.e. it should be able to explain the results of rival models (see Hendry and Richard, 1982, Mizon, 1984). These comparisons may take various forms, as outlined in Mizon (1984)<sup>8</sup>.

To assess a model's ability to satisfy these criteria a number of statistical tests have been devised. A satisfactory model should be able to pass a whole battery of diagnostic tests, and in our estimations in this and other chapters we shall be using (and briefly outlining) a number of these tests. For a reasonably full account of the statistical testing procedures required, see inter alia Judge et al. (1982, 1985), Harvey (1981), Engle (1984), Spanos (1986).

The nature of the first and second steps in the general-to-specific procedure (reparameterization and simplification) is not easy to describe in the abstract and, as Pagan (1987) points out, these two steps are usually blurred into one giant inscrutable step in most of Hendry's work. (In particular see Hendry 1983, 1985, 1986. For a more detailed approach, see Hendry and Mizon 1978, Hendry and Ericsson 1983). Thus, for instance, the general

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equation (2.1) may in itself be reparameterised at the outset to obtain regressors which are more or less orthogonal, but the researcher may also choose to begin with a simplification search, deleting a number of regressors before attempting an appropriate reparameterisation. There do not seem to be any fixed rules about the procedure to be followed, and to a large extent the researcher's 'intuition' and 'artistic flair' plays a great part on the route to be followed (see Hendry, 1986). It is also not particularly helpful to detail every step of the simplification search in the case of any given model, because the steps followed will inevitably vary from estimation to estimation<sup>9</sup>. For example, Pagan (1987) points out that Hendry (1986) only reports the transition from a general equation with 31 regressors to a more specific one with only 14 regressors by stating that:

"These equations....were then transformed to a more interpretable parameterization and redundant functions were deleted; the resulting parsimonious models were tested against the initial unrestricted forms by the overall F-test..." (Hendry 1986, p.29)

This reliance on the F-test in restricting such a large number of regressors may lead to the erroneous exclusion of some regressors, and thus one possible disadvantage of this method would appear to be its haphazard nature<sup>10</sup>. Having said this, in other occasions a more structured simplification search may be less susceptible to this criticism (e.g. the COMFAC procedure advocated in Hendry and Mizon, 1978). Furthermore, any model

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reported in Hendry (1986) still satisfies the criteria for model adequacy listed above.

Overall, it would be fair to say that to some extent the general-to-specific method leaves the process of simplification search sufficiently unconstrained so as to potentially allow the researcher a number of different paths to (possibly different) final specifications of an empirical model. As explained above, this is due to a large extent to the large role played by the data in determining the dynamics of the model. The real question here is whether one should opt for a more structured search (see Hendry and Mizon, 1978, 1985), or whether one should, at the very least, report every step in the search for a 'specific' model (see McAleer et al. 1985, Pagan, 1987). In what follows, we do explicitly examine some of the effects of taking different initial routes from the general model at the outset of a simplification search (see Chapter 3). However, in our present work reporting every step undertaken on the way to the final model would have required the presentation of a large volume of material peripheral to the main issues under scrutiny, which would have been inappropriate given the main aims of the thesis. Furthermore, even when examining a single model, Pagan's criticism of Hendry may be somewhat exaggerated for two reasons: first, there is absolutely no guarantee that two researchers would ever agree on the precise route to take when engaging in a simplification search. Second, as pointed out above, the final

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model must satisfy a number of criteria for model adequacy.

We now complete our review of the general-to-specific methodology by examining in more detail the characteristic dynamic structure of the models obtained by David Hendry and his associates.

### 2.1.3. General-to-Specific and Error Correction

As pointed out in Chapter 1, the final equation proposed by Hendry and Mizon (1978) (see equation 1.9) involved a combination of terms in differences and levels of the variables involved. We saw that equation (1.9), in contrast to the model in differences proposed by Haache (1974) ensures that the model converges in steady state to a long-run static equilibrium which conforms to our theoretical priors about the demand for money. As we shall see in Section two, the debate between those who advocate estimating models in 'differences' and those who propose the use of data in 'levels' is connected with the subject of cointegration.

For the moment, however, we should briefly highlight one feature of the dynamic structure of the general-to-specific models. For instance, let us recall the 'final' model for the demand for M1 in the UK estimated by Hendry (1985) for the sample period 1963(i)-1982(iv):

$$\begin{aligned} \Delta(m - p)_t = & 0.37\Delta y_{t-1} - 0.58R_t - 0.80 \Delta p_t - 0.10(m - p - y)_{t-2} - \\ & (0.13) \quad (0.07) \quad (0.12) \quad (0.01) \\ & 0.28 \Delta(m - p)_{t-1} + 0.041 \\ & (0.07) \quad (0.005) \end{aligned} \quad (2.4)$$

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where the numbers in brackets denote standard errors. In steady state, where all change has ceased in  $m$ ,  $y$ , and  $R$ , and where inflation proceeds at a constant rate<sup>11</sup>, equation (2.4) suggests the following steady state demand for money (where upper case variables denote levels):

$$M = 1.5PY(1 + R)^{-5.6} (1 + \pi)^{-1.9} \quad (2.5)$$

where  $\pi$  denotes the annual inflation rate. Note that the model suggests a unit long-run elasticity of the demand for  $M1$  with respect to the price level and real income. The signs of the long-run coefficients on the interest and inflation rates have the correct signs.

The dynamic structure of (2.4) contains what is known as an 'error correction mechanism' (ECM). If we look at the term  $-0.10(m - p - y)_{t-2}$ , we see that the dynamic structure implies that economic agents will gradually adjust any short-run divergence between  $M$  and  $PY$  due, say, to a differential rate of growth in the short run between  $M$  and  $PY$ . Thus, terms of this type have become known as ECM's, and have been found to be appropriate in a number of empirical applications as well as the demand for  $M1$ . As we saw in chapter 1, in the case of money  $M3$ , Hendry and Mizon (1978) find such an error-correction term to be significant. Furthermore, such terms appear in wage-price models (see Sargan, 1964), models of house prices (see Ericsson and Hendry, 1985), and models of aggregate consumption (see Davidson *et al.*, 1978, Hendry and von Ungern-Sternberg, 1980).

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It should be noted that there is no inherent reason in why general models of the type illustrated in (2.2) should yield dynamic structures like (2.4) which embody an ECM. This just happens to be a reparameterization which is convenient for two reasons. Firstly, it leads to regressors which are nearly orthogonal to each other, thus removing the problems of multicollinearity present in (2.2). Secondly, as we have just pointed out, the ECM term has a natural interpretation in terms of a 'rational' response by economic agents to disequilibrium states<sup>12</sup>. In fact, as Salmon (1982) points out, the ECM may be interpreted in terms of the literature on the optimal control of dynamic systems. ECM's may be seen as examples of proportional-integral-derivative (PID) control rules (see Phillips, 1954, 1957), or optimal reaction functions derived from optimal control experiments of the LQC-type (Linear model, Quadratic cost function, and Gaussian disturbances). Clearly, the type of ECM which will be relevant will depend on whether the economic variables in question return in steady state to a static equilibrium, a constant growth path, or to a 'dynamic growth' (i.e. an increasing rate of growth) path. We should therefore note that the ECM implies a model of agent behaviour which is not necessarily 'backward-looking', as it is an example of an optimal control rule. In the simple case where in the (hypothetical) steady state economic variables return to a static equilibrium (i.e. all change ceases), models like (2.4) with an ECM have

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'sensible' long-run solutions from the economic point of view (i.e. equation 2.5)<sup>13</sup>.

We now turn to a discussion of the concept of cointegration which, as we shall see, is intimately related to the concept of ECMs, and the methodology of estimating stochastic difference equations.

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### SECTION TWO: COINTEGRATION AND GENERAL-TO-SPECIFIC

#### 2.2.1 The Concept of Cointegration

A stationary series may be broadly defined as a series whose mean and variance are time-invariant (see Escribano, 1987, Harvey, 1981b, for a more precise definition). In contrast, most economic series are non-stationary (i.e. do not satisfy these properties), and require differencing to induce stationarity. Thus consider, for example, a series which follows a random walk:

$$x_t = x_{t-1} + \varepsilon_t \quad (2.6)$$

where  $\varepsilon_t \sim \text{IN}(0, \sigma^2)$

then  $x_t$  is non-stationary, as an innovation has a permanent effect on the value of  $x_t$ , as  $x_t$  is the sum of all previous changes, (i.e. if  $x_0 = 0$ ,  $x_t = \sum_{i=0}^{t-1} \varepsilon_{t-i}$ ). This may be easily seen by noting that  $\text{var}(x_t) = t\sigma^2$ . The random walk tends to drift away from its initial value (though it does not exhibit a particular trend in doing so). We may induce stationarity in  $x_t$  by differencing. Thus,  $\Delta x_t = \varepsilon_t$ , which is white noise and clearly stationary. A series which requires to be differenced  $d$  times to induce stationarity is said to be integrated of order  $d$ , or  $I(d)$ . The random walk is therefore  $I(1)$ , and white noise is  $I(0)$ . Formally, if a series  $x_t$  is  $I(d)$  it has a univariate generating model (moving average representation) of the type:

$$(1 - L)^d (x_t - m) = A(L)\varepsilon_t \quad (2.7)$$

where  $\varepsilon_t$  is a zero-mean white noise process,  $m$  is a constant (the starting value of the series for  $d > 0$ ),  $L$  is the lag operator



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such that  $L^j y_t = y_{t-j}$ , and  $A(L)$  is a polynomial in the lag operator such that  $0 < |A(1)| < \infty$ .

In general, the degree of integration is considered to be an integer, though it may be possible to consider cases where this is not so by defining  $(1 - L)^d$  in terms of a power expansion in the lag operator (see Granger and Joyeux, 1981).

The above concepts relating to integrated series have been known to time series modellers for some time, but their importance to econometric modelling are best seen in the multivariate context, given that econometricians primarily have an interest in the relationship between the time series properties of two or more economic series (usually integrated of order greater than zero). This has recently led to the development of a literature on cointegration, which extends the above concepts to the multivariate context. This literature is mainly based on seminal work by Clive Granger (see Granger, 1983, 1986, Granger and Weiss, 1983, Engle and Granger, 1987).

It is useful to begin with a formal definition of cointegration. Consider a vector  $y_t$  of  $K$  time series, each integrated of order  $d$ . Then, the series are said to be cointegrated (CI( $d, b$ )) if there exists a vector of constants  $\alpha$  (with some of its elements non-zero) such that a linear combination of the elements of  $y_t$ ,  $\alpha'y_t$ , is integrated of degree  $(d-b)$ , where  $b > 0$ . The vector  $\alpha$  is then said to be the cointegrating vector.

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The significance of this concept is best seen with a simple example of a special case where we have two variables and  $d=b=1$ . Consider two economic time series,  $x_t$  and  $y_t$ , both  $I(1)$ . Then, if they are cointegrated, there will be a constant  $\alpha$  (the cointegrating parameter) such that:

$$z_t = y_t - \alpha x_t \quad z_t \sim I(0) \quad (2.8)$$

Essentially if two variables are cointegrated, they share some common features in their long-run behaviour. Any deviation from  $z_t = 0$  would be bounded, and hence  $z_t$  has been dubbed the 'equilibrium error' (see Engle and Granger, 1987). Thus, any economic theory which a priori links  $y$  and  $x$  through a linear relationship, i.e.  $y_t = \alpha x_t$ , will only make sense if the two economic series are cointegrated otherwise the notion of equilibrium has no relevance. In a sense, as Dolado and Jenkinson (1987) point out, cointegration is a statistical definition of equilibrium. In the case of the demand for money, a test of whether a long-run relationship exists between the money stock, prices, real income and the interest rate is whether these variables are cointegrated (see Hendry and Ericsson, 1983).

Before we move on to a discussion of methods of testing whether a set of variables is cointegrated and of estimating the cointegration vector, we have to examine, for sake of completeness, some results relating to the frequency domain properties of cointegrated variables.

### 2.2.2 The Frequency Domain and Cointegration

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Frequency domain analysis has an important place in business cycle theory (see Sargent, 1979), as it provides a way of assessing the contribution made by periodic components of different frequencies to the overall variance of a stochastic process<sup>14</sup>.

At the outset, it is worthwhile to remind ourselves of the definition of a power spectrum for a time series, which is a continuous function  $f(w)$  such that:

$$f(w) = (1/2\pi)\{\tau(0) + 2\sum_{t=1}^{\infty} \tau(t)\cos(wt)\} \quad (2.9)$$

where  $\tau(t)$  is the autocovariance at time  $t$  (for a lag  $t$ ) for the series. (More generally, the spectrum is the Fourier transform of the covariogram, but for a series of real numbers, it reduces to (2.9). One advantage of using the Fourier transform definition is that it shows that the spectrum is best defined over the range of frequencies  $(-\pi, \pi)$  as it is symmetric about  $w = 0$ , and hence 'repeats' itself over certain ranges). Note that, because theoretically all autocovariances for white noise will be zero, the spectrum will be flat at all frequencies  $w$ . White noise, as expected, does not exhibit any cyclical behaviour, and hence its spectrum does not show up any peaks at any frequencies.

Note that, given the definition of the spectrum, a time series integrated of order zero will have a spectrum such that  $0 < f(0) < \infty$ . This is because the integral sum of the spectrum

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over the range  $(-\pi, \pi)$  is equal to the variance of the series:

$$\int_{-\pi}^{\pi} f(w)dw = \tau(0) \quad (2.10)$$

and the variance for a stationary series is finite and time invariant. On the other hand, for series integrated of degree  $d$ , where  $d$  is equal or greater than one<sup>15</sup>, the spectrum has a shape proportional to  $(1 - \cos(w))^{-d}$  which is approximately equal to  $(w)^{-2d}$  at low frequencies (see Granger, 1983, and Engle and Granger, 1987). Thus, the larger the value of  $d$ , the greater the value of  $f(w)$  at small frequencies. Note also, that  $f(0) = \infty$  for these series, since the theoretical variance of non-stationary series will clearly be infinite (as the variance varies over time). The main thing to note is that for trending series low frequencies (overwhelmingly) dominate the spectrum. This phenomenon is clearly accentuated the higher the order of integration.

However, as we noted above it may be possible for a vector of time series to be cointegrated such that the order of integration of a linear combination of the series will be lower than that of any individual series. For the special case where  $d = b = 1$ , this would mean that a linear combination of  $I(1)$  series could produce a series which is  $I(0)$  and therefore stationary. This leads to some interesting frequency domain results highlighted by Granger (1983) and Granger and Weiss (1983).

If two series are cointegrated, the spectrum of the

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resulting 'equilibrium error' will be that of a stationary series, i.e.  $f(w)$  is finite at zero frequency. This therefore suggests that the spectra of the two cointegrated series must in some ways be related at low frequencies to allow the (theoretically) infinite values of  $f(0)$  for the two  $I(1)$  series to 'cancel out'. For two stationary series,  $y_t$  and  $x_t$ , the relationship between them at different frequencies is given by the cross-spectrum,  $f_{yx}(w)$ , which is defined as:

$$f_{yx}(w) = (1/2\pi) \sum_{t=-\infty}^{\infty} \tau_{yx}(t) e^{-iwt} \quad (2.11)$$

where  $\tau_{yx}(t)$  is the cross-covariance for a lag  $t$  between the two series. Thus, the cross-spectrum is defined as the Fourier transform of the cross-covariance function. However, as (2.11) defines a complex series, a more useful insight in the relationship between the two series is given by the measures of gain, phase, and coherence. Our main interest is in the coherence, which measures the strength of the relationship between the two series at different frequencies, and is defined as:

$$\text{Coh}(w) = (|f_{yx}(w)|)^2 / f_x(w) f_y(w) \quad (2.12)$$

For two cointegrated series Granger (1983) and Granger and Weiss (1983) show that  $\text{Coh}(0) = 1$ . This indicates that the very low frequency components of two cointegrated series must be perfectly correlated. We do not reproduce this proof here, but the intuition behind this result should be apparent. Given that the equilibrium error between two cointegrated variables is

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stationary, it is apparent that the zero-frequency components of the two series practically obey a linear constraint, so that any discrepancy between the two has finite variance. The above result suggests a test for cointegration which attempts to examine the properties of time series in the frequency domain (see Granger and Weiss, 1983). We shall return to this in the next section, when we attempt such an experiment. Meanwhile, however, we will briefly survey some of the main statistical tests used to establish whether a set of time series are cointegrated. These tests will then be used in our empirical results in Section three.

### 2.2.3 Testing for Cointegration

Given that cointegration appears to be a requirement for the existence of an equilibrium relationship between a set of economic time series, it is natural that a lot of attention has been dedicated recently to testing for cointegration. At the very least it seems that it is desirable to test for cointegration before estimating a dynamic model of the type outlined in section one. However, the set up is complicated, and cointegration tests are (not surprisingly) related to tests for unit roots in time series (see Engle and Granger, 1987, Dolado and Jenkinson, 1987). A number of tests have been proposed, and the following by no means represent an exhaustive list:

- (i) The Dickey-Fuller Test (see Fuller, 1976, Dickey and Fuller, 1979)

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(ii) The Augmented Dickey-Fuller Test (see Dickey and Fuller, 1981, Said and Dickey, 1984).

(iii) The Cointegrating Regression Durbin-Watson Test (see Sargan and Bhargava, 1983, Bhargava, 1984).

All three tests rely on first estimating a so-called 'cointegrating regression' between the two  $I(1)$  series using OLS:

$$y_t = k + \hat{\alpha}x_t + \hat{u}_t \quad (2.13)$$

where  $k$  is a constant, and  $\hat{\cdot}$  denotes an estimated value. The Dickey-Fuller test involves running a second regression of the type:

$$\hat{\Delta u}_t = -\hat{\beta}u_{t-1} + \varepsilon_t \quad (2.14)$$

The Dickey-Fuller (DF) test statistic is the  $t$ -statistic for  $\hat{\beta}$ , to be compared with critical values reported by Fuller (1976). Essentially the rationale for the test is the following: if  $y_t$  and  $x_t$  are cointegrated, this suggests that the equilibrium error should be white noise. Thus, the estimated residuals, which give us an estimate of the equilibrium error are tested for a unit root using (2.14). Rearranging (2.14):

$$\hat{u}_t = (1 - \hat{\beta})\hat{u}_{t-1} + \varepsilon_t \quad (2.15)$$

From (2.15) it is clear that if  $\hat{\beta}$  is significantly different from zero,  $\hat{u}_t$  will be stationary, and hence  $y$  and  $x$  are cointegrated.

The Augmented Dickey-Fuller (ADF) test is designed to cover those cases where  $\varepsilon_t$  is serially correlated (a case which is probably relevant to simple bivariate examples in economics).

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Given that this affects the estimated standard errors in (2.14), the suggestion here is to allow for a number of lags of  $\hat{\Delta u}_t$  to capture the serial correlation. Thus, the choice between the ADF and DF test statistics will vary from case to case. In general, though, Hallman (1987) recommends the use of the ADF test, in preference to the DF test given the fact that most economic time series will generate a regression equation which exhibits serial correlation.

The Cointegrating Regression Durbin-Watson statistic (CRDW) is simply the DW statistic from (2.13). The null hypothesis of no cointegration is rejected if the DW statistic is 'too large', where the critical values are given in Sargan and Bhargava (1983), Engle and Granger (1987) and Hall (1986). This test is simpler to apply as it does not need an auxiliary regression, and its rationale is again simple to see from (2.14) and (2.15). The DW statistic tests whether the residuals  $\hat{u}_t$  follow a stationary first order autoregressive process. If  $\beta$  is significantly large, then the autoregressive parameter  $(1 - \beta)$  will be small, and the  $\hat{u}$  are likely to be stationary.

There are clearly problems with such tests. In particular, they are not likely to be particularly useful in detecting cointegration when the autoregressive parameter in the residuals is very close to (but still less than) one. In such cases, the power of such tests may be quite low. Engle and Granger (1987) also show that the relative power of these three tests vary



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depending on whether there is serial correlation in the  $\varepsilon_t$  process. In general, they advocate the use of the CRDW as a useful benchmark test, subject to confirmation from a test such as the ADF test, to capture possible serial correlation in the  $\varepsilon_t$  term. In Section three we shall, where appropriate, make use of all three tests reported here.

Before that, however, we turn to the analysis of three related issues. First, how does one set about estimating the cointegrating vector? Second, what is the relationship between this cointegrating vector and the 'long-run' coefficients derived from an estimated dynamic equation (see equations 2.4 and 2.5). Third, what insights does cointegration theory have to offer into the estimation of dynamic models, and particularly those models which embody an ECM?

### 2.2.4 Estimating the Cointegration Vector and ECM's

Equation (2.13) has already offered us a possible answer to the first question. Does an OLS regression of one economic variable on the other provide us with a consistent estimate of the cointegrating vector? Engle and Granger (1987) suggests that this simple OLS equation does in fact give a very good estimate of the  $\alpha$  vector. This is because only one linear combination of the cointegrated series will produce a set of residuals which have finite variance. In fact, as Stock (1984) points out, the estimate of  $\alpha$  produced by the cointegrating regression are 'superconsistent', in that it converges to the true value of the

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cointegrating parameters at a faster rate than in OLS regressions between series that are  $I(0)$  (see also Engle, 1987, Engle and Granger, 1987, Dolado and Jenkinson, 1987, Hallman, 1987). Furthermore, Engle and Granger suggest that one could equally well regress  $x_t$  on  $y_t$  and obtain a consistent estimate of  $1/\alpha$ . This is because the product of the two estimates is the  $R^2$  statistic, and if we have a value of  $R^2$  close to unity, the two estimates of  $\alpha$  will be very close.

However, this rosy picture may be over-optimistic, as Hendry (1986b) points out. First of all, in small samples, there may still be large biases in estimated values of  $\alpha$ . Second, as Banerjee *et al.* (1986) show in a Monte Carlo study, the bias may be large and may vary inversely with the size of the  $R^2$  statistic in the cointegrating regression. Thus, it would appear that cointegrating regressions where  $R^2$  is well below unity may not be particularly useful<sup>16</sup>.

The next question we have to ask is why we are interested in the cointegrating parameter. As we pointed out in subsection 2.2.1, cointegration relates to the equilibrium relationship between a set of time series. Thus, when dealing with a set of economic variables, the cointegrating vector represents the 'long-run' equilibrium relationship between these variables. Thus, it would appear that the cointegrating regression may offer us a method of directly parameterising a long-run equilibrium relationship like, say, the long-run demand for money, without

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necessarily resorting to estimating complex dynamic equations like (2.4) above. The extent to which this is a valid approach is clearly dependent on some of the considerations outlined in the previous paragraph, and we return to a fuller discussion of this issue further on in this section.

The final question which we have to confront at this juncture is whether the cointegration approach can shed any more light on the subject of dynamic specification and models with ECM's which were outlined in section one. It may be shown (see Granger, 1983, Engle and Granger, 1987) that if we have two time series  $y_t$  and  $x_t$ , both  $I(1)$ , which are cointegrated, then there exists an error correction representation for the multivariate time series system.

The argument goes as follows: first, we know that a multivariate vector process  $(y_t, x_t)$  may be given a moving average, (Wold) representation (see for instance Harvey, 1981b):

$$\begin{aligned}(1 - L)y_t &= C_{11}(L)\varepsilon_{1t} + C_{12}(L)\varepsilon_{2t} \\ (1 - L)x_t &= C_{21}(L)\varepsilon_{1t} + C_{22}(L)\varepsilon_{2t}\end{aligned}\tag{2.16}$$

where the  $C_{ij}(L)$  are polynomials in the lag operator, where we assume that  $C_{ii}(0) = 1$ , and  $C_{ij} = 0$ , for  $i, j = 1, 2$ . Suppose further that the two  $\varepsilon_{it}$  are zero-mean white noise series, where  $\varepsilon_1$  and  $\varepsilon_2$  are only contemporaneously correlated (see Harvey, 1981b). Next, Granger (1983) and Engle and Granger (1987) show that it is possible to invert (2.16) to obtain an error correction representation for  $(1 - L)x_t$  and  $(1 - L)y_t$  of the

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type:

$$\begin{aligned}(1 - L)A_1(L)y_t &= (1 - L)B_1(L)x_t - \beta_1(y_{t-1} - Ax_{t-1}) + D(L)\varepsilon_{1t} \\ (1 - L)A_2(L)x_t &= (1 - L)B_2(L)y_t - \beta_2(y_{t-1} - Ax_{t-1}) + D(L)\varepsilon_{2t}\end{aligned}\tag{2.17}$$

where the  $A_i(L)$ ,  $B_i(L)$ , and  $D(L)$  are lag polynomials, and  $\beta > 0$ .

Several points follow from (2.17). First, this result confirms some of the points outlined in section one. That is, it shows that if an equilibrium relationship exists between a number of time series then an 'error correction' representation provides us with a correct characterisation of the time series behaviour of these series. Note, in fact, that the structure of (2.17) closely resembles that of (2.4). The difference terms capture the short-term dynamics of the model, whilst the ECM 'pins down' the relationship between the time series in the long run.

Second, equation (2.17) implies that a reparameterization of an autoregressive distributed lag model at the outset may provide an alternative method to model a dynamic relation. We deal with this point in detail in the next subsection.

Third, equation (2.17) gives us an insight into the relationship between ECM's and 'Granger-causality' in multivariate models. In the case where one of the  $\beta_i$  term is zero, causality will run one way<sup>17</sup>. For example, if  $\beta_1 \neq 0$ , and  $\beta_2 = 0$ , then the low frequency component of the disturbance  $\varepsilon_{2t}$  will drive both  $x_t$ , and  $y_t$ . A corollary of this is that if two variables are cointegrated, Granger-causality must run at least

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one way, as one variable may be used to predict the other in the long-run (see Granger, 1986). It also follows that it is impossible for two variables which are determined in efficient markets to be cointegrated, as one cannot help predict the other if markets are efficient (see Granger and Escribano, 1986).

Fourth, the connection between cointegration and ECM's is closely related to the debate on whether one should estimate demand for money functions in levels or differences, a debate on which we already touched in Chapter 1 and section one of this chapter (see Hacche, 1974, Hendry and Mizon, 1978, Williams, 1978). Hendry and Mizon (1978) pointed out that demand for money equations estimated in differences, like the one presented by Hacche (1974), did not have a static steady state equilibrium, and hence were not consistent with conventional demand for money theory. Williams (1978) argued that the rationale behind Hacche's method lay in the necessity to ensure that all variables are stationary before undertaking any estimations, and quoted Granger and Newbold's (1974) illustration of 'spurious' regressions between non-stationary variables as an example of the dangers of estimating in levels when the series used are non-stationary. However, cointegration theory has shown us that only including differences (i.e. estimating a vector autoregressive (VAR) system in the differences) and excluding levels (i.e. the ECM terms in (2.17)) involves a serious misspecification if the series are cointegrated. Note that the literature on cointegration does not

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invalidate the Granger and Newbold 'spurious regression' result: it remains true that the standard error estimates in the cointegration equations are highly misleading (though the estimate of  $\alpha$  is consistent), and that regressions may still be spurious for cases where the variables are not cointegrated (i.e. if the CRDW is 'too low').

Lastly, Granger and Weiss (1983) and Engle and Granger (1987) suggest that the significance of the error-correction terms in a multivariate VAR model of the differenced series may in itself be used as a test of whether some of the variables are cointegrated. This test, however, is more complex to execute than those described in subsection 2.2.3, and their low power does not make them preferable to the latter.

To conclude this subsection we will show that the result derived by Granger (1983) and Engle and Granger (1987) illustrating the link between long-run relationships between variables and ECMs which was obtained in (2.17) using the concept of cointegration may also be approached from an econometric angle.

Let us begin by assuming that an autoregressive distributed lag model adequately characterises the behaviour between two time series  $Y_t$  and  $X_t$ . That is, our initial model is:

$$Y_t = k + \sum_{i=1}^5 \alpha_i Y_{t-i} + \sum_{i=0}^5 \beta_i X_{t-i} \quad (2.18)$$

where  $k$  is a constant. In what follows we set  $k = 0$  to simplify the notation; this restriction does not affect the nature of the

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result. Note that (2.18) is merely a two-variable version of the demand for money model in (2.2). Again, our results generalise to the  $n$ -variable and  $k$ -lag case. Let us note at the outset that in a static steady state equilibrium (i.e. once all change in  $Y$  and  $X$  ceases, and  $Y = Y_0$  and  $X = X_0$ , say) then from (2.18) the two variables will be related as follows:

$$Y_0 = (\sum_{i=0}^5 \beta_i) / (1 - \sum_{i=1}^5 \alpha_i) X_0 \quad (2.19)$$

which easily generalises to the following expression for the  $k$ -lag case:

$$Y_0 = (\sum_{i=0}^k \beta_i) / (1 - \sum_{i=1}^k \alpha_i) X_0 \quad (2.19')$$

Thus,  $(\sum_{i=0}^k \beta_i) / (1 - \sum_{i=1}^k \alpha_i)$  represents the long-run response of  $Y$  to  $X$ , and for a demand for money equation like (2.2) in logarithms it would represent the long-run elasticity of the demand for money with respect to its determinants (the semi-elasticity in the case of the interest rate).

We may now rearrange (2.18) by first subtracting  $Y_{t-1}$  from both sides to yield:

$$\Delta Y_t = (\alpha_1 - 1)Y_{t-1} + \sum_{i=2}^5 \alpha_i Y_{t-i} + \sum_{i=0}^5 \beta_i X_{t-i} \quad (2.20)$$

By adding the terms  $(\sum_{i=0}^5 \beta_i)X_{t-1}$  and  $(1 - \sum_{i=1}^5 \alpha_i)Y_{t-1}$  from both sides of (2.20) and rearranging we obtain:

$$\begin{aligned} \Delta Y_t = & \sum_{i=3}^5 \alpha_i Y_{t-i} + \sum_{i=3}^5 \beta_i X_{t-i} - \alpha_2 \Delta Y_{t-1} + \beta_0 \Delta X_t + \beta_2 \Delta X_{t-1} \\ & - (\sum_{i=3}^5 \alpha_i) Y_{t-1} - (\sum_{i=3}^5 \beta_i) X_{t-1} - \\ & (1 - \sum_{i=1}^5 \alpha_i)(Y_{t-1} - \{(\sum_{i=0}^5 \beta_i) / (1 - \sum_{i=1}^5 \alpha_i)\} X_{t-1}) \end{aligned} \quad (2.21)$$

Note that (2.21) already embodies an error correction term (the

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last term in the equation). We reach our final equation by adding the following terms to both sides of (2.21):

$$(\sum_{i=j+1}^5 \alpha_i)Y_{t-j}, (\sum_{i=j+1}^5 \beta_i)X_{t-j} \text{ for } j = 2, 3, 4$$

and rearranging. This finally yields:

$$\begin{aligned} \Delta Y_t = & \beta_0 \Delta X_t - (\sum_{i=2}^5 \beta_i) \Delta X_{t-1} - (\sum_{i=3}^5 \beta_i) \Delta X_{t-2} - (\sum_{i=4}^5 \beta_i) \Delta X_{t-3} \\ & - \beta_5 \Delta X_{t-4} - (\sum_{i=2}^5 \alpha_i) \Delta Y_{t-1} - (\sum_{i=3}^5 \alpha_i) \Delta Y_{t-2} \\ & - (\sum_{i=4}^5 \alpha_i) \Delta Y_{t-3} - \alpha_5 \Delta Y_{t-4} \\ & - (1 - \sum_{i=1}^5 \alpha_i)(Y_{t-1} - \{(\sum_{i=0}^5 \beta_i)/(1 - \sum_{i=1}^5 \alpha_i)\}X_{t-1}) \end{aligned} \quad (2.21)$$

Note that (2.21) has the same dynamic structure as (2.17), and hence a general ADL model may be re-expressed in an error-correction form with first differences. As we shall see in the next sub-section, this is by no means the only way to reparameterise an ADL model, and alternative forms may be derived which also embody an ECM (see for instance Bewley, 1979)<sup>18</sup>. The importance of (2.21) is that it shows that, provided (2.18) represents an adequate characterisation of the data generation process (i.e. the residuals are white noise innovations by construction), then the model will have an error-correction form. It also confirms that the cointegration parameter,  $\beta$ , in (2.17) will capture the long run properties of the system. Note also that the error-correction term will only be insignificant in the case where  $(1 - \sum_{i=1}^5 \alpha_i) = 0$ , in which case the model is correctly specified only in terms of differences. Thus, only when (2.18) is unstable will the ECM (and cointegration parameter) not



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appear in the equation. This confirms the link between cointegration and the notion of long-run equilibrium between a vector of economic time series.

As we pointed out above, the importance of cointegration to economics is twofold: firstly, it explains the success of dynamic models which embody ECMs in modelling economic time series, as the latter are often integrated of order greater than or equal to one. It also contributes to the 'regression in levels' versus 'regression in differences' debate, relating these to a statistical notion of the long-run relationship between a vector of economic time series. Secondly, and more importantly, it also opens the way for alternative approaches to dynamic modelling, as it appears that it is possible to obtain direct estimates of long-run elasticities when faced with parameterizations of behavioural equations which are dynamic. It is this second aspect of cointegration which accounts for a huge literature on this subject, and it is to this that we now turn.

### 2.2.5 Dynamic Specification, Cointegration, and the Estimation of Transformed Models.

At this stage it is appropriate to re-examine some of the questions involving dynamic specification and the general-to-specific models selection procedure outlined in Section one in the light of the cointegration results presented in Section two. As we have seen, the general-to-specific procedure involves an a priori indeterminate mix of data-acceptable simplification and

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reparameterization (if possible into a model with an ECM). The search procedure followed is not necessarily very structured and leaves the researcher some freedom in choosing the preferred specification route from the 'general' to the 'specific'.

Following on from their research on cointegrated variables, Engle and Granger (1987) have suggested an alternative 'two-stage' approach to dynamic specification. This involves the initial estimation of a cointegration equation like (2.13) to get initial values of the cointegrating vector  $\alpha$ . These values for the cointegrating parameters are then used to construct an error correction term of the form  $(y_t - \alpha'x_t)$ , which is then used to estimate a first difference model with an ECM term of the type illustrated in (2.17)<sup>19</sup>. A general-to-specific simplification could then be carried out on the short-run dynamics of the model within the bounds of the structure of (2.17). There are two advantages to this procedure: first, one obtains direct (and consistent) estimates of the long-run properties of the system which may then be imposed on the model at the outset. In the conventional 'general-to-specific' approach, we may solve for the long-run elasticities in the final model, and the estimated values will clearly vary between intermediate steps in the specification search. Second, some degree of structure is imposed on the specification search, with an initial reparameterization (transformation) of the general ADL model into a model in first differences with an ECM, in contrast with the rather unstructured

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mixture of simplification and reparameterization of the traditional approach (see Pagan, 1987). The disadvantage of this procedure derives from the fact that, as pointed out in subsection 2.2.3, the estimates of  $\alpha$  will be biased, and the 'superconsistency' property may be irrelevant in small samples. As we noted above, Hendry (1986b) and Banerjee *et al* (1986) argue that the value of the two-stage procedure to dynamic specification may be conditional on whether the value of  $R^2$  is too low (because of possible effects on the bias of  $\alpha$ )<sup>20</sup>, and on the low power of the cointegration tests available (for some evidence on this, see Jenkinson (1986b), Banerjee *et al.* (1987a, 1987b). Having said this, the two-stage estimation procedure has already found applications in the UK in tests on neoclassical theories of labour demand and on aggregate wage data (see Jenkinson (1986) and Hall (1986) respectively).

Once we recognise (see equations 2.18-2.21) that the two-stage estimator proposed by Engle and Granger involves a reparameterization (transformation) of the ADL model (albeit one in which one of the terms is obtained from a first-stage estimation), it is natural to consider other transformations of regression models which may give us further insights into the properties of dynamic models. Bewley (1979) proposed a transformation of the ADL model to yield direct estimates of the long-run elasticities of the model. More recently, Wickens and Breusch (1987) have shown that Bewley's result may be extended to

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yield a whole variety of transformations, each of which may be useful in its own individual application. We now turn to a brief examination of these reparameterizations, following Wickens and Breusch (1987).

Let us begin by considering the model given by (2.18):

$$Y_t = k + \sum_{i=1}^5 \alpha_i Y_{t-i} + \sum_{i=0}^5 \beta_i X_{t-i} \quad (2.18)$$

This model may be transformed by subtracting  $(\sum_{i=1}^5 \alpha_i)Y_t$  from both sides and re-arranging the resulting expression to obtain:

$$Y_t = -(1 / 1 - \sum_{i=1}^5 \alpha_i) \sum_{i=1}^5 \alpha_i \Delta_i Y_t - (1 / 1 - \sum_{i=1}^5 \alpha_i) \sum_{i=1}^5 \beta_i \Delta_i X_t + (1 / 1 - \sum_{i=1}^5 \alpha_i) (\sum_{i=0}^5 \beta_i) X_t \quad (2.22)$$

Breusch and Wickens also point out that the difference terms in  $X_t$  and  $Y_t$  on the right-hand-side of (2.22) may be rearranged and combined linearly in a number of ways, without altering the parameter on  $X_t$ . They also note that formulations like (2.22) differ in several respects with the type of ECM reparameterization found in, for example, Hendry *et al.*, which has the structure:

$$\Delta Y_t = -\sum_{i=1}^4 (1 - \sum_{j=1}^5 \alpha_j) \Delta Y_{t-i} + \sum_{i=0}^4 (1 - \sum_{j=0}^5 \beta_j) \Delta X_{t-i} - (1 - \sum_{i=1}^5 \alpha_i) (Y_{t-5} - X_{t-5}) \quad (2.23)$$

Equation (2.23) is very similar in structure to (2.21), but the ECM term is lagged by the amount of the maximum lag of the initial ADL, and it is implicitly assumed that  $\sum_{i=1}^5 \alpha_i + \sum_{i=0}^5 \beta_i = 1$ . This last restriction would allow us to enter an ECM with a unit restriction on the X and Y variables in (2.21). Note that this will only be valid in the case of demand for money

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models (in logs) when we are dealing with variables with respect to which the demand for money is unit elastic.

An examination of transformations of the ADL model of the type proposed in (2.21)-(2.23) leads us to the following conclusions regarding their usefulness in the estimation of dynamic models. Firstly, equations (2.21) and (2.22) provide direct point estimates of the long-run multipliers of the model (elasticities in the case of a log-linear demand for money model), and an estimate of the corresponding standard errors. In contrast, the cointegrating equation suggested by Engle and Granger (1987) gives consistent estimates of the long-run multipliers, but not of their standard errors, as we pointed out in the previous sub-section. Equation (2.23) imposes a long-run multiplier of unity, and hence may be seen as a restricted version of (2.21). One problem which arises in estimating (2.22) but not (2.21) is that OLS will not produce a consistent estimator because the vector of regressors is now asymptotically correlated with the error term, because of their dependence on  $Y$  (see Bewley, 1979, Wickens and Breusch, 1987). Thus, we need to employ an instrumental variable (IV) estimator with the vector of regressors before the transformation used as instruments. This will yield the same point estimates for the long-run multiplier which would have been obtained by estimating the general ADL model by OLS, and solving for the static steady state (for a proof of this result, see Breusch and Wickens, 1987). Equation

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(2.21) does not encounter this problem, but at the cost of transforming the dependent variable as well. As we shall see in Chapter 3, this becomes of some significance in comparing different approaches.

Secondly, the advantage of transforming the model to one of (2.21) or (2.22) to estimate the long-run multipliers instead of adopting the Engle-Granger two-stage procedure derives from the problem of dynamic misspecification. As we pointed out in the last subsection, the estimates of the cointegrating vector, though super-consistent, were inevitably biased due to dynamic misspecification. It was suggested above, following Banerjee et al., that in small samples the bias could well be large. However, Wickens and Breusch (1987) show that for the case where the cointegrated variables are trend-stationary<sup>21</sup> ignoring the short-run dynamics may not cause large biases in the estimates of the long-run multiplier. Their conclusions, unlike those of Banerjee et al. (1986), derive from a comparison of the OLS estimator of the cointegrating equation and the IV estimator of a transformed equation like (2.22) which specifies the dynamics of the model. They find that the IV estimator of the long-run multiplier is asymptotically less efficient than the OLS one.<sup>22</sup> A further consideration is that generally equations such as (2.22) will be overspecify the short-run dynamics, as they are reparameterizations of overparameterized ADL models. This may again point in the direction of estimating a cointegrating

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equation, depending on the aims of the researcher. Of course, if the researcher is interested not only in estimating the long-run multipliers but also in finding a good forecasting equation, transformed regression models may prove to offer a more useful approach to the Granger-Engle procedure, as they collapse the two-stage process into one.

Thirdly, as we shall see in Section four of this chapter, certain transformations may be particularly useful in cases where we consider steady-state growth paths.

Fourthly, the transformation we put forward in equation (2.21) appears to be a more convenient one than that advanced by Wickens and Breusch in (2.22) because it enables us to use OLS methods to estimate it. On the other hand, (2.22) has the advantage of having coefficients on the difference terms which are proportional to the original distributed lag coefficients of the ADL equation. In contrast, in (2.21) the ADL coefficients have been 'mixed up' somewhat (although they are clearly still retrievable from 2.21). This latter consideration is only significant if the shape of the distributed lag functions of the general model are of particular importance.

Fifthly, if we apply a 'general-to-specific' approach to a transformed equation rather than to the ADL model, we may be able to check the effect on the estimates of the long-run multipliers of eliminating any regressor relating to the short-run dynamics. This may provide an additional diagnostic check on the process of

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dynamic specification. Lastly, by transforming the model at the outset, we immediately remove the problems of collinearity emphasised in Section one.

To conclude, we have observed that there exist a number of possible approaches to the modelling of a dynamic relationship between a number of economic variables:

(i) The conventional 'general-to-specific' approach outlined in Section one. The specification search begins from a general ADL model for the variables involved, and involves a mixture of reparameterization and restriction. The long-run multipliers are obtained indirectly by solving the final equation for its long-run steady-state.

(ii) The Granger-Engle two-stage OLS method which involves the estimation of the cointegrating equation (and hence the long-run multipliers) to derive an equilibrium error, which is then placed in a transformed equation like (2.17). A simplification search may then be carried out on the short-run dynamics of the model to obtain a parsimonious model.

(iii) A procedure which derives from the Wickens-Breusch results. A transformation is applied to the ADL model at the outset to obtain an equation like (2.21) or (2.22) which give direct estimates of the long-run multipliers. We may then attempt to simplify the short-run dynamics of the model to find a more parsimonious representation.

(iv) A procedure which pulls together some of the elements of



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(i), (ii) and (iii), as follows. One may first transform the ADL model to obtain an equation such as (2.21) or (2.22). We then use this to obtain estimates of the long-run multipliers of the model. These are then imposed at the outset on the model, and we may then attempt to simplify the short-run dynamics. This procedure differs from (ii) in that it does not ignore the short-run dynamics in obtaining estimates of the cointegrating vector. As we argued above, this may or may not be an advantage over (ii), depending on whether the dominant issue in the model to be estimated is bias or efficiency in estimating the cointegrating vector. The procedure differs from (iii) in that the long-run multipliers are imposed at the outset, and not left free to vary during the simplification search. The advantage of each procedure should again be clear: in (iii) there may be biases in the final values obtained for the long-run multipliers because of possible dynamic misspecification of the short-run dynamics in the final equation. In (iv), the initial overparameterization may cause the estimates of the long-run multipliers to be inefficient. If the wrong choice is made, the resulting model will then have long-run properties which do not conform to reality.

In Chapter 3 we apply these (and additional) methods to the problem of estimating a demand for M3 function for the United Kingdom. Before we turn to this task, however, we have to investigate the time series properties of the variables to be used in constructing such a model, as the property of

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cointegratedness is a necessary one to ensure the existence of a long-run equilibrium relationship between these variables. Cointegration tests on the data are therefore carried out in Section four below. However, before we turn our attention to this it is necessary, for sake of completeness, to briefly outline some other results regarding cointegration not yet touched upon in this section.

### 2.2.6 Other Properties of Cointegrated Variables: A Digression

In this sub-section, we briefly list some additional properties of cointegrated variables, and problems which may arise in cointegration testing. (For further details, see Granger, 1983, 1986, Granger and Weiss, 1983, Engle, 1987, Engle and Granger, 1987). These properties will be of use in our later work.

First, if two  $X_t$  and  $Y_t$  series are  $CI(1,1)$ , then alternative series produced from the application of linear transformations and linear filters to  $X_t$  and  $Y_t$  will also be cointegrated. Thus, for example, if  $W_t = a + bX_{t-s}$  and  $Y_t = c + dY_{t-r}$ ,  $W_t$  and  $Z_t$  will also be  $CI(1,1)$  (where  $r$  and  $s$  are finite lags and not too large, and  $a, b, c$  and  $d$  are constants). It may also be proved that cointegration in levels implies cointegration in logs, but not viceversa.

Second, in the multivariate case where we are dealing with more than two variables, it is possible that the cointegrating vector is not unique. In this case, the vector  $x_t$  is said to be

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'multicointegrated'. (For an example, see Granger, 1983, Hendry and Von Ungern-Sternberg, 1981). Several equilibrium relations may then govern the joint time series behaviour of the set of variables.

Third, in the three variable case, if two pairs of variables are cointegrated, the third pair must also be. The proof of this is easily given in terms of the low-frequency coherence of a vector of time series (see Granger and Weiss, 1983). However, problems may be caused in the three-variable case where one series is cointegrated with the sum of the other two, but not with any individual component of the sum. Thus, for instance, the log of the nominal money stock may be found to be cointegrated with the log of nominal income, but may not necessarily be cointegrated with the log of the price level, or the log of real income. This may cause problems in cases where some variables are acting as proxies for other, unobservable, variables but, in contrast to the latter, may not be cointegrated.

Fourth, as Dolado and Jenkinson (1987) point out, cointegration may offer a useful guide to determining the functional form of a relationship. The following remarks should therefore be borne in mind throughout the remainder of this thesis. All of the cointegrating regression equations considered so far have been linear in structure (usually in logs) motivated by economic theory rather than by empirical considerations.

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Dolado and Jenkinson suggest that, intuitively, non-linear combinations of  $I(1)$  variables may always be found which yield a series which is  $I(0)$ , even though a linear cointegrating regression would suggest non-cointegration. Though we normally restrict our attention to the linear case for simplicity, could it be that we are thereby obtaining inconclusive results regarding the existence (or statistical foundation) of the theoretical equilibrium?<sup>23</sup>

Fifth, it is also worthwhile to point out that, in the multivariate case, if  $N$  variables are  $CI(1,1)$ , the omission of a single variable may lead to the conclusion that the remaining  $N-1$  variables are not cointegrated. Thus, again we may have to interpret the results obtained from cointegration tests with care, especially when dealing with relationships like the demand for money which include at least 4 (and potentially far many more) variables, and where the precise functional form is not necessarily known a priori.

The last issue covered here is also very important, particularly when dealing with seasonally unadjusted series where seasonal factors are significant. Series which display a marked seasonal pattern will have infinite peaks in the power spectrum not only at zero frequency, but also at seasonal frequencies. Given that cointegration deals with the whole issue of common trends, it is not surprising that the simple results on cointegration required some extensions to cope with the problem

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of seasonality. These are outlined in the context of a model of the sales for an electricity generating industry by Engle et al. (1987).

Let us begin with the definition of a seasonally integrated series: a series  $x_t$  is said to be seasonally integrated of orders  $(d,s)$  (i.e.  $SI(d,s)$ ) if  $d$  and  $s$  the smallest integers which enable us to reduce  $x_t$  to stationarity via the transformation:

$$(1 - L)^d S(L)^s x_t$$

where  $S(L)$  is the lag polynomial  $1 + L + L^2 + \dots + L^{s-1}$ . (Note that  $(1 - L)S(L) = (1 - L^s)$  where  $d = 1$ ).

Most of the economic series we are likely to deal with have a peak at zero frequency, and hence the issue is really whether they also have the additional unit root cause by seasonality. That is, most of these series are likely to be  $SI(1,0)$  or  $SI(1,1)$ . Clearly it is possible to use the conventional methods for detecting unit roots outlined above to detect seasonal unit roots.

The importance of the issue of seasonal cointegration is that it may undermine some of the powerful superconsistency results already mentioned. Basically, Engle et al (1987) outline three separate cases if we consider a vector of three economic variables  $(x_{1t}, x_{2t}, x_{3t})'$  such that  $x_{1t}$  is  $SI(1,1)$ ,  $x_{2t}$  is  $SI(1,0)$ , and  $x_{3t}$  is  $SI(0,1)$ , and a corresponding cointegrating vector  $(\alpha_1, \alpha_2, \alpha_3)'$ .

Firstly, these variables are seasonally cointegrated at zero

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frequency , but not at seasonal frequencies if  $\alpha'x$  is  $SI(0, 1)$  with  $\alpha_3 = 0$ . Secondly, these variables are seasonally cointegrated at seasonal frequencies but not at zero frequency if  $\alpha'x$  is  $SI(1,0)$ . Thirdly, these variables are fully cointegrated if  $\alpha'x$  is  $SI(0,0)$ .

Furthermore Engle et al show that these three different cases have widely differing implications for the consistency of the estimated parameters obtained from a cointegrating regression. Unless the variables are fully cointegrated, the familiar superconsistency result disappears, and may only be reinstated by applying some filter to the data prior to estimation. In the case of cointegration at zero and not at seasonal frequencies, a seasonal filter (i.e  $S(L)$ ) should be applied to the data to restore the consistency result. In the case of cointegration at seasonal but not at zero frequency, a simple difference filter  $(1-L)$  should instead be used.

It should also be apparent that although the example above (following Engle et al.) uses three series which are  $SI(1,1)$ ,  $SI(1,0)$ , and  $SI(0,1)$  respectively, the results easily carry over to several variables all of which are either  $SI(1,1)$  or  $SI(1,0)$ . Furthermore, Engle (1987) shows that the presence of two different roots in the time series involved will lead to a different error-correction formulation from that advanced by Engle and Granger (equation 2.17). In general, the series will yield more than one error correction term, to allow for the

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presence of a seasonal polynomial. This may perhaps explain the success of error-correction terms with lags greater than one in econometric studies which adopt seasonally unadjusted data (see for example Davidson et al., 1978). In our empirical study we choose to use seasonally unadjusted data, and therefore we have to seriously consider whether the extensions provided by the literature on seasonal cointegration are in any way significant to the case of the demand for money. It is worth pointing out, however, that the presence of seasonality in no way implies that a series must be seasonally integrated. To put the matter another way, it is perfectly possible for a series which is  $SI(0,0)$  to display seasonality - providing the seasonality is not dominant.

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### SECTION THREE: EMPIRICAL EVIDENCE FOR COINTEGRATION IN UK DEMAND FOR MONEY DATA

#### 2.3.1 The Choice of Data

As we pointed out at the outset of this thesis, a preliminary issue before we begin to model the demand for money regards the choice of the set of appropriate variables which will be used in constructing a single-equation demand for money model. In the light of the apparent 'breakdown' of estimated demand for money functions in the UK in the 1970s, there have been various attempts to introduce new, additional, explanatory variables to improve model design. For instance, one may recall the emphasis of the role of the own-rate of interest in models of broad money (see for instance Goodhart, 1975), or attempts to model the effect of wealth (see Grice and Bennett, 1984) on the demand for money or the effect of foreign interest rates on money holdings denominated in domestic currency (see for example McKenzie and Thomas, 1985) in the spirit of currency-substitution exchange rate models. Furthermore, one would also expect attempts to model the demand for money over a period from the early 1960s to today to take account of the switch from a fixed to a floating exchange rate regime. Indeed, portfolio theory suggests that a change in the role of exchange rate risk is bound to affect the parameters of the demand for money (and other asset demands, see for instance Branson and Henderson, 1985, Muscatelli et al., 1988). In work along similar lines, Baba et al. (1986) have found



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significant effects on the demand for M1 in the US of changes in the variability of asset yields. It is certainly true that the 1970s and 1980s have proved far more volatile times than the 1960s for many economic time series, and in particular inflation and interest rates, with a more active use of monetary and fiscal policy. Again, we would expect this to impinge on the parameters of conventional demand for money functions which usually assume constant variances for asset yields.

Whilst not wishing to dismiss these studies, in this work we shall pay less attention to the type of work which has emphasised the role of additional variables in single-equation demand for money models. There are several reasons for this. First, given the multitude of different alternative variables involved, a study which attempted to encapsulate all these effects would be of considerable length and complexity. Furthermore, such a study may not answer many questions because any conclusions would inevitably apply only to models of particular definitions of the money stock. In addition, in the case of models of the demand for money in open economies, it is doubtful whether single-equation studies could shed more light on the effect of exchange rate movements on the demand for money than full structural models of the financial sector in an open economy. Second, in this thesis we concentrate on testing the application of alternative approaches to the modelling of the demand for money in a single equation context, and any conclusions may carry over to empirical

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studies on definitions of money other than those used here, or to models in other areas of economics. Third, recent attempts to modelling the demand for money (see Hendry 1979, 1985, 1986, Hendry and Mizon, 1978) have shown that models involving a small number of explanatory variables may still reach satisfactory results.

We begin, in this and the next chapters with an attempt to model the M3 definition of money in the UK which, as noted in Chapter 1, has proved to be the most 'troublesome' for researchers to 'pin down'. Later, in Chapters 4 and 5, we shall also present some evidence on the demand M1 balances, which has apparently proved to be more stable over the 1970s and 1980s (see Hendry, 1985).

### 2.3.2 Testing for Integration of Degree One.

The data used in modelling the the demand for money is the following. Following Hendry and Mizon (1978), we assume that the relevant explanatory variables are real income,  $Y$  (defined as real personal disposable income at 1980 prices), the price level,  $P$  (defined as the implicit deflator of  $Y$ ) and an interest rate on an alternative asset,  $R^1$  (defined as the yield medium-term gilts). In addition, we also investigate the possibility that a short-term interest-rate,  $R^S$  (defined as the Treasury Bill rate), may capture the own-interest effect on the demand for broad money. All the series used were obtained from Bank of England data and various issues of Financial Statistics.

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There are basically two ways of checking whether all the variables involved are integrated of degree one. First, by the informal inspection of the correlogram of each series. The correlogram plots the sample autocorrelations for the series,  $r(\tau)$ , against time, where we define the autocorrelation for the  $\tau$ th period as:

$$r(\tau) = c(\tau)/c(0) \quad (2.24)$$

where  $c(\tau)$  denotes the sample autocovariance for period  $\tau$  and (for a series  $y_t$  with fixed sample mean  $\bar{y}$ ) is defined as:

$$c(\tau) = E\{(y_t - \bar{y})(y_{t-\tau} - \bar{y})\} \quad (2.25)$$

It is apparent from this that  $c(0)$  denotes the sample variance for  $y_t$ , and that the autocorrelation function is dimensionless. Using these definitions, it may be easily shown (see Harvey, 1981b) that, theoretically, for an  $I(1)$  (nonstationary) series the autocorrelations are equal to unity for all  $\tau$ . Similarly, for an  $I(0)$  (stationary) series, the autocorrelations decrease steadily in magnitude as  $\tau$  increases and their sum is finite.

The second (more formal) method of testing for a degree of integration of unity involves using the statistical tests described above, namely the CRDW, DF and ADF test statistics. In the case of the CRDW test, a simple regression has to be carried out of the series on a constant.

All the sample autocorrelations for  $m$ ,  $p$ ,  $y$ ,  $R^S$ , and  $R^L$  (where lower cases indicate natural logarithms) were found to be close to unity for a large number of periods. In contrast, the

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sample correlations for the differenced series are plotted in Graphs 2.1-2.5. These indicate that these series are indeed  $I(1)$ . In particular, all the sample autocorrelations decline rapidly. The sharp peaks in the correlograms for  $\Delta y$  and  $\Delta m$  are an indication of seasonality, whilst the low autocorrelations for  $\Delta R^S$  and  $\Delta R^1$  show that these variables have an autoregressive parameter close to unity (i.e. they probably follow a random walk).

We can confirm these results with reference to the results of the formal tests reported in Table 2.1. The first five rows of the table show that none of the series, as expected, are stationary in the levels, as all the test statistics are not significant. The next five rows show that there is considerable evidence to suggest that all of the series employed are  $I(1)$ , as all the first differences appear to be stationary. Only in the cases of  $\Delta m$  and  $\Delta R^S$  do some of the tests tend to point into the opposite direction. However, on balance, we would argue that taken together with the correlogram evidence, the results in Table 2.1 (and in particular the CRDW statistics) point towards all of these series being  $I(1)$ <sup>24</sup>.

FIGURE 2.1

CORRELOGRAM FOR DELTA (M)

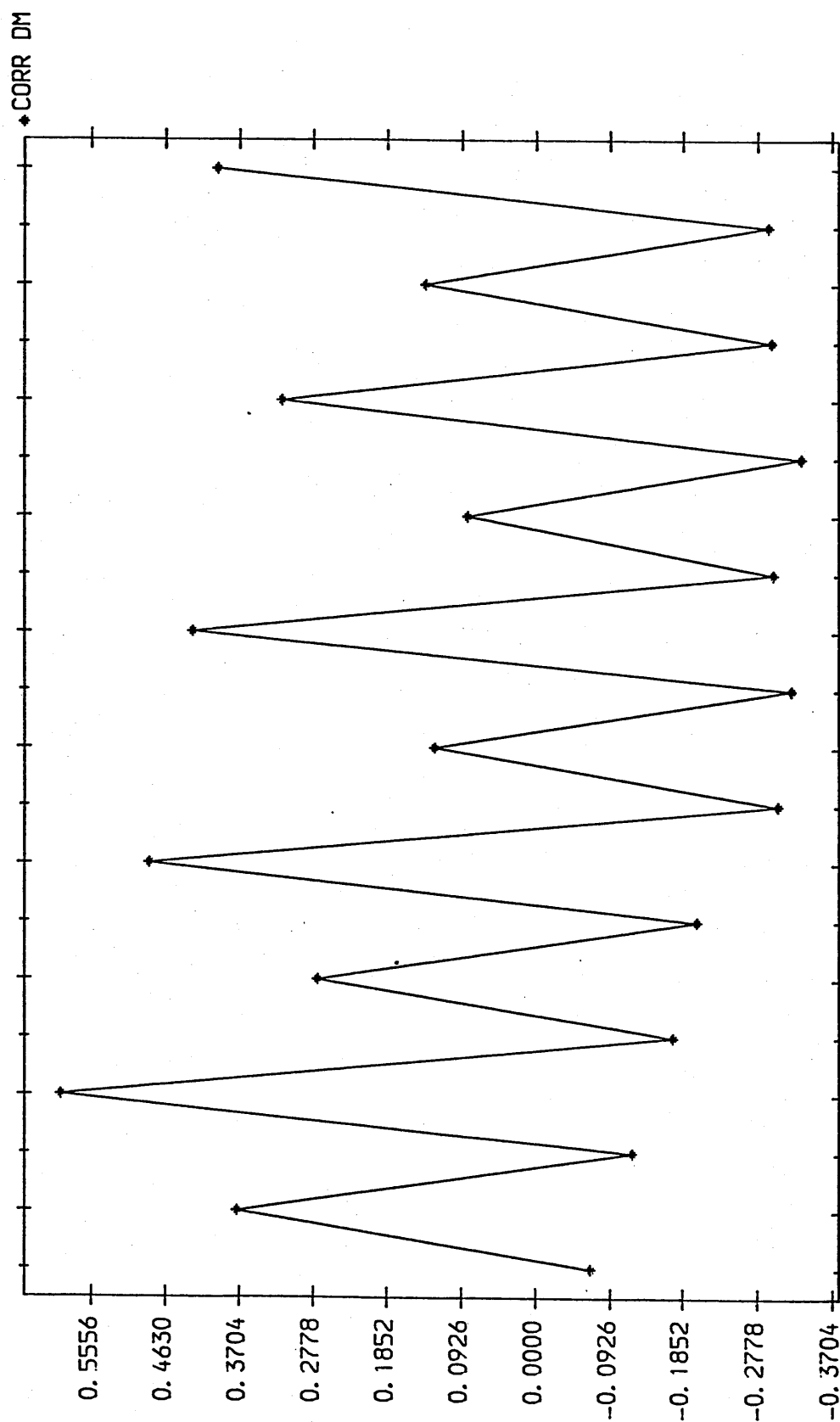


FIGURE 2.2:

CORRELOGRAM FOR DELTA (P)

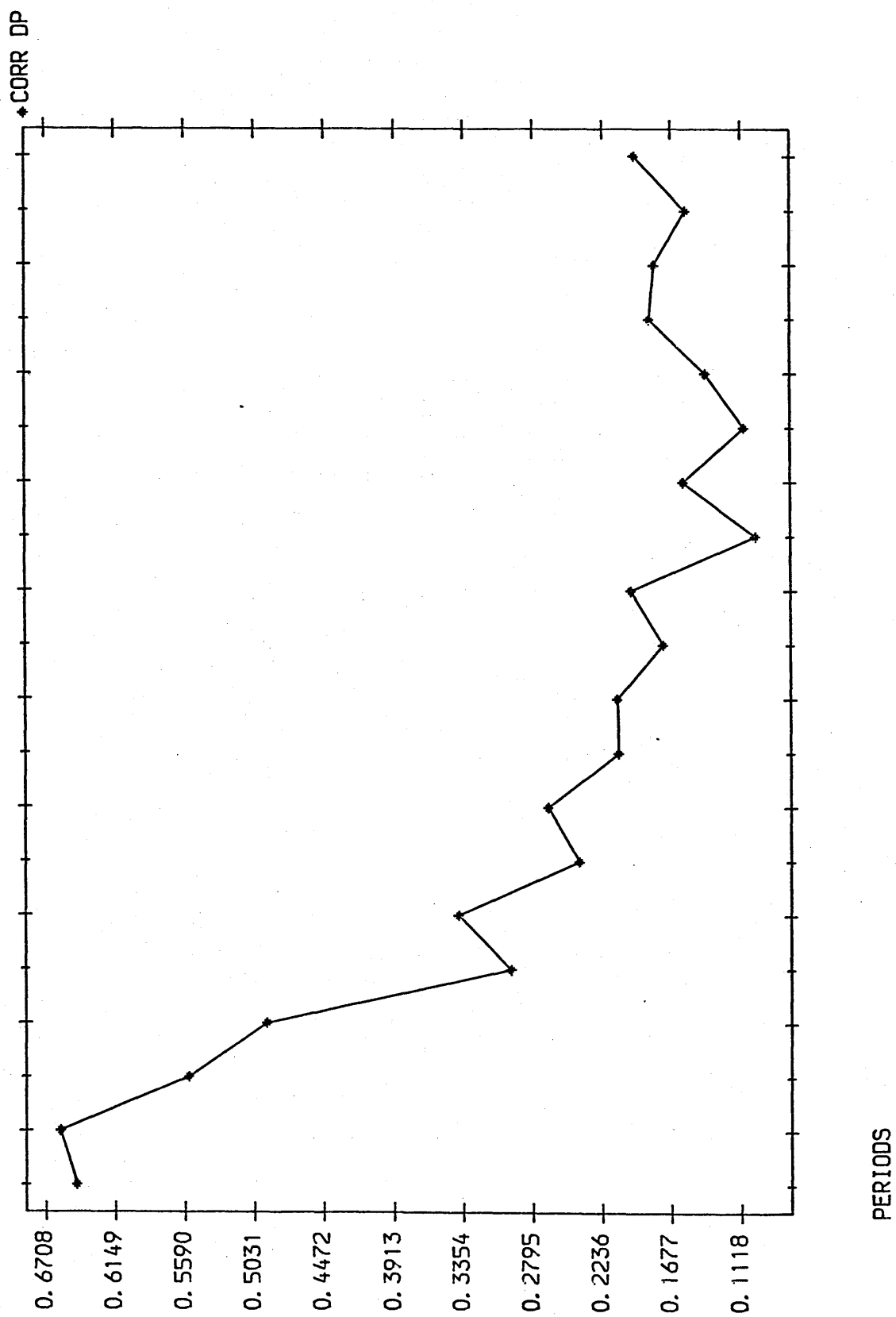


FIGURE 2.3

CORRELOGRAM FOR DELTA (Y)

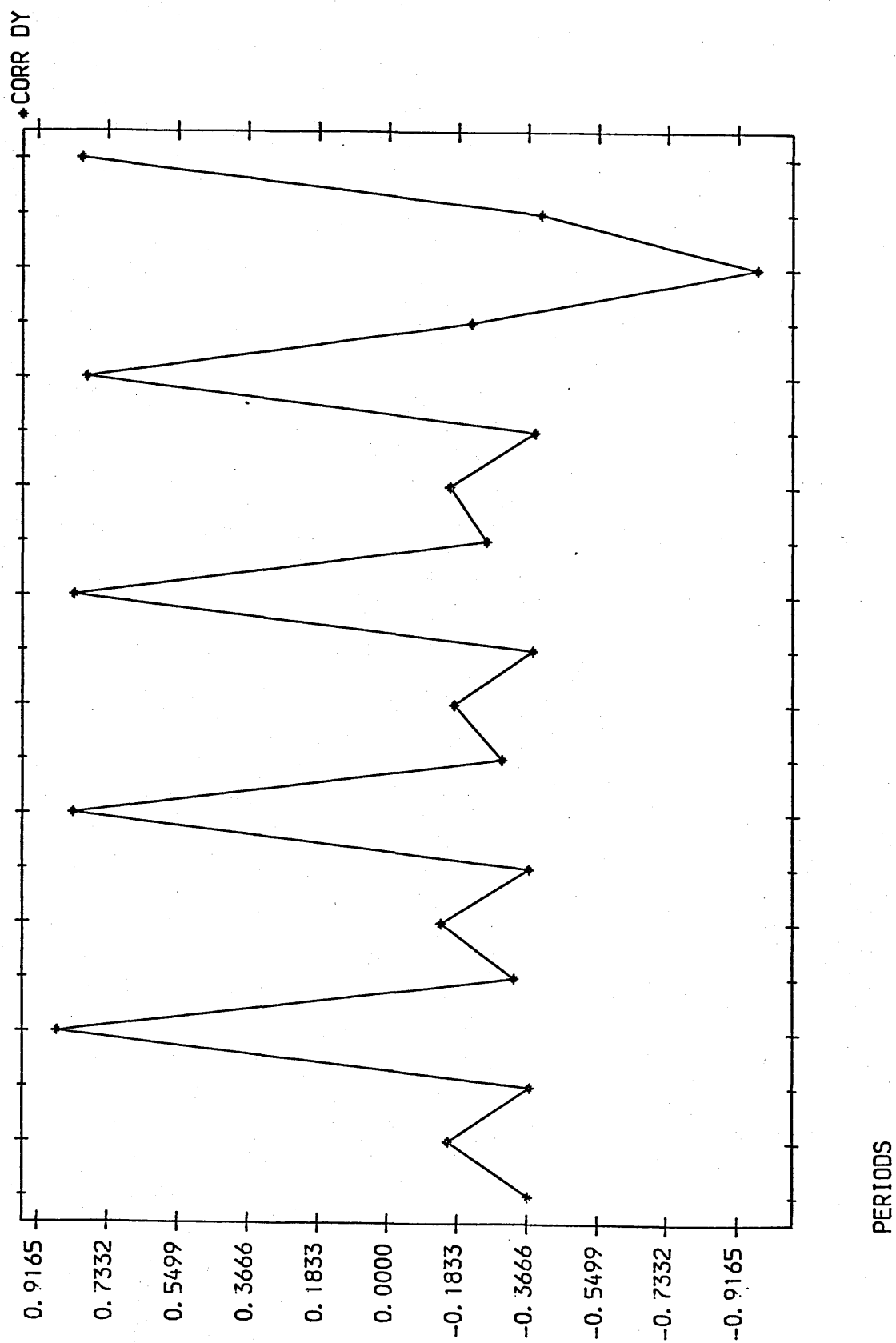


FIGURE 2.4:

CORRELOGRAM FOR DELTA (RS)

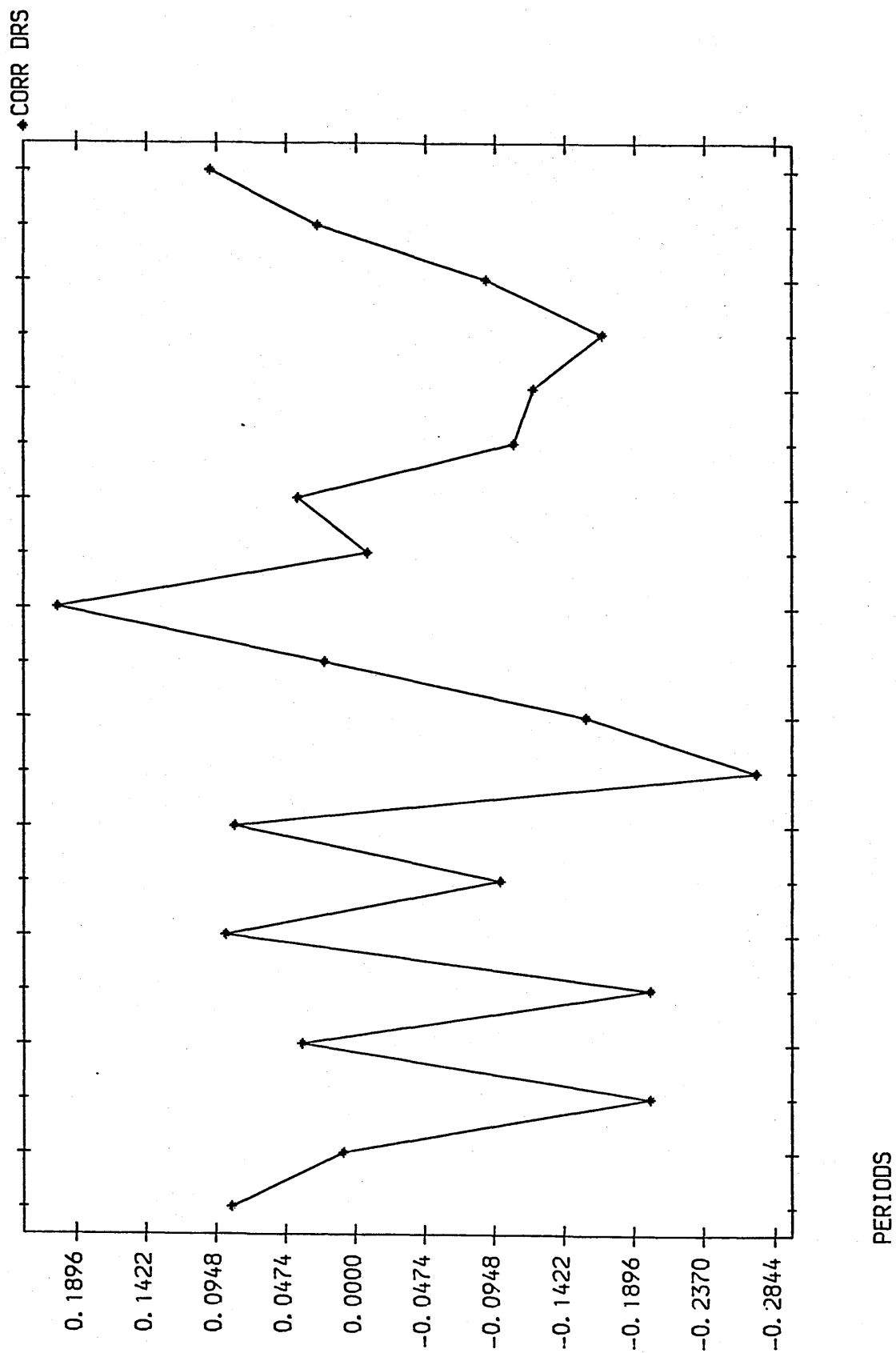
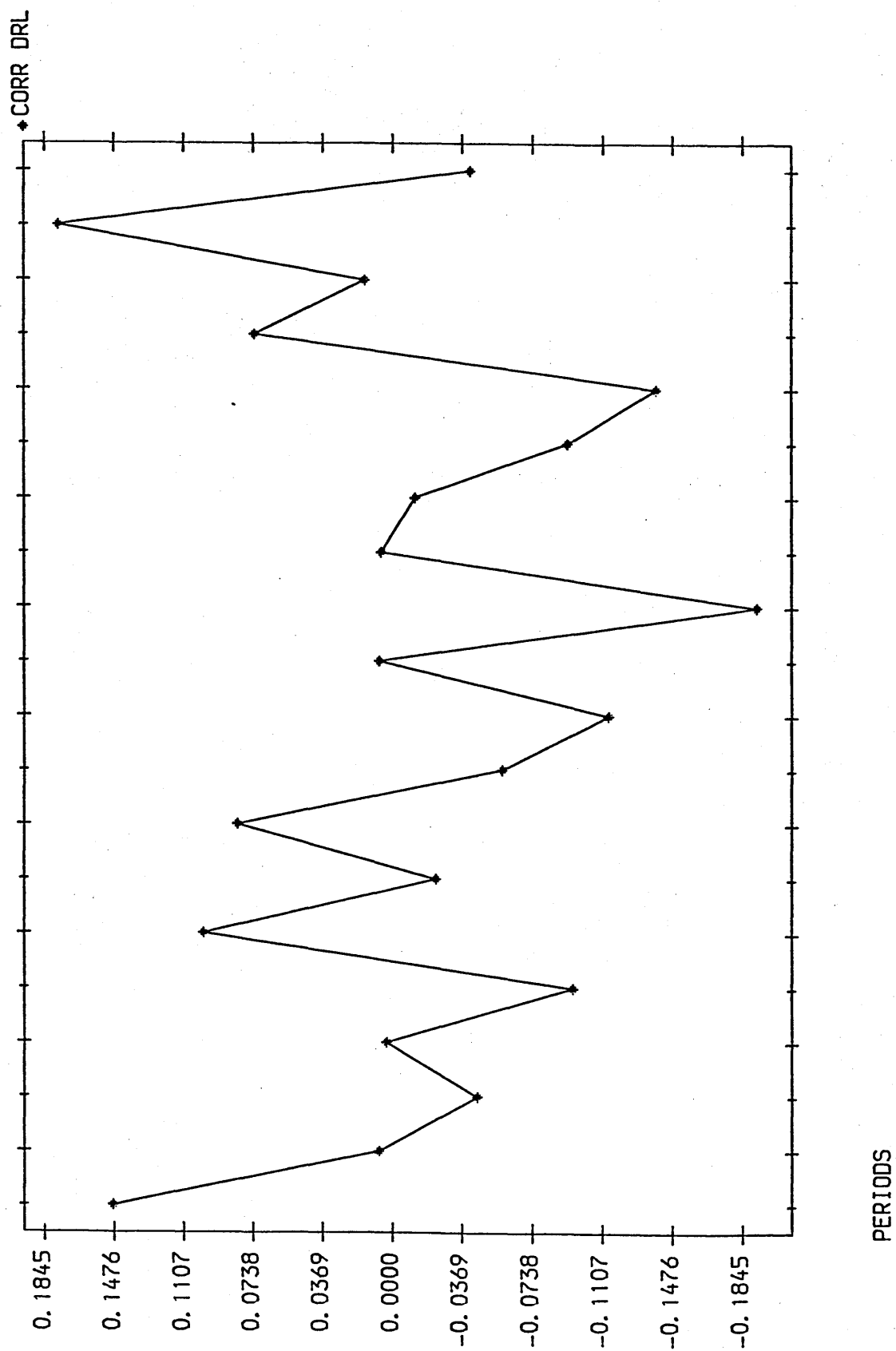




FIGURE 2.5.

CORRELOGRAM FOR DELTA (RL)



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Table 2.1

<u>Variable</u>	<u>DF</u>	<u>ADF</u>	<u>CRDW</u>
m	2.230	0.940	0.002
p	1.319	-0.465	0.002
y	-1.744	-0.904	0.108
R <sup>s</sup>	-2.420	-2.480	0.179
R <sup>l</sup>	-1.740	-1.811	0.067
$\Delta m$	-9.960(*)	-2.398	2.128(*)
$\Delta p$	-4.193(*)	-2.684	0.673(*)
$\Delta y$	-13.907(*)	-4.938(*)	2.725(*)
$\Delta R^s$	-8.552(*)	-5.234(*)	1.839(*)
$\Delta R^l$	-7.963(*)	-4.374(*)	1.709(*)
$\Delta_4 m$	-2.017	-1.966	0.173
$\Delta_4 p$	-1.269	-1.408	0.089
$\Delta_4 y$	-4.621(*)	-3.059(*)	0.829(*)
$\Delta_4 R^s$	-3.255(*)	-3.030(*)	0.471(*)
$\Delta_4 R^l$	-3.026(*)	-2.348	0.420(*)

(\*) indicates a test statistic which rejects the null hypothesis at the 5% significance level.

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In the last five rows of Table 2.1 we test for the presence of seasonal unit roots in the series, by checking whether the fourth differences of the data are stationary. Whilst in the case of the  $\Delta_4 y$ ,  $\Delta_4 R^S$ , and  $\Delta_4 R^I$  the test statistics significant, in the case of the differences of the money stock and the price level are not even close to their 5% critical values. Whilst, because of its marked seasonal pattern one may tend to conclude that  $y$  is  $SI(1,1)$ , it is difficult to argue that this also applies to the interest rate variables. Furthermore, the magnitudes of the test statistics are greater in the first difference case even for the real income variable. In any case, we should bear the possible effect of seasonality in mind when testing for cointegration.

### 2.3.3. Testing for Cointegration in the Case of M3.

An appropriate starting point for the testing of cointegration is to estimate a cointegrating equation which includes all the variables to be incorporated into our model. Given that the OLS estimates of the equation's parameters represent estimates of the long-run elasticities (semi-elasticities in the case of the interest rates), we should expect 'sensible' signs on them. In all the cointegration tests presented in this chapter we will use the full available data set, as the main purpose is to focus on whether these variables are indeed cointegrated. In the later chapters, however, some of the data periods will be retained for the purposes of presenting

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evidence on the fitted models' ex ante forecasting ability.

The following cointegrating equation was estimated over the full sample period (1963(1)-1986(2)):

$$m = -1.162 + 1.003p + 1.149y - 0.006R^S - 0.004R^L \quad (2.26)$$

(2.078) (0.036) (0.191) (0.006) (0.006)

$$R^2 = 0.987 \quad CRDW = 0.216 \quad \hat{\sigma} = 0.0976$$

Apart from the low value of the CRDW statistic which does not reject the hypothesis of no cointegration (see Sargan and Bhargava, 1983), the coefficient on  $R^S$  has the wrong sign for an own-interest variable. We therefore eliminated  $R^S$ , to obtain a long-run specification akin to that of Hendry and Mizon (1978):

$$m = -1.071 + 0.996p + 1.139y - 0.008R^L \quad (2.27)$$

(2.076) (0.036) (0.191) (0.004)

$$R^2 = 0.987 \quad \sigma = 0.097 \quad DW = 0.210 \quad DF = -1.85 \quad ADF = -2.70$$

This cointegrating equation is more promising in that the estimated values of the cointegrating parameters look plausible (e.g. the estimated long-run price elasticity of the demand for money is close to unity). However, none of the statistics presented rejects the null hypothesis of no cointegration at the 5% significance level, which does not augure well for our attempts to estimate a model for M3. However, at this point we examined number of escape routes:

First, it is worthwhile to point out that these tests for unit roots have relatively low power, leading to close-run results in many applications<sup>25</sup>. In the vast majority of the

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applied econometric literature there is an asymmetric treatment of type I and type II errors when dealing with classical hypothesis testing. The significance level is usually arbitrarily fixed at a given level (usually 1%, 5% or 10%). It has been argued elsewhere (see Leamer, 1978, Mizon, 1984) that there may be a case for altering the significance level to reduce the probability of type II errors where tests are known to have low power against the alternative. In our case, if the root of the equilibrium error is very close to unity (though still  $< 1$ ), the cointegration tests may not pick this up. Some applied economists may object to the practice of altering significance levels as it may seem as if we are moving the goalposts to suit our own purposes. Nevertheless, this criticism of low power tests should be borne in mind before dismissing equations like (2.27) whose residuals only narrowly fail the cointegrating tests.

Secondly, we examined whether the problem in establishing cointegration lay in the relationship between any individual pair of variables in the demand for money. It should be recalled that the exclusion of any one variable from the cointegrating equation may cause us to refute cointegration. In the case of the demand for money this problem is particularly serious given the number of variables at the researcher's disposal. In Table 2.2 we reproduce the CRDW statistics for certain pairs of the variables involved. We can see that the problem seems to arise mainly between  $m$ ,  $p$ , and the interest rate. In fact, if we estimate the

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cointegrating equation over other sample periods (say, 1963(1)-1984(2)), we obtain a positive estimate of the parameter on  $R^1$  which is clearly counter-intuitive. This suggests that the results of cointegrating tests are sensitive to the sample period chosen, and the apparent absence of cointegration detected in (2.27) may be due to an unfortunate choice of sample.

For instance, we estimated the following cointegrating equations for various sub-sample periods:

1963(1)-1984(2):

$$m = 0.996 + 0.953p + 0.931y + 0.005R^1 \quad (2.28a)$$

(1.944) (0.034) (0.180) (0.005)

$$R^2 = 0.987 \quad CRDW = 0.162 \quad \hat{\sigma} = 0.0881$$

1963(1)-1978(4):

$$m = -1.667 + 0.725p + 1.134y + 0.029R^1 \quad (2.28b)$$

(1.534) (0.044) (0.140) (0.004)

$$R^2 = 0.989 \quad CRDW = 0.610 \quad \hat{\sigma} = 0.0566$$

1972(1)-1986(2):

$$m = -0.921 + 0.951p + 1.142y - 0.023R^1 \quad (2.28c)$$

(4.301) (0.053) (0.391) (0.008)

$$R^2 = 0.962 \quad CRDW = 0.168 \quad \hat{\sigma} = 0.1133$$

1975(1)-1986(2):

$$m = 8.674 + 1.112p + 0.286y - 0.040R^1 \quad (2.28d)$$

(4.555) (0.062) (0.410) (0.011)

$$R^2 = 0.961 \quad CRDW = 0.235 \quad \hat{\sigma} = 0.0933$$

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Table 2.2

CRDW statistics on pairs of series

	m	p	y	R <sup>1</sup>
m	-	0.077	0.578	0.056
p		-	0.492	0.057
y			-	0.300
R <sup>1</sup>				-

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These equations show that the point estimates of the cointegrating vector vary dramatically over time. This may suggest either that important variables have been omitted from these equations or, alternatively, that the long-run multipliers have not remained constant over the whole sample period. If parameter variation is of some importance, then this may suggest the adoption of estimating techniques which capture this effect. In the case of the United Kingdom, it would be surprising indeed if the institutional changes which have occurred in the financial system over the last 25 years had not influenced people's behaviour in the money market. Given the number of institutional reforms both within the UK's own monetary system (see, for instance Llewellyn *et al.*, 1982, Hall, 1984) and in the international financial system with the move to a world of floating exchange rates there are plenty of reasons why one should doubt that the long-run parameters of the demand for money have remained unchanged. We shall return to this point further on.

It is also interesting that if we take the sample up to 1978(4), the CRDW statistic rejects the null hypothesis of no cointegration. Recall that this was the sample period used in the Hendry-Mizon (1978) study, though of course they did not attempt to estimate the long-run multipliers directly. Recalling the final equation estimated by Hendry and Mizon from Chapter 1, we should note that the interest rate does not enter the ECM, but appears on its own with a lag of four periods. This suggests a



## CHAPTER 2

third explanation for the apparent failure of (2.27), to which we now turn:

The Granger-Engle two-step procedure involves the prior construction of an ECM or 'equilibrium error' term which is then included with a single lag in an estimating equation. However, frequently previous data-based modelling exercises following the 'general-to-specific' approach have yielded final equations where the lag on the levels of  $m$ ,  $p$ , and  $y$  differed from that of  $R^1$ . Given that this discrepancy was obviously suggested by the data, could this be causing some difficulties in estimating our cointegrating equation? We should recall from Section two that by transforming individual variables through the use linear finite-length filters, one does not alter their cointegration properties. Furthermore, it should be apparent that, in considering steady-state equilibria, the lag with which a variable enters a dynamic equation is of no consequence. We therefore examined whether our cointegrating equation gave different results if the interest rate variable appeared in a lagged form. The following results were obtained for different lags of  $R^1$  over the Sample Period 1963(1)-1986(2):

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$$m = -1.704 + 0.996p + 1.200y - 0.0104R_{t-1}^1 \quad (2.29a)$$

$$(2.129) \quad (0.037) \quad (0.196) \quad (0.005)$$

$$R^2 = 0.987 \quad CRDW = 0.224 \quad DF = -97.34 \quad ADF = -42.6 \quad \hat{\sigma} = 0.097$$

$$m = -1.846 + 1.005p + 1.212y - 0.0134R_{t-2}^1 \quad (2.29b)$$

$$(2.118) \quad (0.037) \quad (0.194) \quad (0.005)$$

$$R^2 = 0.987 \quad CRDW = 0.236 \quad DF = -9.51 \quad ADF = -1.63 \quad \hat{\sigma} = 0.096$$

$$m = -2.140 + 1.013p + 1.246y - 0.0164R_{t-3}^1 \quad (2.29c)$$

$$(2.125) \quad (0.037) \quad (0.195) \quad (0.005)$$

$$R^2 = 0.987 \quad CRDW = 0.252 \quad DF = -6.74 \quad ADF = -2.00 \quad \hat{\sigma} = 0.094$$

$$m = -1.921 + 1.026p + 1.229y - 0.0187R_{t-4}^1 \quad (2.29d)$$

$$(2.098) \quad (0.038) \quad (0.192) \quad (0.005)$$

$$R^2 = 0.988 \quad CRDW = 0.261 \quad DF = -5.49 \quad ADF = -70.67 \quad \hat{\sigma} = 0.093$$

These equations are very similar except for the values obtained for the CRDW, DF and ADF statistics. Some of the extraordinary high values for the DF and ADF statistics is probably attributable to the fact that the interest rate can be usually modelled by an autoregressive equation of order two, where the second autoregressive parameter is often less than unity. It could be that lagging the interest rate captures this effect. In any case, for (2.29c) and (2.29d) the CRDW statistic also narrowly rejects the null hypothesis of no cointegration, and this statistic seems less sensitive to the lag with which  $R^1$  enters the cointegrating equation. Thus, an ECM with a lagged interest rate term may be an alternative to the conventional one proposed by Engle and Granger (1987), and it conforms more

## CHAPTER 2

closely with the type of specification found acceptable by, inter alia, Hendry and Mizon (1978) and Hendry (1979, 1985).

The fourth possible way round the failure of (2.27) to detect cointegration is to use alternative tests to those reported above. In Section two we suggested two alternatives which may offer an informal statistical test of the presence or absence of cointegration. The first involves the frequency domain properties of the series under examination, and evidence on these will be presented in the next subsection. The second involves examining the significance of the 'equilibrium error' term in an equation involving only first differences: i.e, we regress  $\Delta m$  on  $\Delta p$ ,  $\Delta y$ ,  $\Delta R^1$ , and the ECM term. In the next experiment we therefore tested models where ECMs have been obtained from equations (2.27), (2.29a)-(2.29d). This will also offer some evidence on the significance of ECMs which include lagged interest rates. The evidence is presented in Table 2.3, where the main statistic of note is the t-ratio on the lagged 'equilibrium error' term. As the reader can verify, a standard ECM obtained from (2.27) does not appear to be significant. In contrast, where the interest rate appears in a lagged form, the ECM term becomes significant. In particular, the results from Table 2.3 seem to favour the adoption of an ECM which includes  $R_{t-3}^1$ .

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Table 2.3

t-values of ECM term with lagged interest rate in general equation (equation 2.21)

Sample used 1963(1)-1986(2)

	<u>t-value</u>
$R_t$	-1.626
$R_{t-1}$	-1.763(*)
$R_{t-2}$	-1.880(*)
$R_{t-3}$	-2.010(*)
$R_{t-4}$	-1.865(*)

(\*) indicates a test statistic which rejects the null hypothesis at the 5% significance level.

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Lastly, there is the issue of seasonal cointegration. As we pointed out above, the real income variable may display a seasonal pattern (though whether it is dominant is open to debate). In any case, we attempted to re-estimate the cointegrating equation on seasonally averaged data. This did not seem to affect the point estimates to any considerable extent, and these results are not reported here. On the other hand, as we saw from the correlograms in Graphs 2.1-2.5 (and as we shall see from the frequency domain results in section 2.3.4), there is evidence of seasonality in some of these variables, even if it may not be dominant. To exclude these effects at higher frequencies, Hallman (1987) has suggested the application of a low-pass filter on the data to eliminate unwanted noise in estimating the long-run relationship between variables. This idea, however, has not been applied yet in the literature. Given the negligible effect of filtering the data with a seasonal filter, we chose to ignore this possibility. However, the more complex structure of the error-correction mechanism in the presence of seasonal effects may militate against the success of the simple Engle-Granger procedure. We return to this issue in Chapter 3.

We now turn briefly to an examination of the frequency domain properties of the time series used in modelling the demand for M3.

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### 2.3.4 Frequency Domain Properties of the Time Series Used

As we noted in section two, the frequency domain properties of time series may offer an indication of whether they are indeed cointegrated. One problem is that stochastic processes which are integrated of degree one have (theoretically) infinite variance. The results on coherence obtained in section two relied on the use of the approximate shape of the spectrum for a non-stationary series. Theoretically, at zero frequency the spectrum of a  $I(1)$  series has infinite power. It is of course true that if we attempt to estimate the spectrum for a series, these theoretical results will not always be confirmed. For instance, the theoretical spectrum of white noise is flat, but in practice a sample generated by a white noise disturbance will not conform exactly to this, but will generally have a jagged appearance<sup>26</sup>.

This implies that in practice we can attempt to obtain estimates of the power spectrum for  $I(1)$  series, and this may offer some information (although the low frequency component will clearly exert a dominant influence). An alternative, and more acceptable approach, would seem to be to make these series stationary and then to estimate their power spectra.

To examine the way in which the properties of the cross-spectrum between two  $I(1)$  series vary depending on whether they are  $CI(1,1)$  or not, let us consider the following data generation process for two variables  $X_t$  and  $Y_t$ :

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$$Y_t = \theta Y_{t-1} - \alpha(Y - X)_{t-1} + u_t$$

$$X_t = \phi X_{t-1} + v_t \quad (2.30)$$

where  $u_t$  and  $v_t$  are zero-mean white noise processes which are uncorrelated with  $Y_t$  and  $X_t$ , and have variances  $\sigma_u^2$  and  $\sigma_v^2$  respectively. Note that if we wish both variables to be  $I(1)$ , we require  $\theta, \phi \geq 1$ . Furthermore, if we require them to be  $CI(1,1)$ , we also require  $\alpha \neq 0$ . Because it is difficult to consider the shape of spectra when  $Y$  and  $X$  are non-stationary, let us derive an expression for the coherence between these variables for the general case, and then examine what happens as  $\theta \rightarrow 1$  and  $\phi \rightarrow 1$ . This will also enable us to confirm some of the results stated, but not proved, in section two.

Let us begin with some simple time series results. First, consider a stationary series  $Z_t$ , and define another series  $Q_t$  in terms of linear time invariant filtering operation on  $Z_t$ :

$$Q_t = \sum_{j=-\infty}^{\infty} \beta_j Z_{t-j} \quad (2.31)$$

where the  $\beta_j$  are weights such that  $\sum_j \beta_j^2 < \infty$ . Next, define the frequency response function,  $B(w)$  as:

$$B(w) = \sum_{j=-\infty}^{\infty} \beta_j e^{-i w j} \quad (2.32)$$

It may then be shown (see Harvey 1981b) that, given the spectrum of  $Z_t$ ,  $f_Z(w)$ , the spectrum of  $Q_t$  may be found by:

$$f_Q(w) = |B(w)|^2 f_Z(w) \quad (2.33)$$

where  $|B(w)|^2$  is sometimes known as the transfer function. This 'trick' enables us to find an expression of the power spectrum of a compound series which is a combination of a number of

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known stationary series without having to derive the autocovariance function for the compound series. This method may be generalised to the case of multivariate spectral analysis, because it enables us to find an equivalent expression for the cross-spectrum,  $f_{qz}(w)$ . Suppose that  $Q_t$  is generated by (2.31) with the addition of a zero-mean white-noise disturbance  $u_t$ , which is independent of  $Z_t$ . We may show, for instance, (again see Harvey, 1981b), that:

$$f_{qz}(w) = B(w)f_z \quad (2.34)$$

Recalling the definition of the coherence of two series given in section two:

$$\text{Coh}(w) = |f_{qz}(w)|^2 / (f_q(w)f_z(w)) \quad (2.35)$$

Using (2.33) and (2.34), we may re-arrange (2.35) as:

$$\text{Coh}(w) = \{1 + (f_u(w)/|B(w)|^2 f_z(w))\}^{-1} \quad (2.35')$$

We may now use (2.35') to derive an expression for the coherence of  $X_t$  and  $Y_t$  in equations (2.30). By noting that (2.30) can be rewritten as:

$$Y_t = (\theta - \alpha)Y_{t-1} + \alpha X_{t-1} + u_t$$

$$X_t = \phi X_{t-1} + v_t \quad (2.30')$$

the application of the above results follows directly by finding the frequency response function of  $Y_t$ . The coherence between  $X_t$  and  $Y_t$  can then be found to be:

$$\begin{aligned} \text{Coh}(w) = \{1 + (\sigma_u^2/\alpha^2 \sigma_v^2)(1 + \phi^2 - 2\phi \cos(w))(1 + (\theta - \alpha)^2 - \\ 2(\theta - \alpha)\cos(w))\}^{-1} \end{aligned} \quad (2.36)$$

Note that (2.36) confirms one of the results stated in



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section two: as  $\phi \rightarrow 1$  and  $\theta \rightarrow 1$ , the coherence of the two series increases at very low frequencies, and decreases at low frequencies, as most of the power of the spectra for the two series is concentrated at the lower frequencies. Note furthermore that, as  $\phi \rightarrow 1$ , the second term in round brackets in (2.36) tends to  $2(1 - \cos(w))$ , and the third term in round brackets tends to  $(2(1 - \alpha) + \alpha^2 - 2(1 - \alpha)\cos(w))$ .

Next let us examine what happens to the coherence when  $\alpha$  tends to zero, i.e. in the absence of cointegration. The third term in round brackets will also tend to  $2(1 - \cos(w))$ , and hence this will tend to increase the coherence at low frequencies. However, this is dominated by the first term in round brackets which becomes very large as  $\alpha$  tends to zero. As a result, the coherence between the two series decreases dramatically. This is not surprising, as when  $\alpha$  is zero, and  $\theta = 1$ ,  $\phi = 1$ , the two series are uncorrelated random walks, and one cannot help predict the other.

The converse obviously applies, and if the two series are cointegrated, their coherence is greater at all frequencies. Furthermore, a large value of  $\alpha$  will tend to increase the coherence at high frequencies.

Though this information is useful, the problem of whether it is testable still remains as our original series are non-stationary. However, if we difference the series involved this will merely attenuate the low frequencies of the spectrum, and

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we should still be able to detect a reasonably high coherence at other frequencies if  $\alpha \neq 0$ . This cannot of course serve as a formal test of cointegration, but it may offer insights into the reasons why some of the demand for money variables appear to be  $CI(1,1)$ , but others do not (see Table 2.2). Furthermore, we shall also examine the estimated spectra for the  $I(1)$  variables, which may also offer some insights into this problem<sup>27</sup>.

Before turning to the results obtained, we should outline the methods used to estimate the spectra (and hence the coherence) of the data series. As we pointed out above, it is important to recognise that the spectra obtained using actual data series will not actually conform to the theoretical values which one would expect from stochastic processes of those types (this is of course also a feature of sample correlograms). The example given above of the estimated spectrum of a white noise process is a case in point. This 'problem' should be borne in mind when analysing our results below.

The estimated spectra in our case are generated using the algorithms available in the RATS econometric program. This involves using the fast fourier transform (FFT) algorithm in conjunction with a flat window for spectrum averaging. The spectrum averaging process is introduced to compensate for the fact that the simple sample spectral density is not a consistent estimator of the power spectrum at any given frequency. (This is in fact what causes the estimated spectrum of stochastic process

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to appear jagged and irregular). The problem here is that the choice of the window in the spectrum averaging process is essentially arbitrary and, as Harvey (1981b) points out, large biases may emerge with the wrong choice of window. We chose a flat window in the case of the differenced data, but if the implicit assumption of a 'flat' underlying spectrum is not correct, this will induce bias in our final estimates. In the case of our data in levels, a flat window was obviously inappropriate, and we instead used the 'tent' window option provided by RATS.

Graph 2.6 shows the coherence of  $\Delta m$  and  $\Delta p$ ,  $\Delta y$ ,  $\Delta R^1$  respectively. Note that the coherence does not show the smooth features displayed by the usual economic time series. To some extent this is not surprising, since the data used is not seasonally adjusted, and hence the seasonal pattern is likely to cause the power spectra of the series to be affected (in particular the estimated spectra of real income and the money stock). This is confirmed by an examination of the estimated spectra of  $\Delta y$  and  $\Delta m$ , shown in Graph 2.7. Note the hump-shaped feature at the  $\pi/2$  frequency for both series. Overall, however, we see that the money stock shows a reasonably high level of coherence with all other series except the interest rate, although the seasonal pattern around  $w = \pi/2$  dominates the coherence with respect to both  $\Delta y$  and  $\Delta p$ . Note also that the coherence at low frequencies is high with respect to  $\Delta y$  and  $\Delta p$ ,

FIGURE 2.6.

COHERENCE OF DELTA (M) WITH DELTA (P), DELTA (Y) AND DELTA (RL)

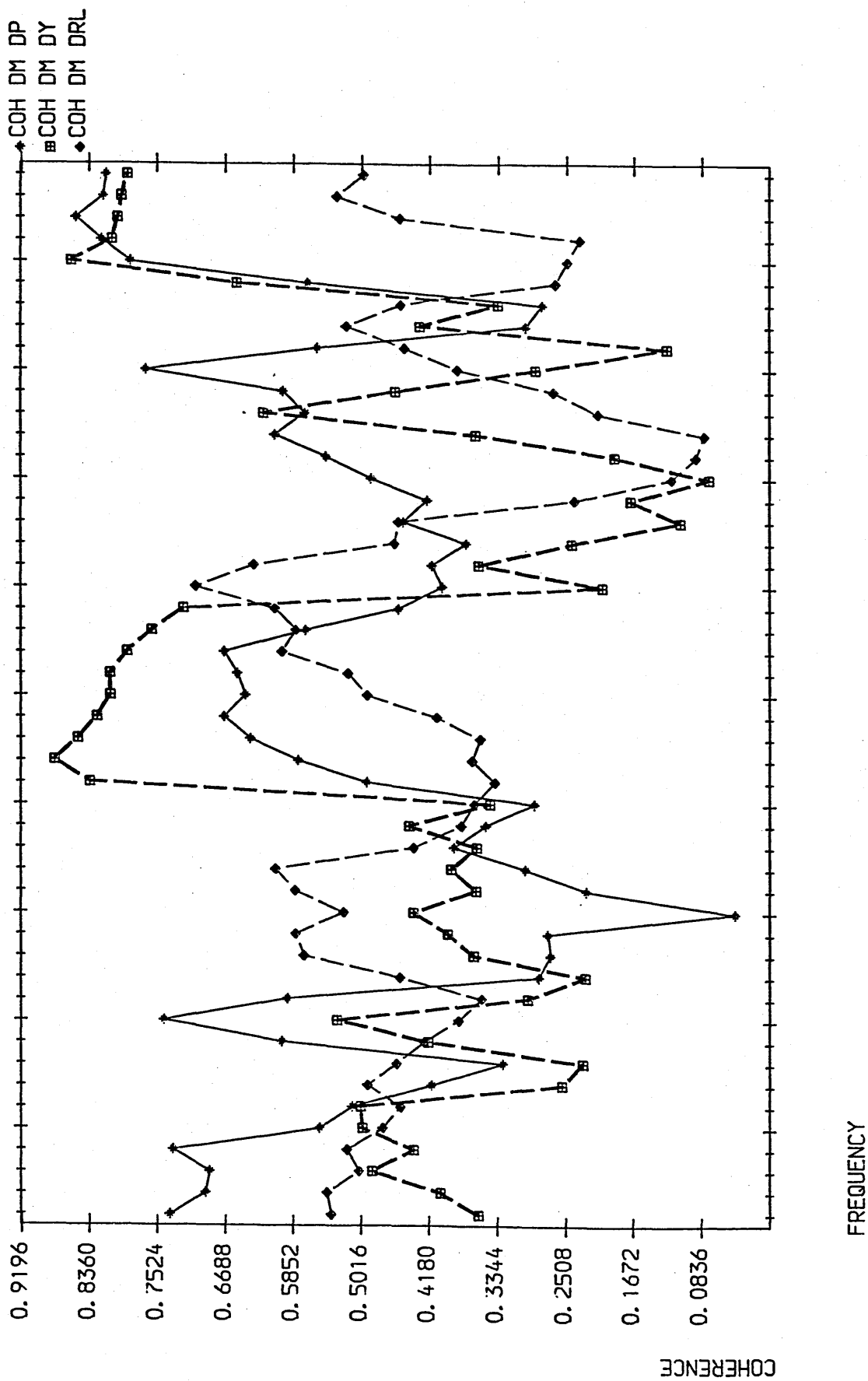
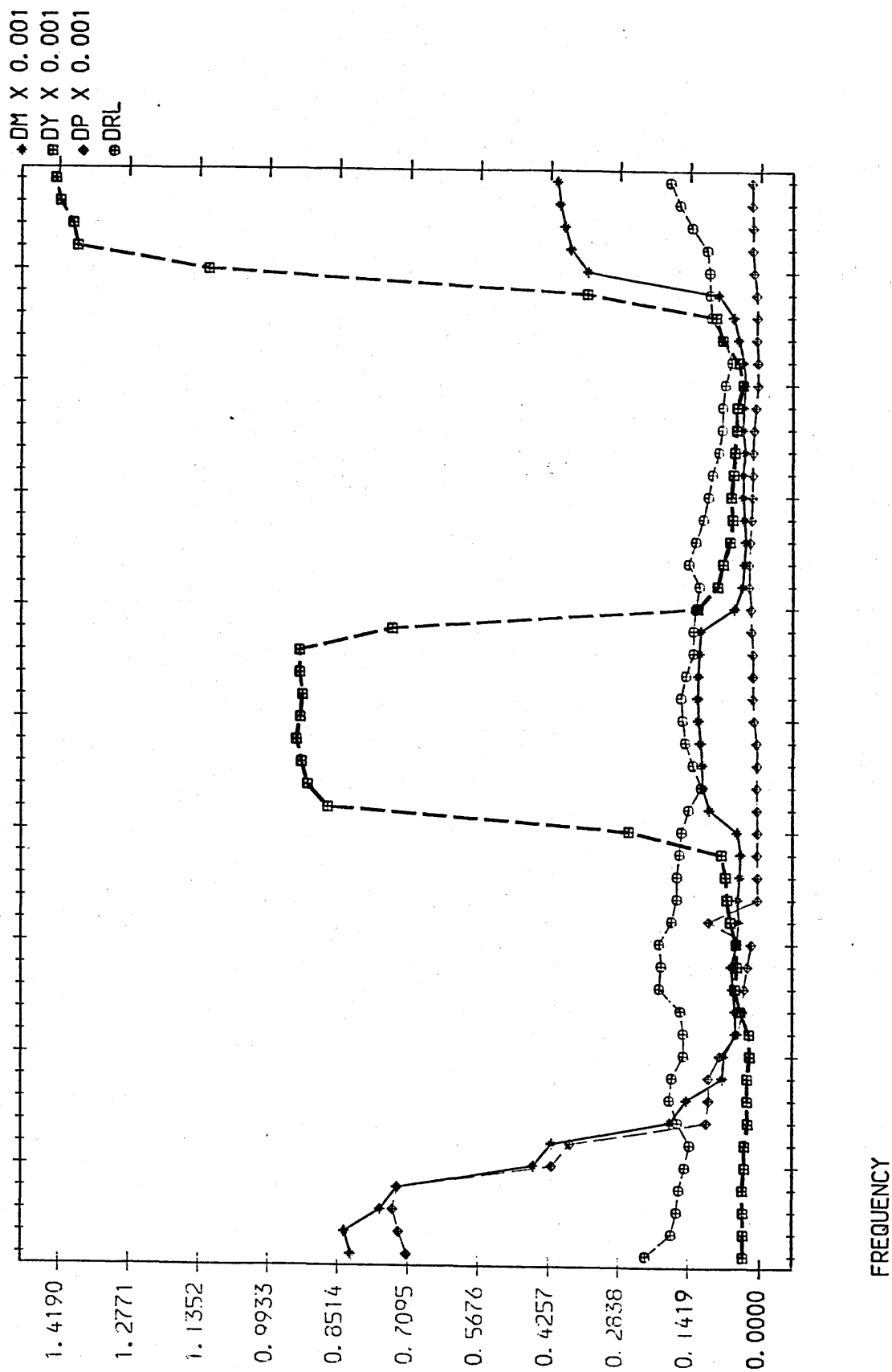


FIGURE 2.7.

ESTIMATED SPECTRA OF SERIES



## CHAPTER 2

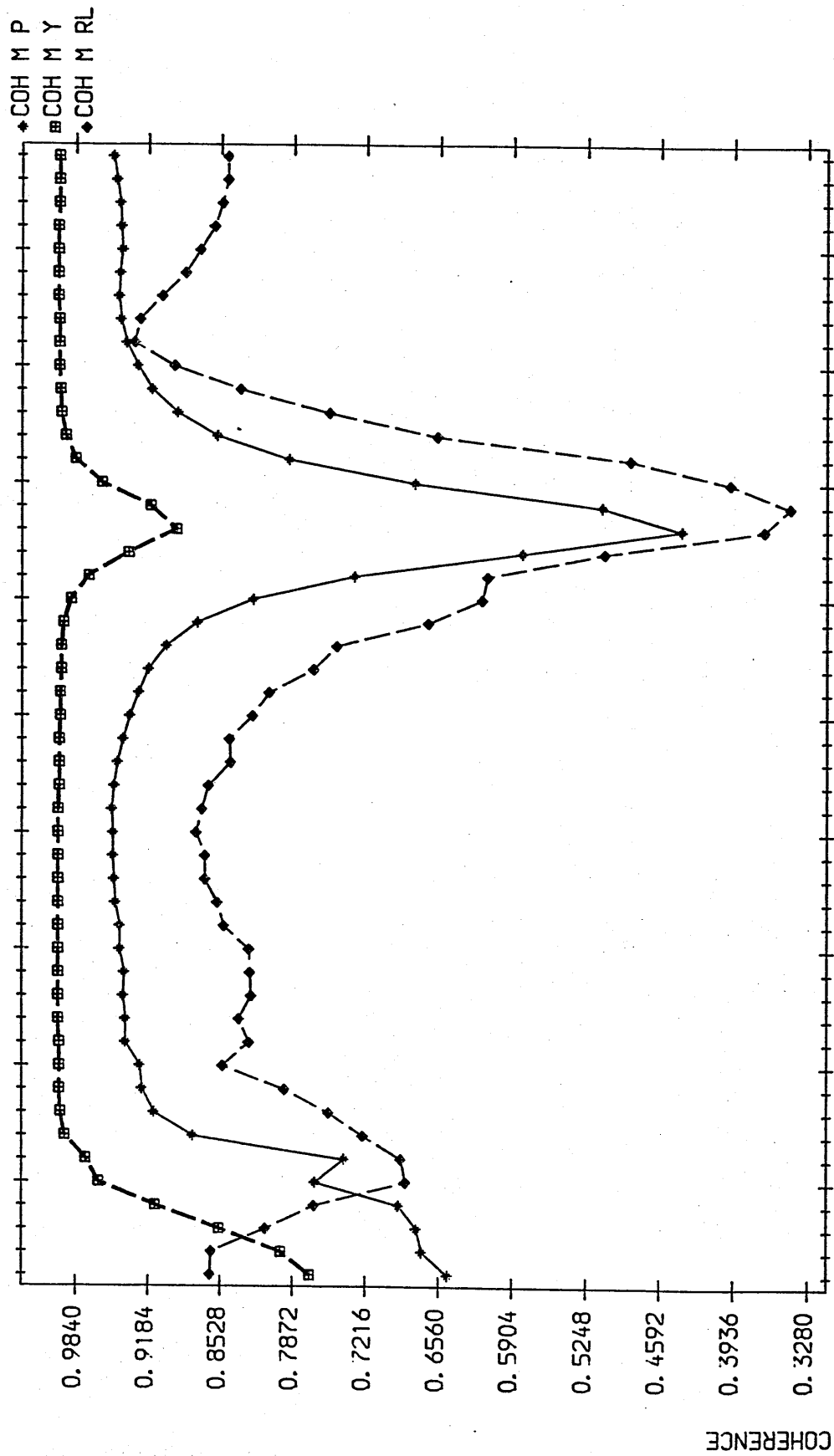
but low with respect to  $\Delta R^1$ . This may reflect some of the difficulties we had in our cointegration tests with regard to the interest rate term.

In Graph 2.8 we have illustrated the coherence between  $m$  and  $y$ ,  $p$ , and  $R^1$  in levels. There are difficulties in interpreting this graph, due to the non-stationary nature of these series, as theoretically the spectrum at zero frequency is infinite. Nevertheless, as a rough guide, it is interesting to note that  $m$  displays a very high coherence with respect to all other three series at all frequencies, though once more the seasonal pattern is apparent<sup>28</sup>. Overall, however, it would be fair to say that the frequency domain analysis cannot offer further insights into the cointegration aspects of the demand for money relationship other than those offered by our conventional time domain analysis of the series. The main problem is the absence of a formal testing procedure with regard to these results.

In the next chapter we shall use some of the above results on cointegration and error-correction to present some estimates of the demand for money ( $M3$ ) in the UK using different approaches. Before doing so, however, we shall present some cointegration results on the demand for money in Italy, to provide a point of contrast with the above UK results. This leads to further questions regarding the Engle-Granger two-step approach.

FIGURE 2.8.

COHERENCE OF M WITH P, Y, AND RL



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### 2.3.5 A Contrast: Cointegration and the Demand for Money in Italy

Again as in the case of the UK data, we begin with an analysis of the time series properties of the individual variables used in a proposed model of the demand for money in Italy. The money stock definition to be modelled is the M2 definition (broad money) which has been the main money stock variable under the monetary authorities' scrutiny since the late 1970s. In addition we use GDP at constant prices for the real income variable, and the GDP deflator for prices. The interest rates used include an own rate of interest (the post-tax average return on M2),  $R^S$ , and a weighted yield on alternative assets (including Buoni Ordinari del Tesoro (BOT), Certificati di Credito del Tesoro (CCT) and other bonds),  $R^1$ . The data used was obtained from the most recent version of the Banca d'Italia econometric model of the Italian Economy and from ISCO (see Caranza, Micossi and Villani, 1983, Banca d'Italia, 1986), and is seasonally unadjusted. More details on the data used may be obtained from the study by Muscatelli and Papi (1988). To some extent the conclusions presented here draw on our joint study. The sample period for this data set is 1960(1)-1986(2), and in all the regressions reported below the full sample is used. Two problems should be highlighted with regard to the data. First, the series for real income have been updated to take into account recent revisions in the Italian national income accounts and are to be taken as provisional. Secondly, the reason for the use of a



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weighted average yield is institutional, and reflects the fact that after 1976 short-term BOTs were regarded as the best substitutes for M2, whilst before 1976 the best alternative option were CCTs.

We first attempted to establish whether all variables involved are  $I(1)$ . The usual test statistics are provided in Table 2.4. The tests again indicate that most of the variables are  $I(1)$ , the only exception to this being the price level, which showed some sign of being integrated of order higher than one. Only the CRDW statistic indicated stationarity for  $\Delta p$ . This is potentially a serious problem, because it renders our search for cointegration pointless. Furthermore, the tests for seasonal integration reported in Table 2.4 proved no more successful for any of the variables.

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Table 2.4

<u>Variable</u>	<u>DF</u>	<u>ADF</u>	<u>CRDW</u>
m	0.75	1.22	0.0033
y	6.72	( - )	0.0048
p	16.97	1.25	0.0021
$R^d$	-0.47	-0.46	0.1054
(m-p)	2.27	0.88	0.02
$\Delta m$	-7.46(*)	-4.03(*)	2.66(*)
$\Delta y$	-5.68(*)	( - )	1.42(*)
$\Delta p$	-2.30	-1.53	0.65(*)
$\Delta R^d$	-10.31(*)	( - )	2.03(*)
$\Delta(m-p)$	-12.88(*)	-2.71	2.61(*)
$\Delta_4 m$	-1.72	-1.70	0.14
$\Delta_4 p$	-1.25	-1.12	0.07
$\Delta_4 y$	-3.03	-3.10	0.34(*)
$\Delta_4 R^d$	-3.26	-2.79	0.43(*)

where (\*) denotes that the statistic rejects the null hypothesis at the 5% level. No value is reported for the ADF statistic (-), where the DF statistic was taken to be the most appropriate of the two tests. This table is reproduced from Muscatelli and Papi (1988).

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Notwithstanding these results, partly due to curiosity, we nevertheless attempted to estimate a cointegration equation for the demand for money, and this yielded the unexpected result that the null hypothesis of no cointegration was rejected given the size of the CRDW, DF, and ADF statistics:

$$m = -12.46 + 0.706p + 1.885y + 0.046R^S - 0.029R^L \quad (2.37)$$

(0.453) (0.019) (0.044) (0.008) (0.006)

$$R^2 = 0.998 \quad CRDW = 0.861 \quad \sigma = 0.053 \quad DF = -5.31 \quad ADF = -5.00$$

Although the size of the price elasticity seems rather implausible, the value of the  $R^2$  statistic is sufficiently large, and all the signs of the long-run multipliers conform to what one would expect in theory.

The question remains as to why one should find these variables to be  $CI(1,1)$  given that some of the tests reported indicated that  $p$  may not be  $I(1)$ . One possible reason for the success of (2.37) may be gauged by checking whether  $(m-p)$  is  $I(1)$ , using an ADF test with two lags:

$$\Delta(m - p) = 0.006\Delta(m - p)_{t-1} - 0.002\Delta(m - p)_{t-2} + 0.001(m - p)_{t-1}$$

(0.007) (0.007) (0.0006)

$$R^2 = 0.071 \quad DW = 2.593 \quad \sigma = 0.044 \quad (2.38)$$

$$\Delta^2(m - p) = 0.001\Delta^2(m - p)_{t-1} - 0.001\Delta^2(m - p)_{t-2} - 0.99\Delta(m - p)_{t-1}$$

(0.010) (0.007) (0.011)

$$R^2 = 0.995 \quad DW = 2.45 \quad \sigma = 0.045 \quad (2.39)$$

These results are corroborated by those obtained by Muscatelli and Papi (1988) using a variety of statistics (these are reported

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in Table 2.4 where more lags are introduced in the regression run to obtain the ADF test) and seem to confirm that  $(m - p)$  is  $I(1)$ , and hence the explanation for the success of the cointegration equation probably lies here:  $(m - p)$  may be cointegrated with the other two variables. The explanation for this result probably derives from Granger's (1983) observation that a non-integer degree of integration may be relevant, and the problem in classifying series as  $I(0)$ ,  $I(1)$  or  $I(2)$  is compounded by the low power of the unit root tests available to us. Furthermore, though many of the results on cointegration focus on difference-stationarity, there is no reason that some series may not in fact be trend-stationary (see Nelson and Plosser, 1982)<sup>29</sup>. Perhaps under the circumstances it is wise not to put too much weight on tests of whether individual series are  $I(1)$ , and we should focus mainly on the results of the cointegration equation.

Even so, as we saw above, the power of our cointegration tests is also limited, giving contrasting results in some cases. These problems also arise with respect to the Italian data. For instance, one may also obtain a successful cointegration equation for Italy over the period 1970(1)-1985(2) using slightly different interest rate data to that used by Muscatelli and Papi (1988):

$$m = -0.554 + 0.731y + 0.954p + 0.052R^S - 0.025R^L \quad (2.40)$$

$$(1.967) \quad (0.218) \quad (0.044) \quad (0.014) \quad (0.008)$$

$$R^2 = 0.990 \quad CRDW = 0.909 \quad \sigma = 0.0781 \quad DF = -4.24 \quad ADF = -3.05$$

However, these results are confounded by the second stage of

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the Engle-Granger procedure, which yields an ECM with a negative but insignificant estimated coefficient. This result is not improved by restricting the initial, overparameterised, equation. The t-value for the ECM term never exceeds -1.4 at any time in the specification process. On the other hand, inverting the equation, and estimating a dynamic equation for  $\Delta p$  yields a significant negative coefficient on the ECM term at the outset (a t-value of -2.73 was obtained). This seems to imply reverse causality in the relationship between  $m$  and  $p$  (see Engle and Granger, 1987). However, paradoxically, a significant ECM is found by Muscatelli and Papi (1988) over a different sample period with the first set of data (1961(3)-1986(2)), implying that some doubts remain about the robustness of the cointegration results.

Furthermore, as in the UK case, the significance of the ECM term could be improved dramatically by changing the lag with which the interest rate enters the cointegrating equation. In terms of cointegration theory, we saw in section two that such a finite-length filter should not alter the cointegration properties of the vector of variables. We shall return to this theme in Chapter 3, when we examine different approaches to dynamic modelling, and where we shall argue that such lags in the ECM term may not be an implausible feature of a dynamic model. In terms of our frequency domain analysis, it is certainly possible that the use of a lag filter may affect the properties of the

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cross-spectrum between the money stock and the interest rate.

To conclude this section, it seems that cointegration tests have to be applied and interpreted with care. This in turn throws some doubt on the Engle-Granger procedure which relies very heavily on the cointegrating equation for its long-run properties. However, cointegration theory has the benefit of shedding additional light on the whole practice of dynamic model estimation, and may in some cases provide pointers to the failure of certain equations. We have also touched on whether the literature on cointegration and transformed equations leads to alternative model selection procedures to the standard 'general-to-specific' approach applied to a standard ADL model. We explore this theme in more detail in Chapter 3, when we evaluate the relative merits of these various (and related) approaches to modelling.

Lastly in this chapter, however, we have to confront one additional issue, which has thrown up a considerable literature in the 1980s, and which has not so far been surveyed in this thesis.

### SECTION FOUR: STEADY-STATE GROWTH SOLUTIONS AND THE DEMAND FOR MONEY

The 'problem' associated with steady-state growth solutions in the case of the demand for money was first confronted by Currie (1981). Earlier in this chapter we illustrated steady-state solutions in terms of a static long-run where  $m$ ,  $p$ ,  $y$ , and

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$R^1$  returned to a constant level. (In some cases, e.g. equation (2.4) relating to Hendry (1985), we allowed  $p$  to grow at a constant rate in steady state and solved for the long-run inflation effect in the demand for money). However, as Currie (1981) points out, a steady-state with no growth is scarcely believable in the real economic world: it is far more interesting to consider a dynamic long run in which there is steady growth. In this case, some interesting results emerge from the dynamic models estimated using the 'general-to-specific' method, due to the usual practice of reparameterising these models in terms of levels and differences.

It is useful to illustrate this 'problem' by examining an existing model. Following Currie (1981), let us examine the final equation presented by Hendry and Mizon (1978) for the demand for  $m3$ :

$$\begin{aligned} \Delta(m - p)_t = & 1.61 + 0.21\Delta y_t + 0.81\Delta r_t + 0.26\Delta(m - p)_{t-1} - \\ & (0.65) \quad (0.09) \quad (0.31) \quad (0.12) \\ & 0.40\Delta p_t - 0.23(m - p - y)_{t-1} - 0.61r_{t-4} + 0.14y_{t-4} \\ & (0.15) \quad (0.05) \quad (0.21) \quad (0.04) \end{aligned}$$

$$R^2 = 0.69 \quad \sigma = 0.0091 \quad (2.41)$$

where  $r_t \equiv \log(1 + R_t)$ . Next, instead of assuming that  $x_t = x_{t-1} = \bar{x}$  in steady-state equilibrium, consider a dynamic equilibrium where  $\Delta m = \pi_0$ ,  $\Delta y = \pi_1$ ,  $\Delta p = \pi_2$ . We do not consider a steady-state growth path for the interest rate for economic reasons. This suggest a long-run dynamic steady-state solution of the

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type:

$$(m - p) = (7.0 - 6.1\pi_1 - 1.7\pi_2) + 1.6y - 2.6r \quad (2.42)$$

The problem, as Currie (1981) points out, is the interpretation of an equation such as (2.42). It suggests a negative relationship between money demand and the inflation rate which may be justified in terms of a 'flight from money' argument (see Friedman, 1956, Cagan, 1956). However, the negative effect on the demand for money of real income is far more difficult to explain in terms of economic theory. Hendry and Mizon themselves offer no explanation. Nor is this phenomenon typical of only one particular study. As Currie points out, the NIESR model at that time also had a negative real income growth effect (see Savage, 1978), as did Coghlan's (1978) model of M1 demand.

Before turning to examine some explanations of this phenomenon and the possible implications for modelling, we have to turn our attention to some necessary preliminary results regarding difference equations in general.

Currie begins by considering a general long-run equilibrium model of the type:

$$Y = a_0 + \sum_{i=1}^k a_i X_i \quad (2.43)$$

Given that the variables  $Y$  and  $X_i$  ( $i = 1, \dots, k$ ) follow a dynamic adjustment of the general ADL type:

$$Y_t = \alpha_0 + \sum_{j=0}^n (\sum_{i=1}^k \alpha_{ij} X_i) + \sum_{j=1}^n \beta_j Y_{t-j} \quad (2.44)$$

The static long-run equilibrium parameters are given, as usual by:  $a_0 = \alpha_0 / (1 - \sum_{j=1}^n \beta_j)$  and  $a_i = \sum_{j=0}^n \alpha_{ij} / (1 - \sum_{j=1}^n \beta_j)$ .



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Next, assume that

$$(1 - \sum_{j=1}^n \beta_j) \pi_0 = \sum_{i=1}^k (\sum_{j=0}^n \alpha_{ij}) \pi_i \quad (2.45)$$

where we define  $\pi_0 \equiv \Delta Y$  and  $\pi_i \equiv \Delta X_i$ . This assumes that all the variables follow given rates of growth given by the  $\pi_i$  terms, and that these growth rates are connected in a similar way to the levels. That is, in dynamic equilibrium the relationship between the rates of growth matches that between the levels in (2.43):

$$\pi_0 = \sum_{i=1}^k a_i \pi_i \quad (2.45')$$

We may then obtain the long-run dynamic relationship between  $Y$  and the  $X_i$  variables:

$$Y = a_0 + \sum_{i=1}^k a_i X_i - \sum_{i=1}^k (1/(1 - \sum_{j=1}^n \beta_{ij})) \{ (\sum_{j=0}^n j \alpha_{ij} + a_i \sum_{j=1}^k j \beta_{ij}) \pi_i \} \quad (2.46)$$

Equation (2.46) suggests that, in this case, in the long-run steady-state, the dependent variables also depends on the rate of growth of the driving variables, the  $X_i$ . This is the so-called 'problem' noted above in the case of the demand for money.

One way to resolve this apparent difficulty is by adopting Currie's suggestion that one could test whether the parameters on the growth rates in (2.46) are significant, by testing the implied non-linear test in terms of the basic estimated parameters of the ADL equation. In this case, one could constrain the resulting dynamic equation such that unwanted growth effects do not appear. It is stressed by Currie that such restrictions must not be imposed without testing their validity, because otherwise the resulting dynamic properties of the model will be

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seriously misspecified. However, the problem with this suggested approach is that it involves testing non-linear restrictions, and also that, as Patterson and Ryding (1984) point out, the restricted dynamic equations may have considerably different dynamic properties (judged by examining the roots of the difference equation) compared with the unrestricted model.

A second approach is to find some theoretical explanation for the growth effects for income. One could argue, for instance, that the negative effect of income growth on money demand may be due to technical developments leading to greater economy in money holdings, and that income growth is really a proxy for technical developments in the financial system in the case of the demand for money function.

A third point to note is that, the types of dynamic steady-state solutions imposed on single-equation models are those implied by the static solution to the single equation. Thus, in (2.45') we assumed that  $\pi_0 = \sum_{i=1}^k a_i \pi_i$ . This will not necessarily be the case in practice given that there are parts of the economic system not considered in the single equation model. In practice the appropriate dynamic solution to be considered should be consistent with the multipliers of a full macroeconomic model, and focusing on the problems with dynamic steady-state solutions may be stretching these single-equation models too far.

We conclude this section by noting that direct estimates of the mean lag may be obtained by an appropriate transformation of

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the ADL model. We have shown above that one may obtain direct estimates of the long-run multipliers of the ADL model from a transformed equation model, and this result is extended by Breusch and Wickens (1987) who suggest alternative transformations suitable to either directly estimate mean lags between the dependent and an independent variable, or the long-run multipliers when the steady-state solution contains both levels and rates of growth of the independent variables (e.g. the effects of the price level and the rate of inflation in the long run demand for money).

### SECTION FIVE: CONCLUSIONS

We conclude this chapter by summarising some of its main conclusions, and by mapping out the direction of the research programme which we will follow in Chapter 3. So far we have surveyed the recent literature on dynamic modelling, and illustrated the links between error correction mechanisms which are often obtained in dynamic models and the literature on cointegration. We have also pointed out that the tests for cointegration are not particularly robust, and that the results presented for broad money aggregates in the UK and Italy are somewhat mixed. However, there is some evidence to suggest that a long-run relationship exists between the M3 definition of money and the price level, real income and the interest rate in the UK. The main problem is that the direct estimation of long-run elasticities via the cointegration equation does yield consistent

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estimates, but this may not be relevant because of the relatively small samples involved. In these circumstances ignoring the short-run dynamics may cause us to obtain distorted estimates of the long-run elasticities. The significance of these distortions will be analysed in Chapter 3. On the other hand, there are alternative methods to estimate the long-run multipliers at an early stage of the specification search through the use of transformed regression models. Overall, therefore, the researcher has several options at his disposal if he wishes to obtain a dynamic model of the demand for money:

(i) He may directly estimate the cointegration equation, and impose these long-run elasticities on the dynamic equation at the outset by incorporating an error-correction term which consists of the lagged residuals of the cointegration equation. He then undertakes a specification search on the short-run dynamics, but his final equation will clearly have the long-run properties implied by the cointegration equation estimated at the first-stage of the modelling process.

(ii) The classic 'general-to-specific' approach followed by David Hendry and his associates involves undertaking a specification search on an ADL model, reparameterising as one goes along. The long-run properties of the model may be gauged at any point by solving for the steady state, but no direct estimates for the standard errors of the long-run elasticities may be obtained. In general, the researcher only checks these steady-state properties

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at the end, and they may change during the specification search. This method may also prove advantageous when seasonally unadjusted data is used, given the fixed error-correction structure of method (i) (see Engle, 1987).

(iii) An intermediate procedure to (i) and (ii) is to estimate a transformed model which enables the researcher to 'keep an eye' on the long-run multipliers at every stage of the specification search. Standard errors for these long-run effects are estimated directly.

Within these three broad categories, further alternatives may be identified from some of the results obtained above. First, as we saw above, the interest rate may not enter in the ECM with the same lag as the other variables in (i). Second, in (iii), the long-run solution may be imposed at the outset by constructing an ECM term, or the variables capturing the long-run effects may be left on their own. These alternatives will also be explored in detail in Chapter 3, where a detailed analysis of alternative routes to building a model of M3 demand in the UK is presented.

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### Footnotes to Chapter 2

(1) Essentially the issue here is whether there are common factors in the lag polynomials (see Hendry and Mizon, 1978). See Chapter 1 and Hendry and Mizon (1978) for more details of the procedure followed in the COMFAC analysis of autoregressive errors proposed by Sargan (1975).

(2) It is worth stressing that at no stage in all of this is the researcher required to assume that his model is in any way an accurate representation of the data generation process which remains unobservable. In this context a strict distinction must be drawn between the model (the statistical generating mechanism) and the data generation process.

(3) For further details on the various concepts of exogeneity, see Engle et al. (1983).

(4) In Chapters 4 and 5 we shall analyse some attempts to link empirical dynamic models with theoretical developments in dynamic optimisation. It is worthwhile to point out here, however, that these attempts do not, by any means, provide a full integration of economic theory and dynamics.

(5) That is, we investigate an equilibrium where  $m_t = m_{t-1} = \bar{m}$ ,  $p_t = p_{t-1} = \bar{p}$ ,  $y_t = y_{t-1} = \bar{y}$ , and  $R_t = R_{t-1} = \bar{R}$ . This may appear unrealistic in a real world where these four variables are non-stationary, and the issue of dynamic steady-state paths is taken up in more detail in section 4 of this chapter.

(6) See Trundle (1982) for a rationale for the use of the level

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rather than the logarithm of the interest rate in our studies.

(7) There are a number of surveys of Hendry's econometric method from other authors' perspectives. To some extent these methods are already described in basic texts (see Harvey, 1981a, Spanos, 1986). For other survey papers, see Gilbert (1986), Pagan (1987).

(8) Encompassing in fact constitutes an approach to econometric modelling, embodying a number of nested and non-nested tests. For further details see Mizon (1984).

(9) There are some examples of estimations which present a detailed account of the simplification search followed (see for example Hendry and Mizon, 1978, McAleer et al., 1985, Molana, 1987). These studies are usually the exception though, partly because of the difficulties involved in assessing the significance level of sequential F-tests (see footnote (10) below), and partly because of the large volume of description which would be required for each model. In this thesis we follow the usual inadequate compromise of partly describing the simplification search followed where possible. In this sense, it follows Hendry's prescriptions for modelling (see Hendry and Mizon, 1985). Pagan's objection to this approach seems to be that:

"....Hendry's attitude seems to be that how a final model is derived is largely irrelevant; it is either useful or not useful, and that characteristic is independent of whether it comes purely from whimsy, some precise theory, or a very structured search..."

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Pagan (1987), p.7

(10) The 'general-to-specific' search procedure would seem to have certain optimal properties with regard to power (see Anderson, 1971, Harvey, 1981a) and the incremental test statistics are independent in large samples (for an example see Harvey, 1981a, p.185.) On the other hand, it is not always made clear that the true significance levels of a simplification search following a nested sequence are not easy to compute. For instance, if one tests two nested hypotheses about a general model, each with a significance level of 5%, the nominal significance level of the most restricted model against the least restricted will not be 5%. Although there are ways to compute these significance levels (see Mizon, 1977), some researchers have attempted to criticise the simplification search procedure on this basis (see Hill, 1986).

(11) See Hendry (1985). This is a strange, hybrid case, since there is no reason to believe that the price level is the only growing variable of the four under scrutiny. In section 4 below we will present a more detailed analysis of dynamic steady-state paths.

(12) The 'rationality' of ECM will become apparent in Chapters 4 and 5, where we will show that ECMs may on fact be generated from (forward-looking) dynamic optimisation exercises (see Hendry et al., 1984, Nickell, 1985).

(13) By 'sensible' we usually mean that the price elasticity of



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the demand for money is unity. It would seem strange if a value other than one were found for any particular model. Economic theory may also have some other things to say regarding income and interest-elasticities, as we saw in Chapter 1, but these statements tend to be more controversial.

(14) These methods have always been used in engineering and physics, and its applications to business cycle theory has more recently been recognised in economics (see Sargent, 1979, Vines et al., 1983, Maciejowski and Vines, 1983).

(15) Strictly speaking, non-stationarity requires  $d \geq \frac{1}{2}$  (see Granger, 1983), but we only consider cases where  $d$  is an integer here.

(16) see for example Campbell and Shiller (1986).

(17) Viceversa the detection of cointegration does not have any implications for causality in models, except for the fact that causality will at least run one way (see Granger and Weiss, 1983, Engle and Granger, 1987). We shall see a possible example of this in the Italian data analysed in section 3 below.

(18) As we shall see further on, though there are advantages with (2.21) in that OLS may be used for both stages of the estimation process, there may be disadvantages in that the lag distribution of the original ADL has been 'scrambled' somewhat. Other reparameterisations may avoid this nuisance.

(19) The lagged residuals of the cointegration equation may be as the lagged ECM term, although as we shall see in Chapter 3,

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there are some problems in doing this if one wishes to do some ex ante forecasting with the model.

(20) This can have serious repercussions for the second stage of the estimation process, since these long-run multipliers are imposed on the model at the outset. The intuitive reason for wanting a high  $R^2$  may be shown in the bivariate case as follows: the coefficient of multiple correlation is consistent to unity will be consistent to unity under cointegration, and this reflects the fact that the product of the cointegration parameter and the inverse of the cointegration parameter will equal unity. Furthermore, Banerjee et al., 1986 show that  $R^2$  converges to unity at the same rate as the bias (see Dolado and Jenkinson, 1986 for more details).

(21) For an illustration of the difference between the concepts of difference-stationarity and trend-stationarity see Nelson and Plosser (1982).

(22) Their results are however restricted to the particular case analysed (cf. the results of Banerjee et al., 1986 described above).

(23) Only limited attention has been given so far to the nonlinear case. For an example, see Escribano (1986).

(24) Again one must stress the problems involved in using these low-power tests, given that one must really know the type of non-stationarity one is testing against (see Dickey and Fuller, 1981, West, 1986, Dolado and Jenkinson, 1987). As we shall see in the

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case of Italy below, further problems arise.

(25) See Engle and Granger (1987) for some results on the power of these tests.

(26) This corresponds to the sort of pattern that one obtains in the time domain from the correlogram of an observed sample of realizations from a white noise series.

(27) Again, one must stress that there are problems with estimated spectra of non-stationary variables. Although, some evidence may still be obtained from them because the estimates will not correspond to the theoretical spectrum. This is similar to the correlogram one obtains from a sample obtained from a non-stationary series.

(28) See Sargent (1978) and Harvey (1981b) for an example of the effects on the spectrum of filters and on the spectra of seasonal series.

(29) In fact, as we saw above, the results of Wickens and Breusch (1987) on cointegration specifically relate to trend-stationary variables.

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### CHAPTER 3: MODELLING THE DEMAND FOR M3 IN THE UK USING FEEDBACK-ONLY MODELS

As we pointed out in Chapter 2, there are several approaches which one may take in estimating a dynamic model of the demand for money. In this chapter we put these model selection procedures to the test by constructing a model of the demand for M3 in the United Kingdom. The data definitions used are the same as those employed in the estimated equations reported in Chapter 2. Sections one to three will outline the different basic procedures adopted, within which different variants of each procedure will be developed. The final models obtained using each procedure will then be compared in section four. Section five concludes this chapter.

#### SECTION ONE: ESTIMATING THE DEMAND FOR MONEY USING THE ENGLE-GRANGER TWO-STAGE PROCEDURE

In the previous chapter we examined the time series properties of the individual series, and also tested whether the money stock, real income, price level, and interest rate definitions used are cointegrated. It should be recalled that we reached the following conclusions.

First, the variables used in our models do not seem to be seasonally integrated (with the possible exception of real income) despite the fact that seasonally unadjusted data is being used in our study, though most of them appear to be  $I(1)$  in the case of the UK data.

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Secondly, the tests used to determine whether the variables are  $CI(1,1)$  were not wholly conclusive. Two approaches were used. The first, which consisted of estimating a conventional cointegrating equation led us not to reject the null hypothesis of non-cointegration, except over a restricted sample period. The second approach, where a lagged interest rate term was used in the cointegrating equation was found to be more successful. This procedure was justified on the grounds that the Engle-Granger two-stage approach forces the researcher to decide on the vector of cointegrated variables at the outset, thus possibly omitting important long-run effects, in addition to the bias caused by the omission of the dynamics in the static first-stage equation. Furthermore, whilst the variables are not all seasonally integrated (in the sense that seasonality is a dominant factor), seasonality may still be present, and it could be that a simple departure from the Engle-Granger procedure may offer more satisfactory results.

Thirdly, we noted that the point estimates of the long-run elasticities obtained using the cointegrating equation varied considerably between different sub-sample periods, leading us to conclude that perhaps parameter constancy may be a problem in these models.

These considerations lead us to construct models for the demand for M3 in the UK using the following variants of the Engle-Granger procedure. First, we adopt the classic two-step

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procedure outlined in Chapter 2, with current values of all the variables used in the cointegrating equation, and where the second stage consists of estimating a model in first differences plus an error-correction term obtained from the first stage. Secondly, we estimate a variant of this method, using an ECM term obtained from a cointegrating equation with a lagged interest rate variable. Thirdly, we estimate the cointegrating equation using recursive least squares, and use the residuals obtained from this procedure to form the ECM term in the second stage. The point of this variant is that it allows some variation in the long-run parameters of the model, which may be an important factor. We now turn to each of these three variants of the two-stage procedure in turn.

### 3.1.1. The Standard Two-Stage Procedure

Let us first recall the cointegrating regression estimated over the sample period 1963(1)-1984(2) from Chapter 2, where the last 8 quarters (1984(3)-1986(2)) have been retained to assess the ex ante performance of the models estimated in this chapter:

$$m_t = 0.996 + 0.953p_t + 0.931y_t + 0.005R_t^1 \quad (3.1)$$

where the standard errors are not reported because of the bias present (see Engle and Granger, 1987). Note that, as we pointed out in the previous chapter, the sign on the interest rate variable is perverse. This is one of the disadvantages of this procedure, and we shall return to this issue in the last section of this chapter.

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Using these estimates for the long-run elasticities, we constructed an ECM term<sup>1</sup>, which was then used to carry out the second stage of the procedure, using the reparameterised equation outlined in equation (2.21). The most 'general' equation estimated using this method was:

$$\Delta m_t = 0.017 + 0.231\Delta m_{t-1} + 0.319\Delta t_{t-2} + 0.124\Delta m_{t-3} + 0.076\Delta m_{t-4} +$$

(0.012) (0.131) (0.134) (0.135) (0.135)

$$0.041\Delta p_t - 0.134\Delta p_{t-1} - 0.055\Delta p_{t-2} + 0.135\Delta p_{t-3} +$$

(0.213) (0.237) (0.254) (0.215)

$$0.064\Delta p_{t-4} + 0.087\Delta y_t + 0.097\Delta y_{t-1} + 0.095\Delta y_{t-2} +$$

(0.208) (0.127) (0.132) (0.130)

$$0.022\Delta y_{t-3} - 0.047\Delta y_{t-4} + 0.0026\Delta R_t^1 - 0.0011\Delta R_{t-1}^1 -$$

(0.126) (0.113) (0.0026) (0.0027)

$$0.0026\Delta R_{t-2}^1 + 0.0035\Delta R_{t-3}^1 - 0.0012\Delta R_{t-4}^1 - 0.0509Q_1 -$$

(0.0028) (0.0029) (0.0030) (0.0160)

$$0.0036Q_2 - 0.0076Q_3 - 0.057ECM_{t-1}$$

(0.0161) (0.0154) (0.027) (3.2)

$$R^2 = 0.679 \quad \hat{\sigma} = 0.0178 \quad DW = 2.01 \quad Z_1 = 1.93 \quad E_1 = 1.53$$

$$LM(5) = 0.45 \quad ARCH(5) = 0.25 \quad Z_5 = 0.03 \quad E_4 = 0.38$$

$$RESET(1) = 3.09 \quad RESET(2) = 1.53$$

where the  $Q_i$  ( $i = 1, 2, 3$ ) represent seasonal dummies, and the numbers reported in brackets are estimated standard errors.

The diagnostic tests reported for equation (3.2) in addition to the  $R^2$ , Durbin-Watson statistics and the standard error of the equation, are the following:  $Z_1$  is the so-called 'Hendry Forecast Test' (see for example Hendry, 1979, 1983), which is an

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asymptotically valid test for parameter constancy, and is a useful indicator (though not an absolute measure) of the model's ex ante forecasting performance;  $E_1$  is the Chow (1960) test applied over the ex ante forecast subperiod;  $LM(n)$  is the Lagrange Multiplier test for serial correlation in the residuals of lags up to  $n$  (see Godfrey, 1978, Harvey, 1981a);  $ARCH(n)$  is a test for Autoregressive Conditional Heteroscedasticity in the residuals (see Engle, 1982) which is reported in its F-form;  $Z_5$  tests if the residuals originate from a normal distribution (see Jarque and Bera, 1980), where the actual statistic reported is a combined skewness-kurtosis Lagrange Multiplier test which is distributed as a chi-square with 2 degrees of freedom under the null hypothesis of normality in the residuals; The  $RESET(n)$  statistic tests for departures from the assumption of linearity in the structure of the equation by testing the model against an alternative which includes higher powers of the fitted demand for money from the linear model. Thus the  $RESET(n)$  test is an F-test which tests for the inclusion in the model of the fitted values of the dependent variable to the power 2 up to  $n+1$  (see Ramsay, 1974). Finally  $E_4$  tests the model for heteroscedasticity quadratic in the regressors (see White, 1980). Any restrictions to be tested in the models in this thesis will be carried out using the conventional F-test (see Harvey, 1981a, Spanos, 1986), which is distributed as an  $F(r, T-k)$  statistic under the null hypothesis of the validity of the  $r$  restrictions on a general



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model estimated over  $T$  periods with  $k$  regressors. In general the notation used for the above diagnostic tests will be maintained throughout the thesis. Any additional tests employed will be reported in the text as required.

We should note that, as expected in the case of a 'general' equation, the model is overparameterised in the sense that most of the regressors prove to be insignificant, and the model passes all the reported diagnostics. At this point one employs a conventional model selection strategy which moves from the general to the particular. The merits and drawbacks of such a strategy have already been discussed in Chapter 2, and form the subject of numerous reviews (see for instance Harvey 1981a, Spanos, 1986, Pagan, 1987, Gilbert 1986, 1987). It is important to note, furthermore, that the strategy of moving towards a more parsimonious specification has also been advocated by the proponents of the two-stage procedure (see for instance Engle and Granger 1987, Engle *et al.*, 1987). As a last point before proceeding, we should note that the error-correction term is already significant at this early stage of the specification search, and that this is encouraging from the point of view of the cointegration properties of the variables; we should recall in fact that Granger and Weiss (1983) see the significance of the ECM term as one way of testing cointegration in a VAR system.

Using the usual criteria for the evaluation of these models, we found the 'best' restricted model to be the following

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equation:

$$\begin{aligned} \Delta m_t = & 0.029 - 0.059Q1 - 0.009Q2 - 0.023Q3 + 0.245\Delta_3 m_{t-1} + \\ & (0.005) (0.005) (0.005) (0.005) (0.040) \\ & 0.0027(\Delta R_t^1 - \Delta R_{t-2}^1 + \Delta R_{t-3}^1) - 0.062ECM_{t-1} \\ & (0.0014) (0.021) \end{aligned} \quad (3.3)$$

$$R^2 = 0.653 \quad DW = 2.02 \quad \hat{\sigma} = 0.0163 \quad Z_1 = 2.21 \quad E_1 = 1.96$$

$$LM(5) = 0.59 \quad ARCH(5) = 0.10 \quad Z_5 = 0.004 \quad E_4 = 0.69$$

$$RESET(1) = 6.51 (*) \quad RESET(2) = 3.22 (*)$$

where (\*) indicates a test statistic which rejects the null hypothesis at the 5% significance level.

Note that in general the model performs quite adequately in terms of within sample fit (see figure 3.1), and that its forecasting performance is also adequate when assessed via the  $Z_1$  and  $E_1$  statistics. The only three flaws with this model are the following: first, the long-run properties are counter to those suggested by economic theory, given the positive long-run multiplier associated with the interest rate. On these grounds alone we should therefore reject the model. Nevertheless, this represents the 'best' model which we managed to obtain by the conventional Engle-Granger two-stage method (i.e. without resorting to lags in the cointegration equation or other tricks), and we retained it for comparison with the other models in section four. Secondly, this model does not allow for any negative inflation effect, as one would normally expect in demand for money models (see Friedman, 1956, Hendry, 1985). To some

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extent this may be explained by the fact that the dependent variable in this case is the growth in nominal balances. If we had reparameterised the model in terms of real balances at the outset, we would in all probability have obtained a significant negative inflation effect at the end of the day. A third negative feature of (3.3) is that it fails both RESET(n) tests. This may be attributed to some extent to the fact that the model tends to overpredict the actual evolution of monetary growth (see figure 3.2). This is an unambiguously negative outcome, and it must also be borne in mind when comparing this model with the other competing equations in section four.

#### 3.1.2 The Two-Stage Procedure with a 'Lagged' Interest Rate Term.

We now turn to our second variant of the Engle-Granger procedure. This involves the prior estimation of a cointegration term containing a lagged interest rate term. As we saw in Chapter 2, the cointegration equation which performs best in this respect is the equation containing the interest rate lagged by three periods. Recalling the results from Chapter 2, we noted there that both the ADF and DF cointegration tests performed adequately for this definition, and in addition the error-correction term containing  $R_t^1 - 3$  performed best in terms of initial significance in a reparameterised equation such as (2.21). Recall that the cointegration equation yielded the following estimates, over the sample period 1963(1)-1984(2):

$$m_t = -0.823 + 0.962p_t + 1.109y_t - 0.004R_t^1 \quad (3.4)$$

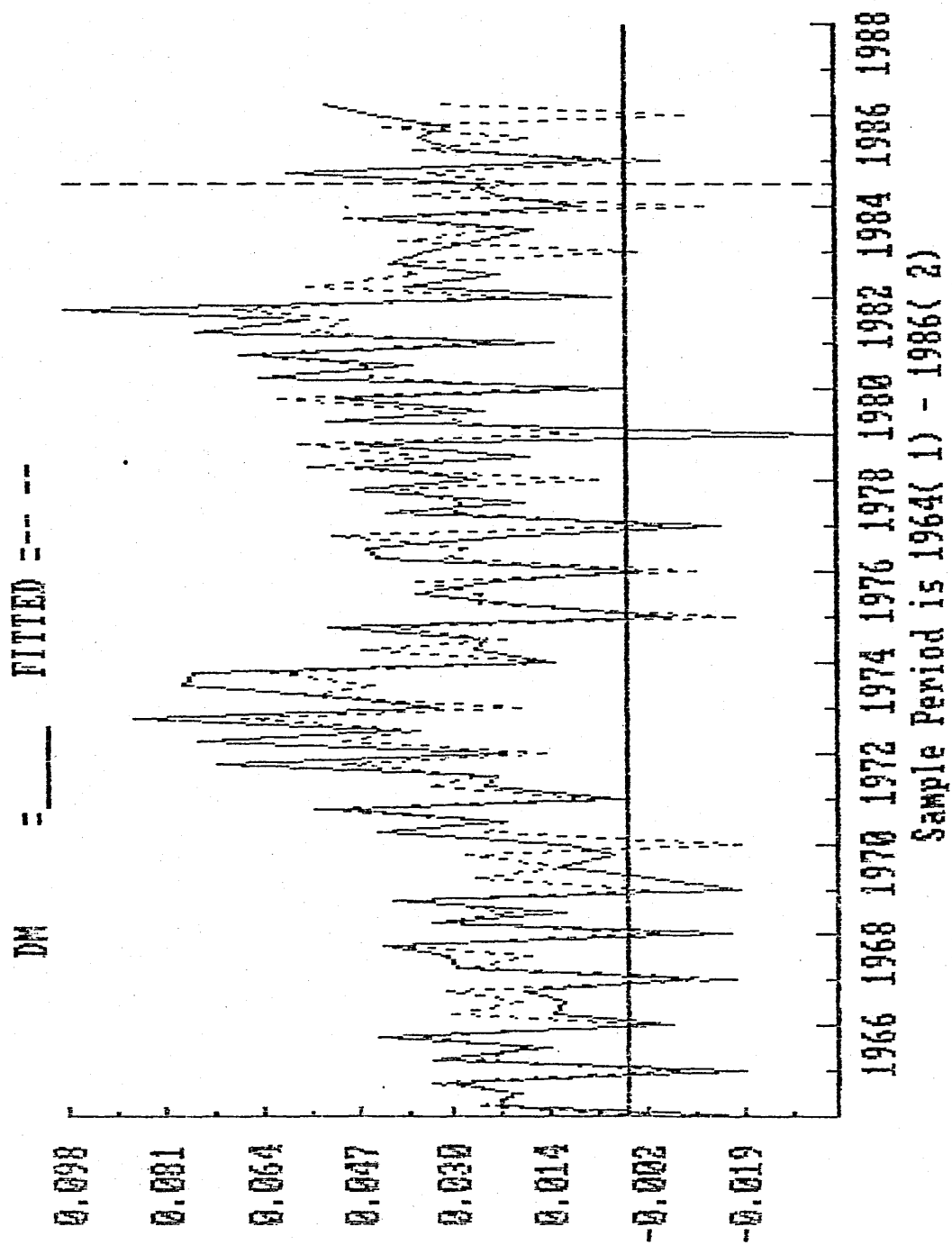


FIGURE 3.1

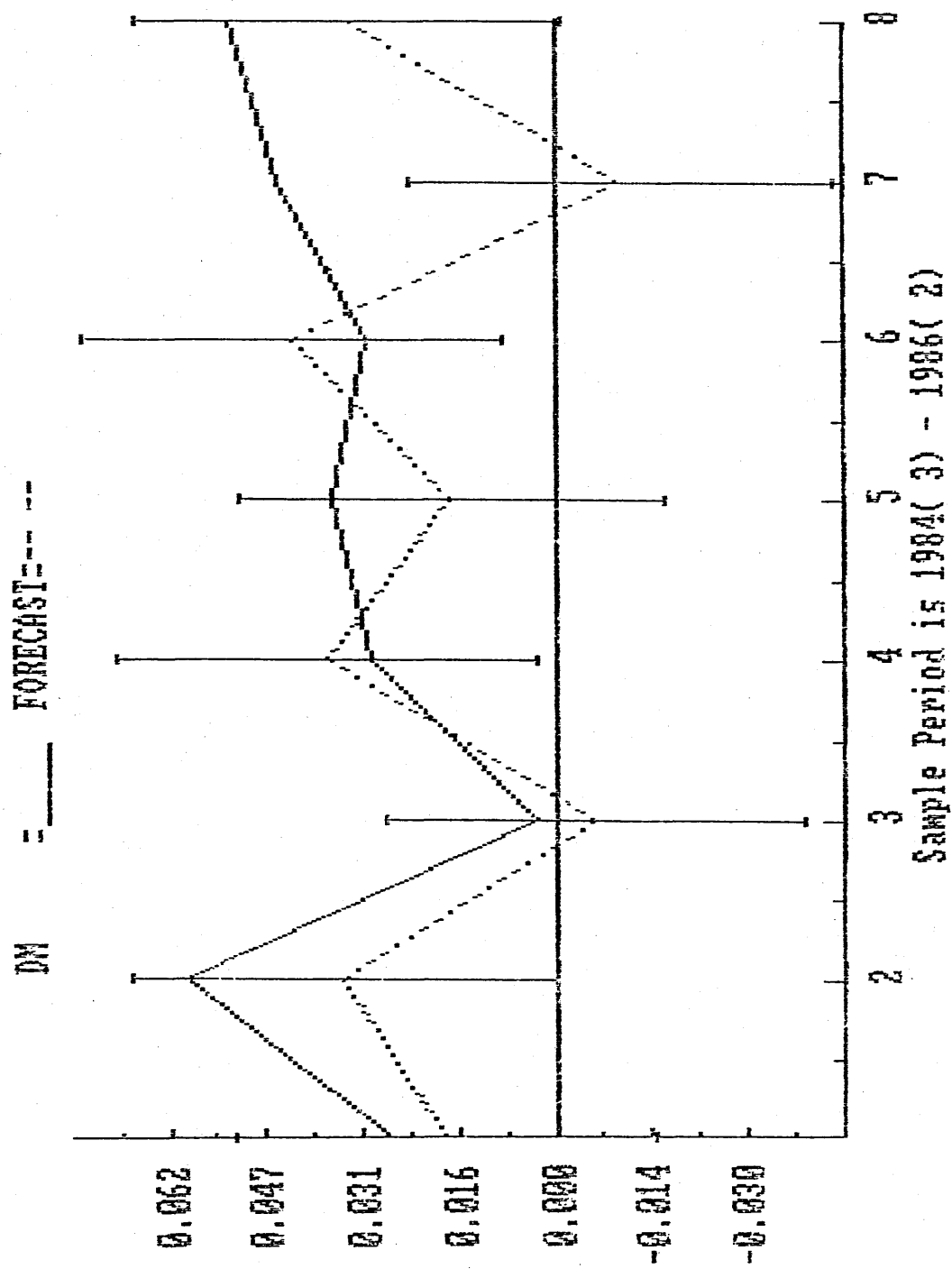


FIGURE 3.2

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where, once more we do not report the standard errors because of the bias present. Note that one advantage of (3.4) over (3.1) is that it yields a point estimate for the long-run interest rate effect on M3 which is negative, conforming to our usual theoretical priors.

Once again we used these estimates to construct an error-correction term to introduce into our general model in first differences. The general model yielded the following estimates over the sample period 1963(1)-1984(2) (with ex ante forecasts reported for the next 8 quarters):

$$\Delta m_t = 0.017 + 0.222\Delta m_{t-1} + 0.317\Delta m_{t-2} + 0.122\Delta m_{t-3} + 0.073\Delta m_{t-4} +$$

(0.012) (0.131) (0.133) (0.134) (0.134)

$$0.145\Delta p_t - 0.133\Delta p_{t-1} - 0.057\Delta p_{t-2} + 0.147\Delta p_{t-3} +$$

(0.212) (0.236) (0.253) (0.214)

$$0.080\Delta p_{t-4} + 0.086\Delta y_t + 0.092\Delta y_{t-1} + 0.094\Delta y_{t-2} +$$

(0.205) (0.127) (0.131) (0.130)

$$0.024\Delta y_{t-3} - 0.044\Delta y_{t-4} + 0.0025\Delta R_t^1 - 0.0017\Delta R_{t-1}^1 -$$

(0.126) (0.113) (0.0026) (0.0026)

$$0.0032\Delta R_{t-2}^1 + 0.0029\Delta R_{t-3}^1 - 0.0009\Delta R_{t-4}^1 - 0.0510Q1 -$$

(0.0028) (0.0028) (0.0030) (0.0159)

$$0.0043Q2 - 0.0076Q3 - 0.056ECM_{t-1}$$

(0.0161) (0.0154) (0.025) (3.5)

$$R^2 = 0.682 \quad \hat{\sigma} = 0.0178 \quad DW = 2.01 \quad Z_1 = 1.76 \quad E_1 = 1.46$$

$$LM(5) = 0.52 \quad ARCH(5) = 0.28 \quad Z_5 = 0.04 \quad E_4 = 0.36$$

$$RESET(1) = 3.20 \quad RESET(2) = 1.59$$

The results from equation (3.5) confirm that the lag imposed

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on the interest rate in (3.4) has had little effect on the short-run dynamics of the model. Note, in fact, that the point estimates on the regressors have changed little from those reported in (3.2). Furthermore, most of the diagnostic tests reported have similar values. The main difference between (3.5) and (3.2) is, of course, the more plausible long-run properties of the former. The importance of the results reported in (3.5) is that this change in the first stage of the estimation procedure does not seem to significantly affect the dynamic properties of the model. This may be seen as a justification for using this new, and rather unorthodox variant of the two-stage procedure. Cointegration theory tells us that the lag with which a variable is entered into the first-stage cointegration equation should not matter for the purposes of estimating the long-run multipliers of the relationship. In addition Equation (3.5) now seems to tell us that this procedure does not significantly alter the short-run properties of the model either.

Following the same procedure as before, we engaged in a simplification search to obtain the following model:

### CHAPTER 3

$$\begin{aligned} \Delta m_t = & 0.030 - 0.059Q1 - 0.010Q2 - 0.023Q3 + 0.246\Delta_{3m}t-1 + \\ & (0.005) (0.005) (0.005) (0.005) (0.040) \\ & 0.0025(\Delta R_t^1 - \Delta R_{t-1}^1 - \Delta R_{t-2}^1 + \Delta R_{t-3}^1) - 0.059ECM_{t-1} \\ & (0.0012) (0.021) \end{aligned} \quad (3.6)$$

$$R^2 = 0.655 \quad DW = 2.02 \quad \hat{\sigma} = 0.0163 \quad Z_1 = 1.96 \quad E_1 = 1.82$$

$$LM(5) = 0.81 \quad ARCH(5) = 0.28 \quad Z_5 = 0.080 \quad E_4 = 0.42$$

$$RESET(1) = 5.46 (*) \quad RESET(2) = 2.74$$

Note the similarities and differences with equation (3.3). First of all, the goodness-of-fit and performance in terms of the reported diagnostic tests is very similar, except that (3.6) seems to have a slightly 'better' ex ante forecasting performance, and does not fail the RESET(2) test. The in-sample and out-of-sample performance of the model is displayed in figures 3.3 and 3.4. However, like equation (3.3), equation (3.6) still fails the RESET(1) test. The dynamic structure is also very similar, and the only difference lies in the interest rate term. In equation (3.6) in fact, a  $\Delta R_{t-1}^1$  term is included in the compound interest rate term, whilst this is absent in (3.3). This difference may in part be due to the different error-correction mechanism employed in (3.6). A more formal comparison of the two variants of the Engle-Granger procedure will be made in section four. However, we should at this stage stress once more that (3.6) has different long-run properties.

#### 3.1.3. Recursive Estimation Methods and the Two-Stage Procedure.

This subsection deals with an (unsuccessful) attempt to



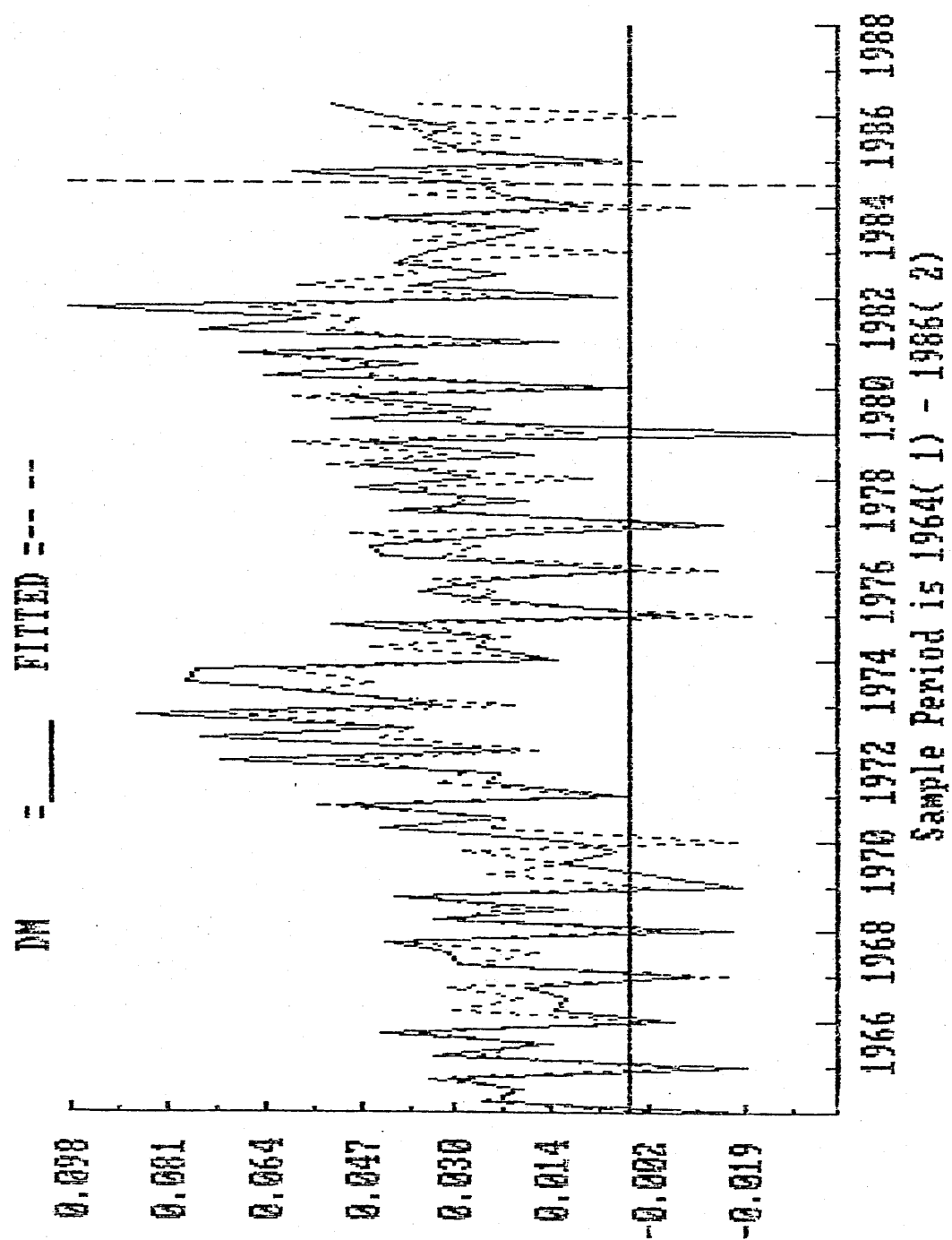


FIGURE 3.3

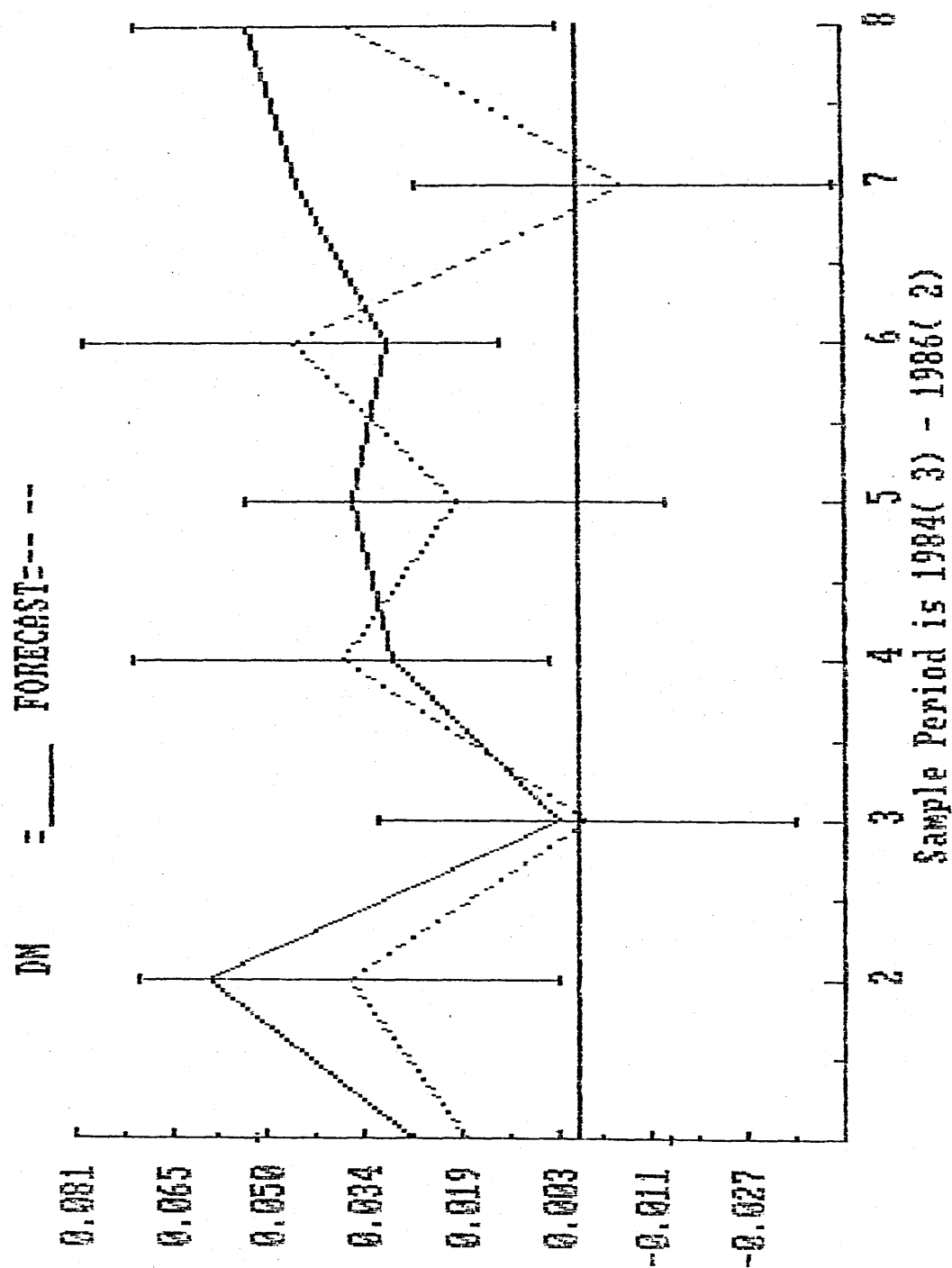


FIGURE 3.4

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model the apparent variation in the long-run parameters of the demand for M3 model, discussed in Chapter 2. The reader will recall from the sub-sample estimates presented in the previous chapter that the point estimates of the cointegration vector displayed some marked variations over the whole sample period. To some economic observers this may not come as a major surprise. Some economists may be sceptical regarding the plausibility of imposing given, fixed long-run properties on a demand-for-money model over a sample period which spans two decades.

Instead of using the conventional Engle-Granger procedure, we attempted to allow for some variation in the cointegration parameters by estimating the cointegration equation using recursive least squares methods, and then using the resulting residuals to construct an error-correction term in the second stage of the estimation procedure as usual. Unfortunately, as we shall see below, these methods were less than totally successful in obtaining a satisfactory model for M3.

Having derived an error-correction term from the recursive residuals, this was embedded in the usual first-difference formulation of our general ADL model. The 'general' model yielded the following estimates:

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$$\Delta m_t = 0.021 + 0.229\Delta m_{t-1} + 0.295\Delta m_{t-2} + 0.044\Delta m_{t-3} - 0.017\Delta m_{t-4} +$$

(0.014) (0.157) (0.152) (0.160) (0.163)

$$0.099\Delta p_t - 0.131\Delta p_{t-1} - 0.033\Delta p_{t-2} + 0.242\Delta p_{t-3} +$$

(0.260) (0.299) (0.293) (0.271)

$$0.114\Delta p_{t-4} + 0.058\Delta y_t + 0.133\Delta y_{t-1} + 0.099\Delta y_{t-2} +$$

(0.249) (0.138) (0.155) (0.148)

$$0.055\Delta y_{t-3} - 0.003\Delta y_{t-4} + 0.0023\Delta R_t^1 - 0.0016\Delta R_{t-1}^1 -$$

(0.152) (0.142) (0.0029) (0.0032)

$$0.0028\Delta R_{t-2}^1 + 0.0039\Delta R_{t-3}^1 - 0.0017\Delta R_{t-4}^1 - 0.0520Q_1 -$$

(0.0033) (0.0033) (0.0033) (0.0182)

$$0.0042Q_2 - 0.0103Q_3 + 0.023ECM_{t-1}$$

(0.0180) (0.0173) (0.061) (3.7)

$$R^2 = 0.630 \quad \hat{\sigma} = 0.0193 \quad DW = 1.98 \quad Z_1 = 1.13 \quad E_1 = 0.91$$

$$LM(5) = 0.09 \quad ARCH(5) = 0.35 \quad Z_5 = 0.41 \quad E_4 = 1.45$$

$$RESET(1) = 1.61 \quad RESET(2) = 0.85$$

Note that although the parameter estimates for the short-run dynamic elements are very similar to those obtained with our other two variants of the Engle-Granger method, there are two main problems with this equation. First, note that the estimated standard error of the equation is 1.93%, slightly greater than that of the corresponding 'general' estimates for the other two models (equations (3.3) and (3.5)). Secondly, the estimated parameter on the error-correction term is positive and insignificant which, recalling our results from chapter two, is a perverse result (implying that  $1 - \sum_{i=1}^n \alpha_i < 0$ ). Furthermore, this problem is not resolved by simplifying the model. All

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restricted versions of this model which performed adequately had an error-correction term which was positive and insignificant. Apart from indicating that the use of recursive methods to capture possible evolutions in the long-run solution may not be feasible, this result illustrates that imposing a long-run solution on the model which displays even small deviations from the 'true' values of the cointegrating vector (such as the consistent estimates obtained from our first-stage static OLS regression) may lead us to find the error-correction term insignificant, and to conclude (incorrectly) that the variables concerned are not cointegrated. This is in line with the suggestion by Granger and Weiss (1983) that one way to test whether the estimated cointegration vector obtained from the first stage of the Engle-Granger procedure is in fact close to the 'true' value is by examining the way in which the estimated variance of the equation and the significance of the error-correction term changes once the model is re-estimated with slightly different values for the cointegrating parameters.

Before turning to the estimates obtained using the 'general-to-specific' method proposed by David Hendry, we specified a 'benchmark' against which we could assess the variants of the Engle-Granger procedure outlined in sub-sections 3.1.1 and 3.1.2. This benchmark model consists of a simple autoregressive model containing only the first differences of the relevant variables. Essentially, it is the single-equation equivalent (the equation

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modelling  $\Delta m$ ) of a general VAR system in differences. If the variables are cointegrated, this type of model will of course be misspecified. It is therefore useful to see if our final models compare favourably with a simple autoregressive model, as we would expect if the variables are indeed cointegrated. Once more, the modelling strategy followed is that of beginning with a general model and moving to a more parsimonious model.

Our final chosen model in fact turns out to be extremely simple in structure, as money M3 growth is found to depend solely on past monetary growth:

$$\Delta m = 0.028 - 0.050Q1 - 0.005Q2 - 0.011Q3 + 0.306\Delta m_{t-1} \\ (0.005) (0.005) (0.006) (0.006) (0.058) \quad (3.8)$$

$$R^2 = 0.600 \quad \hat{\sigma} = 0.0173 \quad DW = 2.02 \quad Z_1 = 1.27 \quad E_1 = 1.23$$

$$LM(5) = 0.24 \quad ARCH(5) = 0.82 \quad Z_5 = 1.10 \quad E_4 = 0.74$$

$$RESET(1) = 3.60 \quad RESET(2) = 2.13$$

Note that this simple atheoretical time series model really performs quite well compared to our models estimated using the two-stage procedure. Note, for instance that the estimated standard error is only 0.1% above that of equations (3.3) and (3.6), and the  $R^2$  is also only about 0.05 below that of these two competing models. An indication of the goodness-of-fit is also given by figure 3.5. This is in some sense illustrative of why simple VAR models have been preferred by some econometricians (see especially Sims, 1980). Furthermore, this simple benchmark model also seems to perform adequately in terms of its ex ante

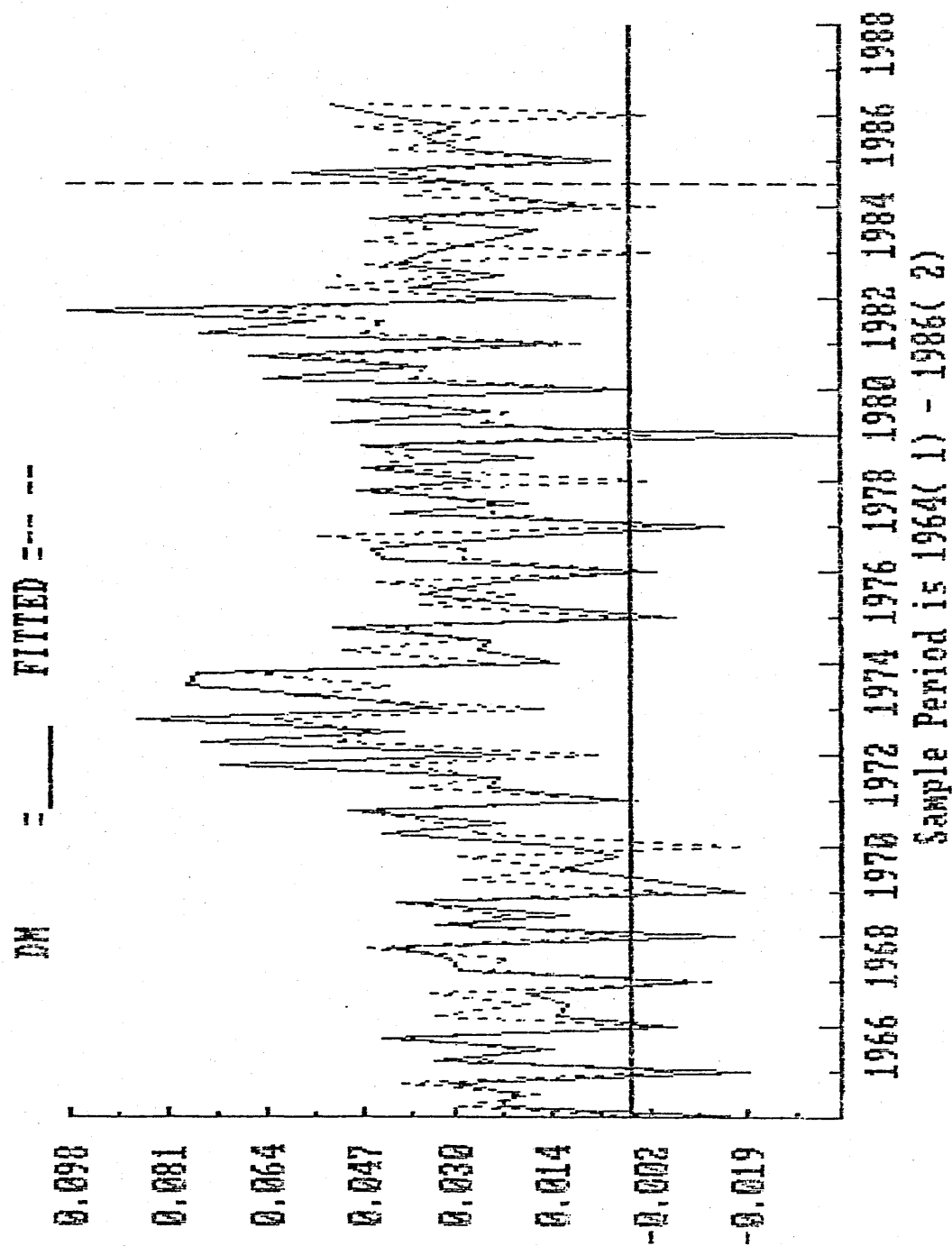


FIGURE 3.5

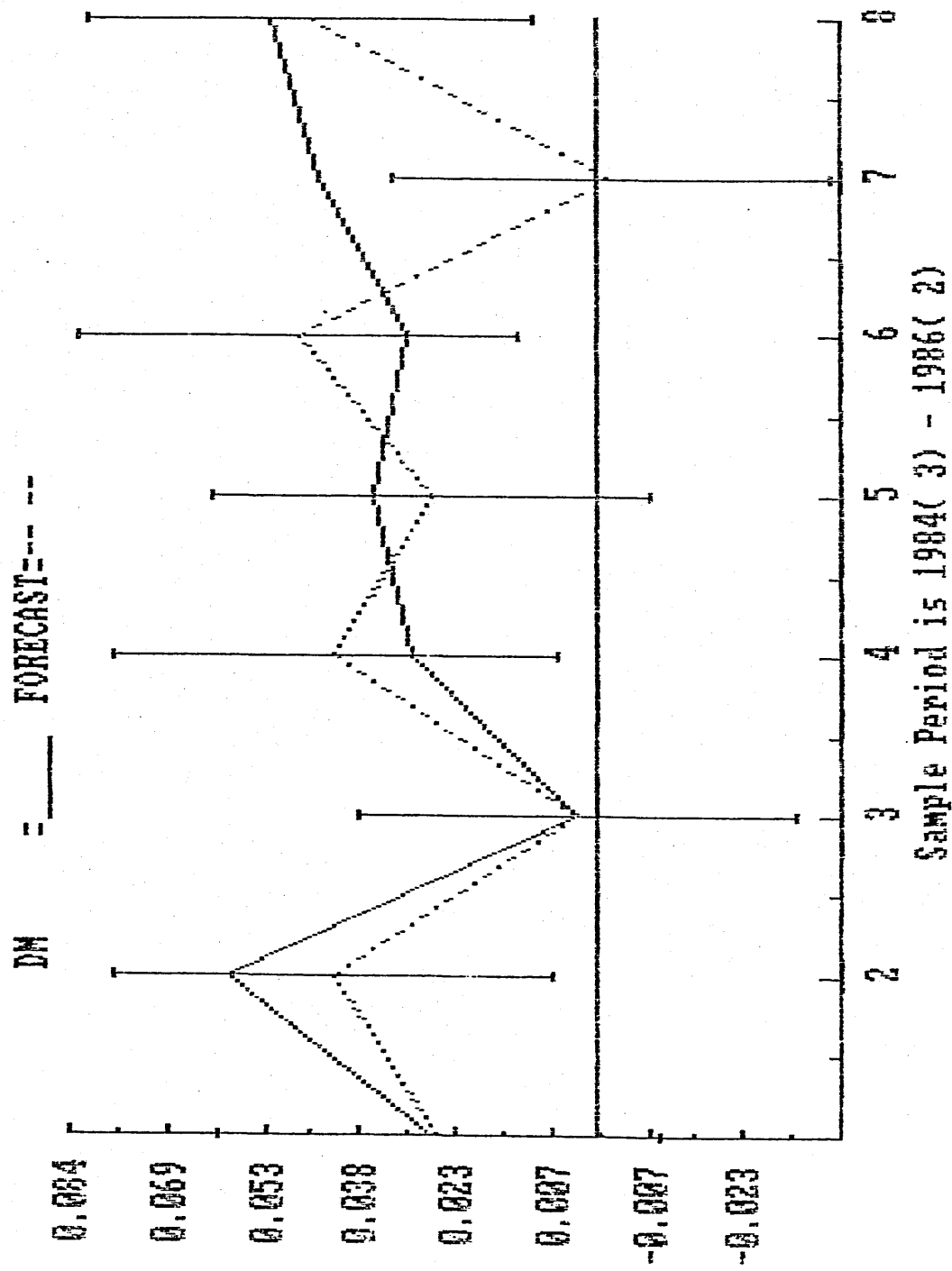


FIGURE 3.6



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forecasting performance, as measured by the  $Z_1$  and  $E_1$  tests, and as displayed in figure 3.6. Only for the 1986(1) quarter does the actual realization of  $\Delta m$  lie outside the forecast confidence interval.

In section four we shall use equation (3.8) as a basis to evaluate our other models estimated in this chapter. Clearly, by adopting a classical econometric approach, we would expect this model to be outperformed by our other theory-based models. Conversely, any theory-based model which does not encompass (3.8), should be regarded as somewhat disappointing.

Meanwhile, though, we turn our attention to the models estimated by applying the 'general-to-specific' methodology, which eschews the imposition of long-run multipliers on the model before a specification search is undertaken.

#### SECTION TWO: ESTIMATING THE DEMAND FOR MONEY USING THE 'GENERAL-TO-SPECIFIC MODEL SELECTION PROCEDURE

To a large extent, as we have already pointed out in this and the previous chapter, there are some common features between the modelling approaches followed in sections one and two. In both cases a general-to-specific search is undertaken, although in the previous section this related solely to the short-run dynamics of the model. As we pointed out in chapter two, there may be some grounds for arguing that any specification search undertaken should follow a certain structure (see Pagan, 1987, McAleer et al. 1985). However, space restricts such a detailed

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report of the path followed in moving to the final chosen models. The arguments for and against a structured search were examined in detail in chapter two. In any case, it is worth reminding ourselves that the main element of any search is a direct comparison of the general with the specific model, evaluating the validity of the restrictions imposed through the use of conventional statistical tests (usually the F-test in the case of the simple linear restrictions imposed for the models in this chapter).

One other thing which should be pointed out at the outset is that, although a major part of the general-to-specific method is the reparameterisation of the general ADL model (as we saw in Chapter 2), in this chapter we are somewhat limited in this regard. This is because the conventional Engle-Granger two-stage procedure applies a transformation in first differences. As we would like to make a direct comparison between these different methods in section four (partly through the use of variance encompassing tests), we are limited in the transformations which we may apply in this section (and in section three for that matter). Throughout the rest of this chapter, we undertake a simplification search which ensures that the dependent variables of our models were compatible with those of the models estimated in section two.

As usual, we begin our search by estimating a general ADL model of the demand for money in levels (including seasonal

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dummies):

$$\begin{aligned}
 m_t = & -0.782 - 0.0516Q1 - 0.0083Q2 - 0.0102Q3 + 1.151m_{t-1} + \\
 & (0.742) (0.0164) (0.0167) (0.0160) (0.140) \\
 & 0.111m_{t-2} - 0.184m_{t-3} - 0.045m_{t-4} - 0.079m_{t-5} + 0.110p_t - \\
 & (0.212) (0.210) (0.210) (0.147) (0.238) \\
 & 0.199p_{t-1} + 0.049p_{t-2} + 0.300p_{t-3} - 0.027p_{t-4} - 0.193p_{t-5} + \\
 & (0.382) (0.370) (0.359) (0.339) (0.233) \\
 & 0.060y_t + 0.069y_{t-1} + 0.025y_{t-2} - 0.031y_{t-3} - 0.046y_{t-4} + \\
 & (0.132) (0.141) (0.132) (0.129) (0.144) \\
 & 0.045y_{t-5} + 0.0009R_t^1 - 0.0036R_{t-1}^1 - 0.0019R_{t-2}^1 + 0.0059R_{t-3}^1 - \\
 & (0.128) (0.0029) (0.0039) (0.0041) (0.0042) \\
 & 0.0041R_{t-4}^1 - 0.0001R_{t-5}^1 \\
 & (0.0043) (0.0033) \quad (3.8)
 \end{aligned}$$

$$R^2 = 0.999 \quad \hat{\sigma} = 0.0181 \quad DW = 2.06 \quad Z_1 = 1.19 \quad E_1 = 0.84$$

$$LM(5) = 2.69(*) \quad ARCH(5) = 0.36 \quad Z_5 = 0.36 \quad E_4 = 0.47$$

$$RESET(1) = 0.17 \quad RESET(2) = 0.08$$

The model is highly parameterised, and the high  $R^2$  merely indicates the trending nature of the regressors. Note that, once more, most of the regressors prove to be insignificant, and that there is a high degree of multicollinearity between them. The model passes all the diagnostic tests, except for the LM(5) test. This may seem surprising, given that we are dealing with an overparameterised model, but this may in part be due to the presence of seasonal dummies. The combination of a general structure and the dummies may be sufficient to produce such an effect, in which case we would expect a simplification of the general model to 'remove' this problem, as the lag structure is

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adapted to fit the pattern suggested by the data.

As we argued in chapter two, the search for a 'specific' model involves two distinct processes: imposing restrictions and reparameterising the model. The first process simplifies the model, whilst the second is helpful in reducing the degree of multicollinearity present in the regressors<sup>2</sup>. As we pointed out in chapter two, however, these processes are often merged into one.

To give some structure to our search (partly in acknowledgement of the criticisms by McAleer *et al*, 1985, Pagan, 1987), we first reparameterised the model in terms of real balances, so that we began from a general ADL of the type:

$$(m - p)_t = k + \text{'seasonals'} + \sum_{i=1}^5 \alpha_i (m - p)_{t-i} + \sum_{i=0}^5 \beta_i y_{t-i} + \sum_{i=0}^5 \gamma_i p_{t-i} + \sum_{i=0}^5 \delta_i R_{t-i}^1 + u_t \quad (3.9)$$

Thereafter, we attempted a reparameterisation of the model, so as to give us a dependent variable  $\Delta(m - p)_t$  and an error-correction term (or, in terms of the terminology of Wickens and Breusch, 1988, a restricted ECM) of the type  $(m - p - y)_{t-1}$ . Thereafter, we imposed the appropriate zero and unit restrictions on the remaining regressors. The result was a chosen model of the form:

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$$\begin{aligned}
 \Delta(m - p)_t = & -0.323 - 0.049Q_1 - 0.005Q_2 - 0.013Q_3 + \\
 & (0.222) (0.006) (0.006) (0.006) \\
 & 0.239\Delta_2(m - p)_{t-1} - 0.708\Delta p_t + 0.0031\Delta_4 R_t^1 - \\
 & (0.064) (0.182) (0.0018) \\
 & 0.0060\Delta_2 R_{t-1}^1 + 0.035y_t - 0.057(m - p - y)_{t-1} \\
 & (0.0025) (0.020) (0.024)
 \end{aligned}
 \tag{3.10}$$

$$R^2 = 0.725 \quad \hat{\sigma} = 0.0171 \quad DW = 1.89 \quad Z_1 = 1.58 \quad E_1 = 1.35$$

$$LM(5) = 0.68 \quad ARCH(5) = 1.63 \quad Z_5 = 0.80 \quad E_4 = 0.92$$

$$RESET(1) = 1.01 \quad RESET(2) = 1.26$$

There are several points to note about equation (3.10). First, we should note that the model passes all the diagnostic tests reported, and hence that the serial correlation 'problem' present in the 'general' model has been overcome. Secondly, we should note that the model fits adequately within sample (see figure 3.7), and forecasts reasonably well (see figure 3.8). Once more, the only quarter for which the forecast lies outside the forecast confidence interval is 1986(1), in common with the equations presented in section one. We should also point out that, in comparing equation (3.10) with our previous models estimated in section one, there are notable differences. The main fact to note is that the fit of (3.10) in terms of the estimated standard error is worse. Furthermore, one negative feature of (3.10) is that it does not allow for a long run interest rate effect on the demand for real balances, which is rather unorthodox. On the other hand, equation (3.3) also has problems in this respect, because the long-term effect imposed on the

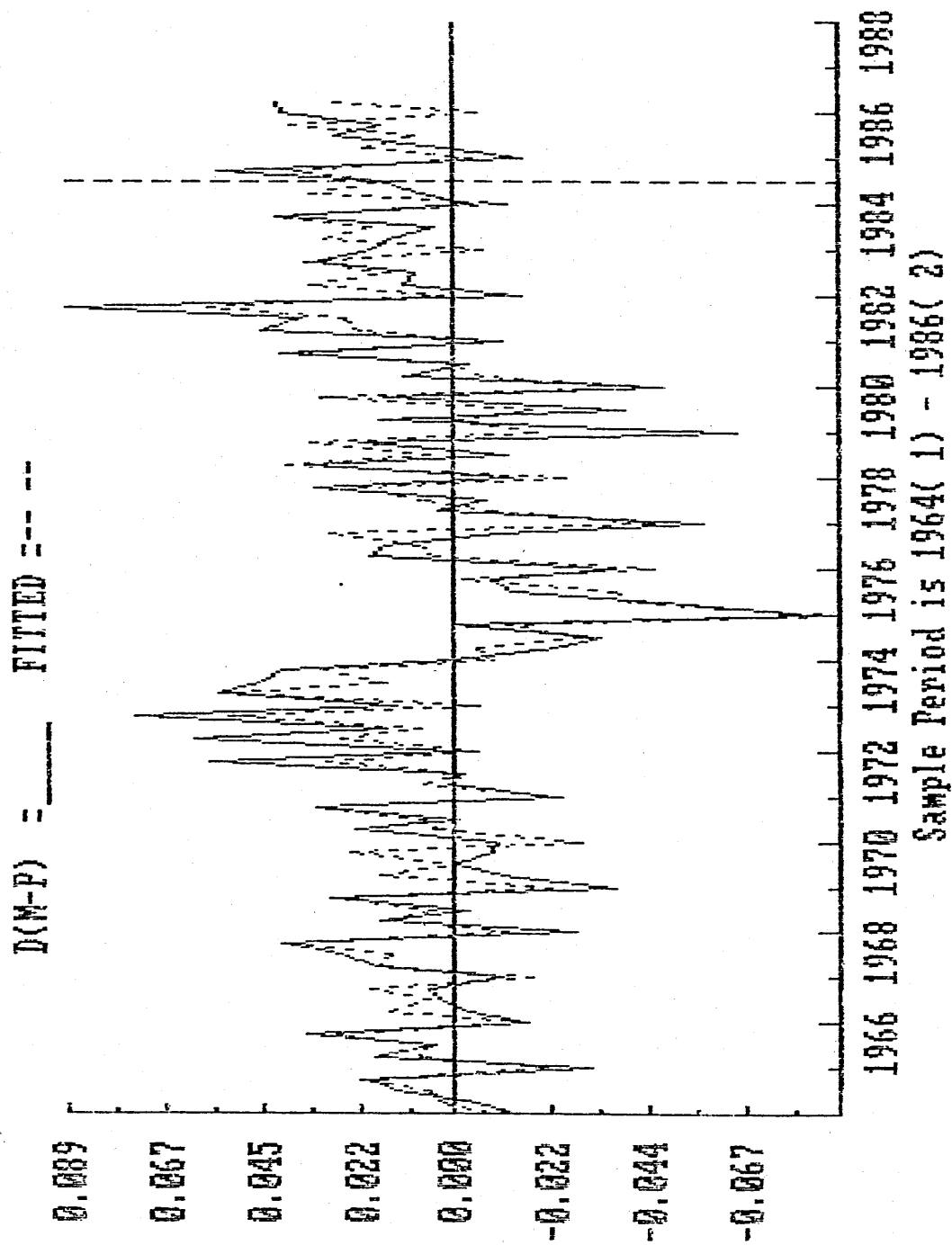


FIGURE 3.7

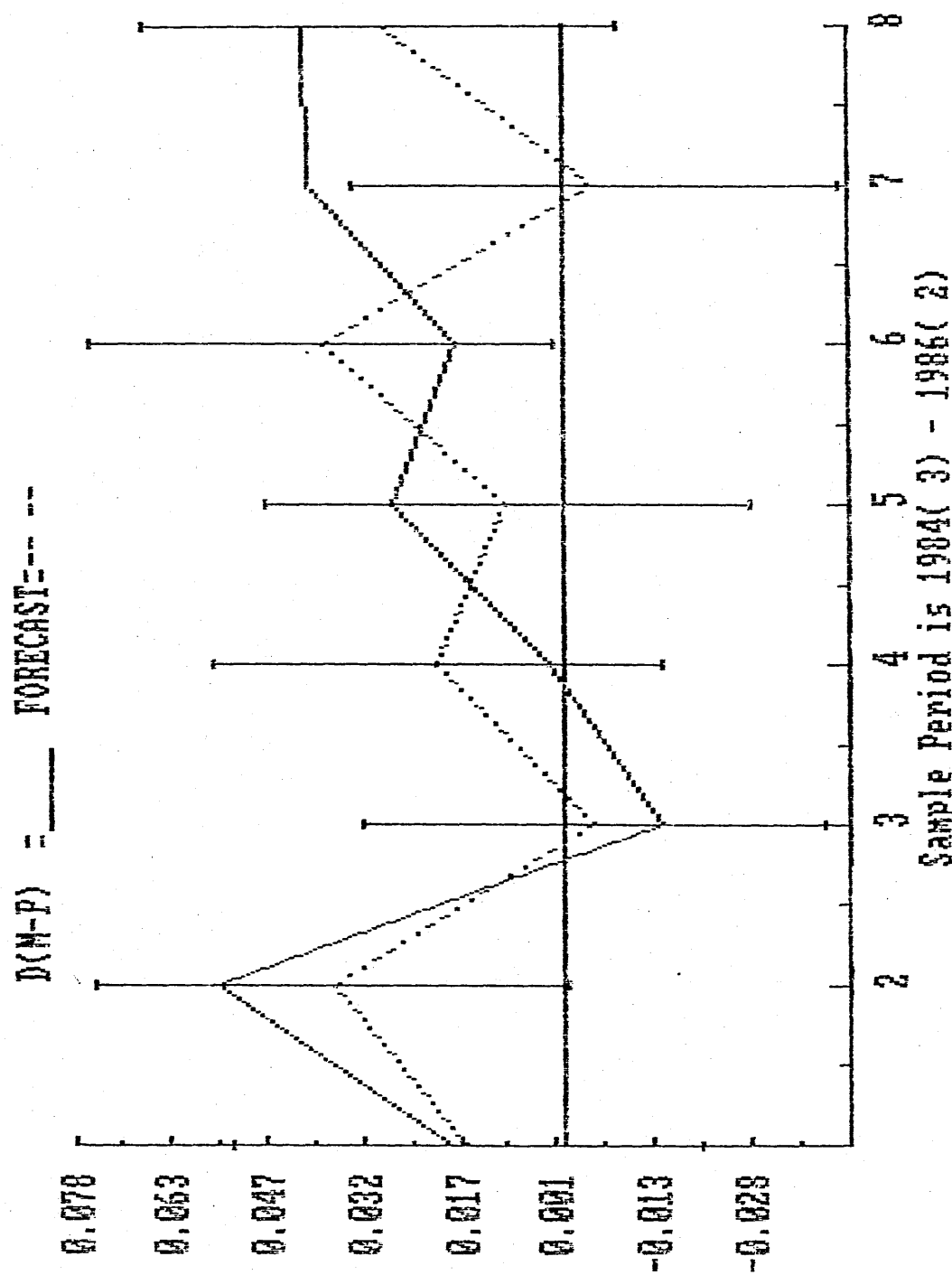


FIGURE 3.8

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model is positive. Only equation (3.6) resolves this problem through the introduction of a lagged interest rate term in the cointegration equation. On the positive side, equation (3.10) does have a negative inflation effect, which we would expect when modelling the demand for real balances. The third point to note is that the error-correction terms in all of (3.3), (3.6) and (3.10) appear with a negative coefficient of approximately -0.06. This consistent result, in spite of the different error-correction terms used, reflects to a large extent the robustness of the error-correction formulation, regardless of whether this has been arrived at via the Engle-Granger two-stage procedure or via a 'general-to-specific' modelling strategy.

In an attempt to re-examine this strange result with regard to the interest rate effect, we re-examined the simplification process, and followed a slightly different route which enabled us to retain a negative interest rate effect in our model:



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$$\begin{aligned}
 \Delta(m - p)_t = & -1.109 - 0.045Q_1 - 0.001Q_2 - 0.010Q_3 + \\
 & (0.520) (0.007) (0.006) (0.006) \\
 & 0.258\Delta_2(m - p)_{t-1} - 0.763\Delta p_t - 0.014p_{t-5} - \\
 & (0.069) (0.200) (0.009) \\
 & 0.0024R_{t-1}^1 + 0.0057R_{t-3}^1 - 0.0036R_{t-4}^1 + 0.106y_t - \\
 & (0.0019) (0.0030) (0.0025) (0.048) \\
 & 0.056(m - p - y)_{t-1} \\
 & (0.025)
 \end{aligned}
 \tag{3.11}$$

$$R^2 = 0.730 \quad \hat{\sigma} = 0.0171 \quad DW = 1.88 \quad Z_1 = 1.42 \quad E_1 = 1.16$$

$$LM(5) = 0.61 \quad ARCH(5) = 1.54 \quad Z_5 = 0.71 \quad E_4 = 0.54$$

$$RESET(1) = 0.59 \quad RESET(2) = 0.92$$

This equation has a very similar in-sample fit to (3.10), and performs slightly better in terms of the diagnostics reported. The important point to note is that the overall long-run interest rate effect is negative, unlike (3.10) (although the reader will readily verify that the point estimate of the total effect is still very small). However, this is achieved at the expense of a non-unitary price elasticity of the demand for money (unlike (3.10)), due to the presence of the  $p_{t-5}$  term in (3.11). The likeness of (3.10) and (3.11) emphasises one of the difficulties of this approach, given the variety of final models which may be obtained by following different branches of the specification tree. These models may have very similar dynamic properties in terms of characterising the data, but have very different implications in terms of their long-run properties. In comparing the different model selection procedures in section four, we should take account of the differences between equations

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(3.10) and (3.11).

However, before turning to such a comparison, we first move to our third model selection procedure, which involves elements of both the procedures outlined in sections one and two.

## CHAPTER 3

### SECTION THREE: ESTIMATING THE DEMAND FOR MONEY USING TRANSFORMED DYNAMIC REGRESSION MODELS

We argued in chapter two that there is a third alternative route to the estimation of a dynamic demand for money model. This involves the application of transformations on the general ADL model before undertaking a specification search. However, as we saw in the previous chapter, there are a number of different transformations which one may consider in modelling the demand for money. Therefore, before we turn to our estimations, it is worthwhile to provide a brief summary of the different alternatives open to us.

#### 3.3.1. Transformed Regression Models.

Let us begin by reviewing some of the arguments of the previous chapter in the context of the search for the demand for money. As we have seen, the transformations are typically applied to a general ADL model of the demand for money of the type:

$$m_t = k + \sum_{i=1}^5 \alpha_i m_{t-i} + \sum_{i=0}^5 \beta_i p_{t-i} + \sum_{i=0}^5 \gamma_i y_{t-i} + \sum_{i=0}^5 \delta_i R_{t-i}^1 + u_t \quad (3.12)$$

In Chapter 2 we pointed out that one possible transformation which may be applied to (3.12) is the one suggested by Breusch and Wickens (1988) based on the earlier work of Bewley (1979):

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$$m_t = k + \sum_{i=1}^4 a_i \Delta_i m_{t-i} + \sum_{i=1}^4 b_i \Delta_i p_{t-i} + \sum_{i=1}^4 c_i \Delta_i y_{t-i} + \sum_{i=1}^4 d_i \Delta_i R_{t-i}^1 + \pi_1 p_t + \pi_2 y_t + \pi_3 R_t^1 + u_t \quad (3.13)$$

$$\begin{aligned} \text{where } a_i &= -\alpha_i / (1 - \sum_{i=1}^5 \alpha_i) & b_i &= -\beta_i / (1 - \sum_{i=1}^5 \alpha_i) \\ c_i &= -\gamma_i / (1 - \sum_{i=1}^5 \alpha_i) & d_i &= -\delta_i / (1 - \sum_{i=1}^5 \alpha_i) \\ \pi_1 &= \sum_{i=0}^5 \beta_i / (1 - \sum_{i=1}^5 \alpha_i) & \pi_2 &= \sum_{i=0}^5 \gamma_i / (1 - \sum_{i=1}^5 \alpha_i) \\ \pi_3 &= \sum_{i=0}^5 \delta_i / (1 - \sum_{i=1}^5 \alpha_i) \end{aligned}$$

We should again re-iterate that the main advantage of transformations such as (3.13) is that they enable us to directly estimate the long-run multipliers of a dynamic regression model without omitting the short-run dynamics as in the cointegration equations used above. The parameters  $\pi_i$  are the long-run multipliers of the system, and we may obtain both point estimates of these and estimated standard errors. In contrast, the simple 'general-to-specific' procedure cannot yield direct estimates of standard errors, because the long-run elasticities are only obtained by finding the steady-state solution for the chosen model. The cointegration equation, as we have seen, may give us consistent estimates of the cointegration vector, but the standard errors obtained from the static cointegration equation are biased and useless for statistical inference. It follows therefore that transformed equations like (3.13) may give us information about the long-run properties of the model without the prior omission of its short-run dynamics. Before we turn to a discussion of how we can use this property of transformed model

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to set up a third model selection procedure, we should examine the other types of transformed models available to us.

There are certain disadvantages with the above transformation proposed by Wickens and Breusch (1988). The main problem is one of convenience: equation (3.13) has regressors which are not independent of the dependent variable,  $m_t$ . Hence, an instrumental variable (IV) estimator is used, which involves using all the regressors before the transformation as instruments. It can be shown (see Wickens and Bruschi, 1988) that the estimates of the parameters and variance-covariance matrix obtained through IV will be identical to that which would have been obtained through OLS in the absence of the transformation (see also Bewley, 1979).

The use of an IV estimator in itself is not problematic given the availability of suitable computer applications packages. However, it does cause some problems of comparability with the other models presented in sections one and two, especially when it comes to the use of variance encompassing tests in section four. Furthermore, a second problem in the application of variance encompassing tests to compare the models of sections one and two with models obtained via equation (3.13) is that the dependent variables are different. This suggests that an alternative transformation to (3.13) which has  $\Delta m_t$  or  $\Delta(m - p)_t$  on the left-hand-side may be more appropriate.

Some of the alternative transformations which are at our

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disposal have already been discussed in chapter two. For instance, let us recall the transformation reported in equation (2.21) as applied to the demand for money:

$$\Delta m_t = k + \sum_{i=1}^4 a_i \Delta m_{t-i} + \sum_{i=0}^4 b_i \Delta p_{t-i} + \sum_{i=0}^4 c_i \Delta y_{t-i} + \sum_{i=0}^4 d_i \Delta R_{t-i}^1 - \Pi(m_{t-1} - \theta_1 p_{t-1} - \theta_2 y_{t-1} - \theta_3 R_{t-1}^1) + u_t \quad (3.14)$$

where  $b_0 = \beta_0$ ,  $c_0 = \gamma_0$ ,  $d_0 = \delta_0$ ,

$a_i = -\sum_{j=i+1}^5 \alpha_j$ ,  $b_i = -\sum_{j=i+1}^5 \beta_j$ ,  $c_i = -\sum_{j=i+1}^5 \gamma_j$ ,  $d_i = -\sum_{j=i+1}^5 \delta_j$  for  $i = 1, \dots, 4$ .

Also,  $\Pi = (1 - \sum_{i=1}^5 \alpha_i)$ ,  $\theta_1 = \sum_{i=0}^5 \beta_i / (1 - \sum_{i=1}^5 \alpha_i)$ ,

$\theta_2 = \sum_{i=0}^5 \gamma_i / (1 - \sum_{i=1}^5 \alpha_i)$ ,  $\theta_3 = \sum_{i=0}^5 \delta_i / (1 - \sum_{i=1}^5 \alpha_i)$

Note that equation (3.14) has the advantage that it may be estimated by OLS, as the regressors are now asymptotically uncorrelated with the error term. In addition, this transformation also yields a direct estimate of the long-run multipliers, and these are given by the estimated coefficient on the  $p_{t-1}$ ,  $y_{t-1}$ , and  $R_{t-1}^1$  variables. Note, however, that unlike (3.13), equation (3.14) does not retain the same distributed lag structure as the original equation (3.12). In (3.14) the  $b_i$ ,  $c_i$ , and  $d_i$  are now sums of the  $\beta_i$ ,  $\gamma_i$ , and  $\delta_i$  respectively. However, this 'scrambling' of the distributed lag structure is not significant unless we are interested, for a priori reasons, in the structure of the short-run dynamics. If, for instance, we believe that distributed lag structures are likely to be smooth, (3.14) may not be an inappropriate transformation, as we are less

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interested in zero restrictions on individual lag lengths of a distributed lag. If the latter are a significant issue, then the lag pattern is likely to be jagged (less smooth), and a transformation such as (3.13) which leaves the structure unaffected may be more appropriate.

As we pointed out in chapter two, it is easy to rearrange a transformed equation such as (3.13) or (3.14) to obtain a slightly different set of regressors on the right-hand-side of the equation. In other words, the range of transformations available to us is vast. For instance, let us consider the following variant of equation (3.14):

$$\begin{aligned} \Delta m_t = & k + \sum_{i=1}^4 a_i \Delta m_{t-i} + \sum_{i=0}^4 b_i \Delta p_{t-i} + \sum_{i=0}^4 c_i \Delta y_{t-i} + \\ & \sum_{i=0}^4 d_i \Delta R_{t-i}^1 - \Pi(m_{t-5} - \theta_1 p_{t-5} - \theta_2 y_{t-5} - \theta_3 R_{t-5}^1) + u_t \end{aligned} \quad (3.15)$$

where  $a_i = (\sum_{j=1}^i \alpha_j) - 1$

$$b_i = \sum_{j=0}^i \beta_j$$

$$c_i = \sum_{j=0}^i \gamma_j$$

$$d_i = \sum_{j=0}^i \delta_j$$

$$\Pi = (1 - \sum_{i=1}^5 \alpha_i), \theta_1 = \sum_{i=0}^5 \beta_i / (1 - \sum_{i=1}^5 \alpha_i),$$

$$\theta_2 = \sum_{i=0}^5 \gamma_i / (1 - \sum_{i=1}^5 \alpha_i), \theta_3 = \sum_{i=0}^5 \delta_i / (1 - \sum_{i=1}^5 \alpha_i)$$

Note that (3.15) has an error-correction term with a lag of five periods and that, as a result, the lag distribution of the transformed equation has changed. This error-correction structure differs slightly from that suggested by Engle and Granger (1987), which is identical to that displayed in (3.14).

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As we saw in the previous chapter, recent developments in cointegration have suggested that the simple structure suggested by Engle and Granger (1987) may have to be modified if a cointegrating vector of polynomials is more appropriate as opposed to a vector cointegrating constants, as is usually assumed (see Yoo, 1986, Engle, 1987). One example of this is provided by models which are built on the property of seasonal cointegration (see for instance Engle et al., 1987). As Engle (1987) points out, if a cointegration polynomial is appropriate, then the error-correction structure may be more complicated, possibly incorporating different error-correction terms.

Let us examine this argument in more detail. Consider a  $(k \times 1)$  vector of time series  $x_t$ . If these are seasonally cointegrated, then, as we saw in chapter two, to achieve a finite moving average representation, we have to take fourth differences if the data is quarterly to reduce these series to stationarity:

$$\Delta_4 x_t = A(L) \varepsilon_t \quad (3.16)$$

where  $A(L)$  is a  $(k \times k)$  matrix of lag polynomials, where  $L$  is the lag operator, and  $\varepsilon_t$  is a  $(k \times 1)$  vector of white noise disturbance terms. Let us re-express (3.16) in a more general form involving two general lag polynomials  $\xi_1(L)$ ,  $\xi_2(L)$ :

$$\xi_1(L) \xi_2(L) x_t = A(L) \varepsilon_t \quad (3.16')$$

where for our seasonal cointegration case the lag polynomials are defined as  $\xi_1(L) \equiv (1 - L)$  and  $\xi_2(L) \equiv (1 + L + L^2 + \dots + L^{s-1})$ , where  $s$  is the order of seasonality.



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Before proceeding with the derivation of a cointegration representation for (3.16'), we need to engage in a brief digression which will enable us to derive an autoregressive representation for our system. To achieve this, Engle (1987) uses the form for polynomial matrices adapted by Yoo(1986) from earlier work by Kailath (1980). This form, which is known as the Smith-Macmillan-Yoo form, is based on the idea that a rational polynomial matrix such as  $A(L)$  may be converted to a finite order polynomial matrix, by appropriately pre- and post-multiplying the matrix. Thus, following the lemma proposed by Yoo (1986)<sup>3</sup>, we may write:

$$A(L) = U^{-1}(L)M(L)V^{-1}(L) \quad (3.17)$$

where  $M(L)$  is diagonal and all the roots of  $\det M(L)$  lie on or within the unit circle, and all the roots of  $\det U(L) = 0$  and  $\det V(L) = 0$  lie outside the unit circle.

Returning to (3.16'), equation (3.17) suggests the Smith-McMillan-Yoo form:

$$\xi_1(L)\xi_2(L)x_t = U^{-1}(L)M(L)V^{-1}(L)\epsilon_t \quad (3.18)$$

where, for the lag polynomials  $\xi_1(L)$  and  $\xi_2(L)$ , the diagonal  $M(L)$  matrix takes the form:

$$M(L) = \begin{bmatrix} I_{k-m-n-r} & & & 0 \\ & \xi_1(L)I_m & & \\ & & \xi_2(L)I_n & \\ 0 & & & \xi_1(L)\xi_2(L)I_r \end{bmatrix}$$

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where  $I_i$  is an  $(i \times i)$  identity matrix, and 0 is a matrix of zeros.

We may pre-multiply both sides of (3.18) by  $U(L)$  to obtain:

$$\xi_1(L)\xi_2(L)U(L)x_t = M(L)V^{-1}(L)\varepsilon_t \quad (3.19)$$

Note that given the definition of  $M(L)$  the last  $r$  rows of (3.19)

have  $\xi_1(L)\xi_2(L)$  on both sides. Defining  $M^t(L)$  as:

$$M^t(L) = \xi_1(L)\xi_2(L)M(L)$$

we may then rewrite (3.19) as:

$$M^t(L)U(L)x_t = V^{-1}(L)\varepsilon_t \quad (3.20)$$

By premultiplying (3.20) by  $V(L)$ , we reach our desired autoregressive representation:

$$V(L)M^t(L)U(L)x_t = \varepsilon_t \quad (3.21)$$

We may express (3.21) into an error-correction form, as for our simpler cointegration systems. Partitioning  $U(L)$  and  $V(L)$  conformably with the partitioning of the  $M(L)$  matrix shown above:

$$U(L) = [U_1(L), \alpha_1(L), \alpha_2(L), \alpha_3(L)]$$

$$V(L) = [V_1(L), \beta_1(L), \beta_2(L), \beta_3(L)]$$

where the  $\alpha_i$  and  $\beta_i$  are cointegrating polynomials. The error-correction form of (3.21) is then:

$$\begin{aligned} V(L)U(L)\xi_1(L)\xi_2(L)x_t = & \beta_1(L)\alpha_1(L)\xi_2(L)(1 - \xi_1(L))x_t + \\ & \beta_2(L)\alpha_2(L)\xi_1(L)(1 - \xi_2(L))x_t + \\ & \beta_3(L)\alpha_3(L)(1 - \xi_1(L)\xi_2(L))x_t + \varepsilon_t \end{aligned} \quad (3.22)$$

Consider the implications of this result for the special case where  $\xi_1(L) \equiv (1 - L)$  and  $\xi_2(L) \equiv (1 + L + L^2 + \dots + L^{s-1})$ ; that is, when the series are seasonally cointegrated. Equation (3.22) implies that there is more than one error-correction term,

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and that, instead of our usual vector of cointegrating constants, we now have a cointegrating polynomial for each term. Engle (1987) shows that whilst some of these polynomials may reduce to a vector with constant coefficients, there is in general no such constant cointegrating vector where there is more than one root to be eliminated by the cointegrating vector. This is of course the case where we have seasonal cointegration.

Turning to the error-correction form, we will then have, for this special case:

$$V(L)U(L)(1 - L^4)x_t = \beta_1(L)\alpha_1(L)(1 + L + L^2 + L^3)x_{t-1} - \beta_2(L)\alpha_2(L)(1 - L^3)x_{t-1} + \beta_3(L)\alpha_3(L)x_{t-4} + \varepsilon_t$$

(3.23)

Note that this system has three error-correction terms. The first is an error correction term in the data which has been 'seasonally averaged', by virtue of having been premultiplied by the  $(1 + L + L^2 + L^3)$  polynomial. The second term is an error-correction term in the 'detrended' data, as the data vector has been premultiplied by the  $(1 - L)$  term. The third error-correction term is in the raw data, and combines with the other two to reduce the system to stationarity.

There are several things to note about the above analysis. Firstly, when we are dealing with economic data, especially if it shows a seasonal pattern, a simple error-correction representation such as the ones adopted in this chapter may not be appropriate, and a more complicated form may have to be

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applied. However, operational testing and inference procedures still have to be developed for these more complicated cases, and therefore they will not be considered here.

However, if the series display a seasonal pattern, and if we restrict our attention to systems with a single error-correction term we would expect the choice of lag on the error-correction term to be rather important, as we are approximating a significantly more complex form with a simple model. Thus, in the case where the variables are really  $CI(1,1)$ , and we are modelling the system as if it were simply  $I(1)$ , the choice between equations (3.14) and (3.15) may matter. Although the forms given in (3.14) and (3.15) are equivalent if the variables are indeed  $I(1)$ , any possible misspecification in this regard may lead us to favour one or other form. Thus, the a lag of four periods on the error-correction term has often been found appropriate when modelling with data which is strongly seasonal (see for instance Davidson et al., 1978, Hendry and Von Ungern-Sternberg, 1980).

It is also worthwhile pointing out in this context that there is yet another transformation of the simple ADL equation (3.12), which is even closer to the form suggested by (3.23), in that it has not only an error-correction term of a lag equal to the seasonal periodicity, but also expresses the dependent variable in fourth differences:

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$$\Delta_4 m_t = k + \sum_{i=1}^4 a_i \Delta m_{t-i} + \sum_{i=0}^4 b_i \Delta p_{t-i} + \sum_{i=0}^4 c_i \Delta y_{t-i} + \sum_{i=0}^4 d_i \Delta R_{t-i}^1 - \Pi(m_{t-4} - \theta_1 p_{t-4} - \theta_2 y_{t-4} - \theta_3 R_{t-4}^1) + u_t \quad (3.24)$$

$$\begin{aligned} \text{where } a_i &= \sum_{j=1}^i \alpha_j \quad \text{for } i = 1, \dots, 3 & a_4 &= \alpha_5 \\ b_i &= \sum_{j=0}^i \beta_j \quad \text{for } i = 1, \dots, 3 & b_4 &= \beta_5 \\ c_i &= \sum_{j=0}^i \gamma_j \quad \text{for } i = 1, \dots, 3 & c_4 &= \gamma_5 \\ d_i &= \sum_{j=0}^i \delta_j \quad \text{for } i = 1, \dots, 3 & d_4 &= \delta_5 \\ \Pi &= (1 - \sum_{i=1}^5 \alpha_i), \theta_1 = \sum_{i=0}^5 \beta_i / (1 - \sum_{i=1}^5 \alpha_i), \\ \theta_2 &= \sum_{i=0}^5 \gamma_i / (1 - \sum_{i=1}^5 \alpha_i), \theta_3 = \sum_{i=0}^5 \delta_i / (1 - \sum_{i=1}^5 \alpha_i) \end{aligned}$$

Alternatively, one could also propose a transformation which combined some aspects of (3.24) and (3.15), by introducing an error-correction term with a five-period lag (the maximum lag of the ADL):

$$\Delta_4 m_t = k + \sum_{i=1}^4 a_i \Delta m_{t-i} + \sum_{i=0}^4 b_i \Delta p_{t-i} + \sum_{i=0}^4 c_i \Delta y_{t-i} + \sum_{i=0}^4 d_i \Delta R_{t-i}^1 - \Pi(m_{t-5} - \theta_1 p_{t-5} - \theta_2 y_{t-5} - \theta_3 R_{t-5}^1) + u_t \quad (3.25)$$

$$\begin{aligned} \text{where } a_i &= \sum_{j=1}^i \alpha_j \quad \text{for } i = 1, \dots, 3 & a_4 &= \sum_{j=1}^4 \alpha_j - 1 \\ b_i &= \sum_{j=0}^i \beta_j \\ c_i &= \sum_{j=0}^i \gamma_j \\ d_i &= \sum_{j=0}^i \delta_j \\ \Pi &= (1 - \sum_{i=1}^5 \alpha_i), \theta_1 = \sum_{i=0}^5 \beta_i / (1 - \sum_{i=1}^5 \alpha_i), \\ \theta_2 &= \sum_{i=0}^5 \gamma_i / (1 - \sum_{i=1}^5 \alpha_i), \theta_3 = \sum_{i=0}^5 \delta_i / (1 - \sum_{i=1}^5 \alpha_i) \end{aligned}$$

The question we face now is: of these many transformed regression models which may be used, which one serves our purpose best? There are several considerations in selecting an

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appropriate transformation: first, we have to note that, though a transformation in fourth differences, where the error-correction term lag takes somehow into account the possible seasonality of the data is appealing. As we have seen above, Engle (1987) has shown that where more than one unit root is present, the simple Engle-Granger transformation is not really valid, because there may be other error-correction terms which should enter the regression equation. On the other hand, our experiments in chapter two did not suggest that all the variables used here are seasonally integrated<sup>4</sup>. Thus, although there may be advantages in using a transformation such as (3.24) or (3.25), this may be a leap in the dark, and at the very least we should compare the results obtained from such regressions with those which could be obtained from (3.13)-(3.14).

Secondly, the estimation method to be used vary between different transformations. We noted above that equation (3.13) had to be estimated using IV methods, though Wickens and Breusch (1988) show that the resulting IV estimator of the transformed model is equivalent to the OLS estimator of the original model. All the other equations may be estimated using OLS methods. Again, it is worth mentioning that our preference for OLS is not motivated by computational considerations, given that nowadays one may obtain easy access to sophisticated computing packages and facilities. However, to permit an easy comparison of the model proposed here with those estimated in sections one and two,

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we chose to discard 'Bewley'-type transformations such as (3.13).

A third consideration, which led us to discard (3.24) and (3.25) for the purposes of the estimations presented in this chapter, was the question of comparability in terms of the dependent variable. The choice of a transformation with  $\Delta m$  as the dependent variable will facilitate our experiments with variance-encompassing tests in section four.

Left with a choice between (3.14) and (3.15), we chose the former, on the grounds that it was essentially the transformation adopted in the Engle-Granger two-stage procedure (except of course for the fact that the long-run multipliers are not imposed on the model at the outset), thus again making any comparisons easier. It should be again pointed out, however, that (3.15) may have its own advantages, namely the fact that it allows for a different error-correction lag without the use of a different dependent variable (which may or may not be appropriate in our case, depending on the possible seasonal effects present).

Thus, in what follows, we adopt the transformation given in (3.14). It would have been interesting to examine the results using some of the other models, especially (3.24) or (3.25) but, for reasons of space, these experiments are not pursued here.

Even by sticking to a single transformed model, we are still faced with two possible routes to the final specification, which leads us to estimate two variants of (3.14) in the remainder of this section. The first is essentially the one

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suggested by Wickens and Breusch (1988), and consists of estimating an unrestricted version of (3.14). This model is then simplified to reach a more parsimonious model. The estimated long-run multipliers may be checked at every stage of the simplification process, and hence this may provide the researcher with another check on whether the 'correct' restrictions are being imposed on the short-run dynamics. This is one advantage of this modelling strategy over the simple 'general-to-specific' method followed in section two. One advantage over the Engle-Granger procedure is clearly that we do not impose (possibly biased) estimates of the long-run multipliers on the model at the outset, and that it is perfectly feasible for the researcher to eliminate variables in levels which are insignificant and therefore do not seem to have any long-run effect on the demand for money. In contrast, we cannot use standard methods of statistical inference to decide which variables to include or exclude from a cointegration equation, because the standard errors are biased. One must decide a priori which variables should be included in the first stage of the Engle-Granger procedure, and these long-run effects are then imposed on the final model. It should be clear, therefore, that this variant of the 'transformed model procedure' may potentially lead us to very different results from those obtained using the methods employed in sections one and two.

The second variant of the 'transformed model procedure'



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which we explore here entails, once more, the unrestricted estimation of the general model, equation (3.14). At this stage, however, we depart from the previous route, by imposing the estimated long-run multipliers on the model from this moment onwards. The difference with the first variant and the 'general-to-specific' model selection procedures should be apparent: this method imposes the long-run solution on the model at the outset, and hence this conditions the rest of the simplification procedure in a similar way to the Engle-Granger two-stage procedure. On the other hand, this method differs sharply from the Engle-Granger procedure in two important respects: first, the estimated long-run elasticities are obtained from an equation which does not exclude the short-run dynamics. In fact, the short-run dynamics are overspecified, thus removing the problem of bias present in the first stage of the Engle-Granger procedure. The estimates may be somehow imprecise due to the overparameterisation of the general equation (3.14), but any statistical inference regarding the significance of individual long-run effects is perfectly valid.

We now present the estimated models which have been obtained from these two variants of the modelling procedure followed in this section.

#### 3.3.2. Estimating the Transformed Model using the Wickens-Breusch Variant.

The first variant was estimated by applying OLS on equation

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(3.14). The sample period used was the same as that adopted throughout this chapter. The estimated 'general' model was found to be:

$$\Delta m_t = -0.782 + 0.197\Delta m_{t-1} + 0.307\Delta t_{-2} + 0.123\Delta m_{t-3} + 0.079\Delta m_{t-4} +$$

(0.742) (0.138) (0.140) (0.143) (0.147)

$$0.110\Delta p_t - 0.129\Delta p_{t-1} - 0.080\Delta p_{t-2} + 0.220\Delta p_{t-3} +$$

(0.238) (0.252) (0.260) (0.229)

$$0.194\Delta p_{t-4} + 0.060\Delta y_t + 0.058\Delta y_{t-1} + 0.031\Delta y_{t-2} +$$

(0.233) (0.132) (0.165) (0.152)

$$0.001\Delta y_{t-3} - 0.045\Delta y_{t-4} + 0.0089\Delta R_t^1 + 0.0019\Delta R_{t-1}^1 -$$

(0.150) (0.128) (0.0029) (0.0032)

$$0.0017\Delta R_{t-2}^1 + 0.0041\Delta R_{t-3}^1 + 0.0000\Delta R_{t-4}^1 - 0.0516Q_1 -$$

(0.0032) (0.0031) (0.0033) (0.0164)

$$0.0083Q_2 - 0.0102Q_3 - 0.045m_{t-1} + 0.040p_{t-1} + 0.123y_{t-1} -$$

(0.0166) (0.0160) (0.031) (0.031) (0.074)

$$0.0029R_{t-1}^1$$

(0.0021) (3.26)

$$R^2 = 0.680 \quad \hat{\sigma} = 0.0181 \quad DW = 2.06 \quad Z_1 = 1.19 \quad E_1 = 0.84$$

$$LM(5) = 2.69 (*) \quad ARCH(5) = 0.36 \quad Z_5 = 0.36 \quad E_4 = 0.35$$

$$RESET(1) = 4.27(*) \quad RESET(2) = 2.11$$

Once more as for all our general 'starting-point' models, this model is overparameterised, with most of the regressors insignificant. Even so, it is gratifying to see that the regressors in levels, which capture the long-run effects, have the highest t-values, even at this stage, again pointing to the fact that the variables are, in all probability, cointegrated.

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However, note that the model fails two diagnostic tests, even at the early stage. However, as in the case of one of our previous 'general' models, the failure of the LM(5) test for serial correlation and the RESET(1) test may be a temporary aberration due to the interaction of the various regressors, including the seasonal dummies. This should disappear if an adequate parsimonious model is found.

After a specification search, we found the following to be the best model using the transformed equation:

$$\begin{aligned} \Delta m_t = & -0.652 - 0.060Q1 - 0.010Q2 - 0.023Q3 + 0.196\Delta_2 p_{t-3} + \\ & (0.317) (0.006) (0.006) (0.005) (0.092) \\ & 0.0043\Delta R_{t-3}^1 + 0.219\Delta_3 m_{t-1} - 0.033(m - p)_{t-1} + \\ & (0.0024) (0.048) (0.024) \\ & 0.100y_{t-1} - 0.0037R_{t-1}^1 \\ & (0.032) (0.0012) \end{aligned} \quad (3.27)$$

$$R^2 = 0.663 \quad \hat{\sigma} = 0.0161 \quad DW = 2.10 \quad Z_1 = 1.25 \quad E_1 = 1.18$$

$$LM(5) = 2.07 \quad ARCH(5) = 0.73 \quad Z_5 = 1.00 \quad E_4 = 0.79$$

$$RESET(1) = 4.71(*) \quad RESET(2) = 2.32$$

This model has adequate within-sample fit (see figure 3.9), and out-of-sample forecasting properties (see figure 3.10), with only one quarter (the usual 1986(1)) lying outside the forecast confidence interval. However, more seriously, the parsimonious model fails the RESET(1) test, in common with the 'general' equation. On the positive side, the estimated standard error of the equation is lower than that achieved by any other model presented in this chapter. Furthermore, it should be recalled

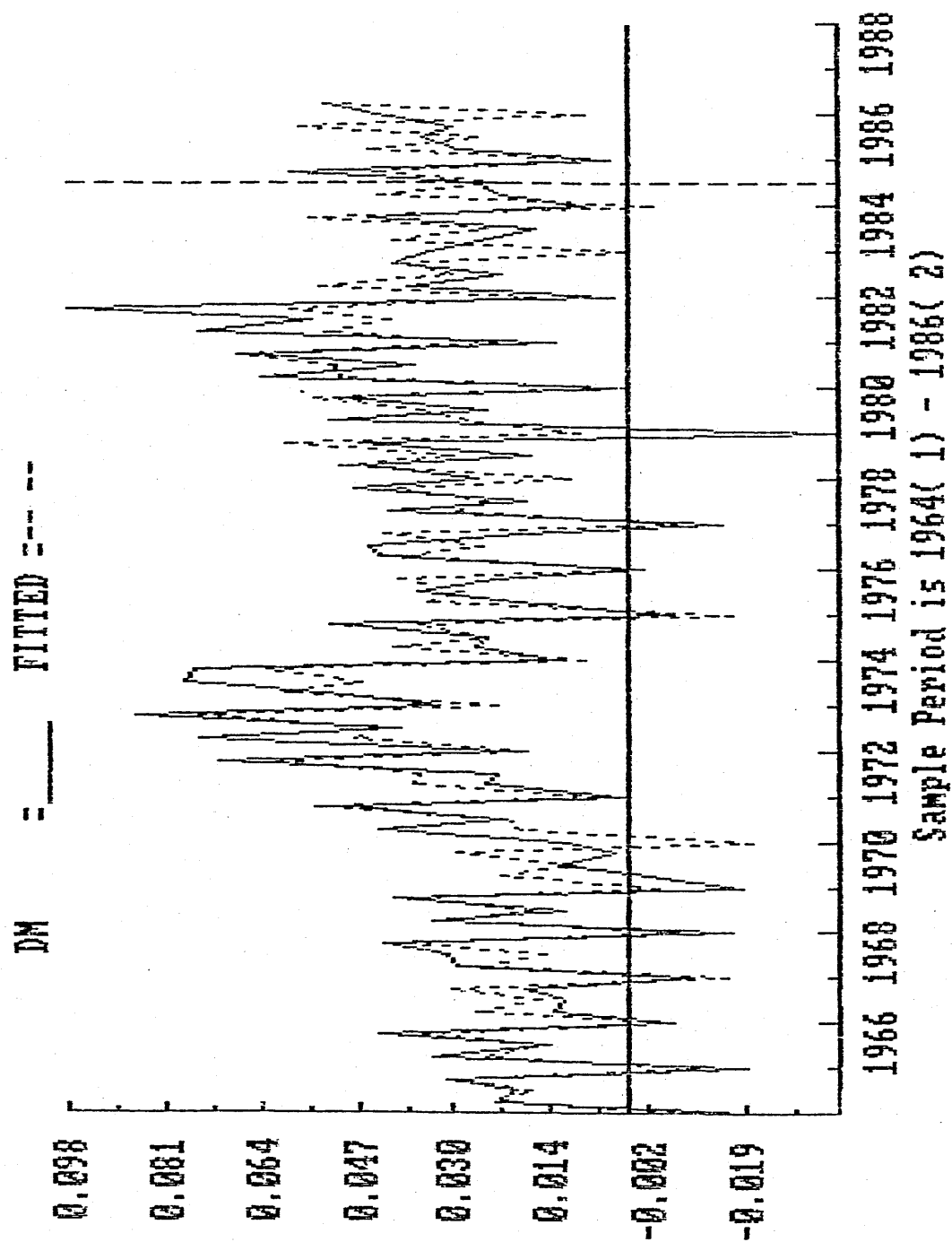


FIGURE 3.9

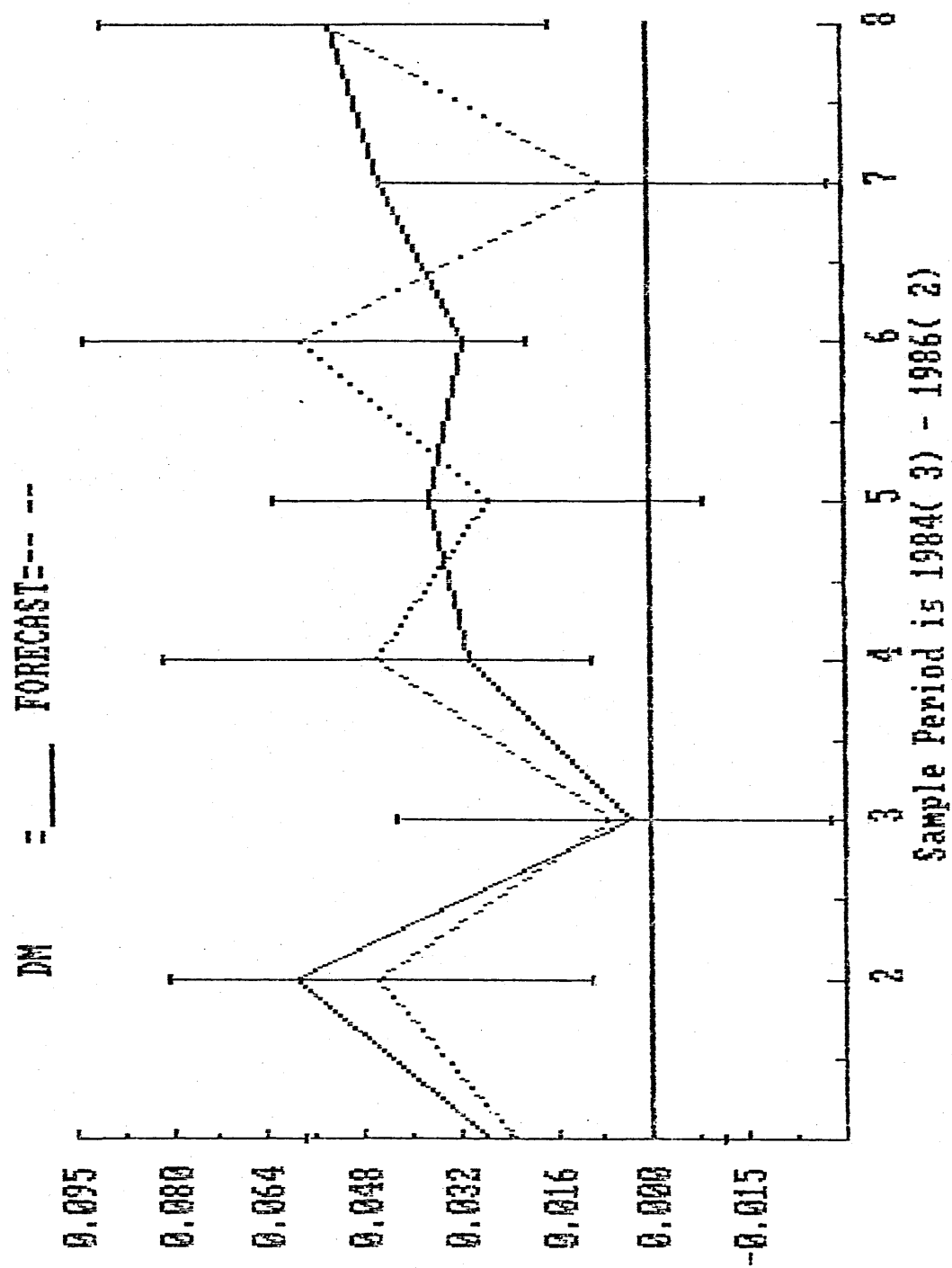


FIGURE 3.10

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that the models obtained using the Engle-Granger two-stage method also failed the RESET tests.

We shall return to a more formal comparison of this model with the others obtained so far in section four. However, we now turn to our final estimation procedure, a variant of that outlined in this subsection.

#### 3.3.3. Estimating the 'Constrained' Variant' of the Transformed Model

As explained above, this variant consists of constraining the long-run properties of the transformed model at the outset. This can be done by taking the point estimates of the long-run multipliers from equation (3.26) and using these to construct an error-correction term which is then used in the rest of the simplification process.

This method led us to estimate the following preferred final model:

$$\Delta m_t = -0.010 - 0.059Q1 - 0.014Q2 - 0.025Q3 + 0.218\Delta p_{t-3} +$$

$$(0.021) (0.006) (0.005) (0.005) (0.087)$$

$$0.237\Delta m_{t-1} - 0.0028ECM_{t-1}$$

$$(0.044) (0.0014)$$

(3.28)

$$R^2 = 0.616 \quad \hat{\sigma} = 0.0169 \quad DW = 2.07 \quad Z_1 = 1.41 \quad E_1 = 1.32$$

$$LM(5) = 1.12 \quad ARCH(5) = 0.22 \quad Z_5 = 0.90 \quad E_4 = 1.07$$

$$RESET(1) = 5.86(*) \quad RESET(2) = 2.90$$

This model is considerably simpler in its short-run dynamic structure compared to (3.27). However, in common with (3.27) it

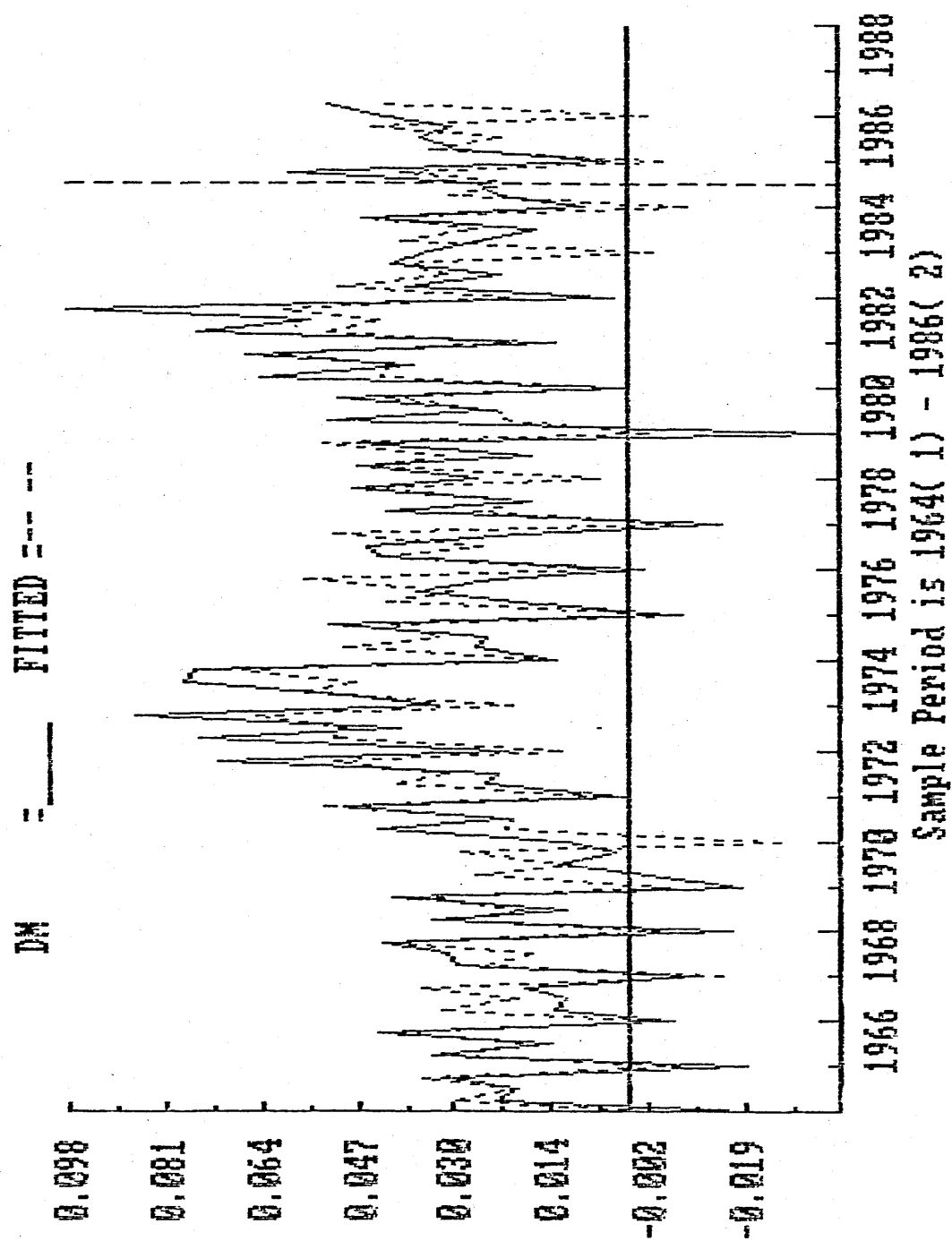


FIGURE 3.11

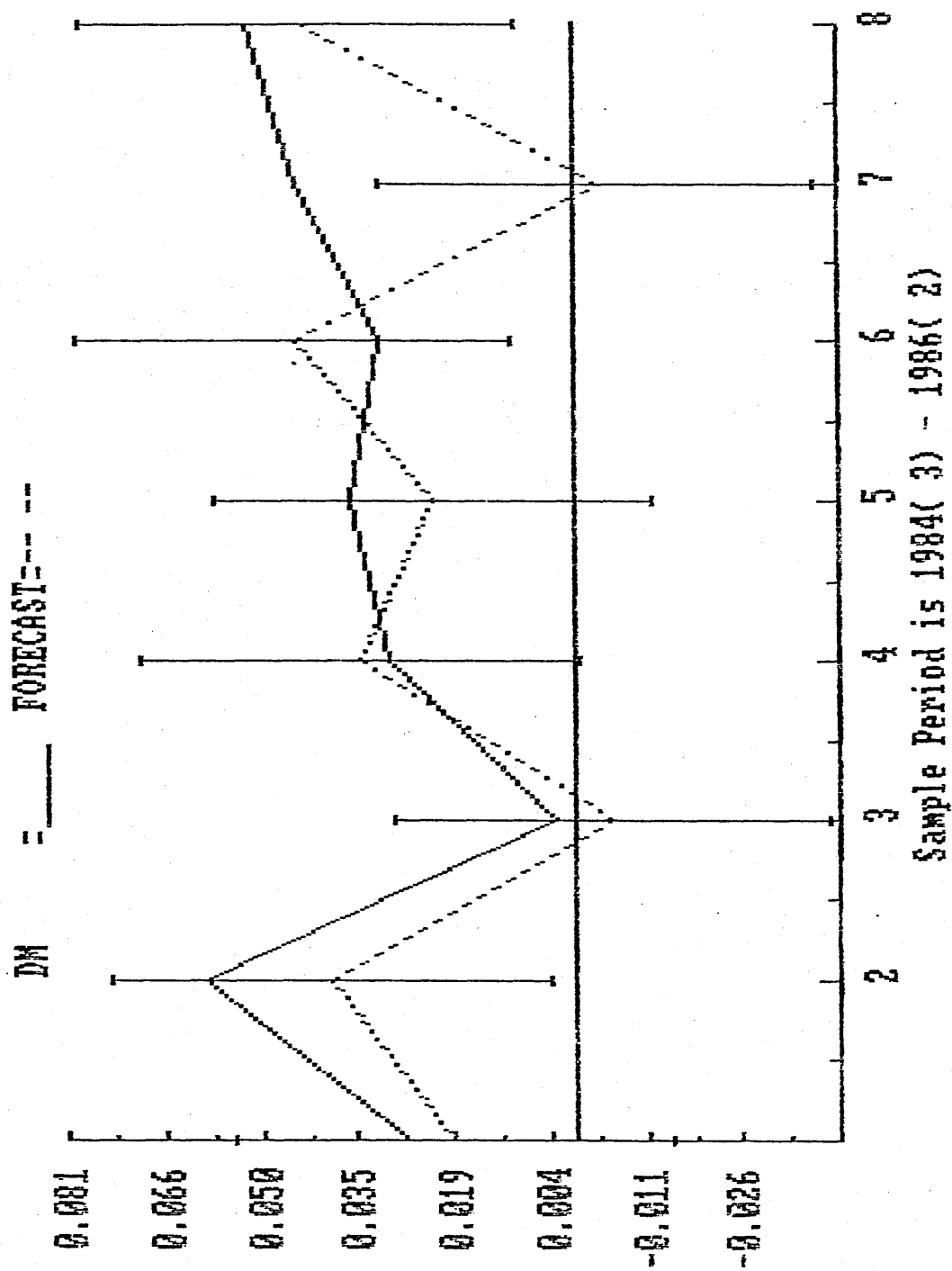


FIGURE 3.12



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still fails the RESET(1) test. The goodness-of-fit of the model is displayed in figure 3.11, and the out-of-sample forecasts in figure 3.12. Figure 3.12 shows us that this model again has problems with the forecast of the 1986(1) quarter, the t-value of the forecast error actually equal to 2.71 in this case.

Both (3.27) and (3.28) do not have a negative inflation effect, though the same explanation applies here as in the case of the other models with  $\Delta m_t$  as the dependent variable. If we were to re-express the model in terms of the rate of growth of real balances, a negative effect would probably appear.

It is interesting, though, that by constraining the long-run properties of the model at the outset such a different equation is obtained. However, (3.28) does have a much poorer fit compared to (3.27) in terms of the estimated standard error, perhaps indicating some misspecification in its short-run dynamics. In fact, its fit is not much better than that achieved with a simple autoregressive model in section one, which is rather worrying.

However, at this stage, after a detailed listing of all the estimated models, it is appropriate to turn to a formal comparison of our preferred equations obtained from all the different modelling strategies followed in this chapter. This is done in the next section and, as we pointed out above, the comparison has three facets to it: first, we engage in a brief superficial comparison of the different models, on the basis of the results and statistics obtained so far. Next, we compare the

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long-run multipliers obtained using the different models. This will enable us to gauge whether the different models used actually lead us to make drastically different inferences regarding the long-run demand for money, or whether we should really be indifferent between the modelling strategies used. Lastly, we will compare the ability of each model to explain the features of the other via non-nested variance-encompassing tests, which examine the relative goodness-of-fit of the different competing models.

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### SECTION FOUR: COMPARING THE DIFFERENT APPROACHES

#### 3.4.1 A Prima-Facie Comparison

Given the volume of evidence presented so far, which modelling approach, or variant of a modelling approach has given us the best results. As we discussed in chapter two, the criteria for selecting a 'good' model are many, and most are of equal importance, so that we cannot freely sacrifice one aspect for the other (see Harvey, 1981a, Hendry, 1983).

One problem is that not all of our models pass even the comparatively small battery of diagnostic or misspecification tests set out in this chapter as simple hurdles. Admittedly, most of them pass the majority of the tests, especially against what we would regard to be the most common alternatives in time series modelling (i.e. serial correlation, autoregressive conditional heteroscedasticity, etc.). However, we have seen that one variant of the transformed regression models in section three comes perilously close to failing the LM(5) test<sup>5</sup>. Furthermore, both models obtained from the transformed regression equation (3.24), and the models obtained using the Engle-Granger two-stage procedure fail the RESET tests. If this is seen as an indication that the usual assumption of linearity is invalid<sup>6</sup> then this is also worrying, as it would indicate our estimates are inconsistent. Non-linearity is usually tackled via transformations on the variables (see for instance Box and Tidwell, 1962, Box and Cox, 1964). However, given that our data

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is already in logarithmic form, it is unlikely that this is likely to resolve the problem. Furthermore, the actual type of non-linearity present is difficult to gauge, as the test has a very general alternative. Furthermore, the failure of the RESET tests may not actually be due to non-linearity but to other problems with the model (see for instance Spanos, 1986), in which case doing nothing may be the second-best strategy, especially in the absence of any further information.

However, the failure of diagnostic tests should really lead us to discard such models at the outset. Here, we still choose to confront them with the best results obtained using other modelling strategies because they represent the 'best' models which we could obtain in the given circumstances, given the particular modelling strategies advocated.

Examining the other statistics available, we see that by comparing the estimated standard error of each equation, we would rank the simple autoregressive model as the worst, which is not surprising. Perhaps surprisingly, the model obtained using the 'general-to-specific' procedure ranks second worse in such a classification, although it does pass all the diagnostic tests: the only model to do so, apart from the simple autoregressive equation.

In terms of their forecasting ability, all the models are somewhat similar, as may be seen by examining the graphs presented so far in this chapter. However, a more interesting

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measure of each model's performance may perhaps be gauged by examining the performance of each model once the sample period is extended over the 8 quarters of the ex ante forecast period. This will yield statistics which to some extent reflect the degree of parameter constancy in the model. The lack of parameter constancy over the forecasting period will yield a far worse equation. Hence, it is worth examining these statistics once the estimates have been extended up to 1986(2). The results are given in Table 3.1 below.

The main point to note from Table 3.1 is that there is a drastic effect on the performance of all the models, which may seem surprising given the reasonable performance reported for all the models in terms of  $Z_1$  and  $E_1$  tests above. However, this is probably due to a large extent to the presence of the 1986(1) data point in the extended sample. As we pointed out above, for all the models this proved to be an outlier in forecasting out of sample.

The deterioration in within-sample-fit has also led, however, to a narrowing of the different models' performances, with most of them performing only narrowly better than the simple autoregressive model in terms of the estimated standard error. However, this observation notwithstanding, the ranking of the models in terms of this statistic remains the same. The unconstrained transformed model is the one exception to the general poorer performance, but this better performance in terms

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of goodness-of-fit is traded off once more by the fact that the model fails the LM(5) autocorrelation test, although the equation now passes both the RESET tests.

Overall, the results of Table 3.1 do emphasise that in the case of M3, although some reasonable results may be obtained, there is no guarantee that the equation estimated for a particular sample period will remain the chosen one for a slightly different sample period. Although our parameter constancy tests are 'passed' in the formal sense, we seem unable to obtain results for the demand for M3 which are as robust as those which may be obtained for the demand for M1 in the UK (see Hendry, 1985). Having qualified our results, however, one must say in their defence that their performance (for some of the modelling strategies used) is adequate when assessed by the usual tests which the researcher has at his disposal to engage in model selection.

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Table 3.1

Statistics for Estimated Models - Sample up to 1986(2)

(1) Simple Autoregressive Model

$$R^2 = 0.567 \quad \hat{\sigma} = 1.747\% \quad DW = 2.07 \quad LM(5) = 0.31 \quad ARCH(5) = 0.87$$

$$Z_5 = 1.31 \quad E_4 = 0.924 \quad RESET(1) = 3.58 \quad RESET(2) = 2.25$$

(2) Engle-Granger, ECM with Unlagged Interest Rate

$$R^2 = 0.598 \quad \hat{\sigma} = 1.705\% \quad DW = 2.00 \quad LM(5) = 0.46 \quad ARCH(5) = 0.31$$

$$Z_5 = 1.07 \quad E_4 = 0.737 \quad RESET(1) = 7.27(*) \quad RESET(2) = 3.62(*)$$

(3) Engle-Granger, ECM with Lagged Interest Rate

$$R^2 = 0.610 \quad \hat{\sigma} = 1.700\% \quad DW = 1.98 \quad LM(5) = 0.56 \quad ARCH(5) = 0.64$$

$$Z_5 = 0.80 \quad E_4 = 0.483 \quad RESET(1) = 3.89 \quad RESET(2) = 1.93$$

(4) 'General-to-Specific'

$$R^2 = 0.706 \quad \hat{\sigma} = 1.737\% \quad DW = 1.92 \quad LM(5) = 0.81 \quad ARCH(5) = 1.09$$

$$Z_5 = 0.34 \quad E_4 = 1.031 \quad RESET(1) = 0.53 \quad RESET(2) = 0.61$$

(5) Transformed Model - Wickens-Breusch - Unconstrained

$$R^2 = 0.635 \quad \hat{\sigma} = 1.629\% \quad DW = 2.23 \quad LM(5) = 2.79(*) \quad ARCH(5) = 0.74$$

$$Z_5 = 1.12 \quad E_4 = 0.802 \quad RESET(1) = 2.45 \quad RESET(2) = 1.26$$

(6) Transformed Model - Constrained ECM

$$R^2 = 0.580 \quad \hat{\sigma} = 1.714\% \quad DW = 2.08 \quad LM(5) = 0.67 \quad ARCH(5) = 0.33$$

$$Z_5 = 1.32 \quad E_4 = 1.230 \quad RESET(1) = 5.68(*) \quad RESET(2) = 2.86(*)$$

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One aspect of a 'good modelling strategy', namely the consonance of empirical results with theoretical priors, has not yet been addressed. If any of the models obtained so far yield long-run elasticities which are implausible, then clearly that must cast a shadow over the use of the particular modelling strategy which yielded the model. One other question which we have to tackle here is whether the long-run properties of the models obtained using different model selection strategies differ. If they do not differ substantially, then it suggests that if one is interested in only the long-run properties of the demand for money, the simplest route to obtaining estimates of the long-run elasticities should be followed. If there are big differences, then a certain amount of caution must be adopted in interpreting these long-run coefficients. In this context, a comparison of the long-run elasticities obtained from the Engle-Granger procedure with the other models may give us a pointer regarding the amount of bias present in the cointegration equation estimates with a sample of 94 periods.

#### 3.4.2 A Comparison of the Long-Run Elasticities

The long-run price, income and interest rate elasticities for the demand for money are reported in Table 3.2 below. In the case of the simple autoregressive model there is no clearly static long-run solution, and hence no elasticities are reported. For the two variants of the Engle-Granger solution, these elasticities are merely the point estimates obtained from the



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Table 3.2

Long-Run Elasticities for Estimated Models - Sample up to 1984(2)

(1) Engle-Granger, ECM with Unlagged Interest Rate

$$\epsilon_p = 0.953 \quad \epsilon_y = 0.931 \quad \epsilon_R = 0.050$$

(2) Engle-Granger, ECM with Lagged Interest Rate

$$\epsilon_p = 0.962 \quad \epsilon_y = 1.109 \quad \epsilon_R = -0.040$$

(3) 'General-to-Specific': Equation (3.10)

$$\epsilon_p = 1.000 \quad \epsilon_y = 1.614 \quad \epsilon_R = 0.000$$

(4) 'General-to-Specific' : Equation (3.11)

$$\epsilon_p = 0.750 \quad \epsilon_y = 2.892 \quad \epsilon_R = -0.054$$

(5) Transformed Model - Wickens-Breusch - Unconstrained

$$\epsilon_p = 1.000 \quad \epsilon_y = 3.030 \quad \epsilon_R = -1.120$$

(6) Transformed Model - Constrained ECM

$$\epsilon_p = 0.889 \quad \epsilon_y = 2.733 \quad \epsilon_R = -0.604$$

(7) Hendry-Mizon (1978)

$$\epsilon_p = 1.000 \quad \epsilon_y = 1.600 \quad \epsilon_R = -2.600$$

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cointegration equation. In the case of the Wickens-Breusch variant, the elasticities are obtained from the lagged terms in levels, where the lagged term on  $m_{t-1}$  yields the value of  $(1 - \sum \alpha_i)$ , and the sum of the distributed lag parameters from the original ADL are given by the lagged independent variables. In the case of the second variant of the transformed model, the long-run elasticities are those which have been imposed at the outset, and may be read off from equation (3.26). For the 'general-to-specific' model we follow the same procedure described in chapter two. We solve the final model for a static equilibrium, in which all the variables (including the price level) return to a constant equilibrium value. Two sets of elasticities are reported for the 'general-to-specific' case: they are derived from equations (3.10) and (3.11) respectively. The first is the model which we shall use formally in all our variance encompassing experiments in the next subsection. However, as we pointed out in section two, this equation has the unfortunate implication that the long-run demand for M3 is interest-inelastic. The second equation (equation 3.11) on the other hand implies a negative (though very small) long-run interest rate effect, and has an equally good fit as (3.10).

Finally, we should point out that the interest rate elasticities reported in Table 3.2 are at an interest rate level of  $R = 10\%$ . Recall that the interest rate enters our models in levels, not logarithms, and hence the estimated coefficients are

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semi-elasticities, which may then be converted to elasticities at a particular interest rate level, as in Table 3.2.

The first aspect one should note about the results presented in Table 3.2 is the wide disparity between the different point estimates obtained for the demand for money elasticities. The implication of this is of course that it matters a great deal which modelling strategy one uses in modelling the demand for M3.

Let us examine some of the figures more closely. Some of the estimates obtained clearly do not conform to the theoretical priors. We have already pointed to the fact that the standard Engle-Granger procedure yields a positive interest rate elasticity for the demand for M3, and this not only runs contrary to economic theory, but is also in conflict with all the other results obtained. Furthermore, the lack of a long-run interest rate effect in equation (3.10) is also in conflict with theory, and the main body of evidence.

From economic theory one should also expect a unit elasticity with respect to the price level. This is confirmed by most of the models. The only model which produces a point estimate which is widely at odds with a value of unity is equation (3.11), and even here we get a value of 0.75. All the other estimates are sufficiently close to unity.

Unfortunately, economic theory offers little further guidance, except for some view regarding the signs of the long-run elasticities, and the fact that  $\epsilon_p$  should be equal to one. As

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a benchmark for our studies we therefore reproduce the elasticities estimates presented by Hendry and Mizon (1978)<sup>7</sup>. Let us compare these values initially with those from the unconstrained transformed equation (equation 5 in Table 3.2). Apart from a lower interest rate elasticity (which may be due to data discrepancies), the price elasticity is very close to unity in equation (5), though the real income elasticity is lower than in Hendry-Mizon (1978). To some extent we could regard the estimates presented in equation (5) as a good indication of the long-run elasticities, because they are a direct estimate of the implied long-run multipliers from the general ADL. Unlike the simple cointegration equations, they are unlikely to suffer from omitted variable bias, although given that the dynamics are overspecified, they are likely to be rather inefficient estimates.

If these estimates are regarded as accurate, this would lead us to conclude that the estimates obtained using the Engle-Granger method are biased on the low side, despite the rather large sample at our disposal<sup>8</sup>. The same applies to the equations obtained using the 'general-to-specific' method, although the real income elasticities obtained using the latter method are reasonably close to those obtained from equation (5).

If we are to believe the long-run elasticities from equation (5), we should be able to explain the differences between these estimates and those reported by Hendry and Mizon (1978). To some

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extent the differences may be accounted for by the fact that different data is used here. Another explanation is of course the fact that the long-run elasticities may have changed over the last decade. The demand for M3 may have indeed become less interest elastic, although it is difficult to explain why the real income elasticity should have increased, unless this phenomenon is capturing some other omitted effect.

We will attempt to draw some further conclusions regarding the relative merits of the various modelling methods presented in this chapter in the concluding section. Before this, however, we attempt to discriminate between the various models using variance encompassing tests.

### 3.4.3. Variance Encompassing Tests

We have already encountered the notion of 'encompassing' in a number of guises. The notion of 'encompassing' has been popularised by the work of Hendry and Richard (1982), Mizon and Richard (1983), and Mizon (1984). To briefly resume what we have said so far, one may consider encompassing as the ability of a given model to explain the results of rival models. This concept is therefore inevitably complicated, and in this section we shall mostly be concentrating on the more restricted notion of variance encompassing.

To some extent we have already engaged in some exercises in encompassing in the previous sections in comparing the adequacies and inadequacies of the different models which have been

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estimated for the demand for M3. As we pointed out there, the issue as to which model is dominant is in part answered by the fact that some models fail to pass all the misspecification tests. For instance, we would not seriously advance a static equation such as (3.4) as a model for the demand for M3 because it is seriously misspecified, as shown by the inevitable time dependence in the residuals and the estimated parameter vector.

However, as we have seen so far in this section such comparisons based on simple misspecification tests do not always lead to clear-cut results in the case of our preferred models. Provided the dynamic models are designed properly within the framework suggested by each of the approaches explored, then most of the tests are passed by the models. The variance encompassing tests suggested here are merely another piece in the puzzle: they enable a direct comparison between the models on the basis of their explanatory power.

The different models summarised in Tables 3.1 and 3.2 are non-nested. In these circumstances one may no longer apply the usual specification and misspecification test criteria to evaluate whether certain restrictions should be imposed on the general model: one model cannot be obtained as a special case of the other. At this point we may proceed in two ways: we may either attempt to discriminate between the models on the basis of some simple criterion such as the  $R^2$  statistic, or preferably a statistic which makes an allowance for parsimony such as the  $R^2$

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statistic corrected for degrees of freedom (see Theil, 1971), the Akaike information criterion (see Akaike, 1973), or alternative criteria such as the Schwarz criterion (see Schwarz, 1978), or the Sawa criterion (see Sawa, 1978).

A more formal approach to the problem of assessing the relative adequacy of competing models is to attempt to embed the problem within the context of a formal statistical test. A whole class of non-nested tests have been developed to enable the researcher to compare such competing hypotheses. Four of these tests are used in this subsection, and most of them follow from the original work of Cox (1961, 1962). We now briefly describe these tests in turn.

The most common reported non-nested test is the so-called 'Cox test' which is an application of the Cox procedure suggested by Pesaran (1974). The idea behind this test is to modify the usual Neyman-Pearson likelihood ratio, by comparing the difference between the observed log-likelihoods of model one and model two with an estimate of the difference to be expected if model one was the correct model:

$$T_1 = \{\log(L_1) - \log(L_2)\} - E|_{H_1} \{\log(L_1) - \log(L_2)\} \quad (3.29)$$

where  $L_1$  and  $L_2$  are the likelihood functions under the hypothesis that model one is valid,  $H_1$  and the hypothesis that model two is valid,  $H_2$ , respectively.

There are two main problems with this test. First, whilst the first term is observable, the second is not, and the test has

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to be rendered operational. Pesaran's (1974) suggested implementation will be discussed below. For the moment it should be apparent that provided the test can be applied, we would expect  $T_1$  to be close to zero if  $H_1$  is true, as the two terms will cancel out. On the other hand, if  $H_2$  is true,  $T_1$  will tend to be 'significantly' negative. The test is a variance encompassing test, because it is based on the relative log-likelihoods, and hence on 'goodness-of-fit'.

The second problem with the test is that it has one model as the maintained hypothesis, and rejection of  $H_1$  may not be interpreted as a sign that  $H_2$  is acceptable. In fact, we may design another test, which reverses the direction of  $T_1$  by taking  $H_2$  as the maintained model, and  $H_1$  as the alternative against which high power is required:

$$T_2 = \{\log(L_2) - \log(L_1)\} - E|_{H_2} \{\log(L_2) - \log(L_1)\} \quad (3.30)$$

As a result of the set-up, it is quite possible for both  $H_1$  and  $H_2$  to be rejected in favour of the alternative, or for both the maintained hypotheses to be retained. As will become apparent, this difficulty is also present in other non-nested tests. The information at our disposal may just not be sufficient to discriminate between the models in terms of their explanatory power. In any case, for this and other non-nested tests, the small-sample properties are unknown, and hence they must be interpreted with care (see Judge *et al.*, 1985).

In any case, whatever its disadvantages, we believe that the



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Cox test, and the other non-nested tests reported here can give useful insights into the comparison of the alternative models for M3. The Cox test statistic reported here, following Pesaran (1974) is:

$$N_1 = T_1 / \sqrt{\text{avar}(T_1)} \quad (3.31)$$

where the asymptotic variance of  $T_1$ ,  $\text{avar}(T_1)$  may be estimated from the data, and the statistic  $N_1$  is distributed as a  $N(0,1)$  statistic under the maintained hypothesis  $H_1$ . As we noted above, the alternative is one-sided, with  $N_1$  taking on significant negative values if  $H_2$  is true. On the other hand, as Pesaran (1974) notes, a significant positive value for  $N_1$  would indicate a rejection of  $H_1$  against some alternative, but not in favour of  $H_2$ .

In addition to the Cox test, we apply three other non-nested tests. The second test is the instrumental variable version of the Cox test described above, also known as the Ericsson IV test (see Ericsson, 1983). Again, this test statistic is distributed as a  $N(0,1)$  variate under the maintained hypothesis.

A test along slightly different lines is the joint-model F-test, which nests both models within a more general model. The test of  $H_1$  against  $H_2$  is then carried out by testing zero restrictions on the variables which enter the joint model which are particular to  $H_2$  with a conventional F-test. This test is an adaptation of the general formulation proposed by Atkinson (1970) and Quandt (1974) and based on Cox's original (1962) contribution.

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There are a number of additional problems with this approach when we compare it to the Cox test (for a fuller outline, see Pesaran and Deaton, 1978). The first problem is that often there will be problems in estimating the general model because of a high degree of collinearity between the regressors. This is especially the case when, as in this instance, we are dealing with models which differ mainly in their dynamic structure, and not in the basic regressors used.

Secondly, if both of the competing hypotheses are rejected in favour of the general model, this may cause severe problems of interpretation in that the general model, especially when the hypothesis are radically different (for a good example see the tests applied to our M1 models in Chapter 5). Overall, it is fair to say that this type of non-nested test may lead to inconclusive evidence, especially when dealing with models which are close in terms of structure and estimated standard errors. In general, the Atkinson formulation has found most favour amongst those researchers who wish to discriminate between competing models with different functional forms, where the test is designed to have considerable power against a specific alternative (see Anueryn-Evans and Deaton, 1977).

The last non-nested test to be considered in this section is an application of the Sargan (1964) misspecification test. This test was originally developed for checking the validity of instruments for the estimation of the parameters of a single

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equation from a simultaneous system. However, it also has an encompassing interpretation (see Mizon, 1984), as it is a statistic which may be used for testing the hypothesis that some of the variables which enter a competing model should enter the maintained model.

Having discussed the different methods at our disposal to compare the models under scrutiny, we now turn to apply these tests to our results.

#### 3.4.4. Discriminating Between the Different M3 Models.

As we noted in the previous subsection, one method of comparison relies on the calculation of some information criterion for the models. Two of the most popular information criteria are the Akaike Information Criterion (AIC) (see Akaike, 1974), and the Schwarz Criterion (SC) (see Schwarz, 1978), which are defined as follows:

$$\text{AIC} = -2\log(L) + 2k \quad (3.32)$$

where  $\log(L)$  is the value of the log-likelihood function for the model, and  $k$  is the number of estimated parameters. Thus, the model penalises a large number of regressors, and hence lays a certain emphasis on parsimony. The SC statistic is given by:

$$\text{SC} = -\log(L) + \frac{1}{2}k\ln(T) \quad (3.33)$$

where  $T$  is the number of observations. Compared to the AIC the SC tends to impose a greater penalty on the numbers of parameters in a model, favouring a lower-dimensional formulation. This difference between the two statistics will be accentuated as the

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number of observations,  $T$ , rises.

The AIC and SC for the various M3 models are reported in Table 3.3 below. As expected, the ranking is heavily influenced by the numbers of regressors included in each model, given that the estimated standard errors (and hence the log-likelihoods) are very close for all the models. As a result, parsimonious models, such as the simple autoregressive model perform rather well in terms of these criteria, even though it has the highest estimated standard error. Similarly, both the Wickens-Breusch variant of the transformed model and the 'general-to-specific' model<sup>9</sup> perform rather badly, because they contain 10 estimated parameters, even though the former model has the lowest standard error. As expected, the information criteria point to models with intermediate rankings in terms of numbers of regressors and goodness-of-fit such as the conventional Engle-Granger model, and the restricted version of the transformed model which turned out to be the best performers. In general, the SC and AIC statistics point in the same direction. As expected, the simple autoregressive model does far better under the SC statistic. To some extent these slight differences point to the ad hoc nature of some of these information criteria.

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Table 3.3

Information Criteria for the Estimated Models - Sample up to 1984(2)

(1) Engle-Granger, ECM with Unlagged Interest Rate

AIC = -428.41    SC = -7.829

(2) Engle-Granger, ECM with Lagged Interest Rate

AIC = -426.41    SC = -7.949

(3) 'General-to-Specific': Equation (3.10)

AIC = -414.56    SC = -7.747

(4) Transformed Model - Wickens-Breusch - Unconstrained

AIC = -416.49    SC = -7.842

(5) Transformed Model - Constrained ECM

AIC = -430.44    SC = -7.875

(6) Simple Autoregressive Model

AIC = -424.65    SC = -7.940

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Furthermore, as we pointed out above, these statistics only offer a very informal comparison between the competing hypotheses. A more formal statistical framework is provided by the four different non-nested tests described above. We carried out these four tests on the six models listed in Table 3.3, and the results are reported in Tables 3.4-3.7, which display the results from the Cox-test, Ericsson IV test, Sargan (1964) test, and joint-model F-test respectively. For each pair of models, the two competing hypotheses were taken as the maintained hypothesis in turn, so that, for example, the  $(i,j)$ th cell in each table tests hypothesis  $H_i$  against the alternative given by  $H_j$ , whilst the  $(j,i)$ th cell shows the result of the test of  $H_j$  against the alternative  $H_i$ .

The results obtained from the four non-nested tests may be paired in two groups, as the Cox and Ericsson IV tests give very similar results, whilst the joint-model F-test and the Sargan test give very similar results to each other. In general the results from the second pair of tests have been less decisive in discriminating between the competing hypotheses, which is not surprising given the problems of testing the joint model against a restricted alternative, especially when dealing with a large number of regressors, many of which are in common between the models (e.g. the constant term and the seasonal dummies). Nevertheless, taken together, the two pairs of non-nested tests give a reasonably complete ranking of the different M3 models.

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Table 3.4

## Testing the Competing Models Using the Cox Test

vs.	H <sub>1</sub>	H <sub>2</sub>	H <sub>3</sub>	H <sub>4</sub>	H <sub>5</sub>	H <sub>6</sub>
H <sub>1</sub>	-	-0.821	-0.977	-3.911*	-2.415*	-0.565
H <sub>2</sub>	-0.313	-	-1.337	-3.542*	-2.509*	-0.827
H <sub>3</sub>	-2.690*	-2.324*	-	-4.148*	-4.092*	-2.073*
H <sub>4</sub>	-0.854	-0.407	-1.417	-	-0.515	-1.218
H <sub>5</sub>	-6.500*	-7.462*	-4.486*	-121.5*	-	-0.926
H <sub>6</sub>	-4.386*	-4.515*	-3.258*	-3.941*	-2.776*	-

where H<sub>1</sub> = Engle-Granger, ECM with unlagged interest rate

H<sub>2</sub> = Engle-Granger, ECM with lagged interest rate

H<sub>3</sub> = 'General-to-Specific' : Equation (3.10)

H<sub>4</sub> = Transformed Model: Wickens-Breusch

H<sub>5</sub> = Transformed Model: Constrained ECM

H<sub>6</sub> = Simple Autoregressive Model

N.B. the test statistics are distributed as a standard normal variate under the maintained hypothesis. A (\*) denotes rejection of the maintained hypothesis at the 5% significance level.

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Table 3.5

## Testing the Competing Models Using the Ericsson IV Test

vs.	H <sub>1</sub>	H <sub>2</sub>	H <sub>3</sub>	H <sub>4</sub>	H <sub>5</sub>	H <sub>6</sub>
H <sub>1</sub>	-	0.777	0.907	3.566*	2.235*	0.534
H <sub>2</sub>	0.298	-	1.220	3.258*	2.321*	0.778
H <sub>3</sub>	2.339*	2.036*	-	3.516*	3.496*	1.851*
H <sub>4</sub>	0.790	0.379	1.279	-	0.482	1.103
H <sub>5</sub>	5.798*	6.629*	3.868*	108.93*	-	0.869
H <sub>6</sub>	3.879*	3.980*	2.908*	3.388*	2.537*	-

where H<sub>1</sub> = Engle-Granger, ECM with unlagged interest rate

H<sub>2</sub> = Engle-Granger, ECM with lagged interest rate

H<sub>3</sub> = 'General-to-Specific' : Equation (3.10)

H<sub>4</sub> = Transformed Model: Wickens-Breusch

H<sub>5</sub> = Transformed Model: Constrained ECM

H<sub>6</sub> = Simple Autoregressive Model

N.B. the test statistics are distributed as a standard normal variate under the maintained hypothesis. A (\*) denotes rejection of the maintained hypothesis at the 5% significance level.



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Table 3.6

## Testing the Competing Models Using the Sargan Test

vs.	H <sub>1</sub>	H <sub>2</sub>	H <sub>3</sub>	H <sub>4</sub>	H <sub>5</sub>	H <sub>6</sub>
H <sub>1</sub>	-	0.532	2.261	5.153	2.139	0.275
H <sub>2</sub>	0.085	-	3.299	4.168	2.285	0.575
H <sub>3</sub>	5.505	4.812	-	8.924	7.975*	2.903
H <sub>4</sub>	0.704	0.128	1.574	-	0.231	1.537
H <sub>5</sub>	6.849*	7.391*	9.491	9.239	-	0.712
H <sub>6</sub>	10.243*	10.907*	9.472	8.671	6.048	-

where H<sub>1</sub> = Engle-Granger, ECM with unlagged interest rate

H<sub>2</sub> = Engle-Granger, ECM with lagged interest rate

H<sub>3</sub> = 'General-to-Specific' : Equation (3.10)

H<sub>4</sub> = Transformed Model: Wickens-Breusch

H<sub>5</sub> = Transformed Model: Constrained ECM

H<sub>6</sub> = Simple Autoregressive Model

N.B. the test statistics are distributed as a chi-square(n) variate under the maintained hypothesis, where n is the number of regressors in the joint model which are not present in the maintained model. A (\*) denotes rejection of the maintained hypothesis at the 5% significance level.

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Table 3.7

## Testing the Competing Models Using the Joint-Model F-Test

vs.	H <sub>1</sub>	H <sub>2</sub>	H <sub>3</sub>	H <sub>4</sub>	H <sub>5</sub>	H <sub>6</sub>
H <sub>1</sub>	-	0.261	0.357	1.033	1.072	0.273
H <sub>2</sub>	0.041	-	0.644	0.824	1.147	0.572
H <sub>3</sub>	1.905	1.649	-	1.898	2.868*	2.984
H <sub>4</sub>	0.347	0.063	0.299	-	0.229	0.371
H <sub>5</sub>	3.672*	3.995*	1.667	2.495	-	0.709
H <sub>6</sub>	3.791*	4.077*	1.661	1.505	2.104	-

where H<sub>1</sub> = Engle-Granger, ECM with unlagged interest rate

H<sub>2</sub> = Engle-Granger, ECM with lagged interest rate

H<sub>3</sub> = 'General-to-Specific' : Equation (3.10)

H<sub>4</sub> = Transformed Model: Wickens-Breusch

H<sub>5</sub> = Transformed Model: Constrained ECM

H<sub>6</sub> = Simple Autoregressive Model

N.B. the test statistics are distributed as an  $F(m, T-k)$  variate under the maintained hypothesis, where  $T$  is the number of observations,  $k$  is the number of estimated parameters in the general (joint) model, and  $m$  is the number of restrictions to be imposed on the joint model to reach the maintained hypothesis. A (\*) denotes rejection of the maintained hypothesis at the 5% significance level.

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Taking the Cox and Ericsson IV tests first, we note that this produces an ambiguous ranking for some of the models, but it points unambiguously to the Wickens-Breusch variant of the transformed model as the 'best', in variance encompassing terms. This shows the problems which can occur in being guided solely by ad hoc statistics in discriminating between models such as the information criteria reported in Table 3.3. At the other end of the scale, it becomes apparent that the 'general-to-specific' and the 'simple autoregressive' models performed worst of all, but a ranking between them is difficult, as each of these models leads to the rejection of the other when taken as the maintained hypothesis. We would probably rank the 'general-to-specific' model above the 'simple autoregressive' one, on the ground that one of the other models (the transformed regression model with a constrained error-correction term) is rejected in favour of the 'general-to-specific' model, but not in favour of the simple autoregression<sup>11</sup>.

As far as the three remaining hypotheses are concerned, a strict ranking becomes more difficult on the basis of these two non-nested tests alone. The transformed model with constrained ECM rejects the other two models when it is taken as the maintained hypothesis. On the other hand, both variants of the Engle-Granger model are rejected in favour of the transformed model. Neither of the two variants of the Engle-Granger model is rejected in favour of the other. The indecisiveness of all of

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this evidence is probably due to the very similar structure of the three models. Again a stricter ranking is possible if we note that the transformed model is rejected a greater number of times taking the other three models as alternative hypotheses. We would therefore rank both the Engle-Granger variants above the transformed model, without being able to discriminate between the former.

The results of Tables 3.6 and 3.7 are generally less decisive than those in Tables 3.4 and 3.5, as most of the maintained hypotheses are not rejected. The possible reasons for this were mentioned earlier. The lack of decision extends even to the 'clear winner' under the Cox and Ericsson tests (the Wickens-Breusch variant of the transformed model), as the simple autoregressive model is not rejected with it as its alternative.

However, it is worthwhile to note that both the Sargan and joint-model F-test lead to the rejection of the restricted-ECM variant of the transformed model against the two variants of the Engle-Granger procedure, but not viceversa. This leads us to prefer the latter models to the former. Taken together, the results of Tables 3.4-3.7 would lead us to propose the following preference ordering:

$$H_4 > \{H_1, H_2\} > H_5 > H_3 > H_6$$

Overall, the variance encompassing tests must be viewed as adding to the total picture offered by the various diagnostic tests, long-run elasticities, etc. reported in this section. In

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the conclusions which follow we try to pull together some of the common themes of the large volume of empirical evidence at our disposal, and look forward to the models to be considered in the next few chapters.

### SECTION FIVE: CONCLUSIONS

In this chapter, we have considered different approaches to the construction of a dynamic model of the demand for money (M3) in the UK. These approaches differed mainly in terms of econometric methodology rather than in terms of economic theory. All the different models recognise that the demand for money must be modelled using some rather intricate short-run dynamics, and all (except the simple autoregressive model) allow us to deduce (or directly estimate) the long-run elasticities of the demand for M3.

The reason why it is interesting to investigate such a multitude of alternative approaches is that, as has been shown in the empirical results in this chapter, they do not always lead to models with similar short-run (or, perhaps more seriously, long-run) properties. We have seen that a variety of point estimates for the long-run elasticities may be obtained by following different modelling approaches. Furthermore, the short-term tracking and forecasting properties of the models differ somewhat, as may be seen by the goodness-of-fit and statistics, and the ex ante forecast tests<sup>12</sup>. The dynamic structure obtained in each case seemed to depend greatly on the initial

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parameterisation considered (e.g. whether one began from the ADL model in levels or from a transformed model). This is not altogether surprising. In a recent criticism of the cointegration testing and two-step estimation procedure, Sims et al. (1986) pointed out that there may be little point in testing and imposing cointegration constraints because of the 'superconsistency' property when dealing with trending series. They argue for the implementation of vector autoregressions in levels, with the unit roots appearing in the estimated parameters.<sup>13</sup> However, this argument has been rebutted by Engle and Yoo (1986) on the grounds that the downward bias in the autoregressive parameter estimates would militate against the researcher finding many unit roots, leading to bad long-run forecasts. In a Monte Carlo experiment, Engle and Yoo showed that the application of the two-stage procedure (and the consequent transformation of the model) led to better long-run forecasts than the VAR, as it recognised the existence of these unit roots. The validity of these results have been confirmed by Hallman (1987).

Although Engle and Yoo's tests were directed at vector autoregressions in levels, it clearly highlights the treacherous terrain which the researcher faces when modelling integrated variables. It makes quite a difference whether one begins by modelling an ADL (with only levels) or a transformed model, as in the Engle-Granger case, where differences and an error-correction

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term are combined. In the former case one has different variables in levels, and one reparameterises the model whilst simplifying it, imposing (possibly invalid) cointegration constraints. This may lead to models which contain variables with very different degrees of integration, where some estimated parameters may be tending rapidly to zero (see Hallman, 1987). On the other hand, the Engle-Granger model is careful in imposing the cointegration constraints at the outset, leaving us with a model with all the variables integrated of the same order (including, of course, the error-correction term), avoiding these problems. No further 'reparameterisation' is then needed, and in fact such practices are viewed with extreme suspicion (see Hallman, 1987).

However, the argument is never wholly one-sided. The cointegration constraints imposed at the outset of the Engle-Granger procedure may be far from correct: in the first place, the low power of the tests may lead the researcher to the wrong conclusion regarding the cointegratedness of a vector of variables. Secondly, the bias in the first stage of the procedure may impose the wrong cointegration constraints on the model, with the consequence of severely altering its long-term forecasting properties, and leading us to reach wrong conclusions regarding long-run elasticities<sup>14</sup>. Thirdly, in situations where there are more than one unit roots, the appropriate error-correction form may not be the simple one suggested by Engle and Granger (1987). Unfortunately, no applied research has been yet forthcoming on

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the recent more complex structures suggested by, inter alia, Engle (1987).

Perhaps the advantage of the 'intermediate procedure' advanced by, amongst others, Wickens and Breusch (1988), Muscatelli and Papi (1988), is that of first transforming the ADL model and proceeding with the specification search without imposing the cointegration constraints at the outset. This has the advantage of reducing most of the variables to the same degree of integration, leaving only some variables in levels, with the estimated parameters on the latter hopefully converging to the cointegration constraints. This specification search could be preceded by an initial exploratory set of tests which seek to establish whether the variables are indeed cointegrated. The further advantage of this procedure would be that, given that the standard errors are not biased (provided the model is correctly specified!), it would enable us to test whether all the variables in levels are significant. The disadvantage, as in the case of the 'general-to-specific' procedure, is that the cointegration constraints are not actually imposed, which takes us back to Hallman's criticism of the whole idea of 'reparameterisation' if we do not impose the correct cointegration constraints. The real problem here is that the 'correct' cointegration constraint is a rather elusive concept, given the lack of powerful testing procedures for cointegration.

Can the results on variance encompassing in section four



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offer any guidance? They certainly seem to point towards the Wickens-Breusch approach. The difficulty, of course, is that this result may be data-specific, and may not carry over to other applications of the methods. Worse still, the results may be 'researcher-specific': although we have attempted to ensure that the 'best' model was found for each procedure by engaging in a comprehensive search, there is no guarantee that someone else supplied with the same data could achieve different results from those presented here!

However, our main purpose here was to find the best procedure in the context of modelling the demand for money. When we also take into account the long-run elasticities obtained using each model, the procedure we have chosen to name the 'Wickens-Breusch approach' seems to give the best and most plausible results (some reasonable estimated long-run elasticities, except perhaps for  $\epsilon_y$ , a negative inflation effect on the demand for real balances, a low estimated standard error compared to the other models, and a reasonable forecasting performance), as well as variance encompassing the other models. Ideally we should have attempted to confirm these results regarding the ranking of different model selection procedures on another data set (say, the demand for M1 money), but for reasons of space such a comparison was not possible here.

Overall, it would be fair to say that in recent years we have witnessed an explosion in the literature on econometric

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methods, but little has been said on the comparative advantages of each, and little attention has been spent on ways of structuring specification searches. Thus, although the above results may be seen as pointing in one direction, there is no guarantee that other investigations may not contradict these conclusions. Perhaps the most important lesson is that there are a variety of methods open to us, and we should probably investigate more than just a single route to the estimation of a dynamic model. The results of Chapter Two also indicate the importance of a preliminary investigation of the time series properties of the variables employed before actually estimating a structural model.

In the next two chapters, we move in a different direction in considering the empirical modelling of the demand for money. All of the models analysed so far have been dynamic in nature, but no rationale has been offered for these dynamics, except, as we pointed out in Chapter One, the obvious one of the presence of some unspecified adjustment costs. In the next two chapters we analyse a different approach, which focuses on the forward-looking nature of the demand for money. The literature on forward-looking models offers a new perspective on dynamic econometric models but, as we shall see, such a perspective is not entirely unrelated to the methods pursued in Chapters 2 and 3. The two approaches will be compared in detail in Chapter Five. Before that, however, we turn to a detailed survey of different

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forward-looking models which have been advocated in the case of the demand for money.

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### Footnotes to Chapter 3

(1) We did not use the lagged residuals from the cointegration equation to construct the error-correction term unlike Engle and Granger (1987), because we wished to conduct some out-of-sample forecasting for which no observations would be available for the error-correction term if the residuals were used. As a result the error-correction term was constructed using the point estimates of the cointegrating vector.

(2) Hallman (1987) objects to the idea of reparameterisation on the grounds that variables of different orders of integration will appear in the same regression. On the other hand, we have endeavoured to reparameterise the model such that the dependent variable at least is of the same degree of integration as the majority of the regressors. We will return to this issue in section five below.

(3) For a proof of this lemma, see Yoo (1986).

(4) Although the testing procedures which have been used here are still rather primitive. For recent surveys on cointegration tests, see Hylleberg (1987), and Engle, Granger, Hylleberg, and Yoo (1987).

(5) The model fails the LM(5) test when the sample is extended to 1986(2) (see Table 3.1).

(6) This should not be seen as the only alternative, though, as the Ramsay (1974) test has some power against other alternatives, especially the failure of the independent sample assumption.

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(7) Even here we are forced to assume that the study by Hendry and Mizon (1978) in some sense delivers the 'correct' long-run elasticities. There are also data discrepancies between their study and the estimates presented here. These estimates should therefore only be seen as some other frame of reference against which to compare our own estimates.

(8) Although the sample used here is quite large, some would argue that there is no guarantee that the long-run demand for money has remained invariant from the 1960s to the 1980s.

(9) Equation (3.10) is taken as the 'general-to-specific' model because of its greater parsimony compared to (3.11). Some would prefer the latter, because of its apparent consistency with conventional theory. On the other hand, the two equations have similar estimated standard errors, so that it does no harm to use (3.10) as the 'best' general-to-specific equation in our comparisons.

(10) There were problems in applying non-nested tests where the dependent variable differed, as in the case of the general-to-specific model. This model had  $\Delta(m-p)$  as the dependent variable, but because of the presence of  $\Delta p_t$  as a regressor, it was easily reparameterised to yield  $\Delta m_t$  as the dependent variable.

(11) This type of 'transitive ordering' of hypotheses in non-nested tests is not strictly valid from the statistical point of view. But the evidence does seem to point in favour of the 'general-to-specific' model as compared to the simple

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autoregressive model.

(12) These discrepancies are, however, considerably less marked than the long-run properties of the models.

(13) Note that this argument by Sims et al. (1986) also points against the standard Box-Jenkins (1970) differencing procedure when dealing with integrated series in univariate autoregressions.

(14) There is quite a difference between Engle and Yoo's 'correctly specified' two-stage model, and reality, where the cointegration vector is not actually known, and we do not know if the variables are actually cointegrated!

## CHAPTER 4

### CHAPTER 4: FORWARD-LOOKING MODELS OF THE DEMAND FOR MONEY AND THE 'BUFFER-STOCK MONEY' HYPOTHESIS

In the previous two chapters we built models of the demand for broad money (M3) on the basis of recent developments in econometric modelling. The approach followed was purely empirical. The task we set ourselves was that of building an adequate statistical model which adequately characterised the data generation process underlying the economic variables under scrutiny. Economic theory played a rather marginal role, except in that it set the limits within which these confrontations between different econometric approaches could be resolved. Thus, in Chapter 3 we stated that, for a dynamic empirical model of the demand for money to be valid, it had to have long-run properties which to some extent conformed to those which economic theory (or 'conventional wisdom' in the profession) suggested were appropriate.

In the next three chapters, we take a rather different approach, as we survey and develop some of the ideas about the behaviour of the money market which have received considerable attention in recent years. The search for some 'alternative approach' in the construction of demand for money models was propelled by two factors: firstly, questions began to be asked about the nature of the 'adjustment lags' implicit in the dynamic empirical 'feedback' models which we analysed in the previous two chapters. No simple explanations was offered for the lagged

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adjustment of the demand for money, and most econometricians were not particularly interested in offering insights into why these lags were present. As we pointed out in Chapter 2, the precise dynamic structure was seen as outwith the scope of economic theory. In practice, most modellers would agree that general Autoregressive Distributed Lag (ADL)-type models are a generalisation of simple 'partial adjustment' schemes<sup>1</sup>, and that the lags are present due to adjustment costs in portfolio allocation. Part of the motivation behind some of the models surveyed in this chapter is to try to provide further 'theoretical insights' into this short-run dynamic process<sup>2</sup>. Secondly, some economists began to argue that the conventional view of the money market, as usually embodied in the LM sector of the neoclassical-Keynesian synthesis required some refinement in the presence of partial adjustment-type schemes.

These two factors led to the development of what may be broadly classified as the theory of 'disequilibrium money', or 'buffer stock money'<sup>3</sup>. In this chapter we analyse the type of models which may be built by adopting this approach and we shall apply and critically assess some of the proposed modelling techniques in this context. As we shall soon discover, most of these models were built with the ultimate objective of empirical testing in mind. Given the vast (and diverse) literature which has emerged, we shall concentrate mostly on a particular method of modelling 'buffer stock money'. We shall critically assess



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existing models and develop some alternative empirical and theoretical variants of the basic approach.

It is important, however, to try to relate this new material to the econometric models presented in Chapters 2 and 3, and an in-depth comparison of the feedback-only and 'buffer-stock' approaches will be presented in Chapter 5. This debate has acquired a certain importance following recent disputes between advocates of both types of models.

In this chapter, however, we focus solely on the concept of 'buffer-stock'-disequilibrium money. In section one, we discuss the basic idea behind this approach, and on the main insights it offers on the behaviour of the money market. In Section two, we discuss the different ways in which proponents of this idea have attempted to render the concept operational at the empirical level. In section three, we focus in more detail on one particular type of 'buffer-stock' money model, and critically assess its advantages and disadvantages vis-a-vis alternative approaches. As these models are invariably 'forward-looking'<sup>4</sup>, we shall also at this point consider the various estimation methods at our disposal. In section four and section five we then consider two modifications which may be made to the simple 'buffer-stock' model of section three to overcome some of its more apparent shortcomings.

## CHAPTER 4

### SECTION ONE: THE CONCEPT OF 'BUFFER-STOCK'-'DISEQUILIBRIUM' MONEY

#### 4.1.1. A Criticism of the Simple 'Partial Adjustment' Approach.

We begin our survey with an analysis of why the introduction of lags in the demand for money proved to be a controversial issue. We will couch our discussion in terms of the simple first-order partial adjustment model, as the same arguments apply in the case of more general stochastic difference equation formulations.

Consider the simple first-order partial adjustment scheme as applied to the demand for money. This may be defined either in real (equation 4.1) or in nominal terms (equation 4.2), as it does not significantly affect the results:

$$(m^d - p)_t - (m^d - p)_{t-1} = \lambda[(m^* - p)_t - (m^d - p)_{t-1}] \quad (4.1)$$

$$m_t^d - m_{t-1}^d = \lambda(m_t^* - m_{t-1}^d) \quad (4.2)$$

where  $m^*$  is the desired (or 'long-run') demand for money, defined by:

$$m_t^* - p_t = \alpha_0 + \alpha_1 y_t - \alpha_2 R_t \quad (4.3)$$

and where  $\lambda$ , the adjustment parameter, lies between zero and one. Combining (4.3) with (4.1) and (4.2) respectively, we get the following specifications for the short-run demand for money,  $m_t^d$ , under real and nominal partial adjustment:

$$(m^d - p)_t = \lambda \alpha_0 + \lambda \alpha_1 y_t - \lambda \alpha_2 R_t + (1 - \lambda)(m^d - p)_{t-1} \quad (4.4)$$

$$m_t^d = \lambda \alpha_0 + \lambda p_t + \lambda \alpha_1 y_t - \lambda \alpha_2 R_t + (1 - \lambda)m_{t-1}^d \quad (4.5)$$

As we pointed out in Chapter 1, these specifications became popular, especially to deal with quarterly data, in the 1960s

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(see for instance, Chow, 1966), but were later supplanted by more general dynamic adjustment models, as we have seen in Chapters 2 and 3. There are no problems if we consider the demand for money to be the endogenous variable, as (4.4) and (4.5) merely indicate that it adjusts with a lag towards its desired long-run value. However, as has been pointed out in slightly different contexts by Darby (1972) and Laidler (1982), problems arise with these models if the nominal money supply is taken as exogenously determined by the authorities, or if the commercial banking sector can cause money supply shocks by autonomously changing bank lending. In this case, it can be shown that 'overshooting' of some of the demand for money variables can occur, if equilibrium is to be maintained in the money market.

Let us examine this proposition in a little more detail. Suppose that the price and real income levels are fixed in the short run. This is not an unrealistic assumption<sup>5</sup>, if we are considering the market-equilibrium period (i.e. the period in which equilibrium is restored in the money market following an exogenous shock to the nominal money stock,  $m^s$ ). From both (4.4) and (4.5), the short-run effect on the interest rate is found to be:  $-(1/\lambda\alpha_2)$ . This is greater than the long-run effect  $-(1/\alpha_2)$ , as  $0 < \lambda < 1$ , and hence the interest rate 'overshoots' its long-run value with this simple model. Whether this is always the case depends crucially on what happens to  $y_t$  and  $p_t$  in the short-run. To see the impact effect on the interest rate in a fuller model,

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we would have to add an expenditure sector and an aggregate supply sector, and we would have to specify short-run adjustments. As Cuthbertson (1985a) argues, the resulting effect could either be under- or over-shooting of the interest rate, depending on the model parameters.

This result is not entirely surprising. In the exchange rate literature, a whole host of models have followed the example set by Dornbusch (1976) in showing that the short-run dynamics of 'flexible' prices are affected if one introduces a lag or sluggishness in another sector of the economy. However, whilst such 'overshooting' models have been seen as appropriate for the modelling of exchange rate behaviour, they cannot characterise the behaviour of the money market, as interest rates have not, at least in the UK shown 'excess volatility' to the same extent (see Goodhart, 1984). At this point, we are faced with a difficulty: we have seen that the combination of a dynamic demand for money model with the assumption of market-clearing in asset markets and the assumption of sluggish adjustment of prices or real income leads to an overshooting result for the interest rate which does not seem to be observed in practice. Given, furthermore, that dynamic models have been successful in modelling the demand for money, what other explanations can we advance for the absence of the predicted 'excess volatility' in the interest rate?

The following three interpretations have been proposed:

- (a) The demand for money is not really exogenous at all. In the

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United Kingdom, the monetary authorities typically do not operate on the basis of monetary base targets, but manipulate money market rates so as to achieve their desired objectives. In this case, the money stock is endogenously determined by the private sector, and the problem highlighted above disappears. This explanation, however, has not proved to be acceptable to economists such as David Laidler, of a traditionally monetarist persuasion (see Laidler 1982, 1983). Furthermore, it does not explain why overshooting may not occur in response to occasional money supply shocks caused by, say, a sudden bank credit expansion, or an increase in the PSBR not offset by bond sales to the non-bank private sector.

(b) Financial markets could be characterised by a structure which is inherently more stable than the simple 'overshooting' model suggests. Consider, for instance, an economy in which goods prices adjust sluggishly, and where the monetary authorities seek to manipulate the monetary base so as to control the total money stock. Suppose furthermore that the commercial banking sector does not conform to the simple 'money multiplier' model, but that it seeks to adjust bank lending gradually in response to any shock to cash reserves. In this case supply shocks may be attenuated in the short-run by the behaviour of the banking sector. If this model is a correct representation of reality, the inconsistency between the lack of observed interest rate 'overshooting' and the success of general lag formulations of the

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demand for money is explained by the fact that, in this hypothesis, the commercial banking sector is effectively acting as a 'shock-absorber' to compensate for the sluggishness in the short-run demand for money in the face of shocks to some components of the total money stock.

(c) We could abandon the usual assumption that the money market has to clear at all times. In other words, a state of 'disequilibrium' may occur in the short run in the money market. This state of disequilibrium occurs because sudden shocks to the money stock do not immediately lead to a portfolio reallocation, and a change in the determinants of the demand for money to restore money market equilibrium. Thus, the money stock acts as a 'buffer stock' in the portfolio, absorbing any unanticipated shocks. This is the approach which we now consider for the remainder of this chapter.

The difference between this interpretation, and the one suggested by (b) is that the non-bank private sector lies at the centre of the 'buffer-stock' approach, whilst in (b) we argued that to some extent the commercial banking system could be absorbing many of the shocks. Option (b) presents us with a rather tricky situation because, if the dynamics of the money stock are governed by factors other than the simple demand for money behavioural equation (e.g. policy reaction functions, some model of the commercial banking system, etc.), then a full model of the financial system is required. The success of the simple

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single-equation stochastic difference models could be put down to a flexible lag structure which can account for a rather more complicated structural model than conventional theory suggests. Not surprisingly, option (b) has not received much attention at the empirical level. We discard option (a) (the hypothesis that there is no real problem because the demand for money is endogenous and there are no independent money supply shocks) for the moment as it is unlikely to hold under all circumstances and for some definitions of the money stock. Thus, we shall focus on the traditional 'buffer stock' approach, namely option (c).

The buffer stock approach has been modelled in a number of different ways, and we shall consider these in more detail in section two. One interesting implication of buffer-stock models is that, as we shall see in Chapter 5, the general lag formulation analysed in Chapters 2 and 3 may prove to be appropriate even if a 'money disequilibrium' or 'buffer stock' model offers the correct interpretation of the short-run behaviour in the money market. Without unduly anticipating our later discussion, it should already be apparent at this juncture that, in the case where feedback models have a sufficiently general lag formulation, then they could adequately characterise the 'monetary disequilibrium' process as, for example, Hendry and Ericsson (1983) recognise:

"As a final point...periods when  $M$  rises sharply are coincident with periods in which (the velocity of money) falls sharply...If

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(the velocity of money) is simply a derived variate - the resultant of plans about  $M$  given  $PY$  - and  $M$  acts as a 'temporary buffer' which agents cannot (or do not find it optimal to) control each period...then the observed behaviour (of velocity) is quite explicable. Thus our model of money demand allows for disequilibria in agents' holdings relative to their ex ante plans (rather than postulating instantaneous adjustment). Such disequilibria are removed...through 'error correction'...(and) the reaction lags of money to changes in its various determinants are allowed to differ for every variate, and are determined from the data..." (Hendry and Ericsson, 1983, p.72)

As the models analysed in the previous two chapters are built on the principle of building an adequate statistical model, and mould together time series and econometric techniques, it is perhaps not surprising that they can adequately characterise the data generation process. Basically, the 'overshooting problem' may be overcome by taking on board the idea that the money market is in disequilibrium in the short run, and that the money stock variable being modelled in autoregressive distributed lag models does not represent 'money demand' but 'money holdings', with the lag formulation embodying in some sense the short-run disequilibrium mechanism. We shall return to the issue of the relationship between the models of Chapters 2 and 3 and the models in this chapter in Chapter 5. We now turn our attention to alternative modelling strategies, which lay a greater and more



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explicit emphasis on monetary disequilibrium.

### 4.1.2. A More Detailed Analysis of the 'Buffer-Stock Money' Concept.

We have already stated that the buffer stock approach to the demand for money takes as its point of departure the idea that the money market is not always in equilibrium. That is, we cannot impose the condition that the money market clears at every point in time.

To some extent, the concept of disequilibrium is rather elusive when we take the money stock (the money supply) as exogenously determined. Clearly, if there is an outstanding stock of money, this must be held, either willingly or unwillingly. In other words, the short-run demand for money may diverge from the long-run demand for money because, either economic agents find it optimal to adjust their demand slowly to their long run desired demand, or because they are in some way 'surprised' by an unexpected change in the money supply, and forced 'involuntarily' to deviate from their desired demand for money. The former will occur, if agents find it optimal to deviate from their long-run desired demand for money because of, say, costs of adjustment. The latter will occur if agents' expectations about the money supply are not fulfilled. As we shall see, aspects of both of these scenarios are present in the 'buffer stock approach'.

In what way does the cost-of-adjustment argument advanced in

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the previous paragraph differ from the implicit costs-of-adjustment rationale of the simple partial adjustment scheme (equations 4.1 and 4.2)? The latter was branded as counterintuitive when combined with an exogenous money supply and a market-clearing condition, as it led to implausible overshooting results. The difference with the buffer-stock approach is that it does not rely on a myopic adjustment mechanism such as (4.1) and (4.2), but assumes that agents engage in an explicit forward-looking optimisation exercise, which does not lead to the same inconsistencies.

The background to the concept of buffer-stock or disequilibrium money was provided by inter alia Laidler (1983), and Goodhart (1984). The point of departure for the treatment of money as a 'buffer asset' is the observation that money plays a special role in economic agents' portfolios. Due to the liquid nature of money, the costs of adjusting money holdings are typically less than the costs involved in changing holdings of real or illiquid financial assets. In an uncertain environment, economic agents are likely to reallocate their portfolios permanently only if they perceive permanent changes in those variables affecting the desired holdings of assets. Conversely, transitory changes in the economic environment are less likely to lead to such portfolio reallocations, with money balances acting as a shock-absorber in such cases.

There are several corollaries to this rather general

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description of money as a 'buffer asset'. First, it implies that sudden shocks to the money supply will only be gradually dissipated throughout the portfolio, through what Laidler (1982, 1983) calls a 'slow real balance effect'. This explains why the approach has proved popular with some economists including Laidler, to the point that the phrase 'buffer stock monetarism' has become common usage in the literature. To some extent, the slow transmission effect following monetary shocks has been seen as a formalisation of Milton Friedman's 'long and variable lags' of monetary policy.

However, the concept of a 'buffering mechanism' is also sometimes associated with the idea of inventories, where stocks of a good are held to allow for sudden changes in demand, if it is costly to suddenly change production. Inventories are then allowed to vary within limits without changes in price and output by the producer.

What we have described so far in this section are different aspects of the 'buffering mechanism', and it shows the way in which a common theme, the general idea that it is costly to reallocate portfolios, may lead us to examine the problem from slightly different angles. In fact, it is fair to say that, following the emergence of the notion of 'buffer stock money' or money market 'disequilibrium', different methods have been advanced of making this concept operational at the empirical level. A range of models have been proposed, and in the next

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section we provide a survey of these. Ultimately, however, the focus will fall on one of these general approaches in the remaining sections of this chapter.

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### SECTION TWO: ALTERNATIVE APPROACHES TO MODELLING 'DISEQUILIBRIUM' OR 'BUFFER STOCK' MONEY

In this section, we consider the different ways in which the concept of monetary disequilibrium has been approached in the recent literature. As we saw above, the difficulties here are that the problem may be approached from rather different angles, all of which offer a partial picture of the whole 'disequilibrium' process. Some models focus on the determinants of short-run money holdings, by considering the different effects which anticipated and unanticipated, and permanent and transitory disturbances have on short-run money holdings. Others focus on the process which follows the money supply shock, that is the slow dissipation effect through the rest of the portfolio, and through to the real sector. Others still, have attempted to build up a fuller picture by considering both of these aspects. We now briefly examine these different strands of the literature.

#### 4.2.1. Costs-of-Adjustment. Expectations. and the Short-Run Demand for Money.

In this subsection we describe three different types of model which have been advanced to model money as a buffer asset. The first two are models of aggregate behaviour and have been designed primarily with an empirical application in mind. They yield some of the properties which we have ascribed to buffer stock models in the previous section. As a third example, we present a theoretical model of individual buffer behaviour,

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which, as we shall see, leads some rather interesting results which may cast some doubts on the validity of aggregate models. This third type of model is not, however, readily verifiable at the empirical level.

The first type of model represents the earliest example of the incorporation of expectations in the demand for money. This model was initially advanced by Carr and Darby (1981) to examine the way in which the demand for money reacts to anticipated and unanticipated money supply shocks. The framework used is the following:

$$(m - p)_t = \beta' X_t + \alpha(m - m^a)_t + u_t \quad (4.6)$$

$$m_t = \delta' Z_t + v_t \quad (4.7)$$

$$m_t^a = \hat{\delta}' Z_t \quad (4.8)$$

where lower cases denote logarithms of variables, and where the vector  $X_t$  in (4.6) contains a vector of variables which usually enter the short-run demand for money function (e.g. real income, the interest rate, etc.) which constitute the planned element of money holdings, and where the vector  $Z_t$  in (4.7) contains variables which are seen as a systematic influence on the money supply, and which can therefore help agents in predicting its path. Furthermore  $\delta$ ,  $\beta$  are suitably dimensioned vectors of parameters, and the parameter  $\alpha$  is such that  $0 < \alpha < 1$ ; also,  $v_t$  and  $u_t$  are white noise disturbances.

What this model sets out to test is the following: equation (4.7) provides a marginal model for the money supply, and the

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fitted values are then used to construct an 'anticipated' money supply series,  $m^a$  (see equation 4.8). The hypothesis in (4.6) is then that current unanticipated money supply shocks ( $m - m^a$ ) will initially lead to increases in the short-run demand for money (the unplanned element of the demand for money), and thus money acts as a 'buffer asset'. This is only a short-run influence, however, as such errors cannot be systematic in such a rational expectations framework.

Carr and Darby (1981) apply this model to M1 data on the countries used in the Mark III International Transmission Model developed as part of a NBER-NFS project (see Darby and Stockman, 1980). The countries used are the the G7 countries with the addition of the Netherlands. Their short-run demand for money model was based on a conventional long-run demand for money of the type:

$$m_t^d - p = Y_0 + Y_1 y_t^p - Y_2 R_t \quad Y_i > 0 \quad (4.9)$$

where  $y^p$  represents 'permanent income'. In addition, a simple partial adjustment scheme was postulated in the short-run, so that the vector of variables  $X$  determining the short-run demand for money contains  $y^p$ ,  $R$ ,  $y^t$  (transitory income), and  $m_{t-1}$ .

On the supply side, Carr and Darby follow the Mark III model in fitting a univariate ARIMA process to the money stock series. This was reported to fit better than alternative formulations including other variables which one would expect to enter a central bank's reaction function. Because of the correlation

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between  $u_t$  and  $m$  in (4.6) (simultaneity bias), the estimates which can be obtained from an OLS estimation of (4.6) are inconsistent. Carr and Darby therefore adopt an instrumental variable estimator which uses the principal components for each country from the Mark III model. Overall, they find that the unanticipated money shock is significant in determining money holdings.

However, these apparently favourable results for the Carr-Darby version of the shock-absorber hypothesis have come under further scrutiny recently. The main critique of their work has come from MacKinnon and Milbourne (1984). The argument put forward by these authors is that the estimator used by Carr and Darby is still likely to perform badly in small samples, and that an alternative procedure yields a far more efficient estimator.

Mackinnon and Milbourne propose a transformation of (4.6) which removes  $m$  from the right-hand side of the equation, thus removing the problem of simultaneity. The alternative estimating equation proposed is:

$$(m - p)_t = \beta'X_t + \lambda(m^a - p)_t + \tilde{u}_t \quad (4.6')$$

where  $\beta = \beta/(1 - \alpha)$ ,  $\lambda = -\alpha/(1 - \alpha)$  and  $\tilde{u}_t = u_t/(1 - \alpha)$ . Note that an OLS estimate of (4.6') will not be subject to simultaneity bias, and that if, as predicted by the Carr-Darby 'buffer-stock' mechanism, the parameter  $\alpha$  is significant and  $0 < \alpha < 1$ , then we would expect  $\lambda$  to be negative and significantly different from zero.



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Furthermore, Mackinnon and Milbourne argue that another fault of the simple Carr-Darby model is that (4.6) only tests part of their hypothesis. As it stands, the model tests whether unanticipated shocks to the money supply initially lead to increases in short-run money holdings. However, one corollary of this hypothesis is that anticipated money supply shocks should not be significant in the model. Thus, we may modify (4.6') to provide a fuller test of the hypothesis:

$$(m - p)_t = \beta'X_t + \lambda(m^a - p)_t + \xi m_t^a + \eta_t \quad (4.6')$$

A full test of the Carr-Darby hypothesis then is whether  $\lambda$  is significant and negative, and whether  $\xi$  is insignificant. These restrictions are rejected by MacKinnon and Milbourne (1984), who in fact find an (implied) estimate for  $\alpha$  of -4.3! This is in sharp contrast with the Carr-Darby hypothesis<sup>7</sup>.

However, other criticisms of the simple model may be advanced. First, Milbourne (1987) criticises the whole approach, indicating that, although Carr and Darby give their model an interpretation where the money supply is exogenous, and the short-run demand for money responds to unanticipated shocks<sup>8</sup>, there is no reason why an alternative interpretation may be given to the buffer-stock model, where the shocks arise from the determinants of the demand for money, and the supply is endogenous. The whole framework does rely on the prior assumption that the money supply is the exogenous variable. Secondly, like most 'rational expectations' models, we are estimating a two-

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equation system here, and there are certain cross-equation restrictions between (4.6'') and (4.7) which arise from the presence of common elements in the X and Z vectors (For more details on these 'rationality' restrictions, see Mishkin, 1983). Cuthbertson and Taylor (1985a, 1986b) test for the validity of these cross-equation restrictions on US and UK data, and find that they do not hold. Thirdly, one should not only focus on the  $\alpha$  parameter in such models, but also expect other 'sensible' parameter restrictions to hold. One example of such a restriction is price homogeneity. MacKinnon and Milbourne (1984) find that these restrictions do not hold in their version of the Carr-Darby model, and therefore suggest that the theory does not really make sense.

However, some evidence in favour of the Carr-Darby approach has been presented by Cuthbertson and Taylor (1985b). The major innovation in this study is the proposal of an alternative method for generating the expected series in the Carr-Darby model. Instead of using conventional fixed-parameter univariate ARIMA models to find a series for the anticipated money stock, Cuthbertson and Taylor adopt an application of the Kalman Filter<sup>9</sup>. Basically, the approach followed is that of assuming that economic agents will adapt their forecasting rule as more information becomes available. As Cuthbertson and Taylor (1985b) aptly put it:

"....using the Kalman filter to generate expectations results in

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a form of 'generalised adaptive expectations' (see Lawson, 1984), where the adjustment parameter itself evolves as agents learn about the process...(providing the state-space form generating the economic series  $x_t$  is known) then the one-step ahead predictors of  $x_t$  are in fact the full rational expectations conditional on information up to the preceding period..." (Cuthbertson and Taylor, 1985b, p.7)

For reasons of space, we do not provide a detailed outline of the Kalman filter procedure here, and full details are provided by Cuthbertson and Taylor (1985b) and Cuthbertson (1986). The difference between this method of obtaining an expected series, and the usual RE methods is that it allows for an approximation to the 'learning process' by economic agents. This is in line with the recent literature which has pointed out the rather unrealistic informational requirements of Muth-rational expectations, and has looked for a greater emphasis on learning and information exploitation by economic agents (see for instance B.Friedman, 1979, Bray, 1982, Frydman and Phelps, 1983).

The main advantage of incorporating expectations obtained from a Kalman filter procedure in the Carr-Darby model is that Cuthbertson and Taylor's evidence appears to validate the Carr-Darby 'buffer-stock' hypothesis on UK data. This is in sharp contrast with the previous results obtained by Cuthbertson and Taylor which we reported above. Problems still remain with this simple model, however, not least because the vast majority of

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empirical evidence still tends to point to its inadequacy. Furthermore, we should also keep in mind Milbourne's scepticism regarding the proper interpretation of an equation such as (4.6).

The second approach to modelling 'buffer stock money' to be considered here overcomes certain shortcomings of the simple Carr-Darby model. Quite apart from the criticisms which can be made of the Carr-Darby approach on econometric grounds, it is arguable that it lacks precise microfoundations. For instance, we are told that economic agents react differently to anticipated and unanticipated monetary shocks, but no argument is provided regarding the determinants of the  $\alpha$  parameter in equation (4.6). To put it another way, no explicit account is given of the costs of adjustment which economic agents face, and hence the model offers a very partial picture of the buffering mechanism. Furthermore, the 'trigger' of the buffer mechanism may not necessarily be an exogenous supply shock, and more attention has to be paid to possible unanticipated shocks in demand-side factors in a prospective buffer-stock model.

An alternative model of the buffer mechanism has been proposed by Cuthbertson and Taylor in a series of papers (see for instance Cuthbertson, 1984, 1988, Cuthbertson and Taylor, 1986a, 1987) which pays more attention to the costs faced by economic agents. The general approach followed by these authors is that of intertemporal optimisation. To some extent, this is an example of 'technology transfer', as similar intertemporal costs-of-

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adjustment models have been popularised by proponents of the new classical macroeconomics, in the context of the theories of labour supply (see Lucas and Rapping, 1969), labour demand (see Sargent, 1978, 1979), and investment (see Lucas and Prescott, 1971).

The model is constructed as follows. We again assume that the individual economic agent has a conventional 'long-run' desired demand for money:

$$m_t^* = a_0 + a_1 p_t + a_2 y_t - a_3 R_t \quad a_i > 0 \quad (4.10)$$

where  $a_1$  is usually set equal to unity, but we do not impose this restriction a priori, as it should be tested empirically. Following our previous argument, we assume that there are costs involved in portfolio adjustment, so that the representative economic agent will attempt to find the optimal path for his actual money balances,  $m_t$ , over his time horizon. Let us assume, for simplicity, that the individual has an infinite time horizon. Then this choice may be characterised by the minimisation of an intertemporal quadratic loss function,  $C$ , conditional on information at time  $t-1$ :

$$C = E_{t-1} \sum_{j=0}^{\infty} \delta^j (a_0 (m_{t+j} - m_{t+j}^*)^2 + a_1 (m_{t+j} - m_{t+j-1})^2) \quad (4.11)$$

where  $a_0$  and  $a_1$  represent the relative weights attached to the costs of being away from the desired long-run holdings of money, and the costs of adjusting money holdings respectively, and  $\delta$  represents a subjective discount rate.

Let us examine this optimisation exercise a little more

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closely. The first thing to note is that the cost function (4.11) penalises adjustments in money holdings, which may seem rather puzzling at first sight given that the buffer-stock approach is based on the costs of adjustment of illiquid assets, and the buffer asset, money, is supposed to be relatively costless to move in and out of. The only way to resolve this apparent inconsistency in the Cuthbertson-Taylor model is to assume that there are only two alternative assets in the portfolio, namely money and bonds. Given that any changes in bond holdings will have a counterpart in changes in money holdings (for a constant wealth stock), it is legitimate to penalise the latter in (4.11). However, this rather special case is unlikely to be applicable in practice for two reasons. Firstly, individuals are likely to hold a whole spectrum of alternative financial and real assets in their portfolios. Secondly, we cannot take the total stock of wealth in the economy as constant in the presence of saving behaviour on the part of economic agents<sup>10</sup>. In the presence of saving, equation (4.11) implies that economic agents find it less costly to adjust their holdings of alternative assets than their money balances, despite the fact that the latter was specifically assigned to be the 'buffer asset'. Therefore, there are some doubts that the simple model in (4.11) can capture the 'buffering mechanism' of money balances in a world where wealth is not constant and there are more than two assets. These criticisms of the simple Cuthbertson-Taylor model have been

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pointed out by Muscatelli (1988a), and we shall deal with possible extensions of the simple model to take these criticisms into account in the latter part of this chapter.

For the moment, let us ignore these problems and return to the problem as set out in equation (4.11). In addition to (4.11), we assume, following Cuthbertson (1984), and Cuthbertson and Taylor (1987), that actual money holdings consist of a planned component,  $m_t^p$ , and an unplanned component,  $m_t^u$ , which will depend on any innovations in the determinants of the demand for money at time  $t$ , once planned holdings have already been chosen conditional on information available at time  $t-1$ :

$$m_t = m_t^p + m_t^u + \varepsilon_t \quad (4.12)$$

where  $m_t^u$  and  $\varepsilon_t$  are assumed to be zero-mean white-noise stochastic processes.

The problem facing the economic agent is therefore that of choosing a sequence  $\{m_{t+j}^p\}_{j=0}^{\infty}$  which minimises (4.11). To find the solution to this problem, we consider the following first-order necessary conditions obtained by minimising (4.11) with respect to  $m_{t+j}$  for  $j = 0, 1, 2, \dots$ :

$$m_{t+j}^p(a_0 + a_1(1 + \delta)) - a_1 m_{t-j-1} - \delta a_1 m_{t+j+1}^p = E_{t+j-1} a_0 m_{t+j}^* \quad (4.13)$$

Equation (4.13) represents a set of second-order stochastic difference equations known as Euler equations (see Sargent, 1979). Let us briefly consider the method of solution which may be applied to (4.13).

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First, let us rearrange and rewrite (4.13) in terms of a polynomial in the lag operator  $L$ :

$$(1 - ((a_0/\delta a_1) + (1/\delta) + 1)L + (1/\delta)L^2)m_{t+j+1}^p = -(a_0/\delta a_1)E_{t+j-1}m_{t+j}^* \quad (4.14)$$

We know that the two roots of the polynomial,  $\lambda_1$  and  $\lambda_2$  must satisfy the relations:

$$((a_0/\delta a_1) + (1/\delta) + 1) = (\lambda_1 + \lambda_2) \quad \text{and} \quad (1/\delta) = \lambda_1\lambda_2 \quad (4.15)$$

These two relations enable us to conclude<sup>11</sup> that one of the roots must be positive and greater than unity, and the other must be positive and smaller than unity (provided  $a_i > 0$ , and  $0 < \delta < 1$ ). The presence of an unstable root is common to dynamic optimisation problems, and the problem may be solved by recognising that economic agents are free to determine the rate of change of their money holdings at any point in time, and hence this may be treated as a non-predetermined variable. However, to ensure equilibrium behaviour we impose the following terminal condition:

$$\lim_{s \rightarrow \infty} E_{t-1} \{ \delta^s [(a_0 + a_1)m_{t+s} - a_1m_{t+s-1} - a_0m_{t+s}^*] \} = 0 \quad (4.16)$$

This condition is also known as the 'transversality condition', and it is common in most dynamic forward-looking (rational expectations) models (see Begg, 1982).

Having established the presence of a stable and unstable root, we may factorise the lag polynomial in (4.14) as follows:

$$(1 - \lambda_1 L)(1 - \lambda_2 L)m_{t+j+1}^p = -(a_0/\delta a_1)E_{t+j-1}m_{t+j}^* \quad (4.17)$$



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where, say,  $\lambda_1$  is the stable root, and  $\lambda_2$  is the unstable root. The Euler equation system may be solved by removing the effect of the unstable root. This may be done by multiplying both sides of (4.17) by  $(1 - \lambda_2 L)^{-1}$ , and then removing  $\lambda_2$  by using the relations in (4.15). This finally yields:

$$m_t^p = \lambda_1 m_{t-1} + (1 - \lambda_1)(1 - \lambda_1 \delta) \sum_{i=0}^{\infty} (\lambda_1 \delta)^i E_{t-1} m_{t+i}^* \quad (4.18)$$

We may then obtain an expression for  $m_t$  by using (4.12):

$$m_t = \lambda_1 m_{t-1} + (1 - \lambda_1)(1 - \lambda_1 \delta) \sum_{i=0}^{\infty} (\lambda_1 \delta)^i E_{t-1} m_{t+i}^* + m_t^u + \epsilon_t \quad (4.19)$$

Finally, we may obtain an estimable equation from (4.19) by substituting for  $m_{t+i}^*$  from equation (4.10). We must also find some measure for the term  $m_t^u$ , and this can be modelled by introducing the current innovations in the determinants of money demand,  $R^u$ ,  $p^u$  and  $y^u$ , which represent any unexpected changes in the targeted variable  $m^*$ :

$$m_t = \lambda_1 m_{t-1} + (1 - \lambda_1) \alpha_0 + (1 - \lambda_1)(1 - \lambda_1 \delta) [\alpha_1 \sum_{i=0}^{\infty} (\lambda_1 \delta)^i p_{t+i}^e + \alpha_2 \sum_{i=0}^{\infty} (\lambda_1 \delta)^i y_{t+i}^e - \alpha_3 \sum_{i=0}^{\infty} (\lambda_1 \delta)^i R_{t+i}^e] + \beta_1 (p - p^e)_t + \beta_2 (y - y^e)_t + \beta_3 (R - R^e)_t + \epsilon_t \quad (4.20)$$

where the superscript  $e$  indicates the expected value of a variable, based on information at time  $t-1$ , and the terms  $(p - p^e)$ ,  $(y - y^e)$ , and  $(R - R^e)$  represent respectively  $p^u$ ,  $y^u$ , and  $R^u$ , the current innovations in prices, real income, and the interest rate.

Models such as (4.20) are claimed to capture 'buffer-stock' behaviour in the following sense: an unexpected shock to income,

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prices or the interest rate will not be reflected in the future expected terms if it is only a temporary, random shock. In this case, money balances will be affected and thus act as a shock-absorber. However, if a shock to these variables constitutes 'news' about the future paths of  $p$ ,  $y$ , and  $R$ , then agents' expectations with regard to these variables will be revised accordingly, and money holdings will rise or fall to a new equilibrium level.

This argument is of course only valid if the money stock is endogenous to movements in  $p$ ,  $y$ , and  $R$ . However, Cuthbertson and Taylor (1987) also argue that their model avoids the awkward 'overshooting' problem usually associated with simple partial-adjustment models. Consider an unexpected random shock to the money supply. To the extent that this is expected to lead to a temporary increase in  $p$ ,  $y$ , or a fall in  $R$ , this will lead to an increase in desired short-run (buffer) holdings. However, if the money supply increase is perceived as permanent, this will lead to a re-evaluation of the future path of  $p$ ,  $y$ , and  $R$ , and hence the demand for money increases permanently, thus reducing any disequilibrium in the money market.

There are, of course, several weak spots in this argument. First, the authors claim that their model provides solid 'microfoundations' to the buffer-stock approach, in contrast to the ad hoc Carr-Darby model. However, despite the appeal of simple quadratic cost functions as they lead to very

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tractable and simple equations such as (4.20), they may not correspond to real-life economic behaviour, and may therefore give misleading results, by constraining the model selection process. This is an argument which is taken up in Muscatelli (1988b) and which we develop at length in Chapter 5, when we compare and contrast forward-looking models such as (4.20), and our models of Chapters 2 and 3 based on ADL equations.

Secondly, as we pointed out above, the simple model above ignores the complexity of a multi-asset world, where wealth is evolving due to saving behaviour. We return to this theme in the latter part of this chapter, when we extend the simple model to take these factors into account.

Thirdly, although the authors claim that the model can take into account situations where the money stock is either endogenous or exogenous to money demand, it does leave much unsaid if the latter scenario is the more likely. If the money stock is subject to exogenous shocks, then a simple single-equation model will not capture the transmission mechanism as it does not focus on the endogenous variables in the system. Some economists have suggested that a more fruitful approach is that of 'inverting' demand for money functions to model these effects, and we turn to these models in the next subsection. Furthermore, most of the estimated buffer stock models have used 'narrow money' definitions (usually M1), and it does seem unlikely that this definition of the money stock may be regarded as exogenous.

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Lastly, it is doubtful to what extent the model proposed constitutes an addition to the 'theory' of the demand for money. It should be emphasised that we assume that economic agents undertake a two-stage process. First, they determine their desired 'long-run' demand for money through our conventional (static) transaction-portfolio models (i.e. via equation (4.10)), and subsequently they determine their optimal speed of adjustment to their long-run equilibrium via a dynamic optimisation exercise. As Cuthbertson and Taylor (1987) point out, a more interesting approach would be to integrate these two stages by constructing a model which determines simultaneously the long-run demand for money, and the speed of adjustment. However, research developments in this direction are all too rare, and any models of the 'short-run' demand for money which have been built along these lines are purely theoretical (see for instance Milbourne et al, 1983). On the other hand, in defence of the Cuthbertson-Taylor approach, it has to be said that the use of dynamic optimisation exercises on the basis of costs-of-adjustment arguments have proved very popular in other macroeconomic applications.

We will return to a fuller discussion of these criticisms later. For the moment, we turn briefly to the estimation procedures adopted for the estimation of 'forward-looking' equations such as (4.20). We will then discuss some of the results obtained by Cuthbertson and Taylor by using this model.

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In such forward-looking models it is natural to assume that economic agents form their expectations rationally. One of the most popular methods applied to the estimation of rational expectations is the so-called McCallum substitution method, also known as the two-stage OLS method (see McCallum, 1976). This involves the construction of marginal models to approximate the data generation process for the expected variables  $p$ ,  $y$ , and  $R$ . These estimated marginal models may then be used to construct forecasts for the expected series. One problem which arises in the case of the two-stage OLS method has already been mentioned when we assessed the Carr-Darby approach. In general Pagan (1984) has shown that there will be a downward bias in the standard errors of the estimates. Despite these problems, we have chosen to adopt this method when estimating forward-looking models in this and the following chapter. This is mainly on grounds of simplicity. However, for sake of completeness, we should also point out that there are alternative methods of estimating these models.

A second method is provided by the 'errors in variables' instrumental variables method (see for instance Wickens, 1982), which relies on including the actual realised values as proxies for expected future variables in an estimating equation. Given that this introduces an 'error' into the model, OLS will not, in general produce consistent estimates, and an instrumental variable estimator has to be used.

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In their most recent work, Cuthbertson and Taylor (1987) prefer to construct marginal models for their expected series, but they also correct for additional econometric problems, in contrast to the simple two-stage OLS approach. First we should note that whilst, in applying the two-stage model later on in this chapter we shall use univariate time series models, Cuthbertson and Taylor (1987) assume that the relevant vector of variables  $z' = (p, y, R)$  may be approximated by an  $n$ -th order vector Markov process. The vector autoregression used<sup>12</sup> is then estimated jointly with the solution to the Euler equation so as to check the cross-equation restrictions implicit in the assumption of rational expectations (see Mishkin, 1983, Hansen and Sargent, 1982). A Three-Stage-Least-Squares estimator is used, with current and four lagged values of the elements of  $z$  as instruments. Overall, this method provides consistent estimates, but is more complex than the two-stage OLS method. As a final point, it should be noted that equation (4.20) is non-linear, and whilst this problem may be overcome by using, say, non-linear least-squares, Cuthbertson and Taylor (1987) simplify the model by imposing a value of 0.99 on  $\delta$ . Whilst this implies an annual rate of time preference of about 4%, which is not entirely unrealistic, they argue that relaxing this restriction does not affect the results drastically. Muscatelli (1988a) reaches a similar conclusion, and in what follows, we shall also impose this restriction thereby avoiding the use of complex estimation

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procedures.

Cuthbertson and Taylor (1986c, 1987) apply the above buffer-stock model to M1 data for the UK, and in Cuthbertson and Taylor (1986b) they attempt to fit a similar model to the M1B definition of the money stock in the United States. (We shall return later to the issue of whether 'narrow' money definitions are likely to provide us with the best testing ground for the buffer-stock hypothesis.) In general, the results presented seem encouraging. In the case of the US data, the 'missing money' episode of 1973-74 was ascribed not to a 'break' in the structural forward-looking demand for money function (4.20), but to a shift in the parameters of the vector autoregressive model used to model expectations<sup>13</sup>. Furthermore, the results seem to indicate a unit elasticity of the demand for money with respect to the expected price level, and all the expected variables were found to be significant. (Although, as we shall see in Chapter 5, this may not be altogether surprising). Their UK results were based on seasonally adjusted data, and they found here that the implied estimate for the ratio of costs of adjustment to costs of deviations from equilibrium,  $(a_1/a_0) = 29.41$ , i.e. costs of adjustment appear to be extremely important in determining short-run money holdings. Furthermore, all the estimated long-run elasticities had the correct signs<sup>14</sup>, and the cross-equation restrictions seemed to hold.

Therefore, the proponents of the forward-looking costs-of-

#### CHAPTER 4

adjustment model present a good amount of empirical evidence in its favour, and, despite some theoretical weaknesses, it does have a number of advantages over the rather ad hoc Carr-Darby model.

As a last example of models which examine buffer-stock behaviour from the money demand side, we now examine a theoretical model of inventory behaviour.

This model differs from the Carr-Darby and Cuthbertson-Taylor models which, whilst considering a representative economic agent, essentially deal with the aggregate demand for money, and automatically assume that what is true for individual behaviour must also apply on aggregate. The model of inventory-behaviour to which we now turn produces rather different results at the aggregate level than may be obtained at the level of the individual economic agent.

Miller and Orr (1966, 1968) originally proposed an inventory-theoretic model of the demand for money with stochastic cash balances (for a more recent inventory-theoretic model see Akerlof and Milbourne, 1980). The setup is the following: the individual economic agent allocates his wealth between money and other assets which yield a higher return. There are however costs in portfolio adjustment, and it is assumed that cash balances are allowed to fluctuate between a maximum ( $h$ ) and a minimum ( $o$ ) threshold. Whenever a threshold is reached, the economic agent returns his money holdings to some intermediate level ( $z$ ) (see



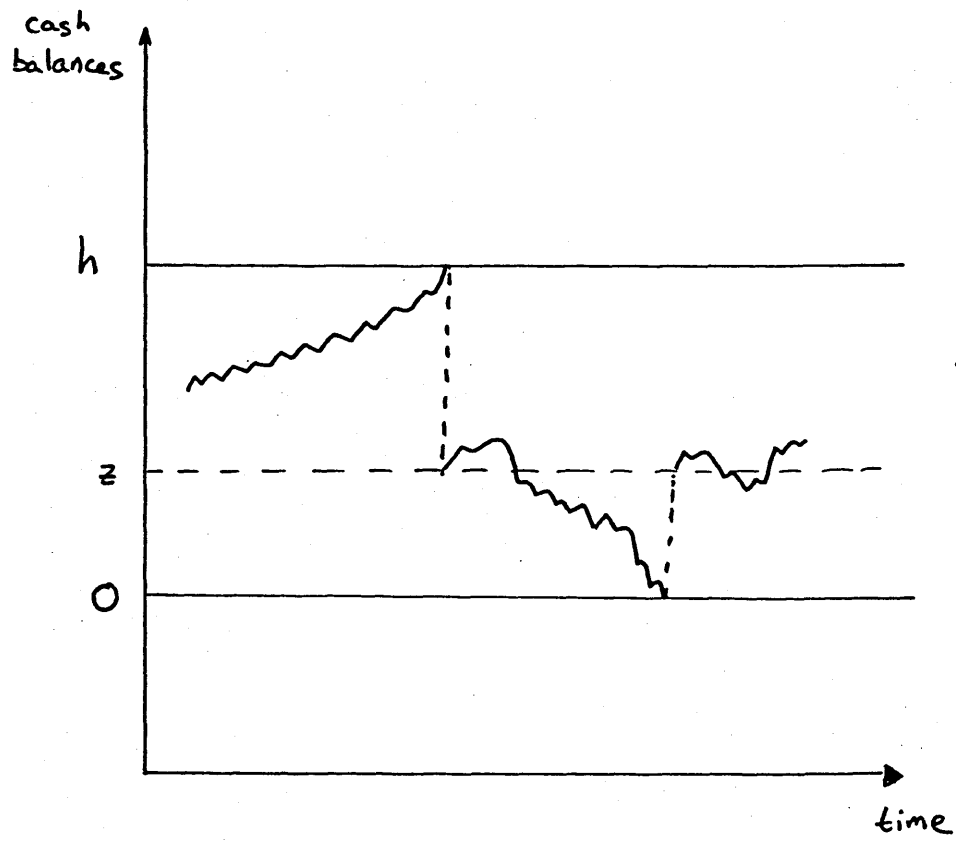


Figure 4.1

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figure 4.1 for an example). Milbourne (1983) in fact proves that such behaviour is optimal, and the values of the thresholds and of the intermediate level may be determined with reference to the stochastic process followed by cash balances, and the variables which usually affect the demand for money (e.g. income, interest rates, etc.).

Milbourne (1987) considers what happens on aggregate if we allow the money stock to suddenly increase in this model. Without entering into the full complexity of Milbourne's argument, it can easily be shown that agents react to a 'helicopter money drop' by attempting to transfer it out of money holdings. Consider a situation where all individuals in the economy behave according to the Miller-Orr model, and suppose that they have initial money holdings such that they are uniformly distributed between 0 and  $h$  in unit amounts. If the helicopter drop consists of giving £1 to each individual, then  $(h - 1)/h$  of the population will incorporate this extra £1 into their inventories and hold on to it, but  $1/h$  of the population will hit the upper threshold, and lower their holdings by  $(h - z)$ . Overall, money holdings increase by only  $(z - 1)/h$ , and if  $z$  is set to, say,  $h/2$ , then overall the increase is  $(1/2) - 1/h$ , which implies that less than half of the increase is retained in inventory money holdings. Milbourne (1987) concludes that:

"...Since buffer stock theorists argue that the price level, interest rates and even income respond slowly, so that thresholds

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do not change, any injection of money will quickly be transferred away in the short run....." (Milbourne, 1987, p.132, emphasis added).

One problem with this conclusion is that, once again, we are left in the dark about what happens to prices, income and interest rates. Milbourne's argument that 'more than half' of the helicopter drop is transferred out is invalid unless  $p$ ,  $y$ , or  $R$  change. Individuals merely attempt to transfer their extra balances out of money to other assets in their portfolio. Whether they actually succeed depends critically on the speed of adjustment of  $p$ ,  $y$ , and  $R$ . One thing which this argument does show, however, is that agents will attempt to get rid of additional balances quickly, thus presumably placing greater pressure on  $p$ ,  $y$ , and  $R$  so as to re-equilibrate the supply and demand for money by altering the thresholds. In this sense, it does cast some doubt on simple aggregate buffer-stock models which lay greater emphasis on the similarities rather than the differences between the circumstances of individual economic agents. By assuming that individuals are uniformly distributed between the thresholds, some aggregate results can be obtained which contradict the simple buffer-stock models.

However, it is not clear whether any of these propositions are testable at the empirical level. The main problem is that many of the predictions of the model clearly depend on the initial distribution of economic agents, and on the stochastic

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process driving cash balances. Nevertheless, this simple model does focus our attention on the limitations of the 'representative economic agent' assumption in buffer stock models. Furthermore, in the case where the money supply is seen as exogenous, the concept of disequilibrium in the money market is seen as relying heavily on the speed of adjustment of the price level, income, and the interest rate. Next we turn to a survey of those models which have tried to model another stage of the transmission mechanism. If we do accept that the 1970s have been characterised by exogenous money supply shocks which have given rise to a state of disequilibrium in the money market due to the slow adjustment of prices, income and/or the interest rate, then it makes more sense to concentrate on the transmission mechanism, by modelling the endogeneous variable(s) in the disequilibrium adjustment process. This involves 'inverting' the demand for money, and we now turn to attempts to model this aspect of disequilibrium money.

### 4.2.2. Disequilibrium Money and 'Inverting' the Demand for Money.

One way of modelling disequilibrium money is to model the gradual adjustment of the price level, real income or the interest rate in response to money supply shocks. An obvious problem in adopting this approach is the following: if we have to model an endogenous variable, which do we choose, the price level, real income, or the interest rate? Clearly if all of these variables are jointly endogenous, we need a fully specified

## CHAPTER 4

multi-equation model. However, many modellers have sought to continue with a single-equation framework, by selecting one of these 'potentially endogenous' variables. We now examine some of these attempts to 'invert' the demand for money equation.

The earliest example in the recent literature is provided by Artis and Lewis (1976, 1981). These authors focus on the interest rate as the 'left-hand-side variable' in their empirical studies. In part this choice to focus on the interest rate is motivated by some interesting results regarding the 'long-run' demand for money (see Artis and Lewis, 1984). Artis and Lewis show that during the early period of the 1970s which was characterised by the apparent 'breakdown' of the demand for broad money function, the increase in the actual stock of money may be interpreted as a movement 'off' the long-run inverse relationship between interest rates and the demand for money (see figure 4.2). Thus, their interpretation of money market disequilibrium following a money supply shock focuses on the relationship between interest rates and the demand for money, and hence the interest rate is chosen as the dependent variable.

The adjustment mechanism assumed for the interest rate is one of simple partial adjustment:

$$\Delta R_t = \lambda(R_t^* - R_{t-1}) \quad 0 < \lambda < 1 \quad (4.21)$$

where  $R^*$  is the 'equilibrium interest rate', that is, the interest rate level which will cause the current money supply,  $m_t$ , to be equal to the long-run demand for money:

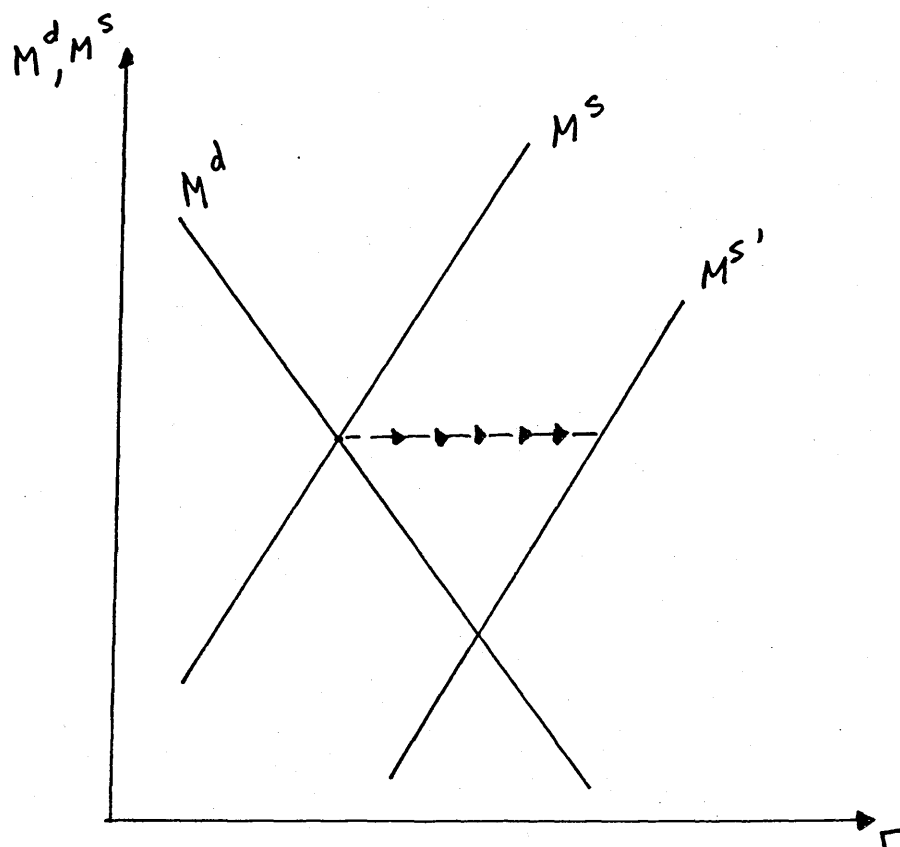


Figure 4.2

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$$m_t = m_t^d = \alpha_0 + \alpha_1(y + p)_t - \alpha_2 R_t^* \quad (4.22)$$

where, for simplicity, the price and real income elasticities are assumed to be both equal to  $\alpha_1$ . Substituting for  $R^*$  from (4.22) into (4.21) we obtain the following estimating equation:

$$R_t = (\lambda \alpha_0 / \alpha_2) + (\lambda \alpha_1 / \alpha_2)(y + p)_t - (\lambda / \alpha_2)m_t + (1 - \lambda)R_{t-1} \quad (4.23)$$

Artis and Lewis estimate this model for broad money for the UK over the period 1963-1973 and find that all the parameters are significant, and that they are more stable over the early quarters of the 1970s. Furthermore, the simple partial adjustment model is found to be stable, in that a point estimate of 0.35 for  $\lambda$  is found. Of course, by inverting the demand for money, the overshooting property of the simple partial adjustment mechanism in the demand for money disappears, because now a sudden money supply shock may be interpreted as causing a slow adjustment in the interest rate to return the money market slowly to equilibrium, where  $m_t$  once more equals  $m_t^d$ .

However, this model is not without its difficulties. First of all, it relies on the interest rate as the 'sluggish' variable, and it is doubtful if financial markets may be regarded as 'sticky-price' markets. For this reason, as we shall see below, many researchers have attempted to invert the demand for money by choosing the price level as the new dependent variable. The fact that the United Kingdom result by Artis and Lewis may be a special case is illustrated by the failed attempts to find a

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similar interest rate equation for the United States (see Laidler, 1980). Secondly, Cuthbertson (1985a) suggests that the adjustment parameter  $\lambda$  in the Artis-Lewis model may be capturing a moving average term in the error term of the model caused by measurement errors if expected interest rates are the appropriate variable (see Hansen and Sargent, 1982), and not the partial adjustment of the interest rate. Thus, the success of the Artis-Lewis model may be explainable in terms of an 'econometric accident'.

The alternative would then seem to take the price level as the dependent variable. However, the results here are also somewhat mixed. Despite several attempts to model the price level in this way (see Laidler, 1982, Kanninen and Tarkka, 1984, MacKinnon and Milbourne, 1986), many of the restrictions implied by the buffer stock models just do not hold. For instance, MacKinnon and Milbourne (1986) attempt to invert the Carr-Darby model, and find that none of the implied restrictions mentioned in the previous subsection hold. Again, one is forced to conclude that whilst focusing on the transmission mechanism may make sense if one adopts a theory of money as a buffer asset, a single-equation framework where the demand for money is simply inverted is likely to provide a very poor picture of the actual disequilibrium transmission mechanism.

In the next subsection we examine some models which have sought to move away from the single-equation framework by



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constructing fuller models of the transmission mechanism.

### 4.2.3. 'Exogeneity', 'Endogeneity', and Full Macromodels

So far we have seen that the whole issue of buffer stock money has been discussed mainly in the context of the single-equation framework. The Carr-Darby model has received much attention at the empirical level which has thrown doubts on its validity as a model of the demand for money. The more recent Cuthbertson-Taylor costs-of-adjustment model has provided some interesting results from the authors themselves but, as we pointed out, there are some awkward theoretical issues which have to be resolved, and which we will examine in detail in sections three and four of this chapter. Inverting the demand for money equation has also thrown up some rather mixed results, and the simple inventory-theoretic model presented in section 4.2.1 has shown us that there are some difficulties in aggregating across economic agents when constructing a buffer-stock model.

At this stage, before proceeding to a further evaluation of the Cuthbertson-Taylor model, we have to consider in some further detail whether the whole issue of 'endogeneity' versus 'exogeneity' leads us to favour the use of full macromodels in preference to single equation models when modelling the demand for money.

Davidson (1984, 1986) has been one of the pioneers of the use of multi-equation models in modelling the monetary disequilibrium process in the UK context. In his 1986 paper he

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again points at problems with the 'alternative' assumptions of exogeneity and endogeneity, which point respectively to the estimation of 'inverted' and 'conventional' demand for money functions. Davidson uses the Akerlof (1973, 1979) distinction between 'autonomous' and 'induced' transactions to analyse the aggregate behaviour of the non-bank private sector in adjusting money holdings. In conventional inventory-type model terminology, the former are transactions which occur through the use of money as the medium of exchange, and they are not intended as a way of changing money holdings. The latter are aimed at altering money holdings, and these occur once money holdings move outside the economic agent's desired thresholds. In addition, Davidson distinguishes between 'inside' transactions, i.e. transactions which do not alter the size of the aggregate money stock, and 'outside' transactions, i.e. transactions which alter the size of the aggregate money stock. In the case of 'broad' money definitions, the former are transactions between the non-bank private sector and other sectors (e.g. the government sector, the overseas sector, etc.), whilst 'inside' transactions are ones which take place between different agents in the non-bank private sector. In the case of 'narrow' money (say,  $M_1$ ), we would have to redefine 'outside' transactions to include shifts between time and demand deposits. Also, when analysing 'not very broad' money definitions, like  $M_3$ , one would have to take into account switches between bank and building society deposits.

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Davidson (1986) argues that the 'degree of exogeneity or endogeneity' of the money stock definition under scrutiny is best seen as the proportion of the induced transactions which are inside transactions. The reason for this is the following: if economic agents are unable to reduce their money holdings via outside transactions following a money supply shock (say, via an increase in the monetary base), then money is a 'hot potato' in the conventional monetarist sense, and adjustment can only come via changes in prices, income and interest rates. If on the other hand, we argue that because of the existence of a modern commercial banking system, some outside transactions are possible (e.g. the running up and paying off of overdraft facilities, see Tobin, 1963), then the money stock is at least to some extent endogenous, as money is no longer a 'hot potato' within the non-bank private sector. The usual argument of why we may regard narrow money as endogenous may be reinterpreted in this context as a situation in which outside transactions are easy to carry out, and rather common, as they involve simple switches between time and demand deposits.

Unfortunately, as Davidson points out, we cannot identify the parameter of interest here, namely the proportion of induced transactions which are inside transactions with reference to aggregate time series. Therefore, the question of exogeneity may have to be tested at the econometric level<sup>15</sup>. As far as the Carr-Darby and Cuthbertson-Taylor models are concerned, their major

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problem is that they seek to model supply-side innovations with what probably are demand-side innovations (innovations in  $p$ ,  $y$ , and  $R$ , the determinants of the threshold levels). Under the circumstances, it may be best to restrict the testing of the Cuthbertson-Taylor model to narrow money data, and interpret it as a model of buffer-stock behaviour in a context where the ('narrow') money supply is endogenous, and most of the innovations are on the demand side, as captured by the  $p^u$ ,  $y^u$ , and  $R^u$  terms in (4.20). This is one of the main reasons why, in the rest of Chapter 4 and Chapter 5 we test the Cuthbertson-Taylor model on M1 data, despite contrary arguments that M1 may not be in fact the best 'buffer asset' available to economic agents.

Davidson's own preferred approach, especially when dealing with a broader definition of money like M3 is to try and distinguish more clearly between supply and demand side innovations by building a multi-equation money disequilibrium model. This would involve not only modelling the non-bank private sector's demand for money, but also the behaviour of the public, banking and overseas sectors. There are a number of examples of this approach, including Davidson's own attempts for the UK (see Davidson and Keil, 1981, 1982, Davidson, 1984). David Laidler has also argued (see Laidler 1982, 1983) that a multi-equation framework provides the best context in which to analyse buffer stock money, and has attempted to build models both for the UK

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(see Laidler and O'Shea, 1980), Canada (see Laidler et al., 1983), and the United States (see Laidler and Bentley, 1983). Jonson and Trevor (1979) have built a disequilibrium model for the Australian case.

The advantage of these complete models over the 'inverted' single-equation demand for money studies (e.g. Artis and Lewis, 1976, Laidler, 1980), is that they do not focus on a single variable as the endogenous variable through which disequilibrium money is dissipated. In contrast, the whole transmission mechanism is modelled. In addition to a long-run demand for money, the supply of money is also modelled (with a number of supply factors from the usual flow of funds identities), and adjustments in the price level, real income, exchange rates, etc. are affected by the excess of money supply over long-run desired demand, via the usual Artis-Lewis partial adjustment mechanism, or via a more general distributed lag formulation.

The advantages of these complete macromodels over single-equation demand for money studies are clear, especially when the definition of the money stock under scrutiny is likely to be subjected to exogenous supply-side shocks. However, despite some rather encouraging empirical results (e.g. the model by Davidson, 1984), there are some advantages in sticking to a single-equation model. For one thing, building a whole macromodel overcomes the whole question of 'identification' vis-a-vis money supply and demand shocks, but it introduces a whole new set of problems

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regarding other series which are brought into the modelling process. For instance, one could ask whether in bringing in the exchange rate in such macromodels, one has to take this as exogenous (thereby leaving out of the model an important part of the transmission mechanism) or whether one treats it as endogenous, in which in building a successful model of the demand for money one has first to build a successful model of the exchange rate! In the latter case, the results obtained vis-a-vis the long-run demand for money parameters will be heavily conditional upon the correct specification of the rest of the macromodel. Perhaps more significantly one could ask whether all of this computational effort is really required in order to obtain some reasonable idea about the nature and length of the adjustment lags and of the long-run elasticities of the demand for money? The use of single-equation models may still lead to reasonable results in this regard.

In this thesis, we have chosen to focus specifically on single-equation models, and in the next two sections we develop the Cuthbertson-Taylor model to see if we may be able to overcome some of the more apparent failings of the simple quadratic costs-of-adjustment model. In Chapter 5 we shall then seek to compare this costs-of-adjustment model with the more ad hoc 'general-to-specific' (general ADL) approach to single-equation modelling examined in Chapters 2 and 3. Unlike the models considered there, however, we shall focus on M1 data, for the reasons detailed

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above. One would expect the Cuthbertson-Taylor model to perform best on this data, on the grounds that it focuses primarily on demand-side innovations<sup>16</sup>.

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### SECTION THREE: SAVING AND THE COSTS-OF-ADJUSTMENT MODEL

#### 4.3.1 Incorporating Saving into the Cuthbertson-Taylor Buffer-Stock Model.

As we pointed out above, the simple Cuthbertson-Taylor model has some questionable characteristics. Let us recall the two main drawbacks of the model: firstly, equation (4.11) penalises changes in money holdings, which makes little sense in the presence of saving behaviour by economic agents. Where the total stock of wealth is not constant, (4.11) implies that economic agents will find it costly to adjust money holdings, and less costly to adjust holdings of alternative financial assets, despite the fact that money was specifically assigned to be the 'buffer asset'. Secondly, the simple model outlined in (4.11) implicitly allows only 'money' and 'bonds' as alternative assets in the portfolio. Once we move away from such a narrow theoretical framework and allow for the existence of many assets, the question arises as to which definition of the money stock (or which aggregate of money and near-money assets) performs the function of a financial buffer best. The first of these issues is addressed in this section, and we treat the issue of multi-asset buffer-stock models in section four.

Consider the following modification of the Cuthbertson-Taylor model. We assume that individuals hold their wealth,  $W$  either in money,  $M$ , or in alternative assets,  $V$ , which we group into a single category for simplicity. The following identity



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therefore holds:

$$W_t \equiv V_t + M_t \quad (4.24)$$

If we denote saving by  $S_t$ , it also follows that:

$$W_t \equiv W_{t-1} + S_t \quad (4.25)$$

Given (4.24), then we may re-express (4.25) as follows:

$$V_t + M_t \equiv V_{t-1} + M_{t-1} + S_t \quad (4.26)$$

Furthermore, given that, in any given time period  $t$ , any deviation from the desired value for  $V$ ,  $V^*$ , must have a counterpart for  $M$ :

$$(M_t - M_t^*) \equiv -(V_t - V_t^*) \quad (4.27)$$

it then follows that:

$$V_t^* + M_t^* = V_{t-1}^* + M_{t-1}^* + S_t \quad (4.28)$$

In what follows, we shall use identities (4.26) and (4.28) to derive our alternative costs-of-adjustment model. We do this by replacing the cost function we used in section two, (4.11), with an alternative intertemporal cost function which penalises both deviations from desired values, and adjustments in non-buffer assets,  $V$ :

$$C = E_{t-1} \sum_{j=0}^{\infty} \delta^j (a_0 (m_{t+j} - m_{t+j}^*)^2 + a_1 (v_{t+j} - v_{t+j-1})^2 - 2a_2 (v_{t+j} - v_{t+j-1})(v_{t+j}^* - v_{t+j-1}^*)) \quad (4.29)$$

where lower case letters indicate natural logarithms. This conversion to a logarithmic form is helpful to our estimation of the model further on. The parameter  $a_0$  represents the weight attached to being away from equilibrium, where  $m = m^*$  and  $v = v^*$ ,  $a_1$  is the weight attached to adjustments in non-buffer assets,

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and  $a_2$  represents the weight attached to changes in the same direction of  $v$  and  $v^*$ , where the latter is treated as a negative cost. The latter element is common to most multi-period quadratic costs-of-adjustment models (see for instance Hendry and Anderson, 1977, 1984, and Nickell, 1985), and although Cuthbertson and Taylor omit such terms on the grounds of simplicity, it is a rather ad hoc omission. The advantage of this term is that if, for example, both  $v$  and  $v^*$  increase at the same time, this will not be unambiguously penalised by economic agents through the second term, as would be the case if the third term were absent.

Following the same procedure as in section two, the problem is to find a sequence of  $\{m_{t+j}\}_{j=0}^{\infty}$  which minimises (4.29). This is found by evaluating  $\partial C / \partial m_{t+j} = 0$ , which yields:

$$\begin{aligned} E_{t+j-1} [a_0(m_{t+j} - m_{t+j}^*) - a_1(m_{t+j-1} - m_{t+j} - s_{t+j}) + \\ a_2(v_{t+j}^* + \delta(v_{t+j}^* - v_{t+j+1}^*) - v_{t+j-1}^*) + \\ \delta a_1(m_{t+j} - m_{t+j+1} + s_{t+j+1})] = 0 \end{aligned} \quad (4.30)$$

for  $j = 0, 1, 2, \dots$

where we have used (4.26) to substitute out  $(v_{t+j} - v_{t+j-1})$ .

It is important to note that in this model we are not proposing that the economic agent simultaneously chooses his preferred sequence of money holdings and his projected saving plan. We continue to incorporate the usual Keynesian assumption which separates the individual's saving and wealth-allocation plan. Thus, we assume that the amount of saving for each period,  $s_{t+j}$ , has already been chosen, and that the agent now attempts to

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find his optimal money holding plan subject to this initial constraint. Again, such a multi-stage decision-making process seems rather far removed from the truth, but it simplifies matters considerably. (For a discussion of the issue of the integration of the saving and investment decisions, see for instance Owen, 1985).

The set of Euler equations in (4.30) could be re-expressed as an estimation equation, but one of the main problems is that it requires knowledge of data on stocks of alternative assets,  $V$ , held by economic agents. To circumvent possible data problems in this regard, we can conveniently rearrange (4.30) using (4.26) and (4.28) so as to obtain an expression containing only terms in  $M$  and  $S$ .

First, substitute for  $(v_{t+j}^* + \delta(v_{t+j}^* - v_{t+j+1}^*) - v_{t+j-1}^*)$  using (4.28), and then divide through by  $-a_1$  to obtain:

$$(b_0L + b_1L^2 + 1)m_{t+j+1}^p = -c_0E_{t+j-1}m_{t+j}^* + c_1E_{t+j-1}m_{t+j+1}^* + c_2m_{t+j-1}^* - c_3E_{t+j-1}st_{t+j} - (c_1 - 1)E_{t+j-1}st_{t+j+1} \quad (4.31)$$

where  $c_0 = (a_0 + a_2(1 + \delta))/a_1\delta$ ,  $c_1 = (a_2/a_1)$ ,  $c_2 = (a_2/a_1\delta)$ ,

and  $c_3 = (a_1 - a_2)/a_1\delta$

and where  $L$  is the lag operator.

As in the case of (4.11), we may consider the following factorisation for the lag polynomial on the right hand side of (4.31):

$$(1 - \lambda_1L)(1 - \lambda_2L)m_{t+j+1}^p = -c_0E_{t+j-1}m_{t+j}^* + c_1E_{t+j-1}m_{t+j+1}^* + c_2m_{t+j-1}^* - c_3E_{t+j-1}st_{t+j} - (c_1 - 1)E_{t+j-1}st_{t+j+1} \quad (4.32)$$

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where the  $\lambda_i$  are the same as in the simple Cuthbertson-Taylor model. Recall that  $\lambda_1\lambda_2 = (1/\delta)$ , and hence we may eliminate the unstable root, say  $\lambda_2$ , by multiplying each side by  $(1 - \delta\lambda_1^{-1}L)^{-1}$ . This term may be re-expressed using a Taylor expansion, as in the simple Cuthbertson-Taylor model (see Sargent, 1979), so that we obtain our final equation:

$$\begin{aligned} m_t = & \lambda_1 m_{t-1} - (a_2/a_1)\lambda_1 m_{t-1}^* + ((a_0 + a_2(1 + \delta))/a_1)\lambda_1 E_{t-1} m_t^* - \\ & [(a_2/a_1)(1 + \lambda_1) - ((a_0 + a_2(1 + \delta))/a_1)\lambda_1]\delta\lambda_1 \sum_{i=0}^{\infty} (\delta\lambda_1)^i E_{t-1} m_{t+i+1}^* + \\ & [(a_2 - a_1)(1 - \lambda_1)/a_1]\delta\lambda_1 \sum_{i=0}^{\infty} (\delta\lambda_1)^i E_{t-1} s_{t+i+1}^* + \\ & (1 - (a_2/a_1))\lambda_1 E_{t-1} s_t + m_t^u + \varepsilon_t \end{aligned} \quad (4.33)$$

As in the case of the simple Cuthbertson-Taylor model outlined in section two, this model displays some appealing theoretical features, with the addition that individuals' expectations of future saving decisions now enter the plan for current money holdings. If we compare the terms on expected current and future saving in (4.33) we note that the signs of their coefficients depend on the relative sizes of  $a_2$  and  $a_1$ . Thus, if individuals attach a large cost to adjusting their holdings of alternative assets ( $a_1$  larger than  $a_2$ ), then current expected saving will appear with a positive coefficient, and expected future saving will have a coefficient with the opposite sign. This result is intuitively plausible, since if economic agents suddenly find that their saving is expected to increase permanently in the current time period, this increased saving will initially be channelled into money balances, and then

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gradually reallocated to other assets. In contrast, if agents attach a large benefit to parallel movements in desired and actual levels of alternative assets, the reverse will apply.

Note also that, whilst in (4.20) individuals were concerned with future values of desired money holdings ( $m^*$ ) and past values of actual money holdings ( $m_{t-1}$ ) alone, in (4.33) past desired money holdings ( $m_{t-1}^*$ ) also enter their plans. This is because we have chosen a different cost function and allowed for saving flows which renders the stock-flow interaction more complex. Thus the divergence of actual and desired money holdings in the last time period becomes a relevant indicator for the economic agent. As we shall see below, this raises some rather difficult problems in estimating (4.33).

The problems associated with the estimation of this model are similar to those encountered in the case of (4.20), but are compounded by the presence of additional terms. We now examine these in turn.

First, the model is once more non-linear, and though non-linear estimation is possible, for simplicity we shall follow Cuthbertson (1984) and Cuthbertson and Taylor (1987) in imposing the prior restriction  $\delta = 0.99$ . Further on we attempt to assess the significance of this restriction.

Secondly, the presence of complex terms involving the  $a_i$  parameters means that the structural parameters of the demand for money cannot be identified by estimating (4.33). That is, we

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cannot identify the long-run price, income and interest rate elasticities. In contrast, this is possible from (4.20), since the value of  $\delta$  is imposed a priori, and the value of the stable root,  $\lambda_1$ , may be found from the estimated coefficient on the  $m_{t-1}$  regressor. Thus, we shall not be able to assess the alternative model by solving for its long-run solution.

Thirdly, additional problems arise due to the presence of  $m_{t-1}^*$  in addition to future expected values of the desired demand for money. Given that future expected values of the desired money stock are found via autoregressive predictions for  $p$ ,  $y$ , and  $R$ , these are likely to be closely correlated with  $p_{t-1}$ ,  $y_{t-1}$ , and  $R_{t-1}$ , leading to multicollinearity in the regressors. This is particularly the case given that we are dealing with series which are trending, and which follow processes close to random walks. (In the case of the interest rate which, as we shall see, is approximated by a random walk, the regressors will be perfectly collinear). To avoid this rather acute problem, we shall exclude  $m_{t-1}^*$  from our estimating equation, on the grounds that the effect of these variables is captured by the other terms of the equation. It should be borne in mind, however, that this leads to problems when testing the appropriateness of restrictions in the forward-looking model.

Fourthly, although we have implicitly treated the economic agents' saving decision as exogenous in this exercise, it is doubtful if the  $s_t$  variables may be regarded as 'weakly

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exogenous' (see Engle et al., 1983), and thus whether OLS remains an appropriate estimation method in the case of this model. We return to this point later, but at this stage it is important to stress that the saving and portfolio decisions of an individual, though treated as separate when deriving the individual's desired demand for money and his intertemporal plan for money holdings, are in practice unlikely to be independent, especially in a dynamic context.

Lastly, given that saving is likely to depend itself on income and interest rates, there is likely to be a degree of multicollinearity between  $s_{t+j}$  and  $m_{t+j}^*$  which, even if we were to be able to make deductions regarding estimated long-run elasticities, would make their interpretation difficult.

Before we turn to report on the estimations which we carried out, we should note that, in the case of both (4.20) and (4.33), the structure of the optimisation exercise leads to a particular pattern of parameter restrictions which may be tested, and which may provide us with a test of the model itself. Note for instance that the parameters on successive future expected values of  $p$ ,  $y$ , and  $R$  in (4.20) should decline geometrically due to the presence of the term  $(\delta\lambda_1)^i$ . Given the estimated value of  $\lambda_1$  from the coefficient on  $m_{t-1}$ , we can impose and test whether these parameter restrictions (also known as 'backward-forward' restrictions) are data-acceptable. If they are not, then this suggests that the functional form of the cost function is not

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appropriate (see Chapter 5, where this issue is taken up in more detail).

As for equation (4.20), there are backward-forward restrictions to be tested in (4.33), though they differ somewhat in that current expected values of  $m^*$  and  $s$  appear with different coefficients from their future expected values in (4.33). However, unlike (4.20), the significance of these parameter restrictions is somewhat lessened due to the presence of such a large number of regressors (and the exclusion of  $m_{t-1}^*$ ).

We shall now attempt to evaluate the performance of our modified model compared to Cuthbertson and Taylor's original equation (4.20). In general, the model presented in (4.33) may be compared with the model described in (4.20) by subjecting both to a variety of diagnostic tests, by comparing their forecast performance, and by testing the significance of the saving variable in (4.33), as it is the only variable which the two models do not have in common.

### 4.3.2 Testing the Two Alternative Forward-Looking Models.

We have chosen to use the two alternative buffer-stock models described in section 4.3.1 to model M1 money demand in the United Kingdom, over the period 1963(1)-1984(4). There are three main reasons for preferring a 'narrow money' definition. First, as pointed out above, it makes more sense in a forward-looking model which stresses the importance of demand-side innovations. Secondly, Cuthbertson and Taylor have chosen to apply their model



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to this definition of money. Thirdly, it will facilitate a comparison of forward-looking and ADL-based models in Chapter 5, given that both these schools of thought have fitted M1 models using their respective methodologies.

The data definitions used for this M1 demand study are the following: M1 is as defined in the Bank of England Quarterly Bulletin. The series used are Bank of England ones which have been adjusted for structural breaks. We use personal disposable income at constant (1980) prices (RPDI) for our real income variable. The price variable is the implicit RPDI deflator, and the interest rate used is the treasury bill rate. The last three series and the saving series have been taken from Financial Statistics, and all the data used are seasonally unadjusted, for the reasons stated in Chapter 3.

Before proceeding to the estimation of the models in (4.20) and (4.33) we estimated autoregressive forecasting equations for  $p$ ,  $y$ ,  $R$ , and  $s$ , and the preferred equations are reported in Table 4.1 (where the numbers in brackets once more denote standard errors). Following Artis and Cuthbertson (1985) we assume that the interest rate follows a random walk, thus making it ex ante unpredictable, and removing the need for a forecasting equation. However, this a priori restriction seemed to be acceptable given the interest rate series used (despite the well-known difficulties in assessing the existence of unit roots). From Table 4.1 we see that the interest rate equation still performs

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rather well in terms of within-sample fit, and that the coefficient on  $R_{t-1}$  is very close to unity, despite the well known downward bias in the estimated parameters in these cases (see for instance Sims et al, 1986). As a further justification for our imposition of a random walk, one should note that the substitution method does not require economic agents to possess full information about the economic environment in which they are operating (see Wallis, 1980).

Note from Table 4.1 that all the autoregressive equations perform adequately in terms of within-sample-fit, and all pass the LM(n) tests for  $n = 1, 4$ , and 8 at the 5% significance level. All the equations are restricted versions of a general autoregressive equation with 8 lags, where all restrictions imposed were data acceptable. Predictably, of all the equations, it is the interest rate one which performs least well. This is not surprising given that this variable is likely to be (or be very close to being) ex ante unpredictable.

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TABLE 4.1

### (1) Price Equation

$$P_t = 1.218P_{t-1} - 0.221P_{t-5} \\ (0.0185) \quad (0.0174)$$

$$R^2 = 0.999 \quad \hat{\sigma} = 0.0137 \quad DW = 1.929 \quad LM(1) = 0.01 \quad LM(4) = 0.19$$

$$LM(8) = 0.13$$

### (2) Real Income Equation

$$y_t = 0.737y_{t-1} + 0.228y_{t-2} - 0.202y_{t-3} + 0.521y_{t-4} - 0.284y_{t-5} \\ (0.111) \quad (0.126) \quad (0.127) \quad (0.124) \quad (0.109)$$

$$R^2 = 0.999 \quad \hat{\sigma} = 0.0223 \quad DW = 1.979 \quad LM(1) = 0.05 \quad LM(4) = 0.39$$

$$LM(8) = 0.89$$

### (3) Interest Rate Equation

$$R_t = 0.996R_{t-1} \\ (0.0142)$$

$$R^2 = 0.983 \quad \hat{\sigma} = 1.231 \quad DW = 1.616 \quad LM(1) = 2.66 \quad LM(4) = 1.33$$

$$LM(8) = 0.95$$

### (4) Saving Equation

$$S_t = 0.143S_{t-1} + 0.231S_{t-3} + 0.623S_{t-4} - 0.255S_{t-7} + 0.161S_{t-8} \\ (0.071) \quad (0.114) \quad (0.117) \quad (0.112) \quad (0.111)$$

$$R^2 = 0.739 \quad \hat{\sigma} = 0.193 \quad DW = 1.843 \quad LM(1) = 0.7 \quad LM(4) = 1.04$$

$$LM(8) = 1.26$$

Notes: (a) All equations were estimated (in logs) over the maximum available data period, given the lag structure. The total data period is 1963(1)-1984(4).

(b) LM(n) refers to the Lagrange Multiplier test against serial correlation in the residuals of order n (see Godfrey, 1978, Harvey, 1981).

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These equations were used to generate the expected data series to be used in estimating equations (4.20) and (4.33) using Wold's chain rule of forecasting. As pointed out above, we restrict  $\delta$  to be equal to 0.99 in our estimations. The models were both estimated over the period 1965(1)-1982(4), so that 8 data periods were kept aside to assess the models' ex ante forecasting performance. We restrict economic agents' horizons to one year in the future when estimating these equations ( $i = 0, \dots, 4$ ). Initially the models were estimated in an unrestricted form (that is, the backward-forward restrictions were not imposed), and subsequently these restrictions were tested, and imposed. Given that the data used was seasonally unadjusted, we included seasonal dummies in our models, in addition to a constant term. This implies that, in modelling the demand for money in a forward-looking manner, we take into account seasonal factors which may affect money holdings. This makes sense, because our simple costs-of-adjustment model does not include such seasonal factors, but they are likely to be a serious matter for consideration by economic agents.

The results of the estimation of the two alternative models are reported in Table 4.2 below. The diagnostic tests reported are the same as those reported in Chapters 2 and 3. The backward-forward restrictions were tested using a conventional F-test (see Harvey, 1981a), where the F-statistic is distributed as  $F(m, T-k)$  under the null hypothesis of the validity of the restricted

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model, where  $m$  is the number of restrictions,  $T$  is the number of observations, and  $k$  is the number of estimated parameters in the unrestricted model. For the simple Cuthbertson-Taylor model (equation (4.20)) we have an F-statistic of 1.905 which lies outside the 5% critical region for an  $F(8, 53)$  distribution. For the alternative model, we found an F-statistic of 1.126, which lies outside the 5% critical region for an  $F(10, 47)$  distribution. Thus, in the case of both our models the backward-forward parameter restrictions are data acceptable at the 5% significance level, though it is a closer-run thing for the simple Cuthbertson-Taylor model.

From Table 4.2 it can be seen that both models perform reasonably well in terms of the reported diagnostic tests, though the simple Cuthbertson-Taylor model performs better than our alternative model (in both the unrestricted and restricted versions) for out-of-sample forecasts. However, three of the four equations reported in Table 4.2 fail the  $LM(4)$  test against serial correlation in the residuals, with only the unrestricted version of equation (4.33) not rejecting the null hypothesis of no time dependence in the residuals at the 5% significance level.

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TABLE 4.2

Equation	Model (4.20) Unrestricted	Model (4.20) Restricted	Model (4.33) Unrestricted	Model (4.33) Restricted
$m_{t-1}$	0.732 (0.079)	0.863 (0.043)	0.732 (0.081)	0.819 (0.072)
$p_t^e$	-2.509 (1.855)	0.045 (0.013)	-2.335 (2.310)	0.268 (0.210)
$p_{t+1}^e$	0.027 (2.004)	-	0.221 (2.415)	-0.042 (0.063)
$p_{t+2}^e$	2.318 (2.027)	-	0.653 (2.338)	-
$p_{t+3}^e$	7.634 (5.610)	-	9.570 (6.244)	-
$p_{t+4}^e$	-7.247 (4.357)	-	-7.895 (5.054)	-
$y_t^e$	-0.256 (1.630)	0.043 (0.018)	-1.400 (2.001)	0.688 (0.269)
$y_{t+1}^e$	-0.035 (0.603)	-	0.143 (0.837)	-0.203 (0.100)
$y_{t+2}^e$	-0.743 (0.961)	-	-1.729 (1.356)	-
$y_{t+3}^e$	-0.605 (1.156)	-	-0.712 (1.567)	-
$y_{t+4}^e$	1.842 (3.085)	-	3.872 (4.015)	-
$R_t^e$	-0.0065 (0.0013)	-0.0021 (0.0004)	-0.0065 (0.0013)	-0.0018 (0.0003)
$s_t^e$	-	-	-0.700 (0.571)	-0.331 (0.415)
$s_{t+1}^e$	-	-	0.528 (0.443)	0.024 (0.021)

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TABLE 4.2 (Cont.)

Equation	Model (4.20) Unrestricted	Model (4.20) Restricted	Model (4.33) Unrestricted	Model (4.33) Restricted
$s_{t+2}^e$	-	-	0.049 (0.025)	-
$s_{t+3}^e$	-	-	0.085 (0.102)	-
$s_{t+4}^e$	-	-	0.067 (0.074)	-
$p^u$	0.464 (0.171)	0.430 (0.173)	0.399 (0.196)	0.509 (0.167)
$y^u$	0.244 (0.112)	0.131 (0.106)	0.398 (0.151)	0.338 (0.140)
$R^u$	-0.0060 (0.0018)	-0.0075 (0.0018)	-0.0063 (0.0018)	-0.0065 (0.0018)
$s^u$	-	-	-0.019 (0.017)	-0.020 (0.016)
Constant	0.755 (0.777)	0.185 (0.601)	0.787 (0.760)	0.171 (0.571)
Q1	-0.038 (0.008)	-0.057 (0.006)	-0.050 (0.021)	-0.041 (0.012)
Q2	-0.022 (0.007)	-0.031 (0.006)	-0.011 (0.018)	-0.026 (0.009)
Q3	-0.016 (0.007)	-0.024 (0.006)	-0.037 (0.018)	-0.022 (0.009)
TEST STATISTICS				
$R^2$	0.999	0.999	0.999	0.999
$\hat{\sigma}$	0.0156	0.0165	0.0150	0.0153
DW	2.16	2.37	2.20	2.33

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TABLE 4.2 (Cont.)

Equation	Model (4.20)	Model (4.20)	Model (4.33)	Model (4.33)
	Unrestricted	Restricted	Unrestricted	Restricted
TEST				
STATISTICS				
$Z_1$	1.73	1.53	2.58	1.70
$E_1$	1.43	1.33	1.79	1.48
LM(4)	1.62 *	2.74 *	1.80	2.83 *
ARCH(4)	0.23	0.63	0.52	0.25
$E_4$	0.674	0.797	0.520	1.14

Note: In the case of the restricted equation of Model 1,  $X_t^e$  is given by:  $X_t^e = \sum_{i=0}^4 (\delta\lambda_1)^i X_{t+i}^e$ , for any variable X. In the case of model 2,  $X_t^e$  is the same as in the unrestricted model, and  $X_{t+1}^e$  is given by:  $X_{t+1}^e = \sum_{i=0}^3 (\delta\lambda_1)^i X_{t+i+1}^e$ , for any variable X. Also note that for the interest rate, given that  $R_t^e = R_{t-1}$ , it follows that  $R^u = \Delta R_t$ . The test statistics denoted by a \* reject  $H_0$  at the 5% significance level.



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There are several points to be noted about these two models. First, on the basis of ex ante forecasting performance, the simple Cuthbertson-Taylor model seems to perform 'better', although one cannot really treat the  $Z_1$  and  $E_1$  statistics as absolute measures of forecasting performance. Furthermore, by computing the t-values of the individual regressors in the two restricted equations, it becomes apparent that whilst for the simple model all the regressors, except for  $y_t^e$  seem significantly different from zero, the same is not true for the saving model, where  $p_t^e$ ,  $p_{t+1}^e$ , and, more importantly, all the saving variables appear insignificant. However, one problem with the simple model is that, by omitting saving, there is time dependence in the residuals.

Secondly, if we examine the signs of the estimated coefficients on the regressors for both of the restricted models, they appear to correspond with what we would expect a priori. In the case of the restricted version of (4.20) the price and income effects have positive signs, whilst the interest rate effects have negative signs. In the case of (4.33), we note that  $p_t^e$  and  $y_t^e$  have positive signs, which we would expect from (4.33), as  $((a_0 + a_2(1 + \delta))/a_1)\lambda_1$  is positive. Note also that  $y_{t+1}^e$  and  $p_{t+1}^e$  have negative signs, which implies from (4.33) that  $(a_2/a_1)(1 + \lambda_1) > ((a_0 + a_2(1 + \delta))/a_1)\lambda_1$ . This does not help us, as we pointed out above, to identify the long-run elasticities of the demand for money with respect to these variables, but the

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results do seem broadly consistent with our theoretical model. However, it must be recognised that the omission of  $m_{t-1}^*$  makes any conclusions in this regard rather tentative.

Thirdly, we have to examine whether the correct parameter restrictions have been imposed by imposing the backward-forward restrictions. In our saving model, we were forced to exclude lagged values of the price level, real income, and the interest rate due to multicollinearity problems between these variables and their future expected values. Thus, for our saving model, we do not really know if we are imposing the correct restrictions. However, similar problems also arise with the simple Cuthbertson-Taylor model due to collinearity problems between the regressors in that model. A casual glance at the point estimates for both models show us that the backward-forward restrictions seem rather unrealistic. The F-tests cannot reject their validity, but this may to a large extent be due to the fact that the unrestricted models are overparameterised. In fact, there are real problems in sticking to a simple quadratic-cost minimisation exercise as a guide to empirical modelling as this may represent a gross simplification of the actual adjustment process, and may lead us to an incorrectly specified forward-looking model. This is an issue which we shall consider in detail in Chapter 5. When we assume to a more complex cost-minimisation exercise, as in the case of our saving model, further problems arise in the specification (such as the presence of the  $m_{t-1}^*$  term), and there

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are even greater reasons for deviating from the path dictated by ad hoc 'theory' (see Hendry and Anderson, 1977, Nickell, 1985).

Given then that (4.33) may be seen as providing only an initial guide to the specification of the model, we sought to improve the results for our alternative 'saving-based' model by considering additional parameter restrictions on the restricted model of Table 4.2. The restriction which we test for the fourth equation of Table 4.2 is  $s_t^e = (1 - \lambda_1)s_{t+1}^e$ , which is suggested to us by (4.33), and also set  $X_t^e = -\delta\lambda_1 X_{t+1}^e$  (where  $X = p, y$ ). This latter restriction implies a restriction of  $(\lambda_1 - 1) = 2$  in terms of (4.33). This is not possible, given that our model suggests that  $0 < \lambda_1 < 1$ , but as we have suggested above, once certain variables have been omitted, we should no longer be hindered by 'theory' in this regard, as the restriction may be data acceptable. The equation which is obtained by imposing these two further restrictions is equation (A) in Table 4.3 below.

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TABLE 4.3

Regressors	Equation	Saving Model Equation (A)	Saving Model Equation (B)
$m_{t-1}$		0.888 (0.044)	0.940 (0.059)
$p_t^e$		0.029 (0.011)	0.017 (0.014)
$y_t^e$		0.022 (0.017)	-0.013 (0.027)
$R^e$		-0.0019 (0.0003)	-0.0023 (0.0005)
$s_t^e$		0.0111 (0.006)	0.027 (0.011)
$p^u$		0.464 (0.172)	0.344 (0.217)
$y^u$		0.207 (0.138)	-0.211 (0.297)
$R^u$		-0.0078 (0.0018)	-0.0093 (0.0022)
$s^u$		-0.006 (0.015)	0.069 (0.047)
Constant		0.109 (0.594)	0.456 (0.737)
Q1		-0.065 (0.010)	-0.098 (0.021)
Q2		-0.040 (0.008)	-0.058 (0.013)
Q3		-0.032 (0.007)	-0.045 (0.011)

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Table 4.3 (Cont)

TEST STATISTICS			
$R^2$	0.999	-	
$\hat{\sigma}$	0.0161	0.0192	
DW	2.42	1.86	
$Z_1$	1.61	1.42	
$E_1$	1.40	-	
LM(4)	2.52	0.51	
ARCH(4)	0.45	0.24	
$E_4$	1.136	1.428	
RESET (2)	0.139	-	
IVS	-	1.68	

## Notes :

(1) The LM(4) test for serial correlation is presented in its chi-square form in the case of equation (B) which is estimated by instrumental variable methods.

(2) The IVS test is the Sargan (1964) test for the appropriateness of the instruments.

(3) The instruments chosen to estimate equation (B) are :  $m_{t-2}$ ,  $m_{t-3}$ ,  $s_{t-2}$ ,  $s_{t-3}$ ,  $s_{t-4}$ ,  $s_{t-7}$ .

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The restrictions imposed to obtain equation (A) from the restricted version of (4.33) in Table 4.2 were tested using an F-test. The value of the F-statistic (distributed as  $F(3,59)$  under the null hypothesis) was calculated to be 2.56 which is not significant at the 5% level. Thus we can accept the restricted version of our alternative model shown in equation (A), Table 4.3. The advantage of this compared to the restricted version in Table 4.2 is that it does not contain more than one compound term for each basic variable. Furthermore, equation (A) shows some 'improvement' in terms of its forecast performance, as judged by the  $Z_1$  and  $E_1$  statistics. It also fits better than the restricted version of the Cuthbertson-Taylor model in terms of the  $\hat{\sigma}$  statistic. More interestingly, the coefficient on the saving term is now significantly different from zero, and there is no evidence of serial correlation in the residuals, indicating that the transformation imposed on the model to obtain equation (A) is sufficient to remove the time dependence present in the models of Table 4.2. This in turn indicates that one reason for the poor performance of the saving model in the previous versions was the imposition of an inappropriate dynamic structure. This highlights once more the problems involved in finding the appropriate dynamic specification whilst sticking closely to the form suggested by a cost-minimisation exercise. As we shall see in Chapter 5, there are severe disadvantages in merely testing backward-forward restrictions in forward-looking models of this

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character. The collinearity problems which arise in the case of the saving model are so severe that we must move to a more flexible dynamic structure at the outset.

Equation (A) also highlights another aspect of the multicollinearity problem which arises from the inclusion of saving in the Cuthbertson-Taylor model, namely the correlation between the  $s$  and  $y$  series. Note in fact that in equation (A) there is a fall in the  $t$ -statistic for the income term.

However, despite some of the positive aspects of equation (A) (i.e. the satisfactory performance in terms of diagnostic tests, the better fit compared to the simple Cuthbertson-Taylor model), it is still slightly disappointing because not all the regressors are significant, though mercifully, all the regressors have the 'correct' signs. In order to see whether we could improve our alternative model further, we considered several other possibilities. First, there is the possibility that OLS estimation may not be valid given the presence of saving in the money demand equation. Although in the theoretical model presented here the individual's saving decision is treated as exogenous to the demand for money decision, in practice we would expect these decisions to be interdependent, with the individual making his saving decision on the basis of the future paths of  $p$ ,  $y$ , and  $R$ . Thus, we may have to estimate the saving model using instrumental variable methods, to allow for possible simultaneity bias. In the case of equation (B) in Table 4.3, we present IV

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estimates of equation (A), where the preferred instruments and their lag structures were chosen according to the goodness of fit of the model. By comparing the point estimates of equations (A) and (B), it can be seen that the estimated coefficients on  $s_t^u$  and  $y_t^e$  have different signs under IV estimation, with the standard errors on the saving and income variables increasing considerably. Furthermore, equation (B) seems to have a poor fit compared to equation (A). Overall, one is forced to conclude that the IV estimation of this particular model has not been too successful to date, though it suggests that simultaneity may be a problem with this model.

A second possibility which we explored was to eliminate seasonal dummies from the estimated models. As we noted earlier, these dummies were included to account for seasonal variations in money holdings. We have already discussed our reasons for not using seasonally adjusted data, in contrast to Cuthbertson and Taylor (1987). In general, when estimating unrestricted stochastic difference equations, one introduces seasonal effects through the lag structure (see Harvey, 1981a). Thus any seasonality is adequately captured by assuming a sufficiently flexible lag structure in the explanatory variables, without necessarily introducing seasonal dummies into the equation. The lag structure will then ensure that the time series properties of the dependent and explanatory variables will be such so as to ensure that the model's residuals will be white noise, and hence



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do not contain moving average elements resulting from seasonality in the dependent variable (see Granger, 1981). We have so far included seasonal dummies in our estimating equations, given that the somewhat rigid lag structure imposed by the forecasting equations, the chain rule of forecasting and the theoretical optimisation exercise may not have accounted for all the seasonal effects in the money stock. Nevertheless, it is appropriate to ask at this juncture whether these seasonal dummies have been correctly included into our models, and what effects their exclusion would have on the other estimated coefficients of the model and on the residuals.

In Table 4.4 equations (C) and (D) represent estimates for the restricted versions of the Cuthbertson-Taylor model and the saving model presented in Table 4.2, but this time excluding the seasonal dummies. The notation used in Table 4.4 for the regressors is identical to that used in Table 4.2. The F-test statistics for equations (C) and (D) testing the zero restrictions on the seasonals are 29.82 and 3.05 respectively, with the F-statistics distributed as  $F(3, 61)$  and  $F(3, 56)$  under the null hypothesis of valid zero restrictions on the seasonals. Thus we reject these restrictions on the seasonals for the simple Cuthbertson-Taylor model, but not for the saving model.

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TABLE 4.4

Regressors	Equation	Simple Model Equation (C)	Saving Equation (D)	Saving Model Equation (E)
$m_{t-1}$		0.743 (0.060)	0.751 (0.070)	0.785 (0.047)
$p_t^e$		0.081 (0.018)	0.364 (0.216)	0.067 (0.014)
$p_{t+1}^e$		-	-0.055 (0.066)	-
$y_t^e$		0.052 (0.027)	1.217 (0.220)	0.068 (0.021)
$y_{t+1}^e$		-	-0.366 (0.090)	-
$R^e$		-0.0022 (0.0006)	-0.0015 (0.0003)	-0.0015 (0.0004)
$s_t^e$		-	-0.078 (0.028)	0.104 (0.023)
$s_{t+1}^e$		-	0.023 (0.015)	-
$p^u$		0.593 (0.246)	0.607 (0.167)	0.688 (0.192)
$y^u$		0.385 (0.150)	0.553 (0.114)	0.574 (0.137)
$R^u$		-0.0032 (0.0026)	-0.0044 (0.0018)	-0.0055 (0.0021)
$s^u$		-	-0.052 (0.012)	-0.064 (0.014)

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TABLE 4.4 (Cont.)

Regressors	Equation	Simple Model Equation (C)	Saving Model Equation (D)	Saving Model Equation (E)
Constant		1.117 (0.881)	0.254 (0.591)	0.423 (0.710)
Q1		-	-	-
Q2		-	-	-
Q3		-	-	-
TEST STATISTICS				
$R^2$		0.998	0.999	0.999
$\hat{\sigma}$		0.0253	0.0164	0.0197
DW		2.53	2.35	2.68
$Z_1$		0.87	1.61	1.13
$E_1$		0.45	1.30	0.88
LM(4)		10.28 *	3.10 *	5.09 *
ARCH(4)		0.19	0.65	0.43
$E_4$		1.290	1.499	1.239
RESET(1)		5.54 *	1.05	3.20
RESET(2)		2.74	1.33	2.65

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Incidentally, it is worth noting that, in comparing equations (C) and (D), the Cuthbertson-Taylor model seems to have a better ex ante forecast performance. However, this is merely an indication that the estimated model (C) tracks the trend behaviour of the future money stock quite well. As it happens, equation (C) consistently underpredicts the demand for money over the forecast period 1983(1)-1984(4). The poor performance of this model is confirmed by the strong rejection of the null hypothesis of no serial correlation from the LM(4) statistic. Also note that the model fails the RESET(1) test. This failure is another indication of its poor forecasting performance (highlighting the need to take a number of forecast performance tests into account when testing a model), and probably derives from the time dependence in the model's residuals. In this context it is worth noting that the LM(4) statistic has also some power against possible MA processes in the residuals, in addition to providing a test against autoregressive processes (see Godfrey, 1978, Breusch and Pagan, 1980).

Note that, in contrast, equation (D) passes both RESET tests, and only narrowly rejects the null hypothesis of no serial correlation in the residuals through the LM(4) test. In addition, this model performs better than the restricted version of the saving model reported in Table 4.2 which contained the seasonal dummies. Furthermore, eliminating the dummies in equation (D) makes the saving terms significant, which confirms the role of

#### CHAPTER 4

saving in this forward-looking demand for money model.

A third possibility which we explored was to change the imposed value of  $\delta = 0.99$  in the model. As pointed out above, technically this requires the adoption of nonlinear least squares methods in order to find the value of  $\delta$  which maximises the likelihood function. This is not a trivial task in itself, and will be an important part of any future research in this field. To quantify the possible effects of restricting  $\delta$  to be equal to 0.99, we re-estimated the equations reported in Table 4.2 for  $\delta = 0.5$ , and  $\delta = 0.25$ . In both cases, the effects on the estimated coefficients and the reported test statistics were not major. In any case, the goodness-of-fit of the models was not improved by changing the value of  $\delta$ , and these results are not reported here.

Overall the equations reported here highlight some major problems in directly estimating forward-looking buffer-stock models. Given that it is desirable, whenever possible, to estimate the models with seasonally unadjusted data (in contrast to Cuthbertson and Taylor, 1987), the restrictive lag structure imposed by this type of buffer stock model makes it difficult to know, a priori, whether such seasonal effects should be explicitly accounted for by including seasonal dummies, or whether the other regressors in the model are sufficient to avoid the presence of moving average processes in the residuals of the estimated models. As it happens, we have shown that our two alternative buffer stock models fall in between these two cases.

## CHAPTER 4

In the case of the simple Cuthbertson-Taylor model, seasonal dummies should be included in the model, whilst in the case of the saving model, the exclusion of these dummies leads to a better model (even though not all the time dependence is eliminated from the residuals).

This implies that, as an econometric model, equation (D) has a lag structure which is too restrictive. This may be confirmed by imposing the same restrictions on equation (D) which were imposed earlier to obtain equation (A). The results of this estimation are reported in equation (E) in Table 4.4. Though all the regressors have coefficients significantly different from zero in equation (E) (except the constant term), this equation shows clear signs of time dependence in the residuals, and its apparent improved forecasting performance in terms of the  $Z_1$  and  $E_1$  tests (as in the case of equation (C)) hides an almost consistent underprediction of the demand for money. However, the values of the RESET statistics may indicate that the presence of saving in the model ensures that the forecasts reflect more accurately the seasonal pattern of the demand for money. Thus whilst equation (C) consistently underpredicts the demand for  $M_1$  and also fails the RESET tests, this is not the case for equations (D) and (E). However, it should be noted that we are not advancing equation (E) as the preferred equation for our saving model, because the restrictions imposed to obtain equation (E) were found not to be data acceptable.

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Given the problems involved in allowing for seasonal effects in this type of model, and in discovering an appropriate lag structure once theory ceases to be of guidance (as in the case of the saving model), the advantages in estimating these types of buffer stock models seem to be limited. As we shall see in Chapter 5, in contrast, the flexible lag structure allowed for by applying the general-to-specific model selection procedure on backward-looking models allows us to circumvent some of the problems highlighted above, despite the problems involved with the general-to-specific procedure itself.

### 4.3.3 Conclusion on Saving and the Cuthbertson-Taylor Model

In this section we have sought to extend the Cuthbertson-Taylor costs-of-adjustment model to incorporate saving behaviour. We have shown that this can lead to a more complex buffer-stock model than envisaged by their simpler set-up. However, this greater complexity does introduce some difficulties in the estimation procedure. There is little doubt that the simple Cuthbertson-Taylor model is unsatisfactory at a theoretical level, as it presents a rather simplistic view of the portfolio adjustment mechanism. However, our saving model has not performed totally satisfactorily at the empirical level. It is true that we have found the saving variable to be significant in a number of versions of the model, which is encouraging. Also on the positive side, the saving model seemed to have a more robust design than the simple Cuthbertson-Taylor model.

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On the other hand, the collinearity problems which arise between the real income and saving series have made conclusions regarding structural parameters difficult to reach. In part this is also a difficulty raised by the more complex cost-minimisation exercise entailed by the inclusion of saving behaviour. This makes a full evaluation of the saving model difficult, but the questions raised in this section raise some doubts regarding the validity of the simple Cuthbertson-Taylor model.

In the next section, we examine another possible extension to the Cuthbertson-Taylor model, namely the inclusion of more than two assets in the portfolio. Once again, this complicates the cost-minimisation exercise. Although we shall not attempt to test this third alternative model at the empirical level, we shall offer some indications on the technical difficulties which may be encountered in this regard.



## CHAPTER 4

### SECTION FOUR: MULTI-ASSET MODELS AND BUFFER-STOCK MONEY

#### 4.4.1. Existing Multi-Asset Adjustment Models.

Since the emergence of multi-asset portfolio theory in macroeconomic models (see for instance Tobin 1969, 1982), some economists have emphasised the importance of examining the interdependent nature of financial markets in the process of portfolio adjustment. Most portfolio models emphasise this interdependence at the comparative static level, through the standard wealth constraint. However, when it comes to modelling the dynamics of short-run adjustment, the tendency of single-equation asset demand studies, like the studies on the demand for money, has been to ignore the possible dynamic interdependence between asset demands.

One notable exception to this is provided by Brainard and Tobin (1968). Brainard and Tobin basically generalise the simple partial adjustment process to depend on disequilibria in other asset markets. Thus, for any asset  $A_i$  ( $i = 1, \dots, n$ ) in the portfolio, short-run demand follows an adjustment process of the type:

$$(A_{it} - A_{it-1}) = \alpha_{i1}(A_{1t}^* - A_{1t-1}) + \alpha_{i2}(A_{2t}^* - A_{2t-1}) + \dots + \alpha_{ii}(A_{it}^* - A_{it-1}) + \dots + \alpha_{in}(A_{nt}^* - A_{nt-1})$$

(4.34)

where  $A_i^*$  is the ('long-run') desired holding of asset  $A_i$ . From our usual partial adjustment model, we know that  $\alpha_{ii} < 0$ , but we cannot sign  $\alpha_{ij}$  a priori, unless we consider some explicit cost-

#### CHAPTER 4

minimisation process which yields (4.34)<sup>17</sup>. Thus, the point of (4.34) is to argue that adjustment in any one market depends on disequilibria in other markets.

Though this makes intuitive sense, one criticism of the simple Brainard-Tobin interdependent adjustment scheme is that it is too ad hoc, with no convincing account of the way in which these partial adjustment mechanisms in different asset markets are linked. Even if one believes that it is impossible to rationalise the nature of dynamic adjustment through the use of economic theory, simple partial adjustment clearly will not do, just as it failed to be appropriate in the single equation demand for money studies. At the very least one should use a more general lag specification for each asset demand, along the lines described in Chapters 2 and 3. Further refinements of the Brainard-Tobin idea have been few and far between. B.Friedman (1977) has postulated a slightly more complicated partial adjustment mechanism which recognises that it is less costly to channel new saving flows into asset holdings than to re-arrange existing asset holdings. The model assumes that the economic agent allocates new savings so as to maintain the long-run desired allocations of wealth between different assets (i.e. according to long-run asset demands). That is, the following partial adjustment scheme is assumed:

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$$\begin{aligned}
 (A_{it} - A_{it-1}) = & \alpha_{i1}[(A_{1t}^*/W_t)W_{t-1} - A_{1t-1}] + \\
 & \alpha_{i2}[(A_{2t}^*/W_t)W_{t-1} - A_{2t-1}] + \dots + \alpha_{ii}[(A_{it}^*/W_t)W_{t-1} - A_{it-1}] \\
 & + \dots + \alpha_{in}[(A_{nt}^*/W_t)W_{t-1} - A_{nt-1}] + (A_{it}^*/W_t)(W_t - W_{t-1})
 \end{aligned}
 \tag{4.35}$$

However, whilst this model at least allows for a growing portfolio, in common with our alternative buffer-stock model examined in section three, it is still based on an ad hoc partial adjustment hypothesis.

In this section we explore the possibility of generalising our model from section three to a multi-asset framework, and compare it with the above models. Although we do not actually test the model at the empirical level, we shall examine some of the problems which may arise in this context.

### 4.4.2. A Multi-Asset 'Buffering' Model.

We pointed out when assessing the simple Cuthbertson-Taylor model that a narrow two-asset framework may not adequately characterise the buffering mechanism, if there are variable costs of adjustment across a multi-asset portfolio. This may be illustrated with the following simple example. As we have seen, the role of the financial buffer is that of preventing costly temporary adjustments in illiquid assets in the portfolio. In the simple Cuthbertson-Taylor model an unanticipated temporary shock causes a temporary change in current money holdings (see equation 4.20), which is slowly adjusted via the lagged  $m_{t-1}$  term in (4.20). Thus, the 'buffering mechanism' works through the lagged

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dependent variable once the initial disturbance is reversed. At the time when the unanticipated shock hits the portfolio, money holdings (and pari passu bond holdings) adjust. In a multi-asset portfolio, the story will be slightly different. Following, say, an unexpected temporary increase in income, money holdings will increase, but this initial increase in transactions balances will probably be accommodated via a decrease in near-money assets, without affecting the holdings of other illiquid assets. Thus, in this scenario, money and near-money together perform a buffer action which insulates the illiquid part of the portfolio from transitory shocks. In fact it becomes apparent from this that a true buffer model must consist of more than two assets. In the simple money-bonds world the Cuthbertson-Taylor costs-of-adjustment model merely provides a forward-looking adjustment model, but the scope for insulating bond holdings from temporary shocks is non-existent, especially given that wealth is kept constant. Thus, in the Cuthbertson-Taylor model what happens to money holdings is the reverse of what happens to bond holdings, and money is not really a 'buffer' at all. The analogy with production and inventories made by Cuthbertson and Taylor in their work is also misleading. The reason that inventories in a production-sales model can act as buffers is that there is a stock-flow interaction via the flow of sales and production: inventories are not just about shifting fixed stocks of goods between different warehouses!

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We have already tried to remedy this deficiency in part by allowing for saving in the model section three. However, as we have just seen, this is only part of the story. It is arguable that a more stringent test of the buffer-stock money hypothesis will examine the behaviour of a number of financial assets with regard to innovations in the determinants of asset demands. Single-equation studies only offer a partial picture of the buffering mechanism.

To examine the properties of a multi-asset model, we concentrate on the simplest possible case, namely a three-asset model which in addition to money,  $M$ , incorporates a near-money asset,  $N$ , and bonds,  $B$ . The basic idea is again that of building an intertemporal costs-of-adjustment model, along the lines of the models examined so far in this chapter. We assume once again that money holdings are costless to adjust, whilst near-money and bond holdings are costly to adjust, with adjustments of the former type of asset carrying a lower cost than adjustments of the latter.

The cost function adopted is the following:

$$C = E_{t-1} \sum_{j=0}^{\infty} \delta^j (a_0(M_{t+j} - M_{t+j}^*)^2 + a_1(B_{t+j} - B_{t+j-1})^2 + a_2(N_{t+j} - N_{t+j-1})^2 + a_3(N_{t+j} - N_{t+j}^*)^2) \quad (4.36)$$

$a_i > 0, a_2 < a_1$

Note that we are, unlike (4.29), omitting possible negative costs due to movements in the same direction of desired and actual holdings of illiquid assets. This is done in order to

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simplify the model's structure. Furthermore, now that we have three assets, to ensure a return to long-run equilibrium in a steady state, we have to penalise not only deviations of  $M$  from  $M^*$ , but also deviations of  $N$  from  $N^*$ .<sup>18</sup>

As in section three, we shall utilize the following identities:

$$W_t \equiv M_t + N_t + B_t \quad (4.37)$$

$$W_t \equiv W_{t-1} + S_t \quad (4.38)$$

$$W_t \equiv M_{t-1} + N_{t-1} + B_{t-1} + S_t \quad (4.39)$$

Finding the solution to this dynamic optimisation problem given a sequence of exogenous saving decisions  $\{S_{t+i}\}_{i=0}^{\infty}$  involves finding a sequence of  $\{M_{t+i}\}_{i=0}^{\infty}$  and  $\{N_{t+i}\}_{i=0}^{\infty}$  such that  $C$  is minimised. Once again we find the appropriate set of Euler equations which satisfy  $\partial C / \partial M_{t+j} = 0$  and  $\partial C / \partial N_{t+j} = 0$  (for  $j = 0, 1, 2, \dots$ ). We then obtain the following equations:

$$\begin{aligned} M_{t+j+1} - ((a_0/6a_1) + 1 + (1/6))M_{t+j} + (1/6)M_{t+j-1} = \\ -(a_0/6a_1)M_{t+j}^* - (1/6)S_{t+j} + (1/6)(N_{t+j} - N_{t+j-1}) - (N_{t+j+1} - N_{t+j}) \\ + S_{t+j+1} \end{aligned} \quad (4.40)$$

$$\begin{aligned} N_{t+j+1} - ((a_3/6(a_2 + a_1)) + 1 + (1/6))N_{t+j} + (1/6)N_{t+j-1} = \\ -(a_3/6(a_2 + a_1))N_{t+j}^* - (a_1/6(a_2 + a_1))S_{t+j} + \\ (a_1/6(a_2 + a_1))(M_{t+j} - M_{t+j-1}) - (a_1/(a_2 + a_1))(M_{t+j+1} - M_{t+j}) + \\ + (a_1/(a_2 + a_1))S_{t+j+1} \end{aligned} \quad (4.41)$$

The method of solution which may be applied to (4.40) and (4.41) is the same analysed above in the case of the other

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forward-looking buffer-stock models, except that now we have a pair of simultaneous second-order difference equations.

It should be apparent from the form of the solution in the case of the variants of the forward-looking model presented in section three that if we applied the same method here of factorising the lag polynomial, and eliminating the unstable root in the case of each equation, we would obtain equations for  $M$  and  $N$  in terms of the current and future values of their desired holdings,  $N^*$  and  $M^*$ , as well as the future evolution of saving and  $N$  and  $M$  themselves. This may be illustrated for the demand for money equation for the simple case where  $\delta = 1$ :

$$M_t = \lambda_1 M_{t-1} + \lambda_1 (a_0/a_1) \sum_{i=0}^{\infty} (\lambda_1)^i M_{t+i}^* - \lambda_1 \sum_{i=0}^{\infty} (\lambda_1)^i \Delta S_{t+i} - \lambda_1 \sum_{i=0}^{\infty} (\lambda_1)^i \Delta N_{t+i-1} + \lambda_1 \sum_{i=0}^{\infty} (\lambda_1)^i \Delta N_{t+i} \quad (4.42)$$

where  $\lambda_1$  is the stable root. A similar equation could be found for  $N_t$ . This, however, is not a 'proper' solution to the second-order differential equation system, as the state variables are not purely in terms of the exogenous 'forcing' variables,  $M^*$ ,  $N^*$ , and  $S$ .

This leads us to the first problem which one encounters in estimating such a multi-asset model. Just as in the single equation context one may estimate the Euler equation directly or its solution (as we have done in this chapter), here we may either estimate a forward-looking equation such as (4.42), or the 'proper' solution to the problem. Secondly, if we choose to estimate the full solution, inevitably an appropriate test of the

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model will involve the statistical testing of cross-equation and within-equation restrictions. These restrictions may turn out to be rather complicated in the light of the complex solution to our single equation model with saving which we examined in section three. The third problem which will emerge in an empirical test of this model will be the choice of appropriate aggregates to enter the model. A three-asset model may still be too simple to capture the disequilibrium adjustment process. On the other hand, introducing more assets into the portfolio will complicate the model even more.

Because of these problems (and likely additional estimation problems), we have chosen not to undertake an empirical test of this multi-asset model. Instead we have chosen to examine the dynamic properties of the buffering process following disturbances to the 'forcing' variables. As mentioned above, this requires a proper mathematical solution of the system of second-order difference equations (4.40) and (4.41).

Here again, the analytical solution is likely to be rather complicated, and not very illuminating. On the other hand, given the nature of the parameters involved, we are likely to be able to assign some 'plausible' values to them a priori, which will enable us to obtain a numerical simulation of the model and give us some indication of its dynamic properties.

To simulate the model, we used the PRISM package developed at Queen Mary College, London (see Gaines et al., 1987). The



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dynamic system is set out as follows:

$$B \begin{bmatrix} M_t^* \\ S_{t+1} \\ N_t^* \\ M_t \\ N_t \\ X_t \\ Y_t \end{bmatrix} = A \begin{bmatrix} M_{t-1}^* \\ S_t \\ N_{t-1}^* \\ M_{t-1} \\ N_{t-1} \\ X_{t-1} \\ Y_{t-1} \end{bmatrix} \quad (4.42)$$

where we define  $X_t \equiv M_{t+1}$  and  $Y_t \equiv N_{t+1}$ . It is necessary to define these two new variables, because, as we saw in section three in the case of the single-asset model, we have a model here with two unstable roots, so that we need two non-predetermined ('free' or 'jump') variables to make the model stable. It is assumed therefore that whilst the economic agent is not free to determine his current holdings of N and M, he can certainly determine the values of  $M_{t+1}$  and  $N_{t+1}$ , and therefore these two latter variables are non-predetermined. In addition to  $N_t$  and  $M_t$  which are endogenous and predetermined, we have three exogenous series  $M_t^*$ ,  $N_t^*$ , and  $S_t$ , which are assumed to follow particular time paths so that we may analyse exogenous disturbances to these series. The matrices A and B are the following:

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$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ A_{41} & -1 & 0 & -A_{42} & -A_{43} & 1 & 1 \\ 0 & -A_{52} & A_{53} & -A_{54} & -A_{55} & A_{56} & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} B_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & B_{33} & 0 & 0 & 0 & 0 \\ 0 & -B_{42} & 0 & -B_{44} & -B_{45} & 0 & 0 \\ 0 & -B_{52} & 0 & -B_{54} & -B_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where the  $A_{ij}$  and  $B_{ij}$  are listed in Table 4.5, and that we have defined these matrix elements such that  $A_{ij}, B_{ij} > 0$ . Note that the parameters  $\mu_1, \mu_2$ , and  $\mu_3$  relate to the disturbances which we wish to examine. Thus, when  $\mu_i = 1$ , the variable under consideration is subjected to a permanent disturbance. On the other hand, when we wish to consider a temporary shock, we have  $\mu_i < 1$ .

We analyse the adjustment of the representative agent's portfolio following various disturbances. We also examine the effects of choosing different values for the discount rate and

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the relative weights in the cost function. The parameter values chosen for our 'base' run are the following:

$$a_0 = 1 \quad a_1 = 20 \quad a_2 = 10 \quad a_3 = 1 \quad \delta = 0.99$$

so that whilst money is costless to adjust, bond holdings are twice as costly to adjust as near-money holdings, and the costs of adjustment of near-money holdings are in turn ten times as large as the costs of being away from equilibrium.

We first of all subject the model to some unanticipated permanent shocks. The disturbances we consider are the following:

(1) A unit reduction of  $B^*$  matched by a unit increase in  $M^*$ . This portfolio shift from desired bond holdings to desired money holdings may be due to, say, an increase in income (provided all money transactions balances are assumed to eventually come from bonds and not near-money), or a reduction in the yields on bonds and near-money such that  $N^*$  remains unchanged but  $B^*$  falls and  $M^*$  rises by the same amount.

(2) A unit reduction of  $B^*$  matched by a unit increase in  $N^*$ . This may be achieved by similar means to the shock described in (1) above.

(3) A unit increase in  $M^*$  matched by a unit decrease in  $N^*$ , again achieved by similar means to (1) above.

(4) A unit increase in  $S$  directed entirely towards increasing bond holdings, i.e. matched by an increase in  $B^*$  in every period.

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TABLE 4.5

$A_{41} = (a_0/\delta a_1)$	$B_{11} = \mu_1$
$A_{44} = (a_0/\delta a_1) + (1/\delta) + 1$	$B_{22} = \mu_2$
$A_{45} = (1/\delta) + 1$	$B_{33} = \mu_3$
$A_{52} = a_1/(a_2 + a_1)$	$B_{42} = (1/\delta)$
$A_{53} = a_3/(a_2 + a_1)\delta$	$B_{44} = (1/\delta)$
$A_{54} = (a_1/(a_2 + a_1))((1/\delta) + 1)$	$B_{45} = (1/\delta)$
$A_{55} = a_3/(a_2 + a_1)\delta + (1/\delta) + 1$	$B_{52} = a_1/(a_2 + a_1)\delta$
$A_{56} = a_1/(a_2 + a_1)$	$B_{54} = a_1/(a_2 + a_1)\delta$
	$B_{55} = (1/\delta)$

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We examine the trajectories of  $M$ ,  $M^*$ ,  $N$ ,  $N^*$ ,  $B$  and  $B^*$  following the above shocks. Figures 4.3-4.6 show us the simulations of the model following each of the above four shocks in turn. Figures 4.7-4.10 show us the trajectories of these variables under the four shocks where we have lowered the value of the parameter  $a_2$  to 5. This has the effect of increasing the cost of adjusting bonds relative to the cost of adjusting near-money. We then lowered the value of the discount factor  $\delta$  to 0.5, and the resulting trajectories following the four shocks are illustrated in Figures 4.11-4.14. Lastly, we reverted to our set of 'base' parameter values, and examined the effect of making the four shocks temporary, with a decay parameter  $\mu_i$  equal to 0.5. The trajectories for this experiment are illustrated in Figures 4.15-4.18.

Let us now examine the nature of the adjustment process as illustrated by these various experiments. First of all, let us examine the simulations with our original parameter values listed above. Figures 4.3 and 4.4 show the buffering mechanism in action when the individual has the ultimate aim of switching funds from bonds to money and near-money respectively. The main thing to note here is that, as in the Brainard-Tobin (1968) model, all the asset adjustments are interdependent. This model in fact represents a forward-looking version of the Brainard-Tobin model.

The second thing to note is that the adjustment of bonds is buffered via adjustments in  $M$  and  $N$ , with money acting as the

FIGURE 4.3.

PERMANENT RISE IN  $M^*$  AND FALL IN  $B^*$

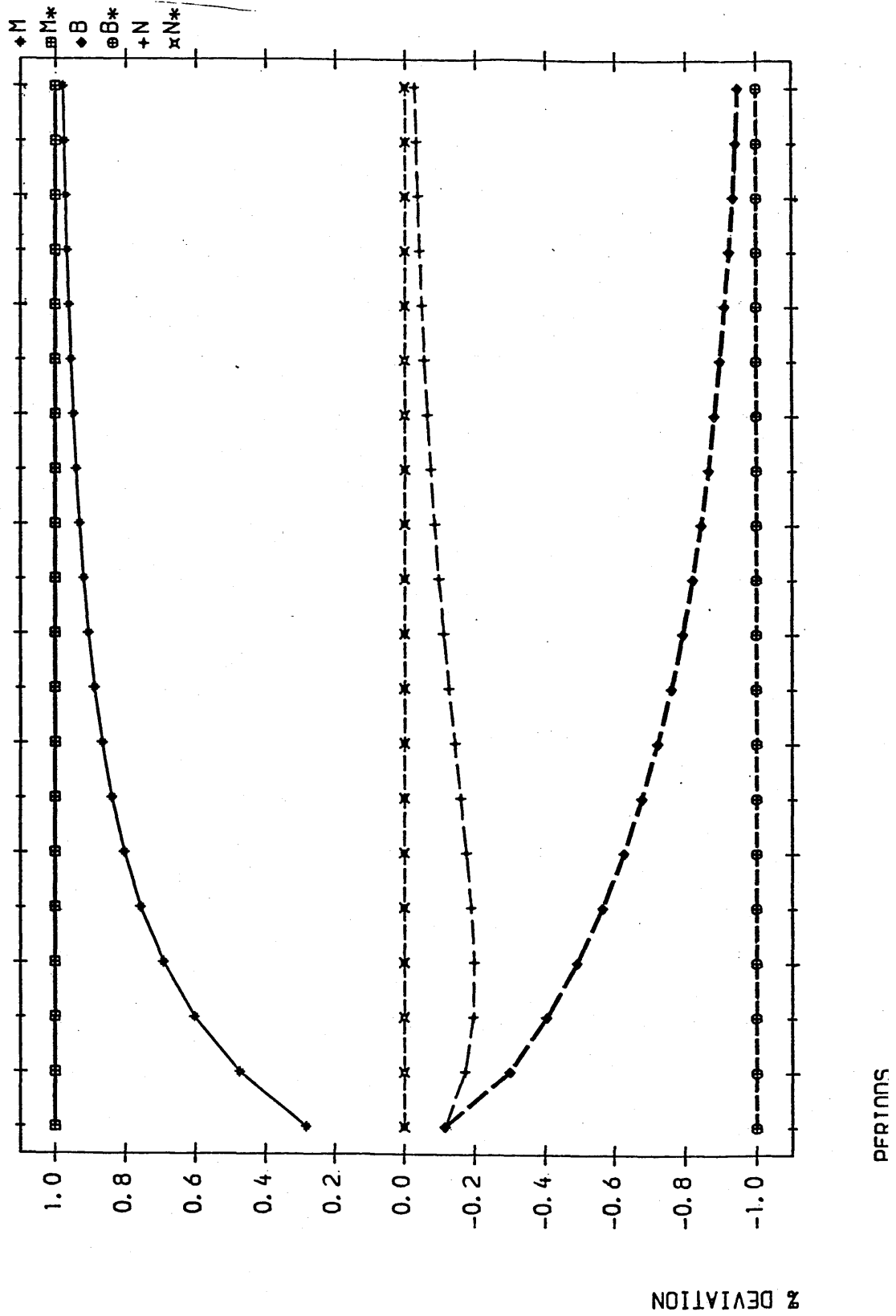


FIGURE 4.4:

PERMANENT RISE IN  $N^*$  AND FALL IN  $B^*$

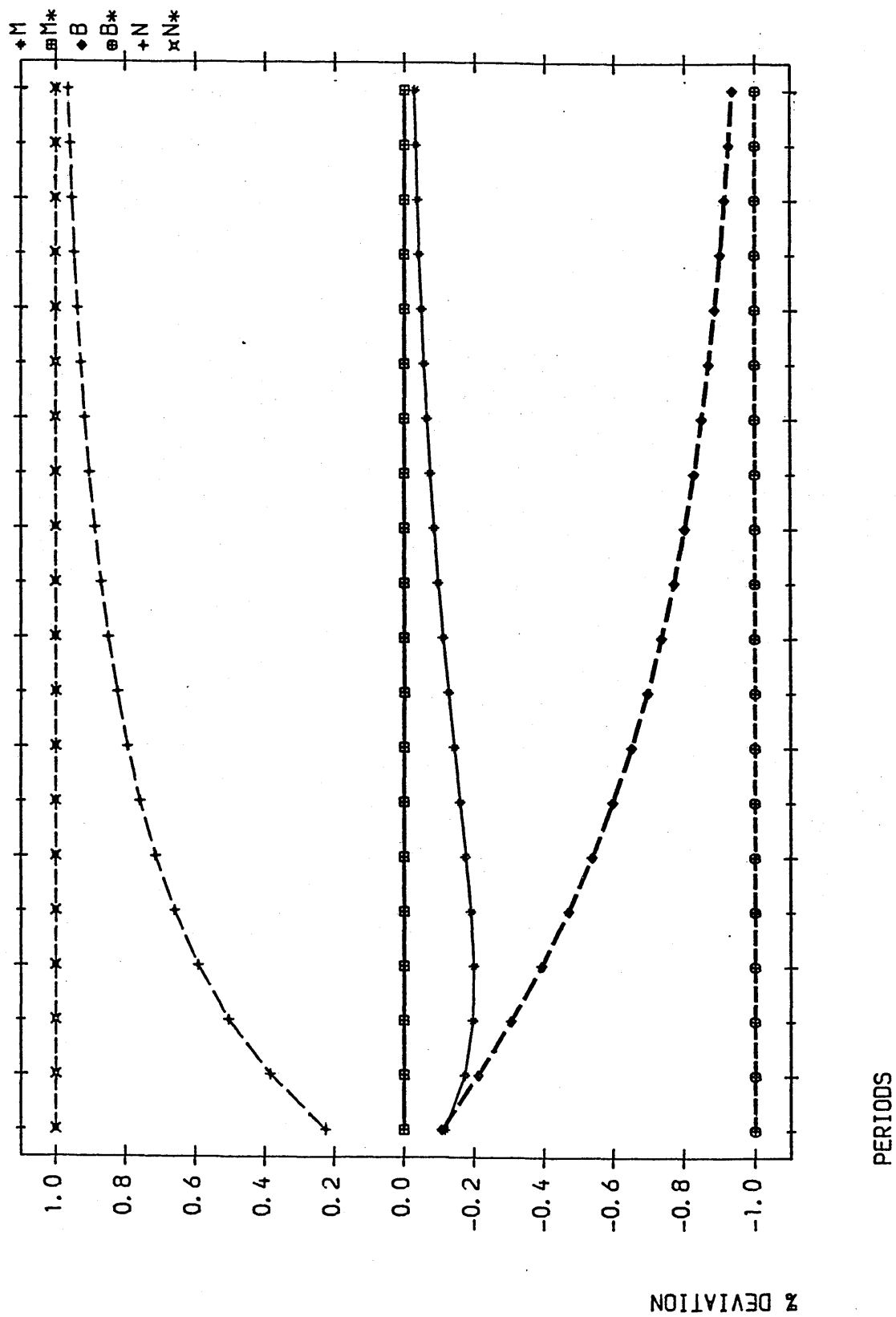


FIGURE 4.5.  
PERMANENT RISE IN M\* AND FALL IN N\*

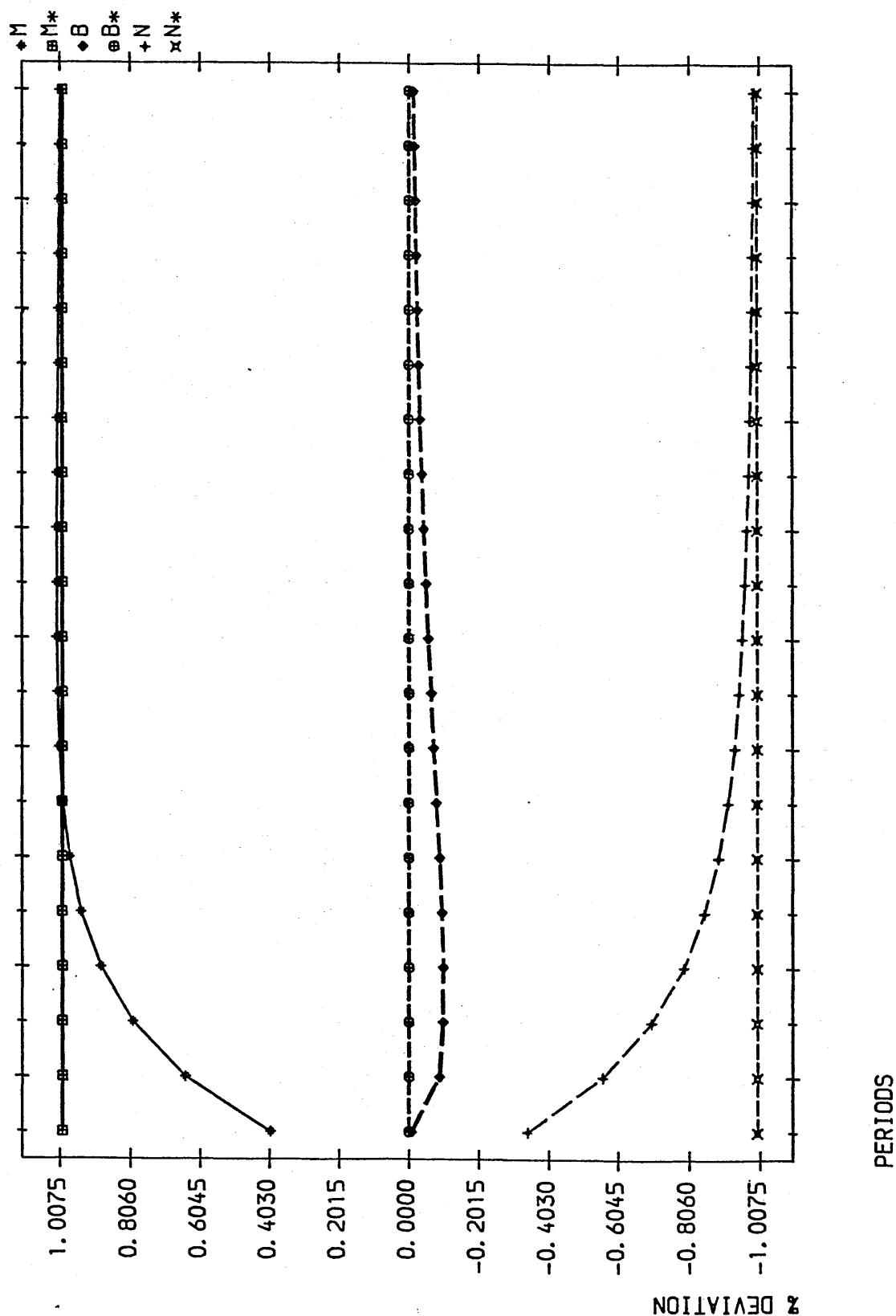




FIGURE 4. 6:

PERMANENT RISE IN S MATCHED BY RISE IN B\*

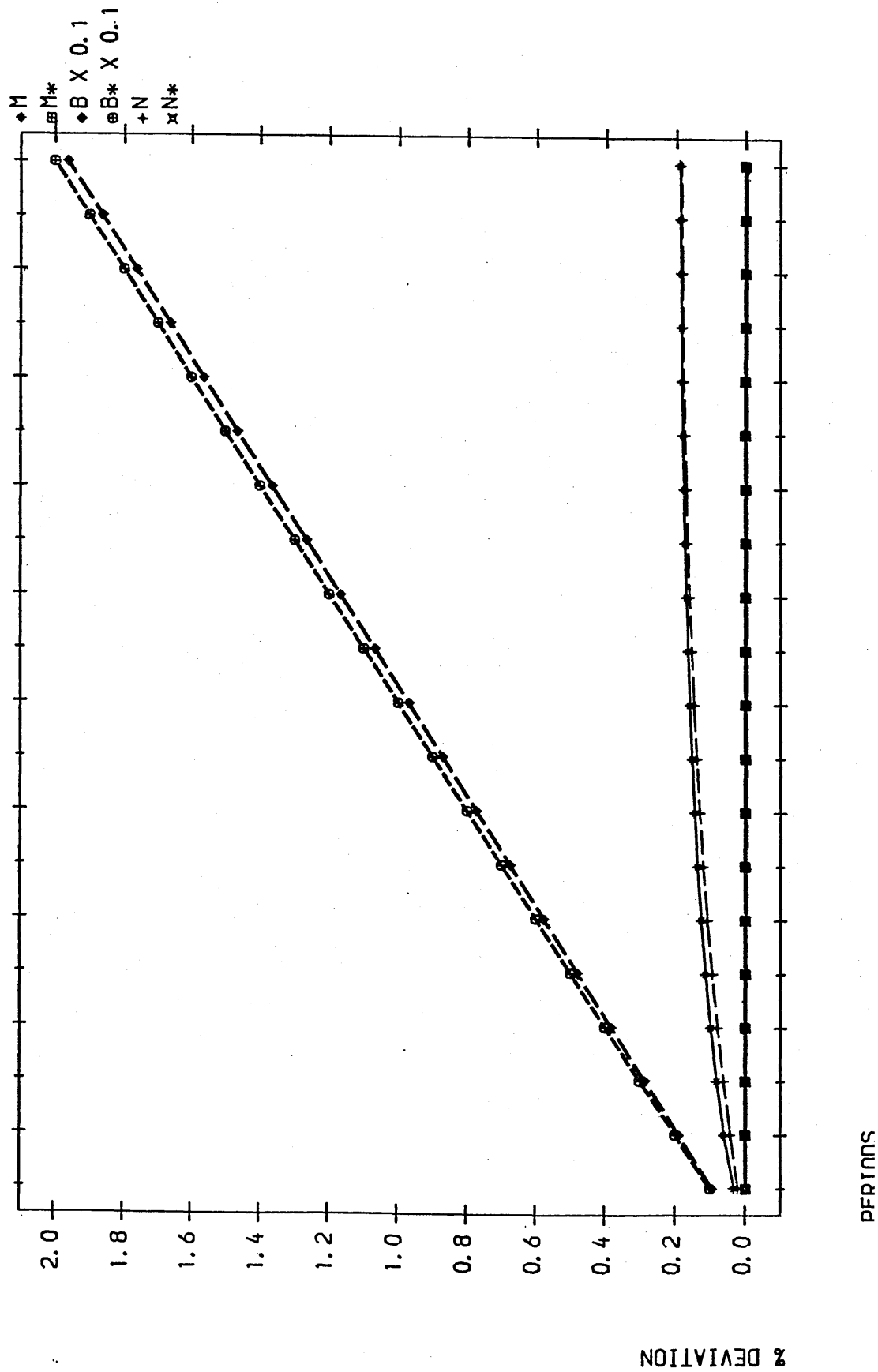


FIGURE 4.7.

PERMANENT RISE IN  $M^*$  AND FALL IN  $B^*$  (WITH  $A_2 = 5$ )

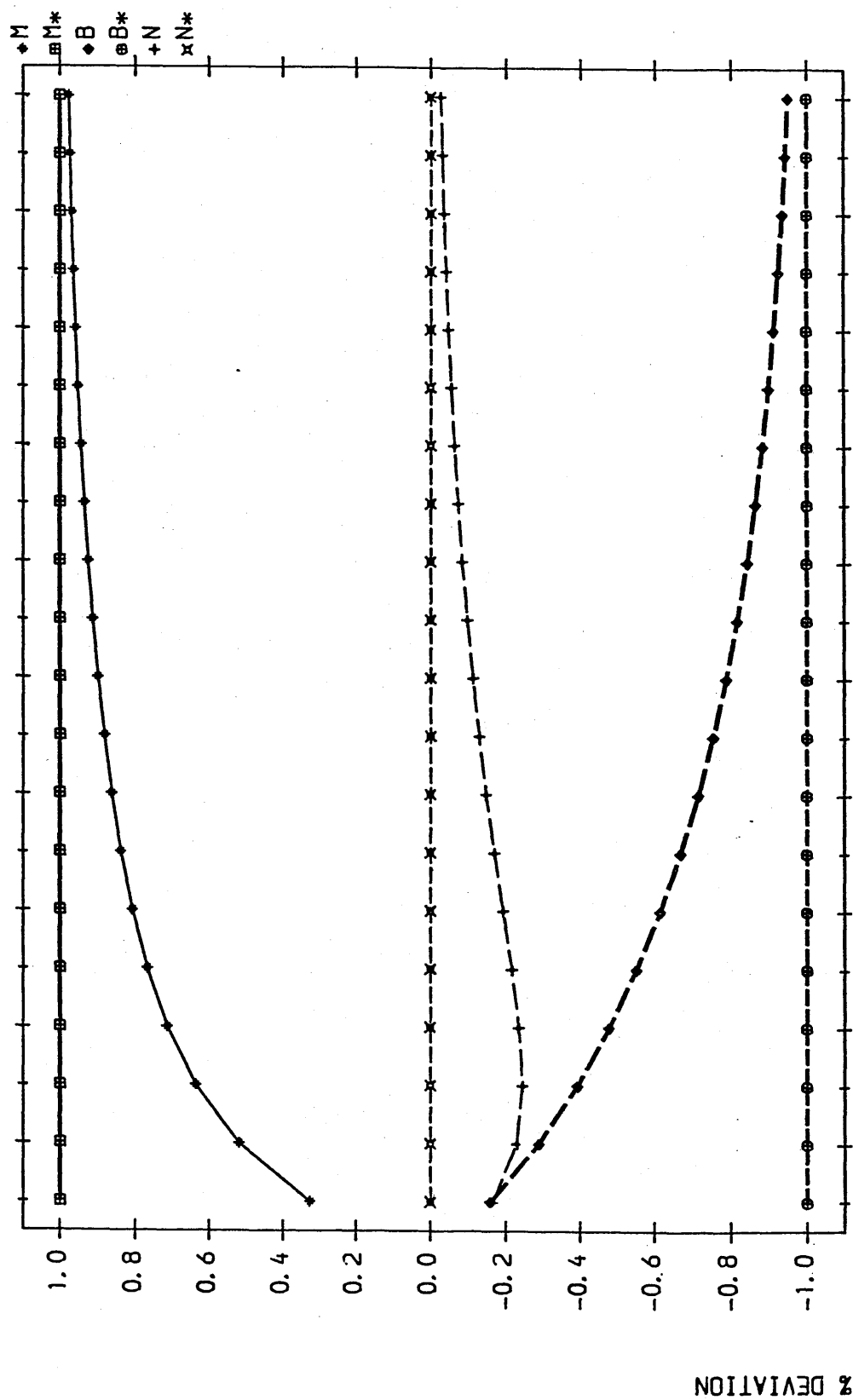


FIGURE 4.8.

PERMANENT RISE IN  $N^*$  AND FALL IN  $B^*$  (WITH  $A_2 = 5$ )

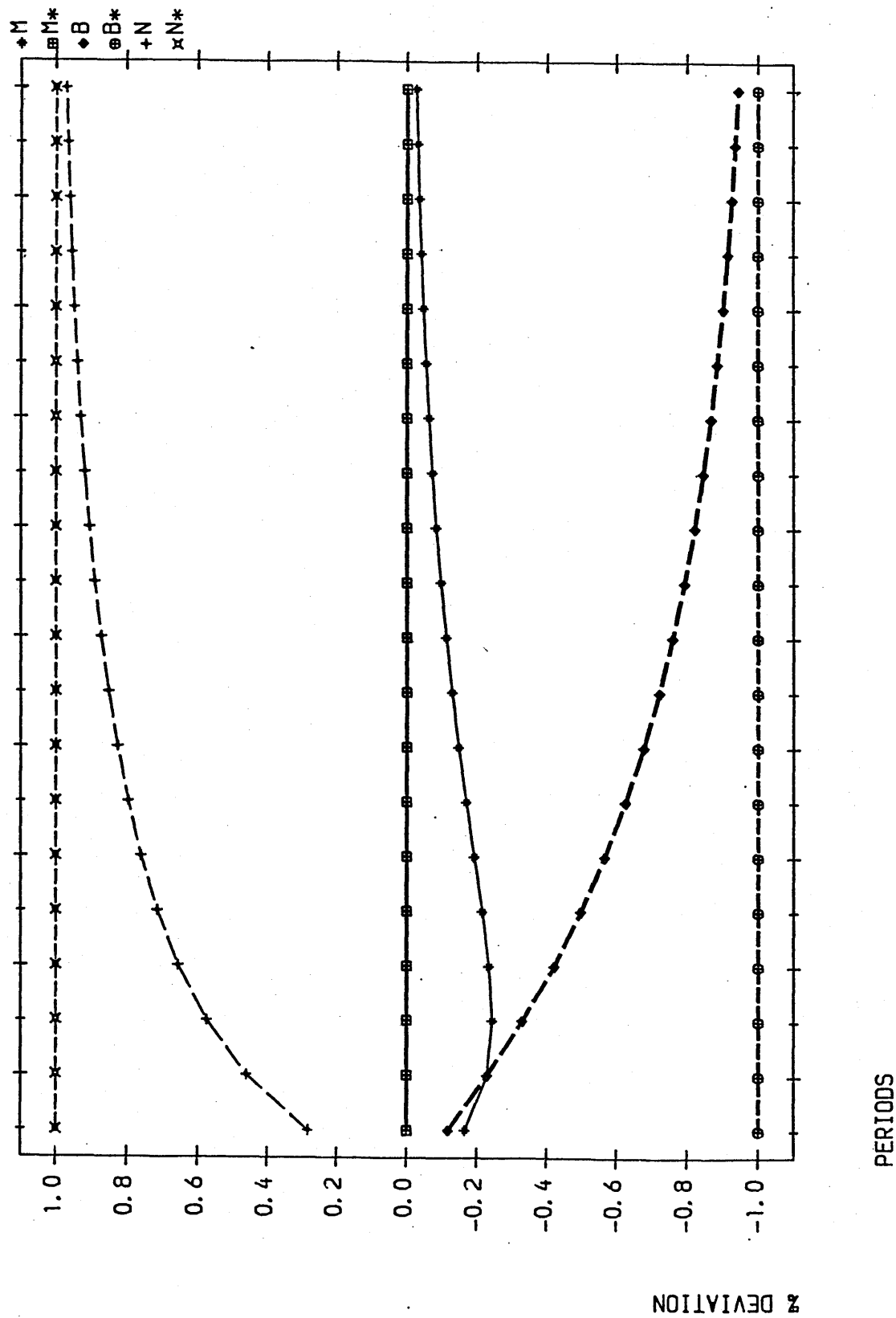


FIGURE 4.9.

PERMANENT RISE IN  $M^*$  AND FALL IN  $N^*$  (WITH  $A_2 = 5$ )

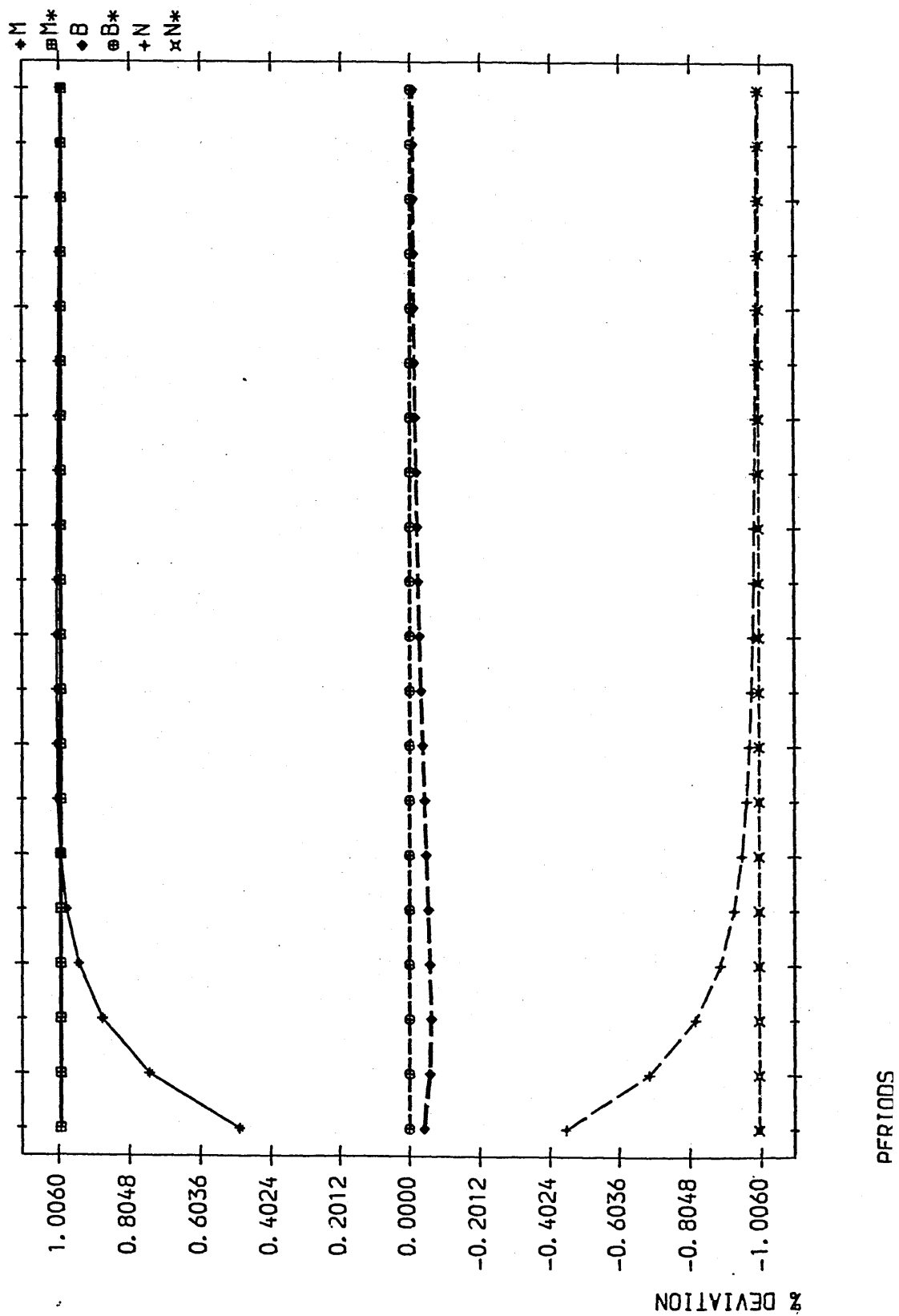


FIGURE 4.10:

PERMANENT RISE IN S MATCHED BY RISE IN B\* (WITH  $\lambda_2 = 5$ )

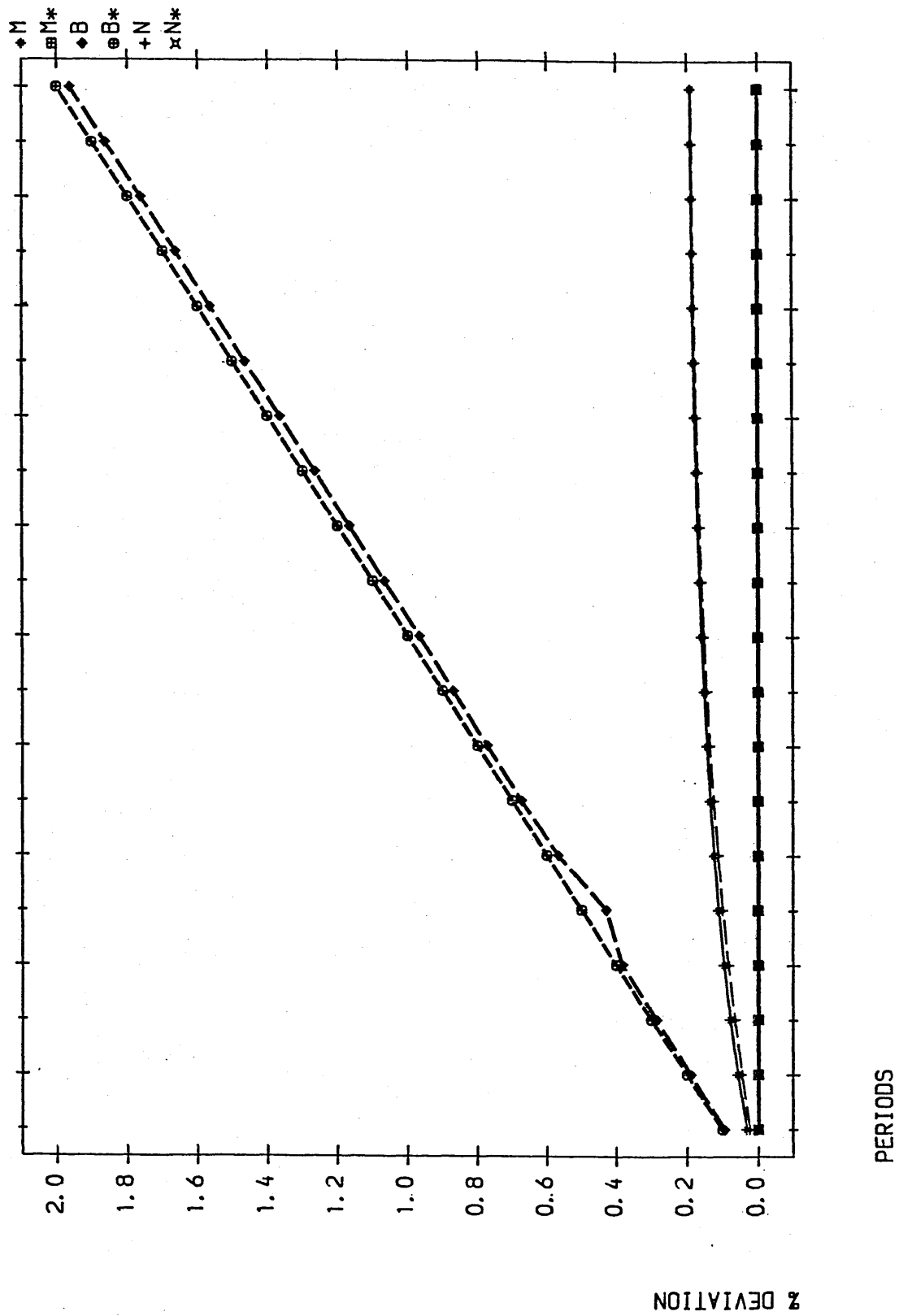


FIGURE 4.11:

PERMANENT RISE IN  $M^*$  AND FALL IN  $B^*$  ( $\Delta T = 0.5$ )

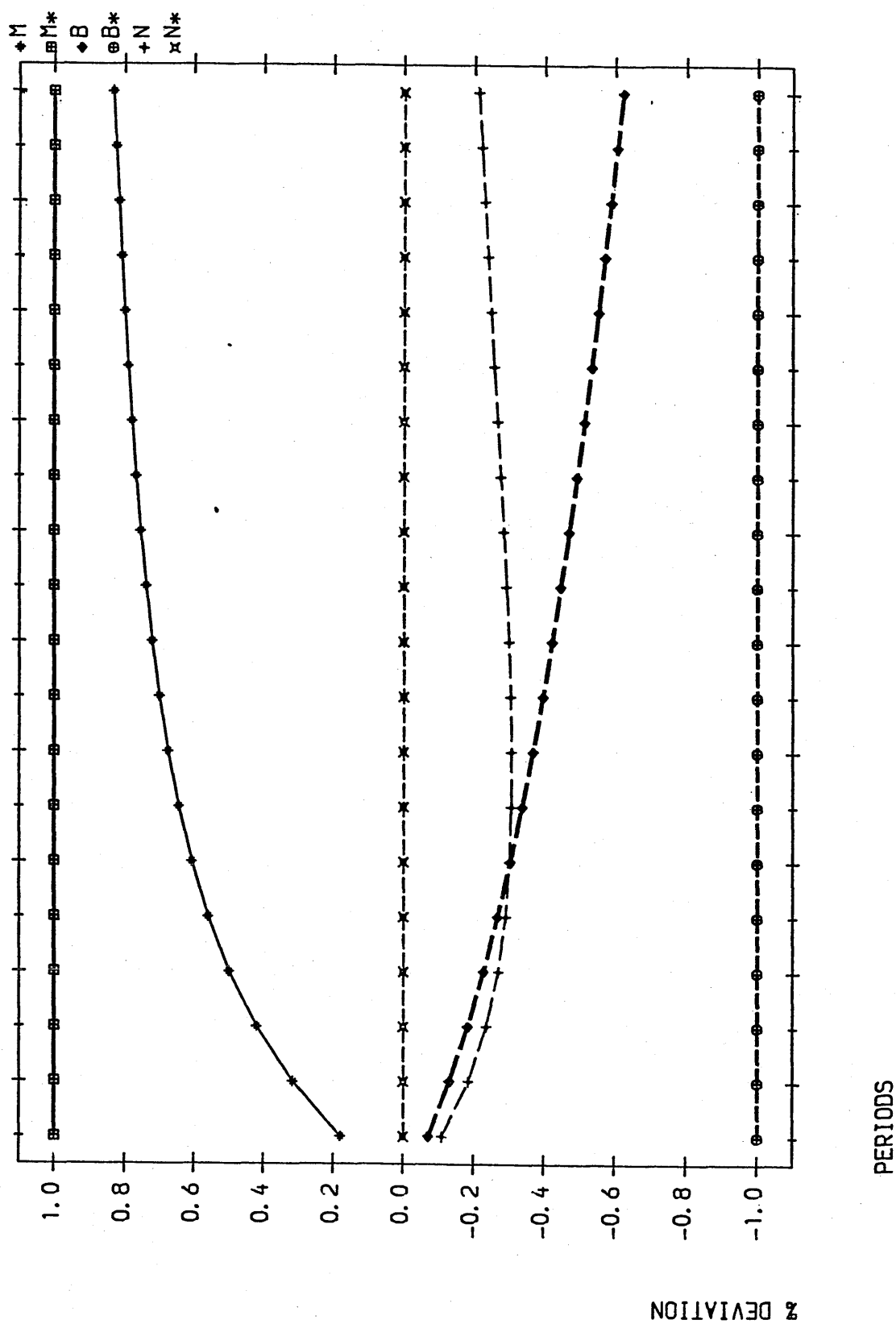


FIGURE 4.12.

PERMANENT RISE IN  $N^*$  AND FALL IN  $B^*$  ( $\Delta = 0.5$ )

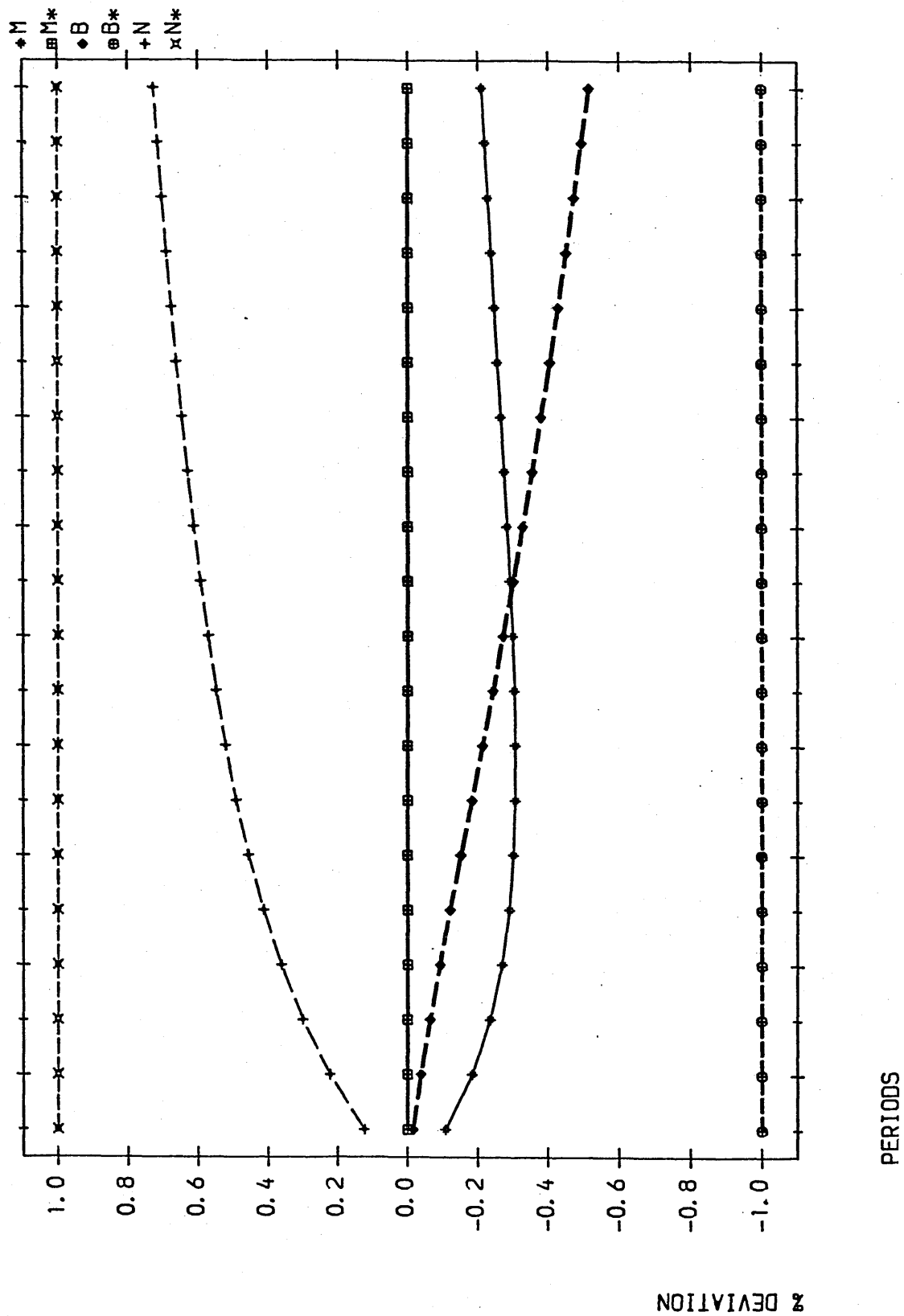


FIGURE 4.13:

PERMANENT RISE IN  $M^*$  AND FALL IN  $N^*$  ( $\Delta = 0.5$ )

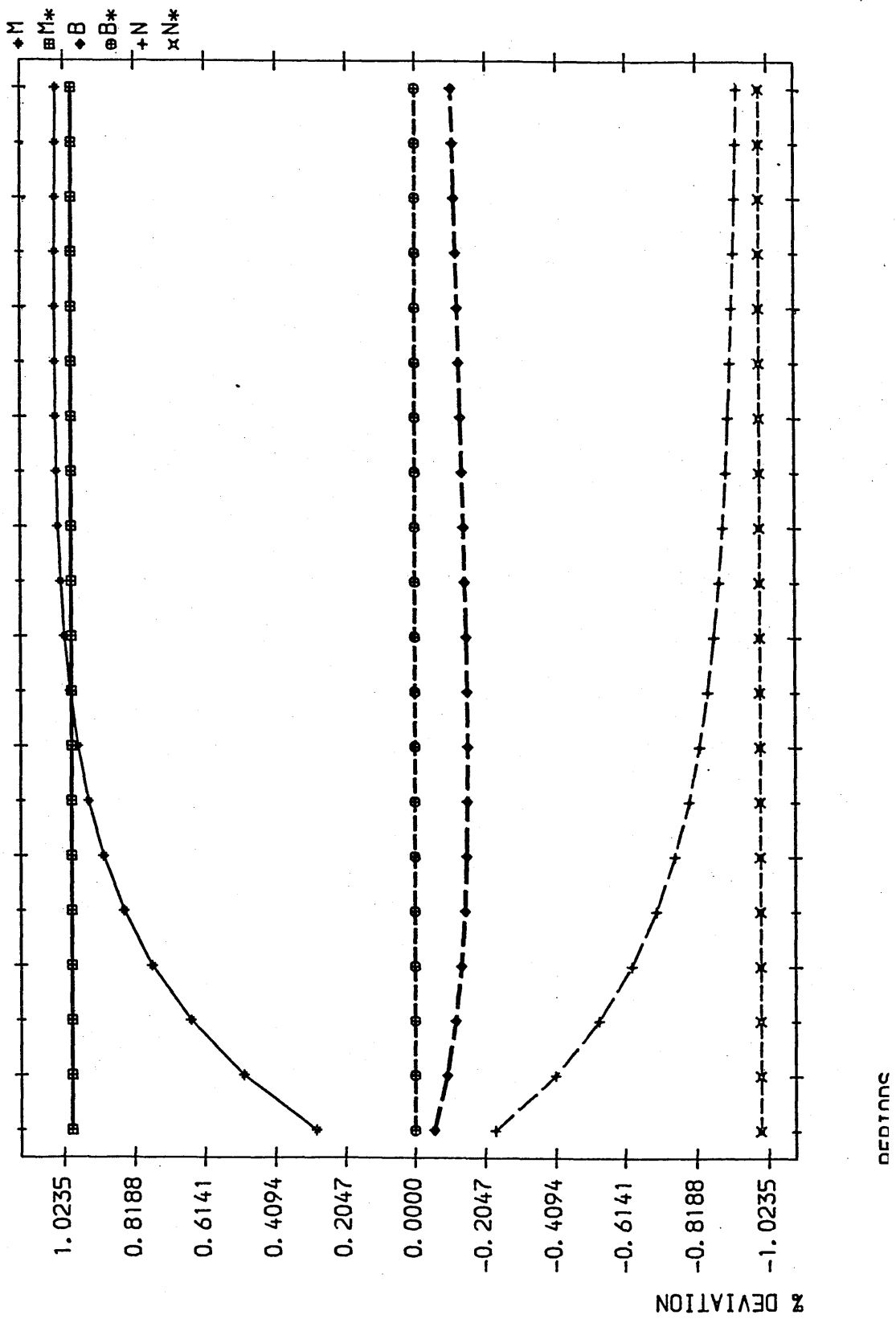




FIGURE 4. 14.

PERMANENT RISE IN S MATCHED BY RISE IN B\* ( $\Delta = 0.5$ )

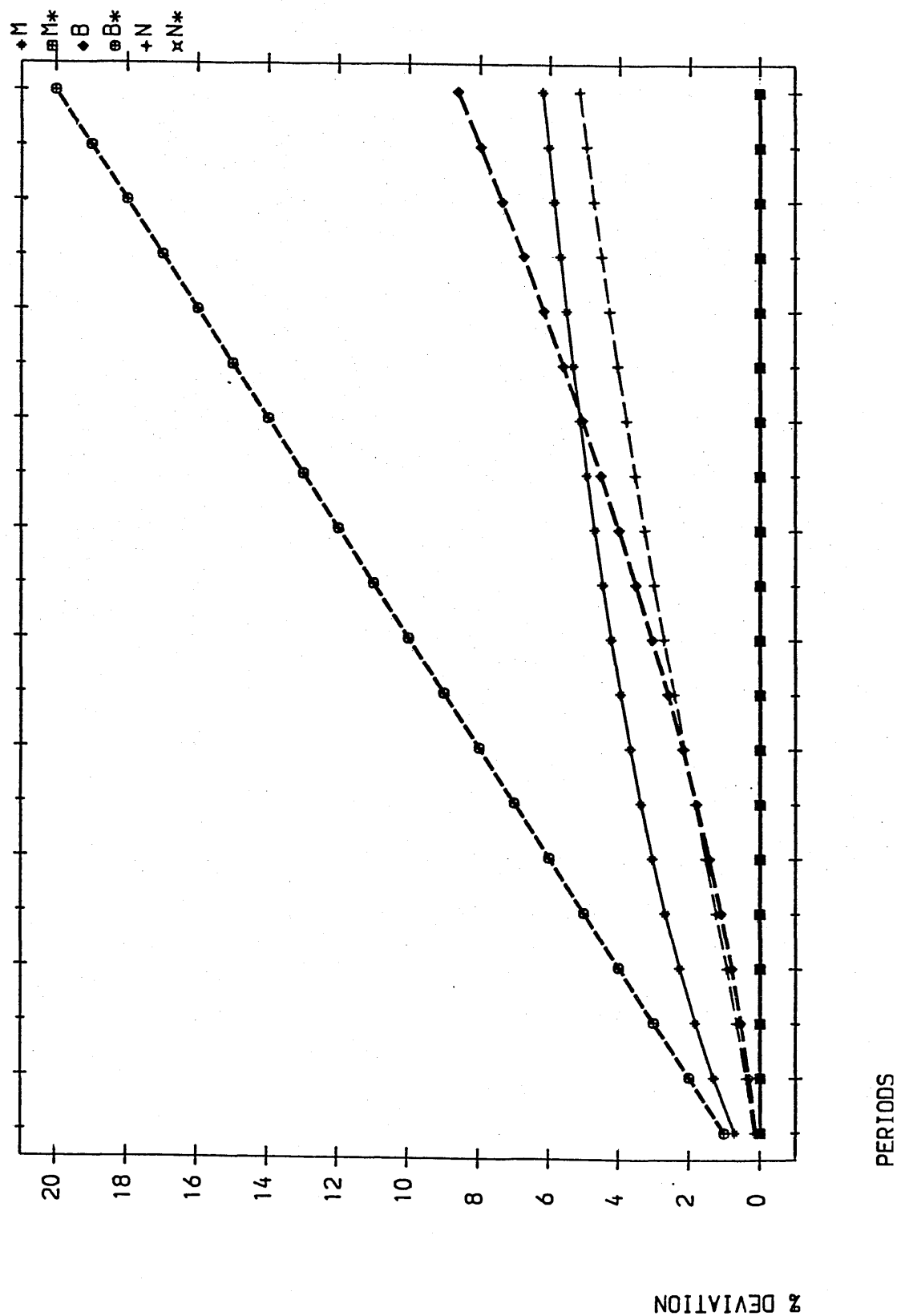
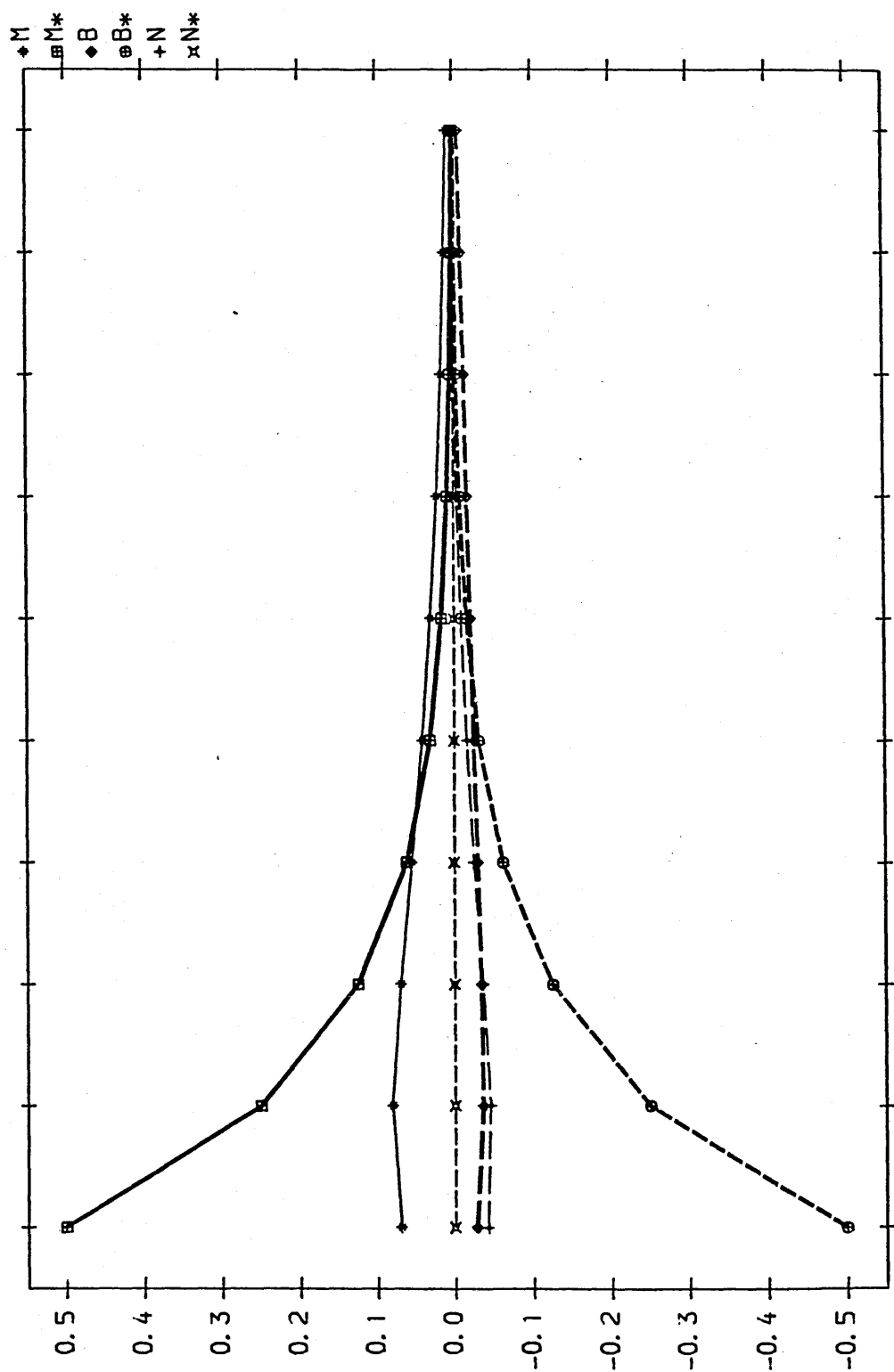


FIGURE 4.15

TEMPORARY DECREASE IN  $B^*$  MATCHED BY A RISE IN  $M^*$



z DEVIATION

FIGURE 4.16

TEMPORARY FALL IN  $B^*$  MATCHED BY A RISE IN  $N^*$

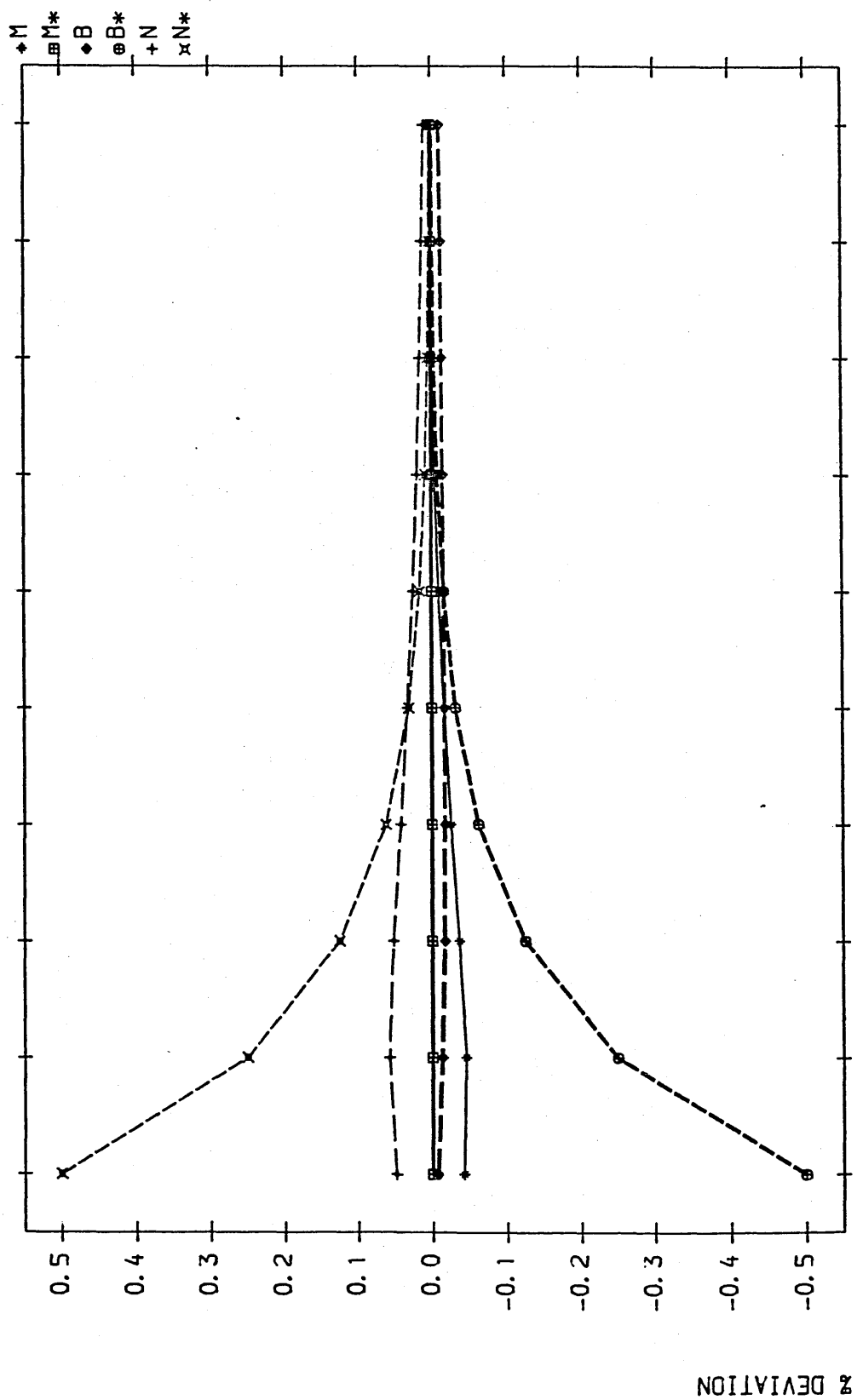


FIGURE 4.17.

TEMPORARY FALL IN  $N^*$  MATCHED BY A RISE IN  $M^*$

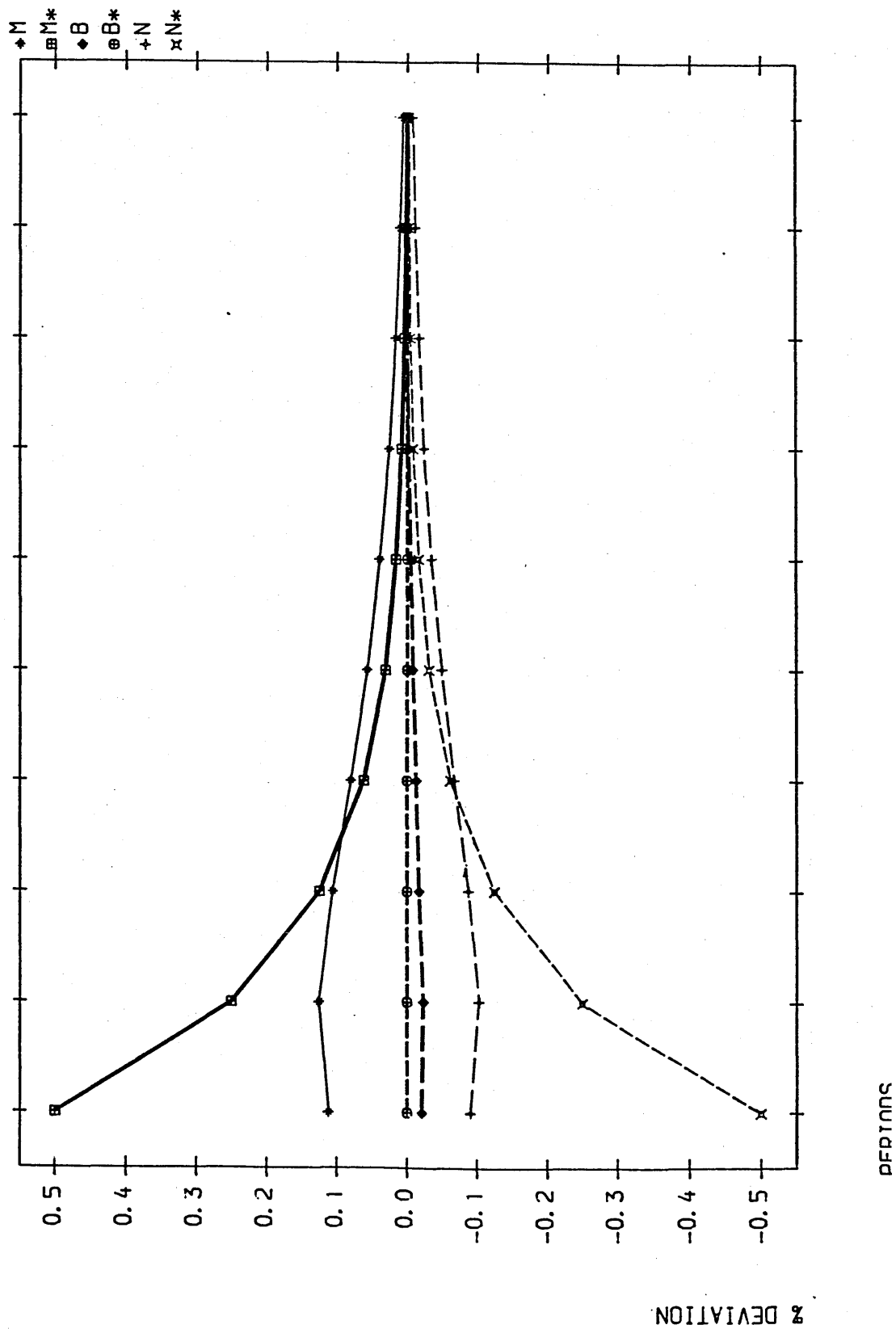
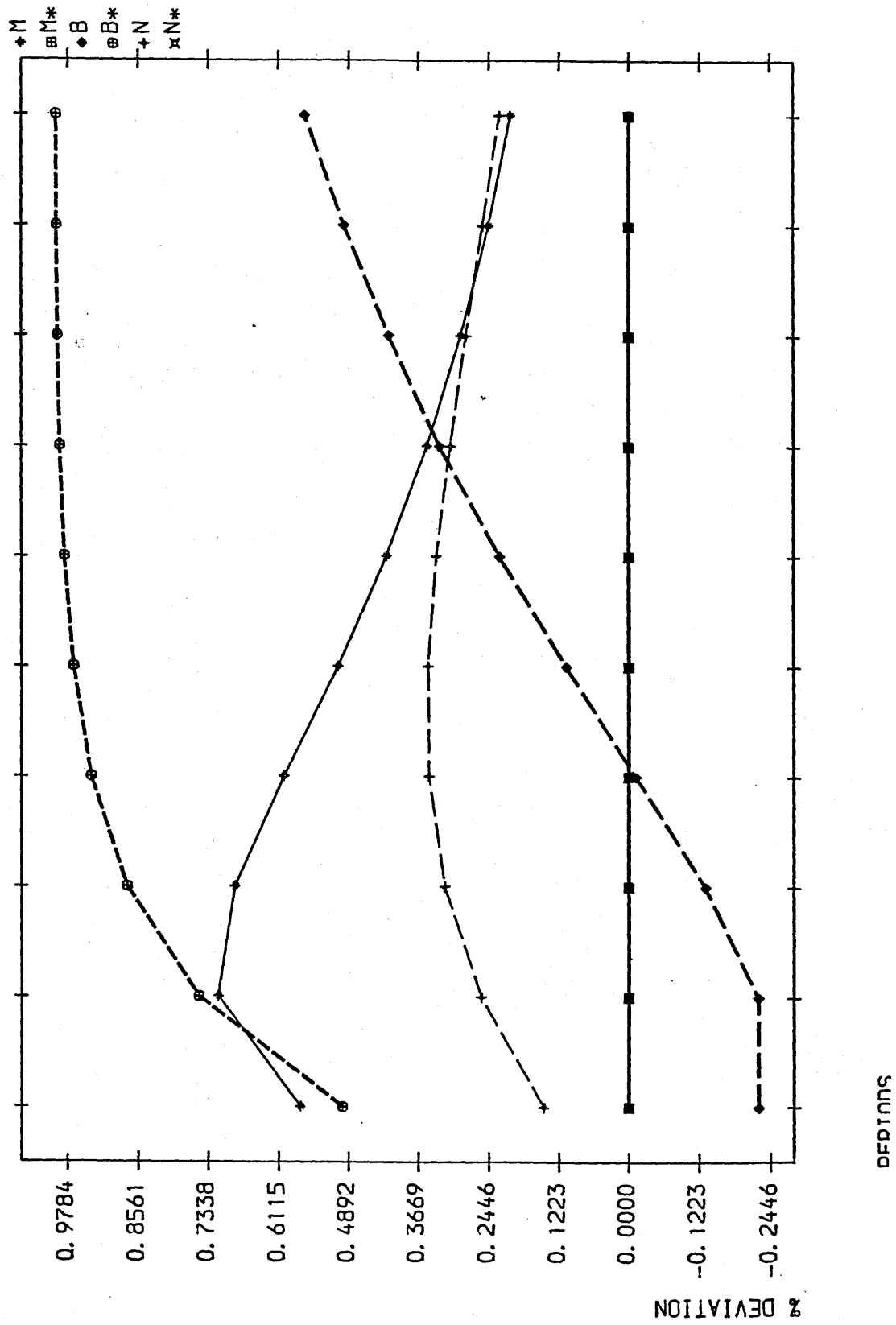


FIGURE 4. 18:

TEMPORARY RISE IN S MATCHED BY A RISE IN B\*



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primary buffer asset.

The third point to note is that even though money is costless to adjust, near-money plays an important part in the buffering mechanism, and thus any proper description of the buffer-stock process must take into account assets other than money. This is even more apparent from Figure 4.5, where in the face of a desired switch of wealth from N to M the adjustment carried out by leaving bond holdings nearly unaffected, which is clearly not the case in a simple two-asset model such as the one examined by Cuthbertson and Taylor. From figure 4.6 we can see that the implications of the costs-of-adjustment hypothesis for a growing portfolio are even more startling. In this example the additions to wealth are matched by a desire to increase bond holdings. However, given the costs of adjusting B, these funds are channelled initially into money and near-money, with the former taking more of the strain of adjustment in the first few periods. Eventually, though, because the costs of adjustment are such that the individual never quite manages to channel all of the additional funds into bonds, the portfolio remains in 'disequilibrium' even in the short-run. Mathematically, it is obvious why this state of permanent disequilibrium is reached: one of the forcing variables (wealth) has a growing time path, and hence the simple quadratic cost function used in our model cannot restore the system to equilibrium. If we find the solution suggested by this model rather unrealistic, then we must

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reformulate the control problem so as to ensure that portfolio equilibrium is restored. One way of doing this would be to design the cost function so that an integral control term is included in the solution (see Phillips 1954, 1957)<sup>19</sup>. This once again highlights the difficulties which exist with simple buffer-stock models - how can they possibly characterise the adjustment process in an adequate manner in a world where wealth is not constant?

Let us now examine how these results change once we change the values of some of the key parameters. First of all, we lower the costs of adjustment of near-money so that the costs of adjusting bond holdings relative to that of adjusting near-money is correspondingly greater. From Figure 4.7 we can see that this makes the adjustment process easier, in that more funds are channelled from N to M than in Figure 4.3, which reduces the disequilibrium in the money market. In later periods, as the adjustment has to involve reductions in bond holdings, the paths in Figures 4.3 and 4.7 become very similar. Thus, a reduction in the costs of adjustment of N improves the buffering process between M and N. This may also be seen by comparing Figures 4.4 and 4.8, except that the short-run switch of funds from M to N is easier. The reduction in the adjustment cost of near-money will clearly make the adjustment to the third disturbance (a switch from  $N^*$  to  $M^*$ ) more rapid, and this is confirmed by comparing Figures 4.5 and 4.9.

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Turning now to the disturbance to saving, this parameter change does not affect the long-run equilibrium properties discussed above (since we have not altered  $a_1$ , the costs of changing bond holdings this is not surprising). It merely means that  $N$  will take more of the strain of short-run adjustment, as more new savings are channelled into it. This may be seen by comparing the trajectories of  $N$  and  $M$  in figures 4.6 and 4.10.

Turning next to our third variant of the simulated model, where the discount rate ( $\delta$ ) is lowered to 0.5, we would expect this to lengthen the process of adjustment as a lower relative weight is placed on the future. This is indeed the case, as may be seen from Figures 4.11 and 4.12 where after 25 periods, a greater degree of disequilibrium persists following switches from  $B^*$  to  $M^*$  and  $N^*$  respectively. This is even more apparent from Figure 4.14, where we see that new funds are channelled in greater quantities to  $M$  and  $N$ , and where the portfolio is approaching a long-run steady state where a greater degree of disequilibrium persists.

To complete our analysis, we examine the case corresponding to Figures 4.3-3.6 where this time the shocks are of a temporary nature. The main effect of this is to remove the 'permanent disequilibrium' result of Figure 4.6, as saving flows only increase temporarily. In Figure 4.18 we see that whilst funds are initially channelled into  $M$  and  $N$ , these gradually return to their long-run desired levels. The other main effect is of course



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that of leaving B nearly unaffected throughout the simulation periods following the disturbances. This is because the agent anticipates the fact that the initial disturbances slowly decay, and takes this into account when adjusting his portfolio. The case illustrated in Figures 4.15-4.18 is that where the  $\mu_i$  are equal to 0.5. If we were to set an even smaller value, say 0.1, this would illustrate even more clearly the nature of the buffering mechanism when the individual's portfolio is hit by temporary unexpected shocks. The advantage of the three-asset model is that bond holdings are unaffected, whilst money and near-money take most of the adjustment.

### SECTION FIVE: CONCLUSIONS

In this chapter we have examined some recent developments in the theory and empirical modelling of the demand for money (as well as the demand for alternative assets in the portfolio). Central to these recent developments has been the notion that money in some sense acts as a 'buffer asset' in portfolios. As we have seen, there are basically two reasons for believing that money acts as a buffer. First, in the tradition of inventory-theoretic models, there are assumed to be costs involved in the continuous monitoring of money balances, particularly in the presence of stochastic disturbances to money holdings. However, as we have seen, there are problems of aggregation which may arise in this type of model, if one wishes to use it as a model of buffer-stock behaviour at the aggregate level. A second reason

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is that there are just costs of adjustment in the portfolio especially in adjusting holdings of illiquid assets. In this case money acts as a financial buffer, taking some of the strain of adjustment following disturbances to the portfolio. This financial costs-of-adjustment argument is more in the spirit of the empirical models proposed by Brainard and Tobin (1968) and B. Friedman (1977), except that most of the models set out here assume forward-looking, optimising economic agents. This, however, leads to some rather intricate problems of empirical testing which we have outlined in this chapter.

In this chapter we have extended and tested the Cuthbertson-Taylor model of buffer-stock money on M1 data. We found that there are a number of difficulties with this type of model, which relate to the form of the estimating equation which the optimisation exercise yields, and the restrictions which have to be imposed. The main problem with the Cuthbertson-Taylor model is that it appears to be rather unrealistic at the theoretical level. However, extending it to incorporate saving behaviour does remove some of the difficulties. We have presented some evidence to suggest that saving plays an important part in the forward-looking demand for money equation, but there are some severe problems of multicollinearity which makes it difficult to interpret all our results. We have also examined some of the dynamic properties of a possible alternative three-asset model. However, as we have seen, attempting an empirical application of

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this multi-asset model is likely to involve even more complicated econometric issues.

One issue which remains to be confronted in detail is how all this material relates to the 'econometric approaches' to the dynamic modelling of the demand for money examined in Chapters 2 and 3. We have seen in this chapter that the reason that the buffer-stock money hypothesis was put forward is that long lags in the demand for money were seen as incompatible with the existence of asset market equilibrium at all times because this gave rise to some rather implausible overshooting results. Once one accepts that the money market may be in disequilibrium, three different general modelling strategies may be followed. First, if one believes that the money supply is essentially exogenous, the proper approach is to 'invert' the demand for money to model prices, real income or the interest rate in a single-equation framework or to build a full model of the disequilibrium and transmission mechanism, as suggested by James Davidson and David Laidler. A second possible approach is to retain some belief in the endogeneity of money and to stick to single-equation demand for money studies, but to build in a forward-looking buffer stock component, as in the Cuthbertson-Taylor model. This is admissible, if one recognises that the innovations in their model are essentially demand-side innovations. A third strategy is to retain the conventional autoregressive-distributed lag approach used in Chapters 2 and 3,

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whilst accepting the notion of monetary disequilibrium. The ad hoc dynamics are deemed to approximate the disequilibrium adjustment process, and as in the Cuthbertson-Taylor model, the regressors used must be regarded as (weakly) exogenous.

We have not so far compared the Cuthbertson-Taylor approach in detail with the approach followed in Chapters 2 and 3. This is the task which we now turn to in the next chapter.

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### Footnotes to Chapter 4

- (1) The way in which simple 'partial adjustment', 'general ADL' and 'buffer-stock' models are related is an issue which is confronted at length in Chapter 5. The main issue here is what type of adjustment costs the individual agents face, and the the type of cost-minimisation exercise they are assumed to undertake.
- (2) To some extent parallel developments have taken place in the field of the theory of consumption expenditure (see for instance Hall, 1978, Wickens and Molana, 1983). The extent to which the models outlined in this chapter actually enhance our theoretical understanding of short-run dynamics in the demand for money is not entirely clear, as we shall argue further on.
- (3) Some authors (notably Cuthbertson 1985a) choose to differentiate between these two concepts. Whether this is justified or not is really a question of semantics. Here we choose not to differentiate between simple 'disequilibrium money' models and those forward-looking buffer-stock money demand models which are based on an explicit intertemporal optimisation exercise.
- (4) Again, there are parallels here with other areas of economics, notably consumption, investment, and labour demand theories, as the Rational Expectations theory spread to affect all components of aggregate demand.
- (5) This is also not a necessary assumption, but it merely simplifies matters by allowing the interest rate to take all the

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effect of the money supply change. Some overshooting will always occur providing that all the determinants of the demand for money do not adjust instantaneously so as to restore money market equilibrium.

(6) For an exception see Brunner and Meltzer (1964).

(7) One problem with the MacKinnon-Milbourne estimates is the one raised by Pagan (1984). Estimates obtained from a two-stage OLS estimation procedure of an RE model will yield consistent parameter estimates, but inconsistent standard error estimates, thus invalidating most of our popular statistical inference procedures. However, as Cuthbertson (1985a) points out, this does not affect the argument in this particular case.

(8) If the money stock is exogenous it is arguable that one should 'invert' the money demand equation to model some endogenous variable (e.g. the price level). This alternative approach of 'turning the money demand on its head' is examined in the next subsection.

(9) For a detailed exposition of the Kalman Filter approach, see Harvey (1981b), Cuthbertson (1986).

(10) The importance of new saving flows in financial models has been emphasised by , inter alia, Bain (1973), and B.Friedman (1977).

(11) This conclusion follows directly from Sargent (1979), p.197-198, esp. Fig. 4.

(12) Cuthbertson and Taylor (1987) find that a fourth-order

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vector autoregression adequately characterises the behaviour of the vector  $z$ .

(13) This is of course related to the Lucas (1976) critique. We shall return to this issue in detail in Chapter 5.

(14) Though some of the values appear somewhat doubtful at first sight (see our discussion in Chapter 5).

(15) Even at the econometric level the issue of the 'exogeneity' or 'endogeneity' of money (esp. with regard to nominal income) has not been resolved (see for instance Sims, 1972, Goodhart et al., 1976).

(16) On the other hand, as we shall see further on and in Chapter 5, the M1 definition of money may not be a good 'buffer asset', and for this reason we may expect the Cuthbertson-Taylor model to fail when applied to this data.

(17) See for instance Christofides (1976) for an example of how this partial adjustment rule may be derived from a single-period optimisation process. The link between simple partial adjustment and single-period cost-minimisation is also outlined in Chapter 5.

(18) It follows that by penalising deviations of  $M$  and  $N$  from their long-run desired values,  $M^*$  and  $N^*$ , we are automatically penalising deviations of  $B$  from  $B^*$ .

(19) This problem is related to that of different orders of ECMs (see Salmon, 1982). For an example of why disequilibrium may persist in buffer-stock models see Molana and Muscatelli (1986),

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who apply optimal control techniques to more complex objective functions to obtain richer dynamics in the money stock.



## CHAPTER 5

### CHAPTER 5: FORWARD-LOOKING VERSUS FEEDBACK-ONLY MODELS OF THE DEMAND FOR MONEY

#### SECTION ONE: INTRODUCTION

In the previous three chapters we have examined two different basic approaches to the modelling of the demand for money in the context of single-equation studies. In Chapters 2 and 3 we examined different modelling procedures which may be used to construct an ADL-based model. This approach which involves building models on the basis of a general lag specification is sometimes referred to as a 'feedback-only' or 'backward-looking' one in the narrow sense that only lagged values of the explanatory variables enter the equation. In contrast, in Chapter 4 we examined how we could construct 'forward-looking' models by extending 'buffer-stock' theories of the demand for money.

In this chapter we compare these two approaches. There are a number of questions which we seek to address. First, we have to examine what the relationship is between these two types of estimating equations. Secondly, on the basis of this we shall predict which we should expect to yield a better model of the demand for money. These two issues will be discussed in section two. Thirdly, we examine which data set may best be used for such a comparison between these two approaches. This issue will be addressed in section three. Lastly, in section four we shall compare the two approaches by estimating both 'backward-looking'

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and 'forward-looking' models of the demand for  $M_1$ .

### SECTION TWO: THE RELATIONSHIP BETWEEN 'BACKWARD-LOOKING' AND 'FORWARD-LOOKING' MODELS

#### 5.2.1. Observational Equivalence and Forward-Looking Models.

We begin this section by looking once more at the simple partial adjustment mechanism. One justification for this ad hoc dynamic adjustment scheme is that it may be derived from an explicit single-period optimisation exercise. Thus, if the representative economic agent minimises a single-period cost function where he penalises both deviations from his desired long-run equilibrium demand for money,  $M^*$ , and adjustments in the actual money stock holdings:

$$C_t = a_0(M_t - M_t^*)^2 + a_1(M_t - M_{t-1})^2 \quad (5.1)$$

Then setting  $\partial C / \partial M_t = 0$ , we find that the optimal actual money holdings at time  $t$ ,  $M_t$  are given by:

$$M_t = (a_0 / (a_0 + a_1)) M_t^* + (a_1 / (a_0 + a_1)) M_{t-1} \quad (5.2)$$

which may be re-arranged to yield:

$$(M_t - M_{t-1}) = \lambda (M_t^* - M_{t-1}) \quad (5.3)$$

where  $\lambda = (a_0 / (a_0 + a_1))$ , and  $0 < \lambda < 1$ .

Thus, we have shown that one justification for the simple partial adjustment scheme is that it may be derived from a single-period optimisation exercise, where the partial adjustment parameter,  $\lambda$ , is indicative of the relative magnitude of the costs of being away from equilibrium,  $a_0$ , and of adjusting money balances,  $a_1$ . However, one criticism of the partial adjustment

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approach (apart from the fact that it fails at the empirical level) is that (5.1) is myopic. Economic agents are more likely to undertake a multi-period optimisation process, and this is indeed the basis for the Cuthbertson-Taylor model which we examined in Chapter 4.

However, a relevant question here is the following: if a single-period cost-minimisation exercise can lead to a simple 'backward-looking' dynamic model such as the partial adjustment model, can a multi-period costs-of-adjustment approach similarly lead to a more general autoregressive distributed lag specification?

As it happens, we can show that this is indeed the case, which leads to the interesting conclusion that our forward-looking model of Chapter 4 is observationally equivalent to our feedback-only (or 'backward-looking') models of Chapters 2 and 3. This problem of observational equivalence is a common one in 'forward-looking' (rational expectations models), and the issue was first raised in the context of empirical tests of the new classical 'policy neutrality proposition' (see Sargent, 1976, McCallum, 1979, Buiter, 1981).

Before we examine this property of multiperiod cost-of-adjustment models we should however point out that one must not necessarily accept that agents have multi-period horizons to derive an error-correction model, as these may equally be derived from modifications to the single-period cost-minimisation in

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(5.1)<sup>1</sup>. Furthermore, as Hendry *et al.* (1984) point out other rationales for general lag formulations may be found. For instance, they may represent the adoption of simple feedback control rules by individual economic agents (see Phillips, 1954, 1957, Salmon, 1979). Alternatively, individual agents may use simple rules-of-thumb when adjusting money balances in disequilibrium (see Day, 1967, Ginsburgh and Waelbroeck, 1977). These hypotheses may be combined with the question of aggregation across different economic agents with different adjustment rules to yield more complex distributed lags at the aggregate level<sup>2</sup>. In other words, whilst a multi-period cost-minimisation exercise is sufficient to generate an ADL model, it is by no means necessary, and the success of ADL-error-correction formulations in economics may or may not be due to forward-looking behaviour, although this interpretation has its attractions for economists who embrace the concept of the rational, forward-looking, representative economic agent.

Having considered these alternative interpretation of 'backward-looking' models, let us now examine the conditions under which a multi-period costs of adjustment model can indeed lead to an ADL-type model (and hence via an appropriate transformation to an error-correction model).

Nickell (1980, 1985) was one of the first to forge the link between multi-period costs-of-adjustment models and error-correction models. Throughout what follows, we shall again make

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use of  $M^*$  to represent long-run desired money balances, and we shall only 'disaggregate' this into price, real income, and interest rate influences (using the long-run demand for money function) where appropriate to avoid notational complexity. Let us begin with one of the simplest examples of cost function presented by Nickell (1985):

$$C_t = \sum_{j=0}^{\infty} (\delta)^j \{a_0(M_{t+j} - M_{t+j}^*)^2 + a_1(M_{t+j} - M_{t+j-1})^2 - 2a_2(M_{t+j} - M_{t+j-1})(M_{t+j}^* - M_{t+j-1}^*)\} \quad (5.4)$$

where as before, the  $a_i$  represent the weights attached to the various elements of the cost function. Note that, unlike the simple Cuthbertson-Taylor model outlined in Chapter 4, we have a negative cost attached to parallel changes in desired and actual balances,  $a_2$ . This feature was built in the Muscatelli (1988a) saving model which was also discussed in the previous chapter. Applying the usual optimisation procedure to (5.4) we obtain the following set of Euler equations:

$$\begin{aligned} (M_{t+j+1} - (a_2/a_1)M_{t+j+1}^*) - (1 + (1/\delta) + (a_0/a_1\delta))(M_{t+j} - (a_2/a_1)M_{t+j}^*) \\ + (1/\delta)(M_{t+j-1} - M_{t+j-1}^*) = (a_0/a_1\delta)((a_2/a_1) - 1)M_{t+j}^* \end{aligned}$$

(5.5)

for  $j = 0, 1, 2, \dots$

This may be solved by expressing the left-hand side in terms of a lag polynomial and using the usual factorisation of this polynomial. The solution of the Euler equation in this case is:

$$\begin{aligned} M_t = (a_2/a_1)M_t^* + \lambda_1 M_{t-1} - \lambda_1(a_2/a_1)M_{t-1}^* - \\ (1 - \lambda_1)(1 - \delta\lambda_1)((a_2/a_1) - 1)\sum_{i=0}^{\infty} (\delta\lambda_1)^i M_{t+i}^* \end{aligned} \quad (5.6)$$

There are a number of things to note about equation (5.6).

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First, as we saw in the saving model of Chapter 4, once we introduce a more complex cost function by including our negative cost term, the solution to the Euler equation becomes far more complicated, including  $M_{t-1}^*$  among the variables in the estimating equation. Only in the case where this term disappears (i.e. where  $a_2 = 0$ ) do we return to the simple case proposed by Cuthbertson and Taylor. We saw in Chapter 4 that there are problems in estimating equations such as (5.6) as the  $M_{t+i}^*$  depend on  $M_{t-1}^*$  in a rational expectations framework. This once again emphasises the reliance of the Cuthbertson-Taylor model on the very simplest of cost function, which may not offer a very accurate description of the buffering mechanism (see Muscatelli 1988a). Secondly, it should even at this stage be apparent that if lagged prices, incomes and interest rates will be used to generate the expected series needed to substitute for the  $M_{t+i}^*$  term, (5.6) will yield an estimating equation which is observationally equivalent to an ADL-based 'backward-looking' model.

Nickell (1980, 1985) demonstrates this second point formally by considering two alternative data generation processes for the  $M_t^*$  series (we do not, for notational simplicity, decompose this into price, income and interest rate terms). First, let us consider the case where  $M_t^*$  follows a random walk with drift, so that:

$$M_t^* = \mu + M_{t-1}^* + \varepsilon_t \quad (5.7)$$

where  $\varepsilon_t$  is white noise. We may then substitute for  $M_{t+i}^*$  in (5.6)

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using the following forecasting formula:

$$M_{t+1}^* = \mu + M_t^* \quad (5.8)$$

This then yields the following 'backward-looking' error-correction model:

$$\Delta M_t = \{(1 - \lambda_1)(1 - (a_2/a_1))\delta\lambda_1\mu\}/(1 - \delta\lambda_1) + (1 - \lambda_1 + (\lambda_1 a_2/a_1))\Delta M_t^* - (1 - \lambda_1)(M_{t-1} - M_{t-1}^*) \quad (5.9)$$

There are several interesting points to note about equation (5.9). First, it confirms that forward-looking models will be observationally equivalent to ECM-type models. Hence empirical support for either specification may not necessarily shed light on whether the demand for money may or may not be given a 'forward-looking' interpretation. Secondly, the constant term in (5.9) contains the drift parameter  $\mu$  from the marginal model in (5.7), and hence represents what Nickell calls an 'integral correction mechanism'. This is of particular importance in a growing economy, especially in the light of our comments in the last section of Chapter 2, where we illustrated the apparent significance of 'growth effects' in the demand for money. Thirdly, if we revert to a Cuthbertson-Taylor type cost function by setting  $a_2 = 0$ , equation (5.9) would reduce to a simple partial adjustment model.

However, this last property does not hold in all cases. For instance, Nickell also considers the case where  $M_t^*$  follows a simple second order autoregressive process with drift:

$$M_t^* = \mu + \beta M_{t-1}^* + (1 - \beta)M_{t-2}^* + \varepsilon_t \quad (5.10)$$

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where again  $\varepsilon_t$  is white noise. We may then use the following forecasting formula to substitute out for  $M_{t+i}^*$  in (5.6):

$$M_{t+i}^* = (1/(2 - \beta))\{\mu_i + M_t^* + (1 - \beta)M_{t-1}^* - (\beta - 1)^{i+1} \Delta M_t^*\} \quad (5.11)$$

This yields the following slightly different error-correction model:

$$\begin{aligned} \Delta M_t = & \{(1 - \lambda_1)(1 - (a_2/a_1))\delta\lambda_1\mu\}/\{(1 - \delta\lambda_1)(2 - \beta)\} + \\ & \{(a_2/a_1) + [(1 - \lambda_1)(1 - (a_2/a_1))/(1 + \delta\lambda_1(1 - \beta))]\}\Delta M_t^* - \\ & (1 - \lambda_1)(M_{t-1} - M_{t-1}^*) \end{aligned} \quad (5.12)$$

Again, we should note a number of points about this version of the model. First, this model does not degenerate into a partial adjustment model, even if  $a_2 = 0$ . This confirms that even a simple cost function may help us to generate an error-correction model provided the  $M^*$  series follows an appropriate generating process. In any case it is worth remembering from Chapters 2 and 3 that general ADL models may be easily transformed into error-correction models, and therefore that unless a strange combination of cost function and data generation process is chosen, error-correction-ADL models will generally provide an appropriate reduced form for a forward-looking model.

Secondly, in both (5.9) and (5.12), the error-correction term has a single-period lag. However, a more general ECM may be obtained (including a four-period lag to capture seasonal behaviour in  $M_t$ ), by either incorporating seasonal behaviour in the agents' cost minimisation process, or by selecting an



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autoregressive scheme of at least order 4 to describe  $M_t^*$ , and then applying an appropriate transformation.

Thirdly, note that both (5.9) and (5.12) fall somewhat short from being general ADL-error-correction models because they only contain a single-period lag in the  $M_t$  term. This, as Nickell points out cannot be remedied by assuming a more complex autoregressive generating mechanism for  $M^*$ . The reason for this feature has already been discussed in Chapter 4, where we saw that all our models contained only a single lag of actual money balances,  $M_{t-1}$ . This is due to the nature of the cost function used. Does this therefore indicate an identification restriction which will enable us to overcome the observational equivalence problem? If this were the case a test of the forward-looking model versus a feedback-only interpretation would be to test the significance of longer lags of  $M$  in the estimated model. Unfortunately, there are two reasons why this does not resolve our identification problem.

First, as Nickell indicates, one can always devise an even more complex cost function which will allow longer lags in  $M$ . For instance, one could penalise not only changes in money balances ( $M_{t+j} - M_{t+j-1}$ ), but changes in the rate of growth of  $M$ , namely ( $\Delta M_{t+j} - \Delta M_{t+j-1}$ ). Secondly, one could devise multivariate marginal models for the individual components of  $M^*$  which would allow us to generate models with further lags in  $M$  (see Muscatelli, 1988b). In particular, one could include lagged

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values of  $M$  in the generating models, as we show below.

Consider the simple Cuthbertson-Taylor version of (5.6), where we have set  $a_2 = 0$ , and as before we have assumed a logarithmic structure:

$$m_t = \lambda_1 m_{t-1} + (1 - \lambda_1)(1 - \lambda_1 \delta) \sum_{j=0}^{\infty} (\delta \lambda_1)^j m_{t+j}^* + m_t^u + \varepsilon_t \quad (5.13)$$

We then assume the usual structure for the long-run demand for money function to re-express (5.13) as:

$$m_t = (1 - \lambda_1) \alpha_0 + \lambda_1 m_{t-1} + (1 - \lambda_1)(1 - \lambda_1 \delta) \sum_{j=0}^{\infty} (\delta \lambda_1)^j (\alpha_1 p_{t+j}^e + \alpha_2 y_{t+j}^e - \alpha_3 R_{t+j}^e) + m_t^u + \varepsilon_t \quad (5.14)$$

Let us now assume that all the generating models for  $p$ ,  $y$ , and  $R$  have autoregressive representations and that, in addition, at least one, say the price level equation, also contains lagged terms in  $m$ . If the following are assumed to be the forecasting equations:

$$p_{t+j}^e = \sum_{i=1}^m \beta_i p_{t-i+j}^e + \sum_{i=1}^m \tau_i m_{t-i+j}^e \quad (5.15)$$

$$y_{t+j}^e = \sum_{i=1}^m \gamma_i y_{t-i+j}^e \quad (5.16)$$

$$R_{t+j}^e = \sum_{i=1}^m \mu_i R_{t-i+j}^e \quad (5.17)$$

where we may obtain expressions in terms of only past values by using the Wiener-Kolmogorov formula for  $k$ -step ahead linear least squares predictions (see for example Sargent, 1979, pp.263-264). However, this is not necessary for our present purposes. Substituting (5.15)-(5.17) into (5.14) we obtain:

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$$\begin{aligned}
 m_t &= (1 - \lambda_1)\alpha_0 + \lambda_1 m_{t-1} + \\
 &(1 - \lambda_1)(1 - \lambda_1\delta)[\alpha_1 \sum_{j=0}^{\infty} (\delta\lambda_1)^j (\sum_{i=1}^m \beta_i p_{t-i+j}^e + \sum_{i=1}^m \tau_i m_{t-i+j}^e) + \\
 &\alpha_2 \sum_{j=0}^{\infty} (\delta\lambda_1)^j \sum_{i=1}^m \gamma_i y_{t-i+j}^e - \alpha_3 \sum_{j=0}^{\infty} (\delta\lambda_1)^j \sum_{i=1}^m \mu_i R_{t-i+j}^e] + m_t^u + \varepsilon_t
 \end{aligned}
 \tag{5.18}$$

It is obvious from this that the use of a prediction formula on the expected terms in (5.18) will yield an equation containing both lagged terms in  $p$ ,  $y$ , and  $R$ , and lagged terms in  $m$ .

Thus, we have shown so far in this section that, by choosing an appropriate combination of cost function and marginal model(s) for the target variable, one can generate a general ADL model (and by a suitable transformation, an error-correction model). This makes forward- and backward-looking models observationally equivalent, and hence the success of either type of model may not be particularly revealing in terms of offering us indications about the appropriateness of the buffer-stock model. The problems here are potentially more acute than the usual observational equivalence problems which arise in, say, tests of the new classical invariance proposition. The reason for this is that the 'structural' parameters involved here are the cost function parameters, and we have no a priori view of their likely magnitudes (indeed we are not even sure if the costs-of-adjustment theory is valid). Thus finding appropriate identifying restrictions becomes rather difficult.

Nevertheless, there are ways of discriminating between the two approaches, based on the famous 'Lucas critique' of

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econometric modelling. Hendry (1988) has recently advanced a number of ways in which we may resolve the observational equivalence problem. We return to a more detailed analysis of these issues in subsection 5.2.3. First, however, we have to briefly survey some of the different concepts of exogeneity which we shall meet in the forthcoming discussion.

### 5.2.2. A Digression: Econometric Concepts of Exogeneity.

In this subsection we discuss some of the issues covered in Engle et al. (1983). For reasons of space, our treatment is somewhat brief. At the outset, let us recall that in Chapter 2 we pointed out that an adequate single-equation regression model required that the regressors are at least weakly exogenous relative to the parameters of interest, as it is then possible to consider the conditional model alone, without having to specify a model for the weakly exogenous variables. Now consider the joint data density  $D(z_t | Z_{t-1}, \theta)$  where the vector  $z$  contains two variables  $(y, x)$ , and where  $Z_{t-1} = (z_0, z_1, \dots, z_{t-1})$ . Factorising this joint data density as follows:

$$D(z_t | Z_{t-1}, \theta) = D_1(y_t | x_t, Z_{t-1}, \phi_1) D_2(x_t | Z_{t-1}, \phi_2)$$

where  $(\phi_1, \phi_2)$  is an appropriate reparameterisation of  $\theta$ . In this framework, weak exogeneity of  $x_t$  with respect to the parameters of interest requires that these parameters of interest depend on  $\phi_1$  alone, and that both  $\phi_1$  and  $\phi_2$  do not vary. Thus weak exogeneity enables us to validly condition the variable which we wish to model,  $y_t$  on  $x_t$ . We may then model  $y_t$  without reference

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to the marginal model for  $x_t$ .

To illustrate this concept further, let us take the following simple example with two variables ( $y, x$ ). If we wish to model  $y$ , and the true generating process is given by:

$$y_t = \alpha_0 + \alpha_1 x_t + e_t \quad (5.19)$$

$$x_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 y_{t-1} + u_t \quad (5.20)$$

where  $e_t$  and  $u_t$  are independent white noise, then we may estimate the  $\alpha_i$  parameters by estimating (5.19) using OLS, as  $x$  is weakly exogenous with respect to the  $\alpha_i$ .

Whilst weak exogeneity is sufficient for single-equation modelling, it remains true that unless we know the marginal model (5.20), equation (5.19) by itself will not yield an adequate forecasting equation for  $y$ , as lagged  $y$ 's provide relevant information for forecasting the  $x$  series in (5.19) (see Engle *et. al.*, 1983, Gilbert, 1986). Thus, we may require strong exogeneity, which requires that past  $y$ 's do not provide any relevant information in forecasting  $x$ . In other words, strong exogeneity requires both weak exogeneity, and in addition that  $y$  does not Granger-cause  $x$ . In the case of equation (5.20) this would require that  $\beta_2 = 0$ . In terms of our joint data density factorisation, this would require the following to hold:

$$D(z_t | Z_{t-1}, \theta) = D_1(y_t | x_t, Z_{t-1}, \phi_1) D_2(x_t | X_{t-1}, \phi_2)$$

The third and most important concept of exogeneity as far as our present analysis is concerned is that of superexogeneity. In terms of our previous simple example, suppose that  $y_t$  depends on

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current expected  $x_t$ ,  $x_t^e$ :

$$y_t = \alpha_0 + \alpha_1 x_t^e + e_t \quad (5.21)$$

and that the 'correct' model used to generate expectations is a simple autoregressive one:

$$x_t^e = \beta_0 + \beta_1 x_{t-1} + u_t \quad (5.22)$$

where the  $\beta_i$  are policy parameters. If a change in government policy implies a change in the marginal model for  $x_t$ , then (5.21) will be useless for forecasting after the policy change unless we know the new model for  $x$ . This is of course the famous 'Lucas critique' (Lucas, 1976) of econometric forecasting in the presence of rational expectations. Even if  $x_t$  is strongly exogenous in this example, a simple single equation model (5.21) is not sufficient to forecast  $y$  accurately across different policy periods. To enable us to do this, we require the estimated parameters to be superexogenous. In other words, we require not only weak exogeneity, but also the invariance of the model parameters to changes in the marginal distributions of the weakly exogenous variables. In terms of our factorisation of the joint data density, superexogeneity requires the parameter vectors  $\phi_1$  and  $\phi_2$  to be independent.

As should be apparent, given the forward-looking nature of one of our competing models, the major concept of interest to us here is that of superexogeneity. As we shall see in the next subsection, an evaluation of backward-looking and forward-looking models essentially revolves around the issue of superexogeneity

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and the Lucas critique.

### 5.2.3. 'Forward-Looking' versus 'Backward-Looking': Which is likely to Perform Better?

This is the main question which we wish to address in this chapter. Before we attempt to resolve it at the empirical level, we should consider what the main factors are which will determine which type of model will perform better in practice. There are really two independent factors which will determine our preference for one or other type of approach. First, there is the question of whether the Lucas critique is likely to be relevant in the case of the demand for money. This comment applies to any type of forward-looking model, including the ones considered here (i.e. quite independently of whether they are derived from an explicit cost-minimisation exercise or not). Secondly, we have the problem that our forward-looking models are derived from a specific cost function and hence have a dynamic structure which is somewhat restrictive. Whether this dynamic structure is correct or not will influence whether one or other approach will work better in practice.

Let us take these two issues in turn. The first is tackled by Hendry (1988) for the case of the demand for money. Hendry recognises that even though forward-looking models are intuitively appealing at the microeconomic level, as they emphasise the role of intertemporal optimisation by rational economic agents, their importance at the macroeconomic level may

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have been overplayed by the proponents of these models<sup>4</sup>. In contrast, feedback-only models have been successful so far in a number of empirical applications, although if we choose to give them an expectations interpretation, as in section two of this chapter, they raise the spectre of the Lucas critique. Hendry (1988) is therefore concerned with two main issues. Firstly, is there any way in which one of these approaches can account for the results of the other (i.e. can encompass the rival approach)? The second issue is whether the parameters obtained from 'backward-looking' models are indeed superexogenous. It turns out, as Hendry shows, and as we shall see below, that these two issues are interdependent: the issue of superexogeneity can shed light on the question of encompassing, as non-constancy of the marginal models will in general yield evidence in favour of one or other approach:

'....Since each hypothesis entails views about the other which confounds behavioural parameters with the parameters of the marginal models, if the marginal models exhibit enough change at least one hypothesis can be rejected on non-constancy grounds. Thus a symmetric analysis results from examining the encompassing implications of super-exogeneity in a changing world: for expectations hypotheses, the Lucas critique is potentially refutable as well as confirmable... ' Hendry (1988), p.3.

Therefore there are two ways to resolve the question of whether either of the 'forward-looking' or 'backward-looking'



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hypotheses can encompass the other. The first is to examine whether the Lucas critique is at all important in the context of the rival demand for money models proposed. As we shall see this approach is applicable whatever the dynamic structure of the 'forward-looking' model. The second is to engage in a 'practical' exercise in variance-encompassing, along the lines of the experiments with rival models which we carried out in Chapter 3. However, as we shall see in section four, this second approach is bound to test both the relevance of the Lucas critique and the relative merits of the dynamic structures of the rival models.

Let us turn first to the issue of whether the Lucas critique is refutable as well as confirmable, and what procedures are available to us to test whether one hypothesis encompasses the other. Hendry (1988) considers the following simple linear models to illustrate the two competing hypotheses. Firstly, suppose that the feedback/backward-looking model is correct. We then have a relationship between the variable of interest,  $y_t$ , and a vector of  $k$  explanatory variables,  $x_t$ :

$$y_t = \alpha'x_t + u_t \quad (5.23)$$

where  $\alpha$  is a  $(1 \times k)$  parameter vector, and  $u_t$  is a white noise error, and where  $E(x_t u_t) = 0$ . We also have a marginal model for the vector  $x_t$ :

$$x_t = \Gamma z_{t-1} + \omega_t \quad (5.24)$$

where  $z_{t-1}$  is a  $(n \times 1)$  vector of explanatory variables which do not include current period realisations,  $\Gamma$  is a matrix of  $(k \times n)$

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parameters, and  $\omega$  is a matrix of disturbances such that  $\omega \sim (0, \Omega)$ . Note that under the 'backward-looking' hypothesis we are assuming that the  $x_t$  variables are at least weakly exogenous. In fact, to simplify the analysis Hendry makes the additional assumption that  $x_t$  and  $z_{t-1}$  have no elements in common. This assumption is not strictly necessary, though, but merely simplifies the analysis<sup>5</sup>.

Next, let us turn to the 'expectations/forward-looking' approach, which implies the following type of model for  $y_t$ :

$$y_t = \beta'E(x_t | z_{t-1}) + v_t \quad (5.25)$$

where  $\beta$  is a vector of  $(1 \times k)$  parameters, and  $v_t$  is a white noise error. Note that in this model expectations about the  $x_t$  variables are conditioned upon the  $z_{t-1}$  vector. Again, we have a marginal model for the  $z_{t-1}$  variables which is the same as in the 'backward-looking' model:

$$x_t = \Gamma z_{t-1} + \omega_t \quad (5.24)$$

Thus, these two simple linear versions of the two approaches illustrate the principle of superexogeneity discussed in the previous subsection. If the 'backward-looking' approach is correct, then (5.23) and (5.24) will correctly characterise the underlying data generation process (DGP), and we can validly condition on the  $x_t$  variables, and  $z_{t-1}$  is irrelevant. On the other hand, if the 'forward-looking' approach is correct, then we cannot validly condition on  $x_t$ , and  $z_{t-1}$  is relevant. In this latter case, equations (5.24) and (5.25) correctly characterise

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the DGP. In terms of the parameterisation of (5.23) and (5.24), if the 'forward-looking' approach is correct, the parameters  $\Gamma$  and  $\alpha$  are not independent: the property of superexogeneity does not hold.

Thus we suppose that (5.23)-(5.24) and (5.24)-(5.25) constitute our rival hypotheses about the nature of the DGP. Next, let us suppose that the 'backward-looking' model constitutes the correct characterisation of the DGP, so that the 'backward-looking' model encompasses the 'forward-looking' model. Under these circumstances, Hendry argues that the following implications will hold:

$$(i) E(y_t | x_t, z_{t-1}) = \alpha' x_t$$

That is,  $y_t$  is independent of  $z_{t-1}$  when conditional on  $x_t$ .

(ii) Both models may be interpreted in expectations terms, i.e. the problem of observational equivalence. Furthermore, even if the marginal model is sufficiently variable, both types of models should display constant parameters<sup>6</sup>.

(iii) If we have variable  $\Gamma$  and/or  $\Omega$  (denoted by  $\Gamma_t$  and  $\Omega_t$  respectively), then the reduced form estimation will give non-constant parameters, since:

$$y_t = \alpha' \Gamma_t z_{t-1} + w_t \tag{5.26}$$

(iv) The reduced form (5.26) will fit worse than (5.23) since  $E(w_t^2) = E(u_t + \alpha' \omega_t)^2 = \sigma_t^2 + \alpha' \Omega_t \alpha$ , and  $\alpha' \Omega_t \alpha \geq 0$ .

Let us now suppose that the 'forward-looking' interpretation of the DGP is the correct one, so that (5.24)-(5.25) correctly

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characterise the DGP. Then, Hendry argues, the following implications hold:

(v) Conditional on  $z_{t-1}$ , we cannot tell a priori if  $y_t$  is independent of the  $x_t$ . This is because we cannot at the outset say anything about the independence of  $v_t$  and  $x_t$ , whilst  $v_t$  and  $z_{t-1}$  are orthogonal. The exact relationship between the errors and the expected series is an important one in forward-looking models, e.g. 'errors-in-variables', two-stage approaches (see for instance Wallis, 1980, Wickens, 1982). This result derives from the fact that we cannot estimate (5.25) using simple OLS methods.

(vi) Providing  $\Gamma_t$  and  $\Omega_t$  are 'sufficiently' variable, then (5.23) will not display constant parameters. This is the Lucas critique which we have already illustrated above in the case of a simple linear model. It may easily be extended in the case of (5.23) to the case where both  $\Gamma$  and  $\Omega$  are non-constant. The Lucas critique result still holds providing the processes driving  $\Gamma_t$  and  $\Omega_t$  are not identical (see Hendry, 1988).

(vii) Once again, with variable parameters in (5.24), the reduced form will have non-constant parameters, as in (iii).

(viii) There are problems in ranking the reduced form and the structural model (5.23) in terms of error variance. This is because the reduced form will now simply have an error equal to  $\sigma_v^2$  (as (5.25) correctly characterises the DGP, the 'reduced form' is the 'correct' model), whilst simply regressing (5.23) will

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yield additional terms in the error variance due to the variability of the  $\alpha$  parameters which will now enter the error term. However, this will partly be offset by the extent to which  $y_t$  and  $x_t$  are independent, conditional on  $z_{t-1}$  (see point (v)). This is because, to the extent that  $y_t$  and  $x_t$  are not independent, the regressors in (5.23) will provide some information about the behaviour of  $y_t$  not captured by the  $z_{t-1}$  alone in the reduced form. However, providing the marginal model is sufficiently variable, one would expect (5.25) to rank better in terms of error variance than (5.23), in contrast to point (iv).

These implications of the two rival hypotheses can yield testing procedures which will enable us to reject either one or the other, despite the problem of observational equivalence. Hendry's own preference in his (1988) study is to focus on the issue of superexogeneity to discriminate between the two models. Having established whether the Lucas critique is applicable or not in the case of the demand for money the relevant encompassing result then follows.

Thus, Hendry's testing procedure is the following. If we show that both the structural 'backward-looking' model and the 'forward-looking' model display parameter constancy, then, if the marginal models do not display sufficient parameter variation, neither hypothesis can be rejected. On the other hand, if in addition to these findings the marginal model is also found to have non-constant parameters, then point (vi) indicates that the

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model cannot be given a 'forward-looking' interpretation. In contrast, point (ii) indicates why a 'backward-looking' interpretation allows both structural models to display parameter constancy.

How does one implement the concept of 'sufficient' variability in the marginal model? Hendry argues that existing tests of parameter constancy, especially those implemented in a recursive estimation environment (i.e. using recursive least squares and recursive instrumental variable estimation), provide adequately powerful tests of such variability (or its absence). Using this approach, Hendry finds in favour of the 'backward-looking' hypothesis: the marginal models are indeed non-constant, whilst both forward-looking and backward-looking estimated equation pass conventional parameter constancy tests. The Lucas critique is therefore not applicable to M1 models of the demand for money. We do not report these results in detail here, since we shall be applying them to a slightly different data set to provide a more 'neutral' testing ground for the rival hypotheses (see section three below).

Before we turn to this, however, let us consider briefly why Hendry has chosen to focus on points (vi) and (ii) above as the main 'testbed' for the rival models. The main reason for this is that some of the other points suggest tests which are not easy to implement. For instance points (i) and (v) involve the implementation of exogeneity tests, which may well prove

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inconclusive, especially if  $x_t$  and  $z_{t-1}$  have many common elements (or indeed if all elements are in common; see Hendry and Neale, 1987). Points (iii) and (vii) are identical and hence cannot be used for testing. This leaves us with points (iv) and (viii), which is the alternative testing procedure which we seek to implement here, and which is adopted in Muscatelli (1988b).

Hendry's main reason for not concentrating on the variance encompassing tests suggested by (iv) and (viii) are obvious from the above analysis: point (viii) suggests that if the 'forward-looking' interpretation is the correct one, it may still not lead to a reversal of the ranking suggested by (iv), so that a purely symmetrical testing procedure would not be available. Another criticism, which we have already mentioned above, is that such tests will take into account other factors (e.g. the restrictiveness of the dynamic structure of the forward-looking model). Nevertheless, there are good reasons for focusing on variance encompassing tests to discriminate between the models. Firstly, point (viii) suggests that the variance ranking of (iv) is inverted provided  $y_t$  and  $x_t$  are independent, conditional on  $z_{t-1}$ . This is likely to be the case in most of our models, as simple autoregressive models are used so that  $x_t$  and  $z_{t-1}$  will have much information in common. Secondly, the power of the two testing procedures proposed here are related: the power of Hendry's own preferred test is inherently dependent upon there being sufficient variation in  $\Gamma$  and  $\Omega$ . This is also the case for

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variance encompassing tests via point (viii).

We now turn to an implementation of these proposed tests of the rival hypotheses. In section three we shall examine some of the data definitions used, and consider the stability of the marginal models for the regressors to be used in the structural model. In section four we shall construct alternative models of the demand for money, and examine the evidence from variance encompassing tests.

### SECTION THREE: DATA DEFINITIONS AND FORECASTING MODELS

#### 5.3.1. The Preferred Data Set

The first thing we have to consider here is which monetary aggregate to use in our study. There are several factors to consider here, as we have already seen from our discussion in Chapter 4. On the one hand, we argued there that it is unlikely that a narrow money aggregate would be an appropriate 'buffer asset' (see Davidson, 1986, Milbourne, 1987). Thus, if our 'rational expectations' models seek in some way to capture the 'buffering action' of money, it may be best to use a broad money aggregate. On the other hand, for ease of comparison it would be best to employ a narrow money aggregate, such as M1. This is because the use of M1 has led to stable, satisfactory estimates, using both 'backward-' and 'forward-looking' methods. Furthermore, as we saw in Chapter 4 it could be that the model proposed by Cuthbertson and Taylor captures demand-side innovations through the terms  $p^u$ ,  $y^u$ , and  $R^u$ .



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In this study we have chosen to use M1 data to estimate our competing models because it will facilitate a comparison with existing studies (see Cuthbertson, 1984, 1988, Artis and Cuthbertson, 1985, Cuthbertson and Taylor, 1987, Hendry, 1979, 1985). However, to differentiate our estimates from those of other authors, we have chosen to employ seasonally unadjusted data. We have already discussed the rationale for using seasonally unadjusted series in the previous chapters (see also Wallis, 1974, Harvey, 1981a). This is in contrast with many other studies. For instance Hendry (1979, 1985) fitted a relationship for the demand for M1 using deseasonalised data over the period 1963(I)-1982(IV). Similarly, Cuthbertson and Taylor (1987) fit a model for M1 over the period 1963(I)-1983(III), again using deseasonalised data series.

As far as the definitions of the explanatory variables are concerned, we used the Treasury Bill rate for R, but the choice of the real income (and corresponding price) variable involved some difficulties given that we wished to use our estimates for the purposes of comparison with other studies. We had the choice of either data on real personal disposable income (RPDI) (evaluated at constant (1980) prices) for Y, with its corresponding implicit deflator for P, or data on total final expenditure on goods and services (TFE) at constant (1980) prices for Y, and the implicit TFE deflator for P. A priori there are good reasons for choosing TFE, as it may be a better measure to

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capture the transactions demand for money (especially on the part of firms, see Hendry, 1979). However, this causes problems of comparison with other studies (e.g. Cuthbertson and Taylor, 1987) which employ RPD1. In order to ensure that our results are not biased towards one particular approach through the use of data definitions which are exclusive to that camp, we shall report estimates obtained using both TFE and RPD1 data.

The data period used for the study is 1963(I)-1984(IV), which is a longer period than that employed by any other study of either type. Before constructing models for the demand for M1 using the two alternative methods under scrutiny, we first construct marginal models for the explanatory variables. These will serve both in the construction of expected series for our rational expectations models, and in applying the test procedures for parameter constancy suggested by Hendry (1988) and outlined in the previous section.

### 5.3.2 Marginal Models and Parameter Constancy Tests.

We begin by fitting marginal models for all our explanatory variables. As we pointed out in Chapter 4, there are many ways to estimate rational expectations models of the type we are considering here. For the reasons already discussed in Chapter 4, here we have chosen to adopt the two-stage substitution method. In constructing marginal models for the explanatory variables we have also chosen to focus on simple single-variable autoregressive models, as opposed to more complex systems (e.g.

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vector autoregressions). One problem with this is that we could be stacking the results regarding the constancy of the marginal models against the rational expectations interpretation. On the other hand, Hendry (1988) finds that even employing vector autoregressions the constancy of the former is strongly rejected. Thus, we have persisted with the use of scalar autoregressions, also given that at this stage of the proceedings we are doing no more than seeking a confirmation of Hendry's own findings.

In constructing forecasting equations, as in Chapter 4, we estimated autoregressive equations with a maximum of 8 lags for each variable, excluding those lags which proved to be statistically insignificant. These are reported in Table 5.1 below. As in Chapter 4, the interest rate was modelled by a simple first-order autoregression, and was found to be close to being ex ante unpredictable from its own past. Otherwise all other equations performed adequately in terms of within-sample fit, and pass the LM(n) tests against nth-order serial correlation in the residuals at the 5% significance level.

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Table 5.1

### Forecasting Equations

#### (1) RPDI Deflator Equation

$$p_t = 1.218p_{t-1} - 0.221p_{t-5}$$

(0.0185)      (0.0174)

$$R^2 = 0.999 \quad \hat{\sigma} = 0.0137 \quad DW = 1.929 \quad LM(4) = 0.19 \quad LM(8) = 0.13$$

#### (2) RPDI Equation

$$y_t = 0.737y_{t-1} + 0.228y_{t-2} - 0.202y_{t-3} + 0.521y_{t-4} - 0.284y_{t-5}$$

(0.111)      (0.126)      (0.127)      (0.124)      (0.109)

$$R^2 = 0.999 \quad \hat{\sigma} = 0.0223 \quad DW = 1.979 \quad LM(4) = 0.39 \quad LM(8) = 0.89$$

#### (3) TFE Deflator Equation

$$p_t = 1.613p_{t-1} - 0.444p_{t-3} - 0.171p_{t-5} - 0.319\Delta_2 p_{t-2}$$

(0.103)      (0.122)      (0.108)      (0.176)

$$R^2 = 0.999 \quad \hat{\sigma} = 0.0819 \quad DW = 1.864 \quad LM(4) = 0.41 \quad LM(8) = 0.43$$

#### (4) TFE Equation

$$y_t = 0.723y_{t-1} + 0.822y_{t-4} - 0.544y_{t-5}$$

(0.092)      (0.080)      (0.085)

$$R^2 = 0.999 \quad \hat{\sigma} = 0.0212 \quad DW = 1.943 \quad LM(4) = 1.92 \quad LM(8) = 1.83$$

#### (5) Interest Rate Equation

$$R_t = 0.996R_{t-1}$$

(0.0142)

$$R^2 = 0.983 \quad \hat{\sigma} = 1.231 \quad DW = 1.616 \quad LM(4) = 1.33 \quad LM(8) = 0.95$$

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We now turn to an examination of the constancy of these marginal models and, following Hendry (1988), we use tests based on recursive estimation procedures. We re-estimated all the equations listed in Table 5.1 using the recursive least squares method. The advantage of this procedure is that the estimates are updated by adding a data period at a time. A series of summary and diagnostic statistics may then be obtained for each intermediate estimation (see McAleer and Fisher, 1982) and graphed to obtain evidence pointing to non-constancy<sup>7</sup>.

Let us examine each of these forecasting equations in turn. The statistics from the recursive estimates of the PDI deflator equation are graphed in Figures 5.1-5.3. These point against constancy. Note from Figure 5.1 that the worst periods (as would normally be expected) in terms of goodness-of-fit correspond to the periods of accelerating or high inflation (i.e. 1974, 1979). These also correspond to the worst periods from the point of view of forecasting performance (i.e. 1973-74, 1979-80) from the 1-step Chow sequence in Figure 5.3. Figure 5.2 basically confirms the picture given by Figure 5.3 for the early 1970s.

Moving over to the RPD1 equation, this seems to perform more satisfactorily in terms of constancy of error variance (Figure 5.4), except for a period in 1973. Again, in terms of forecasting performance, the 1-step Chow test sequence in Figure 5.6 seems to indicate that this is satisfactory except for the 1973 period, whilst Figure 5.5 gives the impression of a generally more

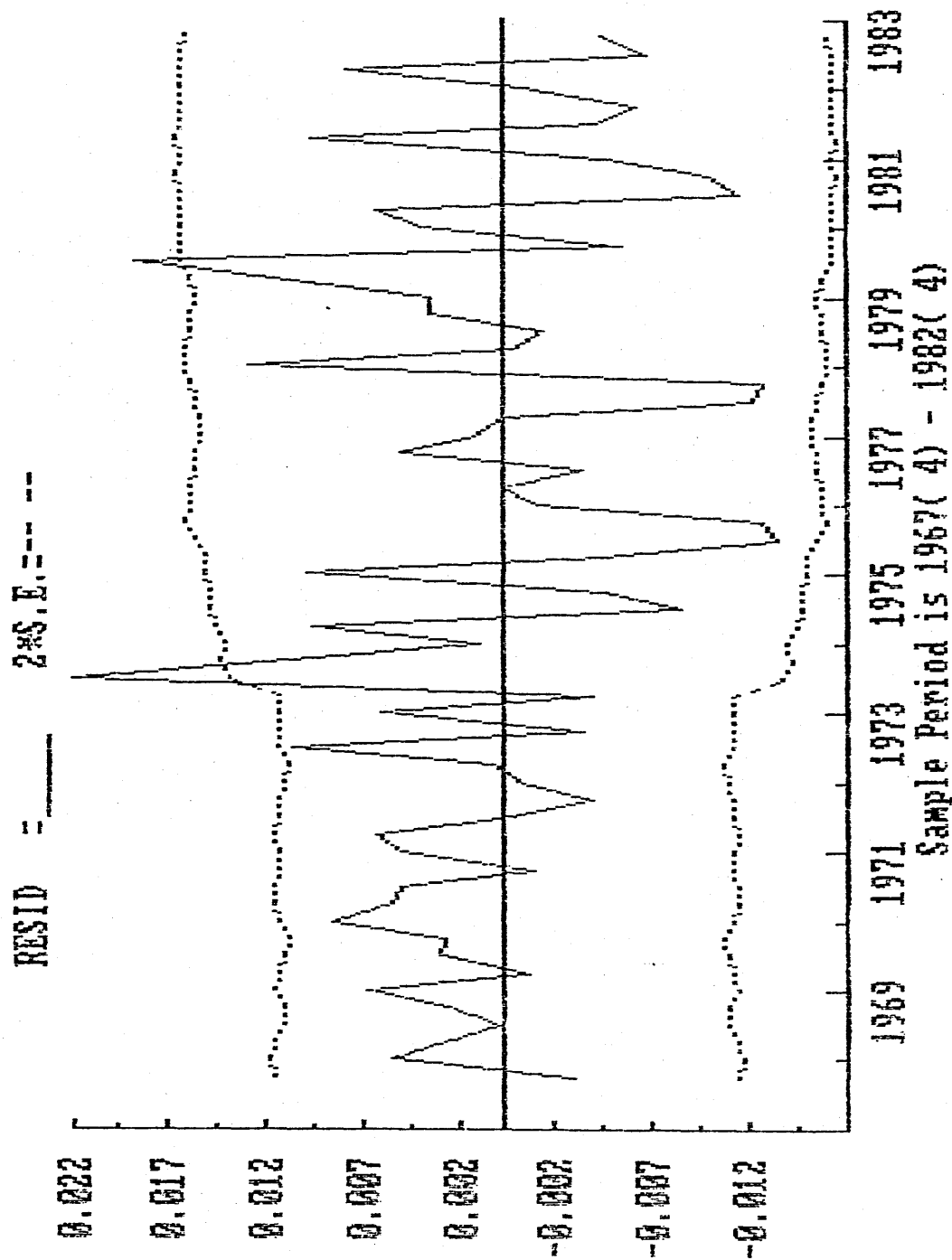


FIGURE 5.1

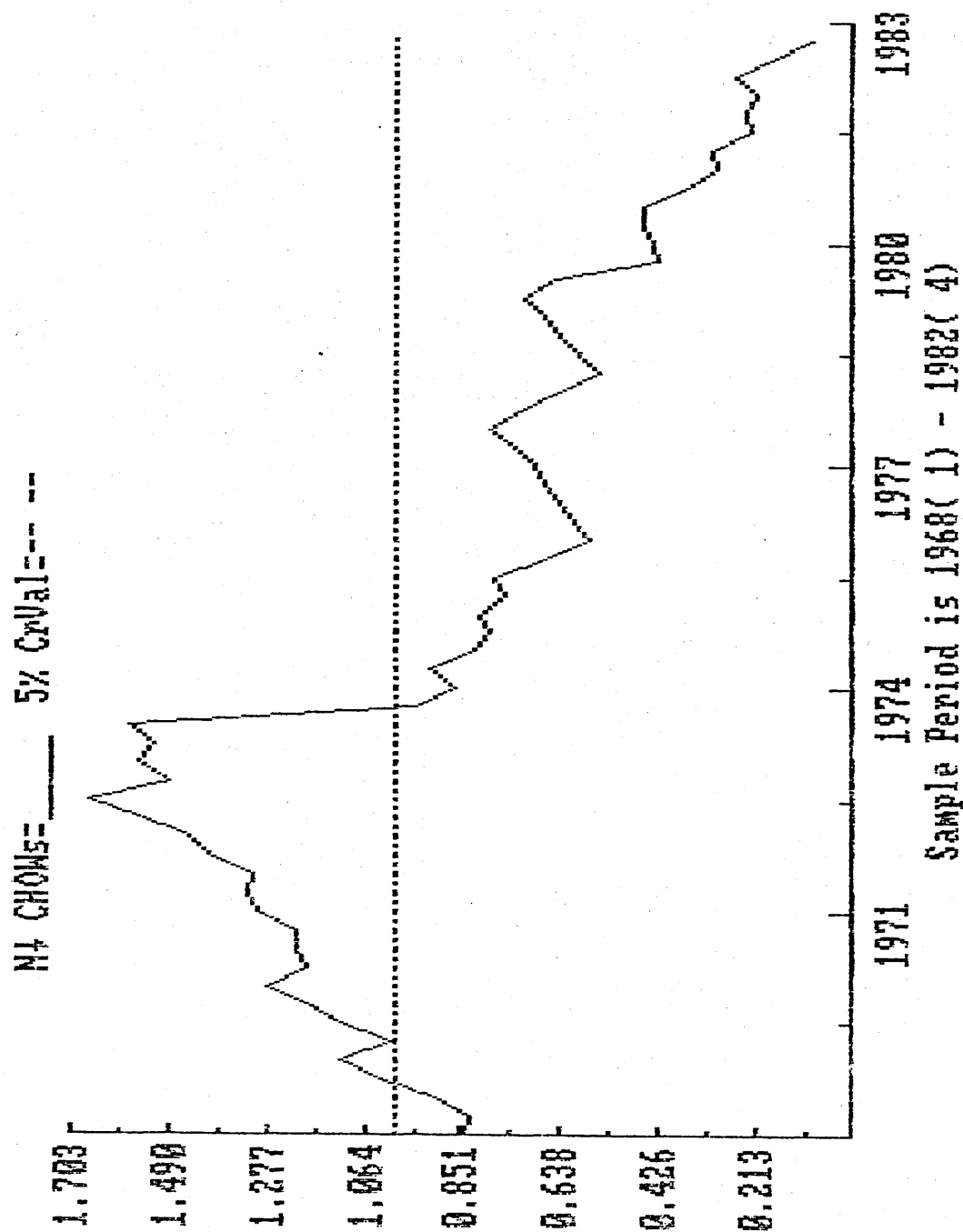


FIGURE 5.2

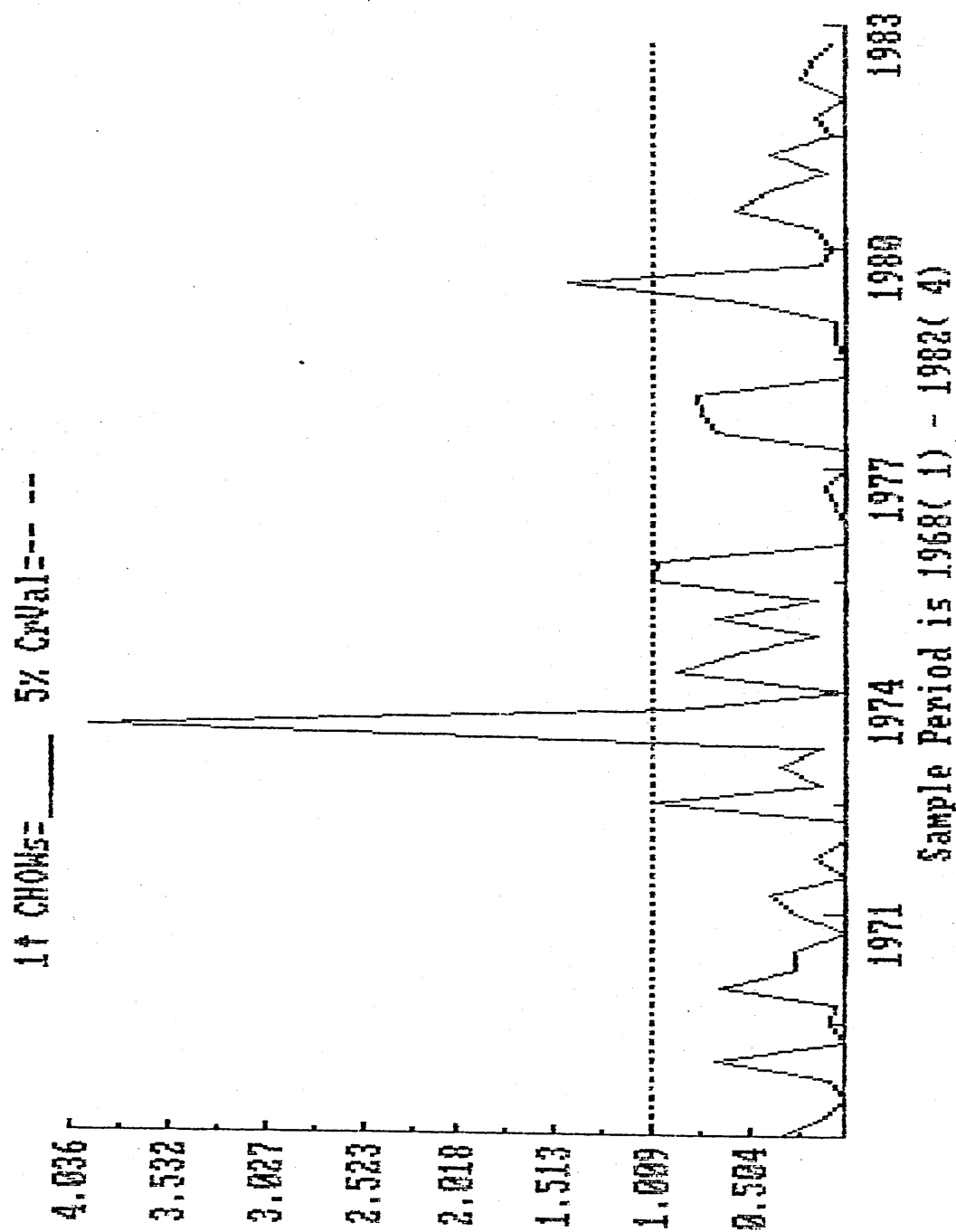


FIGURE 5.3



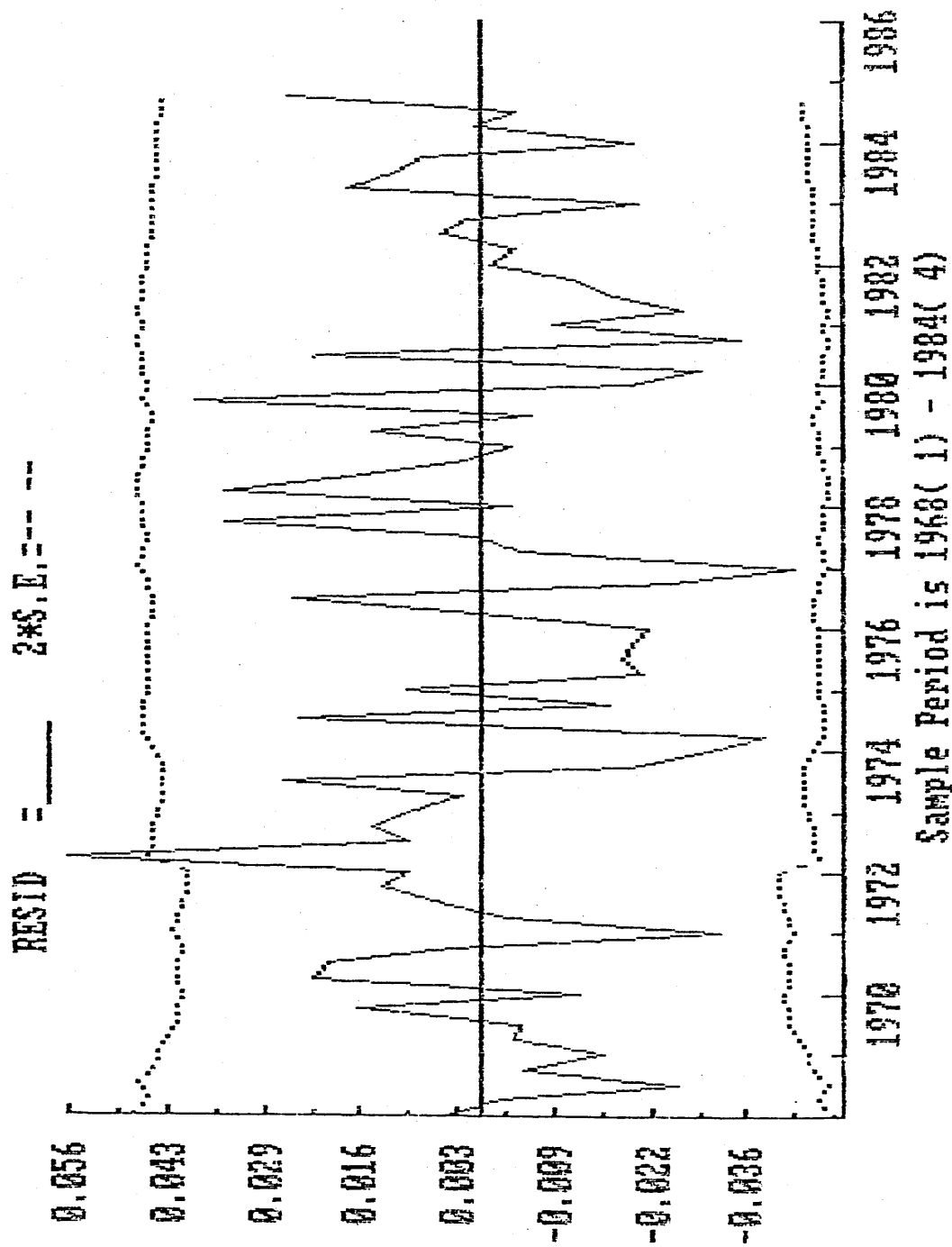


FIGURE 5.4

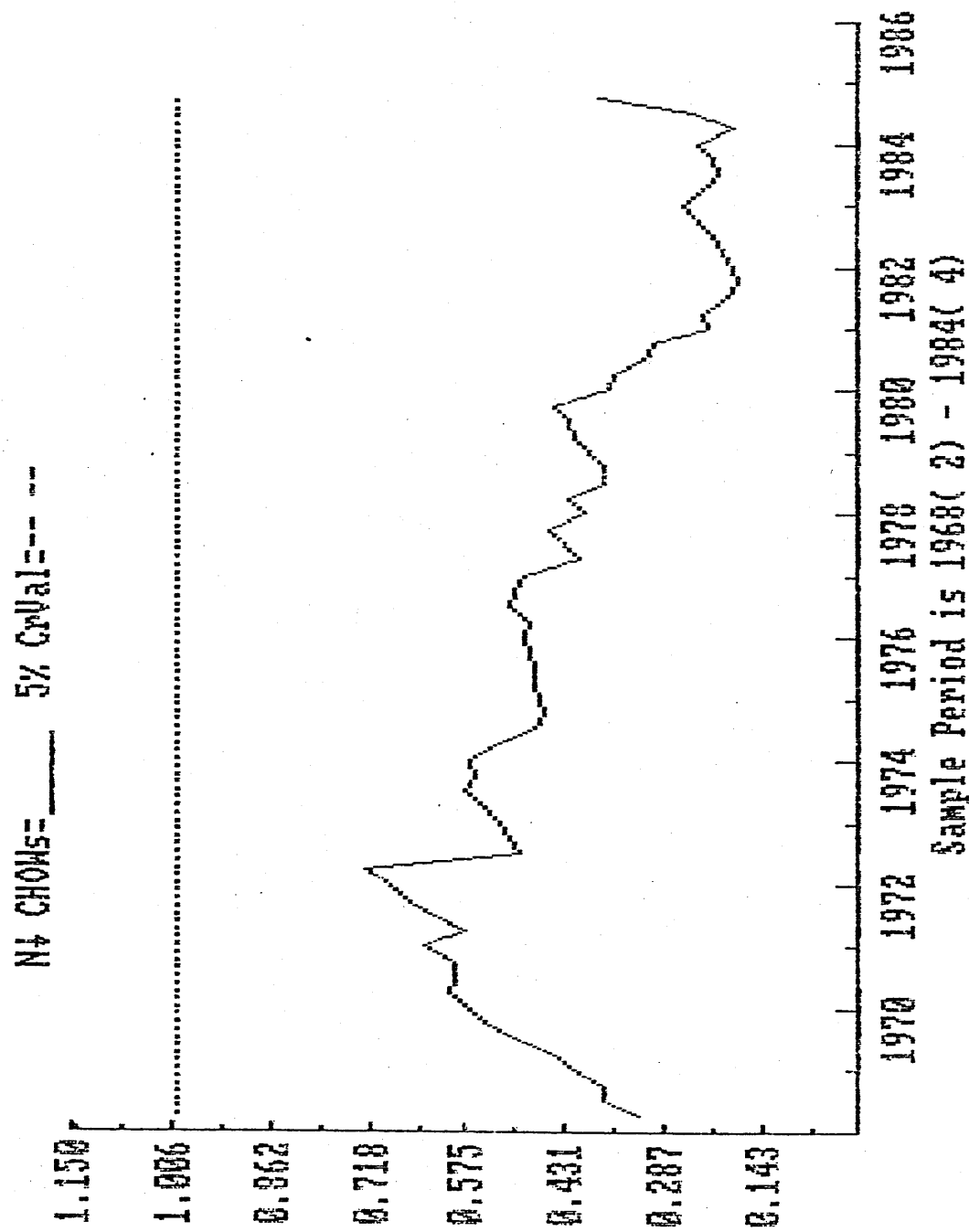


FIGURE 5.5

1↑ CHOWSE= 5% CrVal=-- --

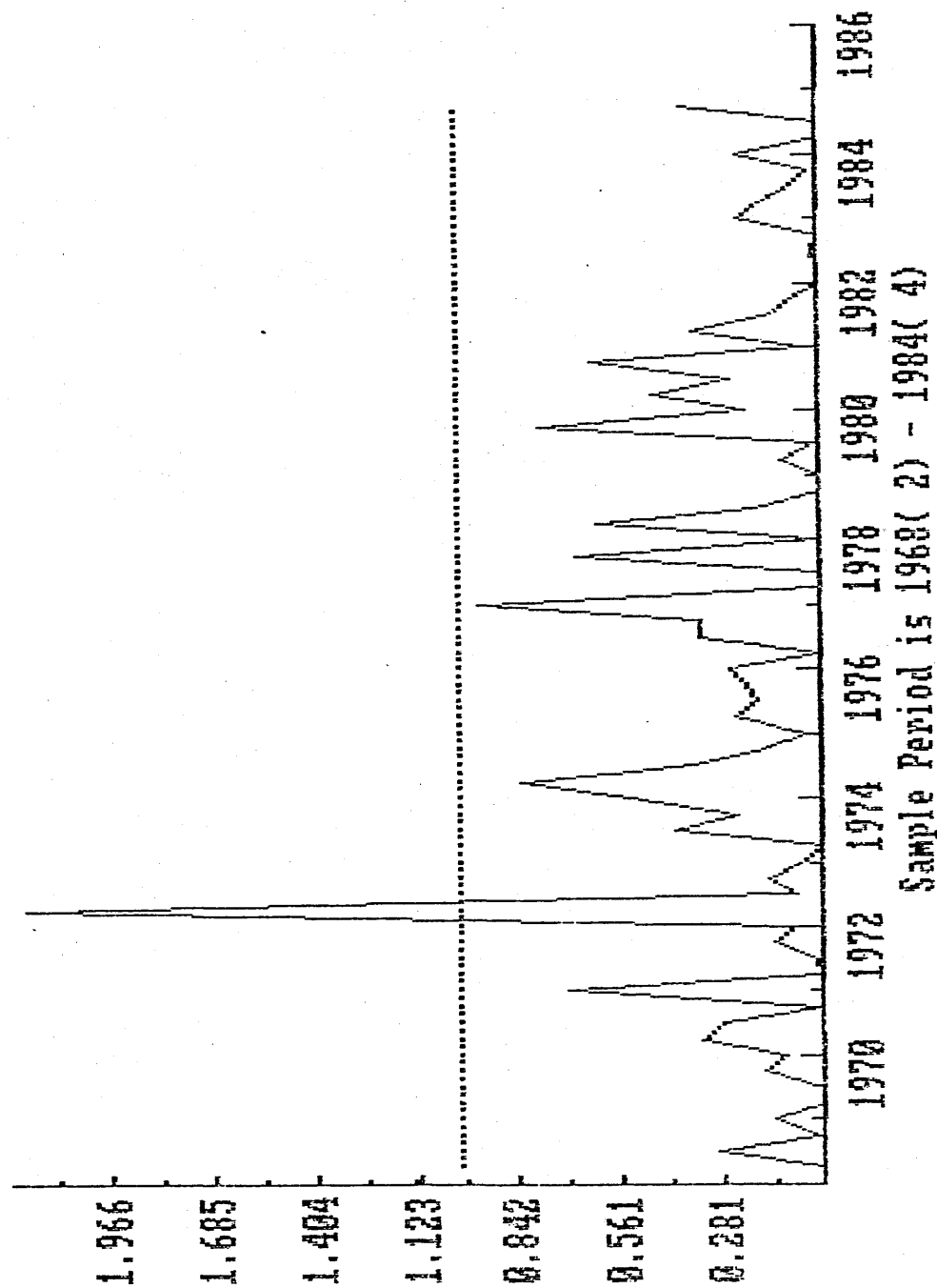


figure 5.6

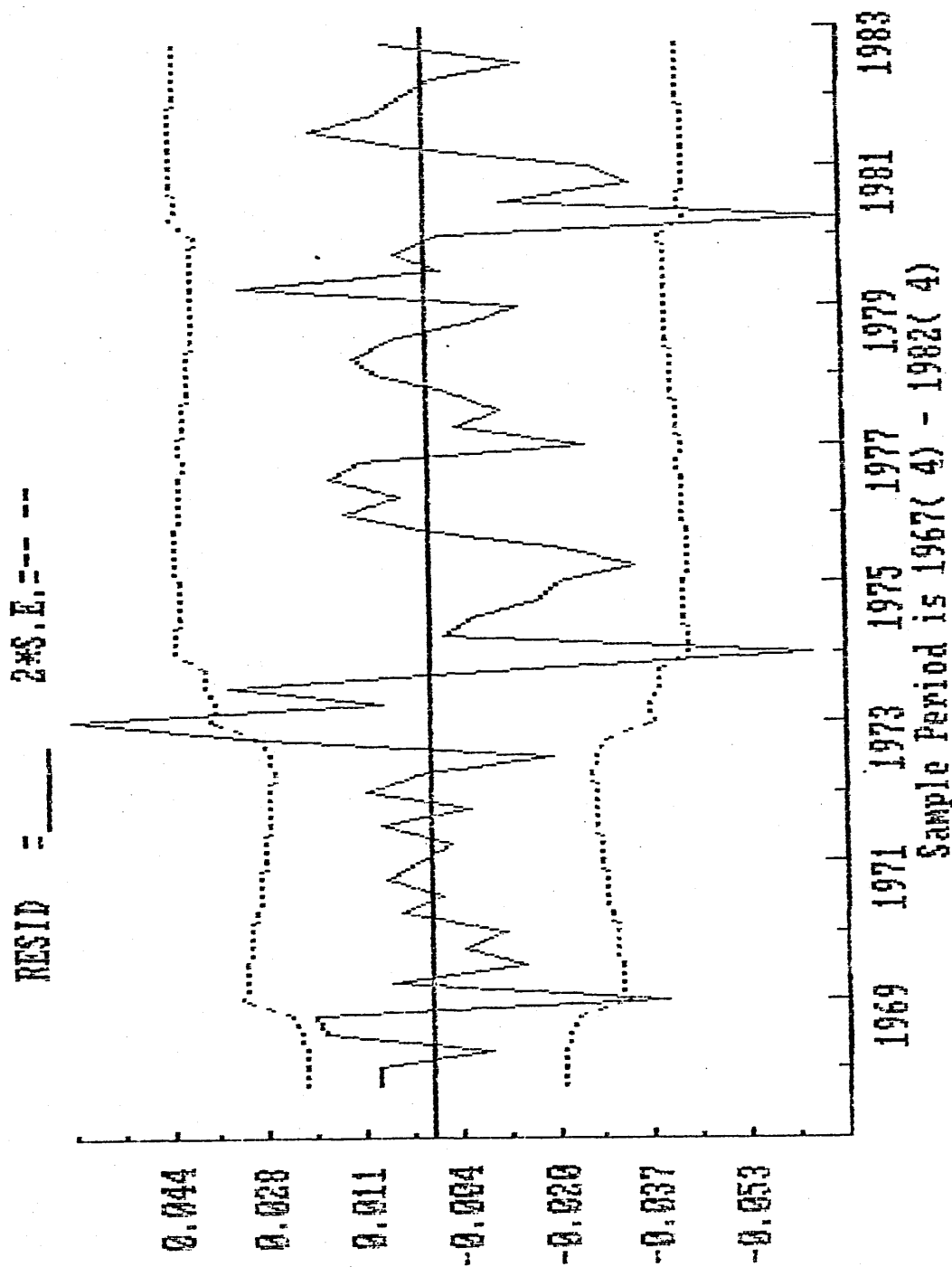


FIGURE 5.7

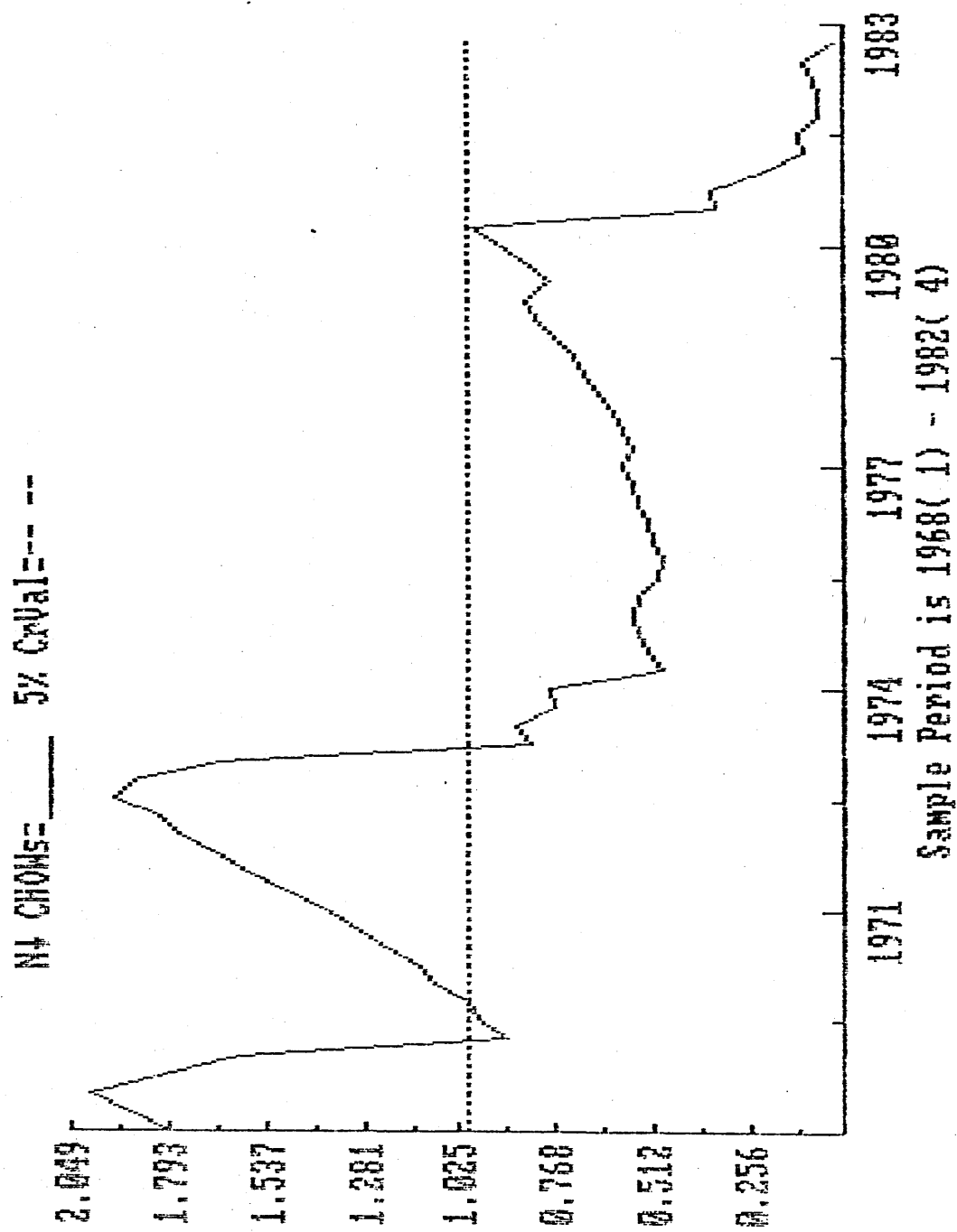


FIGURE 5.8

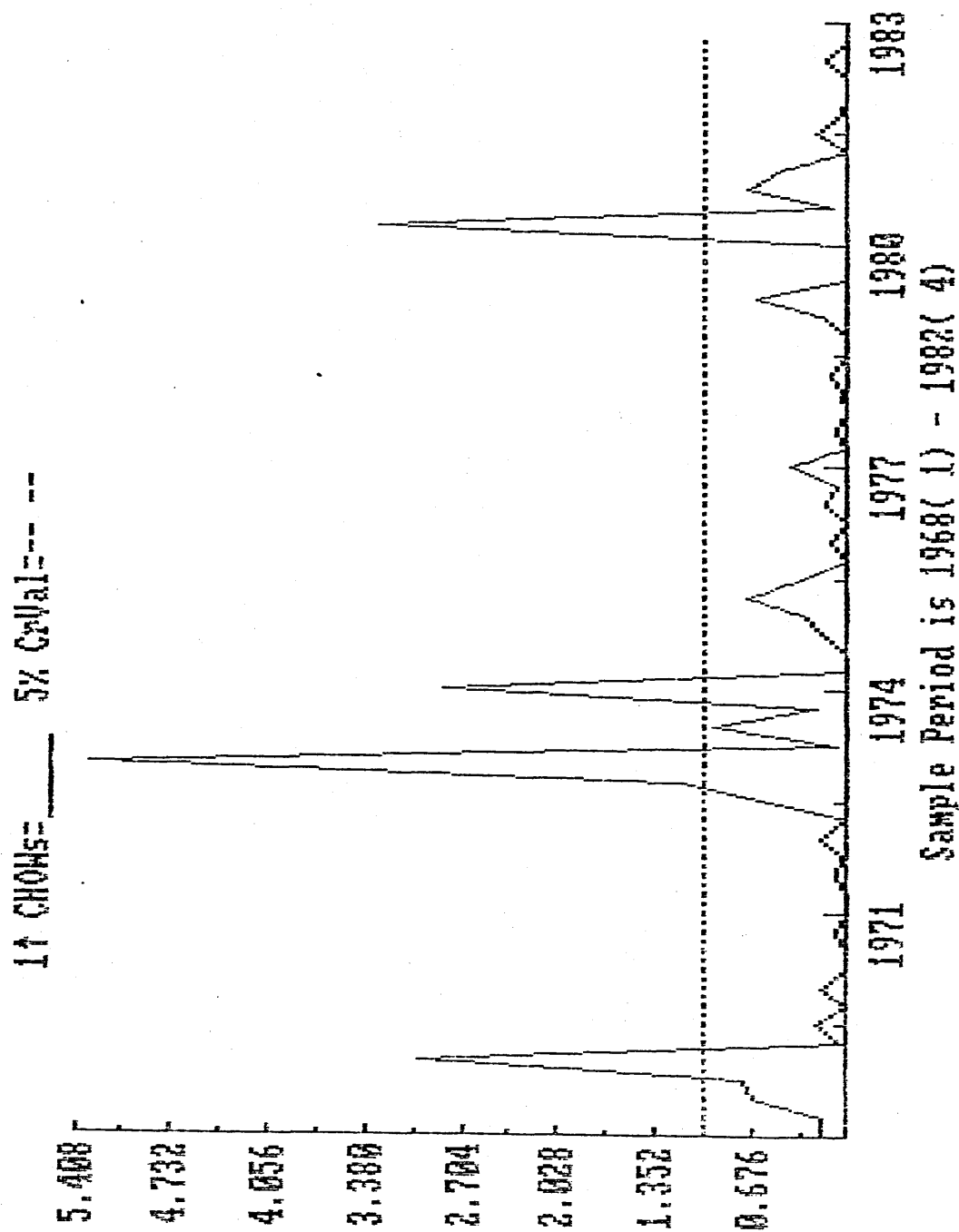


FIGURE 5.9

RESID =        2\*8.E. ---

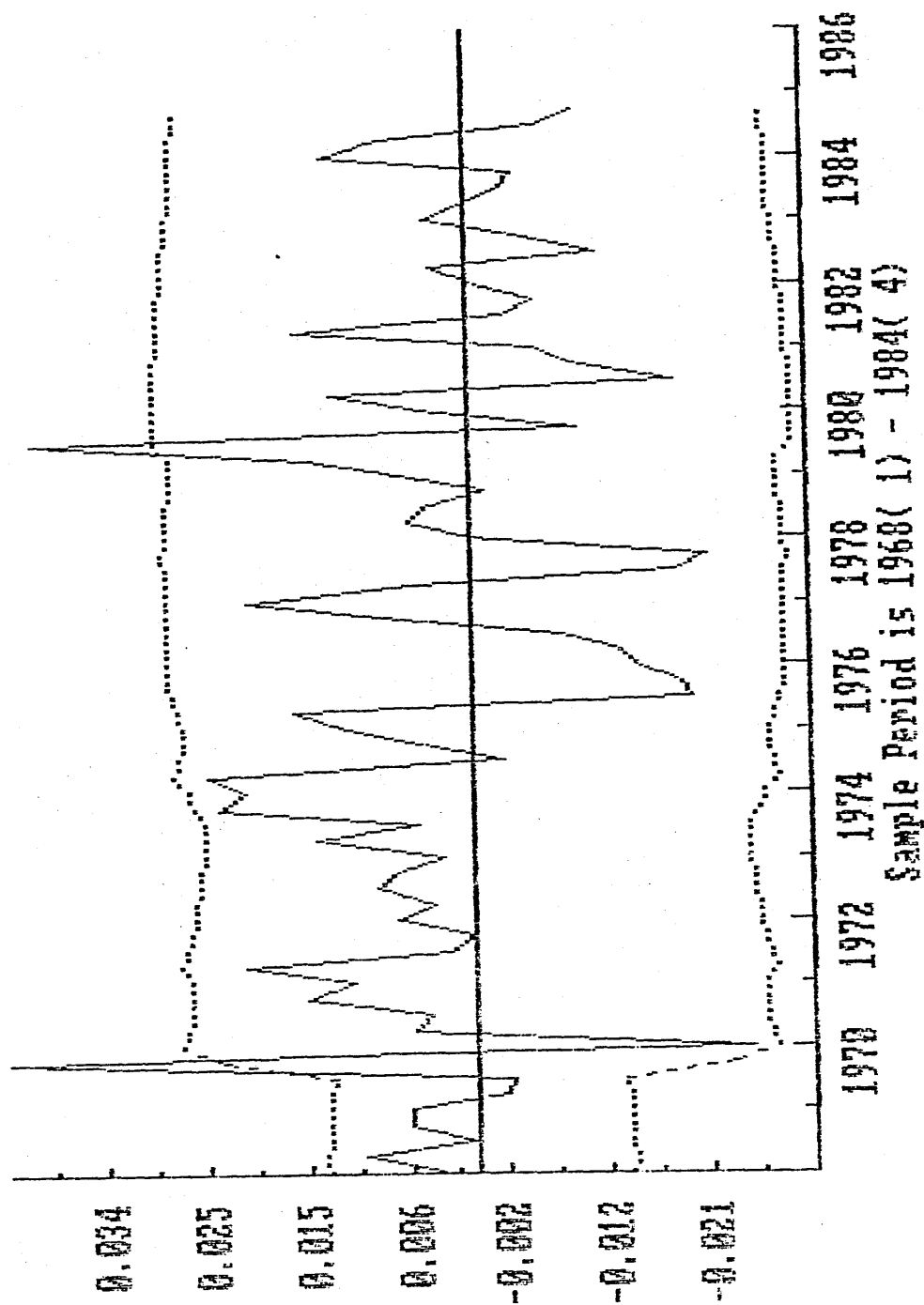
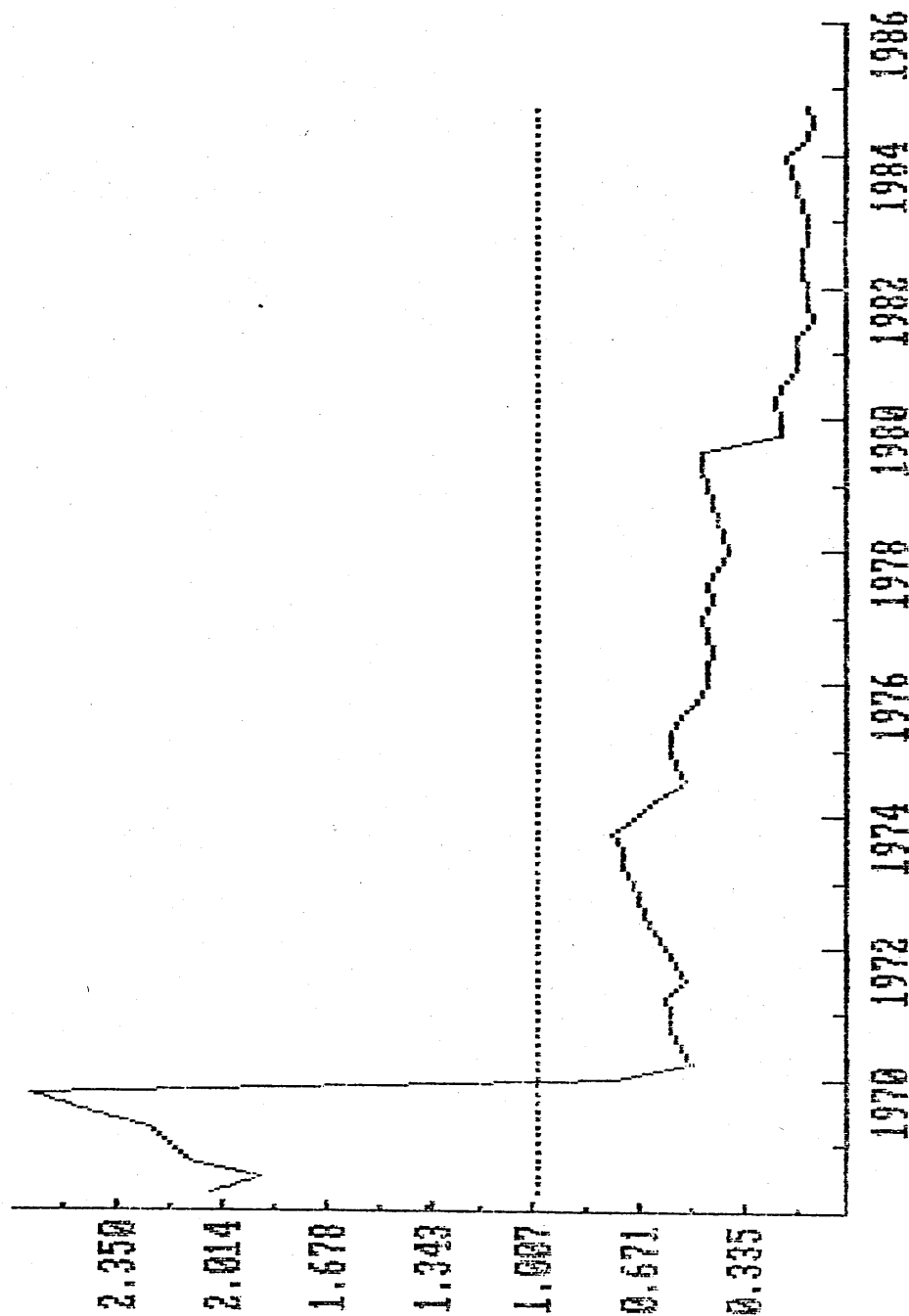


FIGURE 5.10

NT CHOH5= 5% CVAL= -- --



Sample Period is 1968( 2) - 1984( 4)

FIGURE 5.11



1↑ CHON5= 5% CrVal=-- --

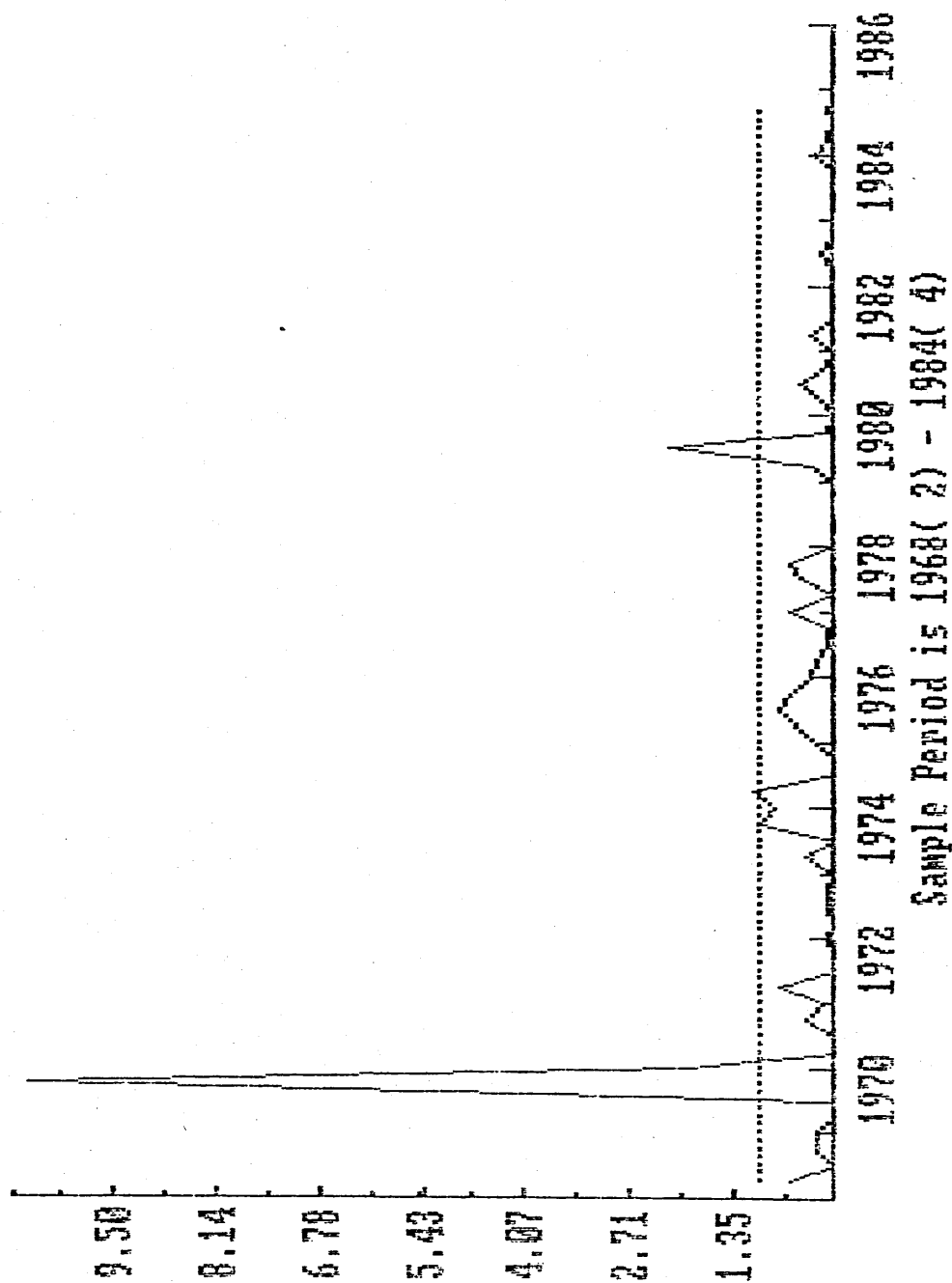


FIGURE 5.12

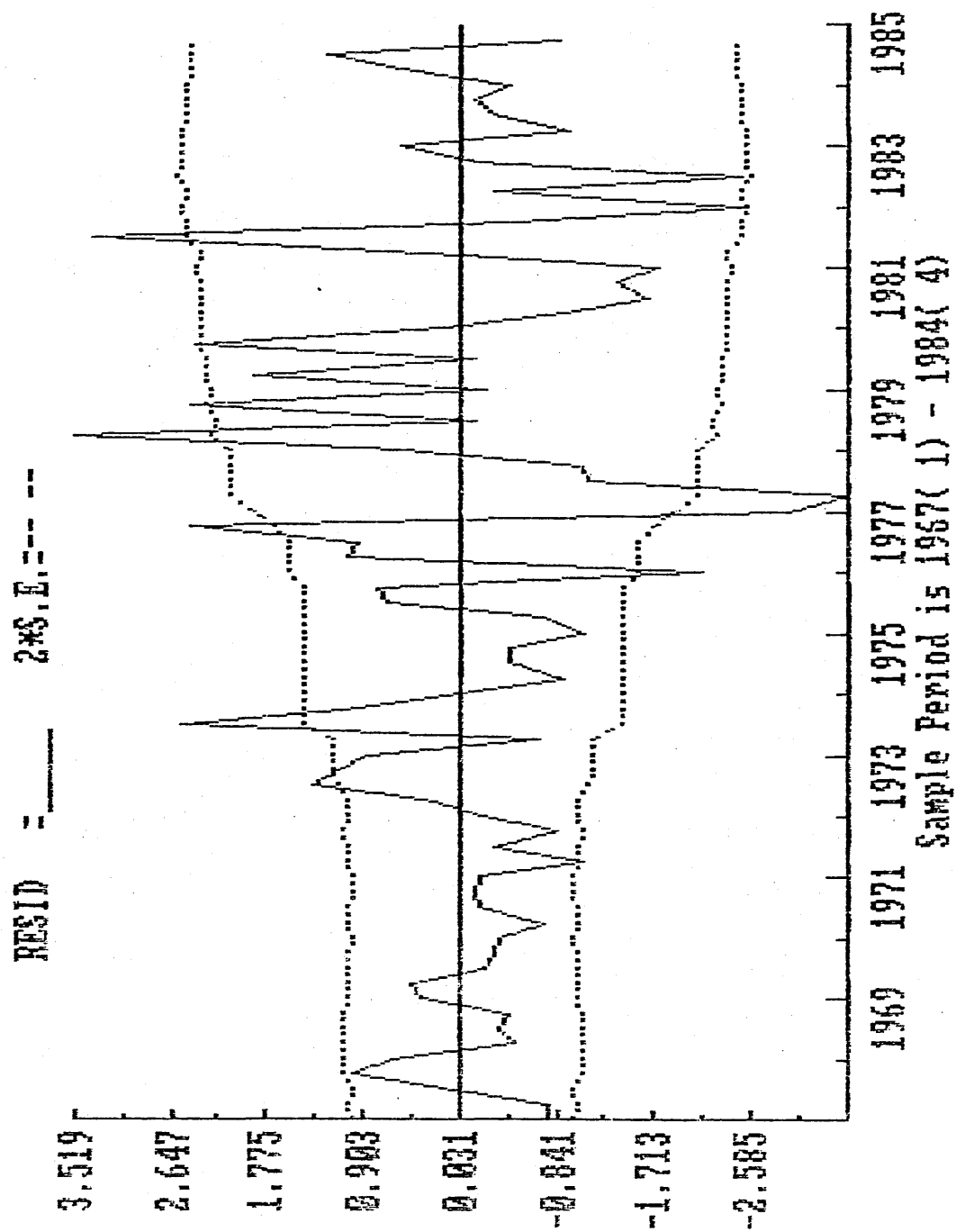


FIGURE 5.13

NI CHOW5= 5% CrVal=

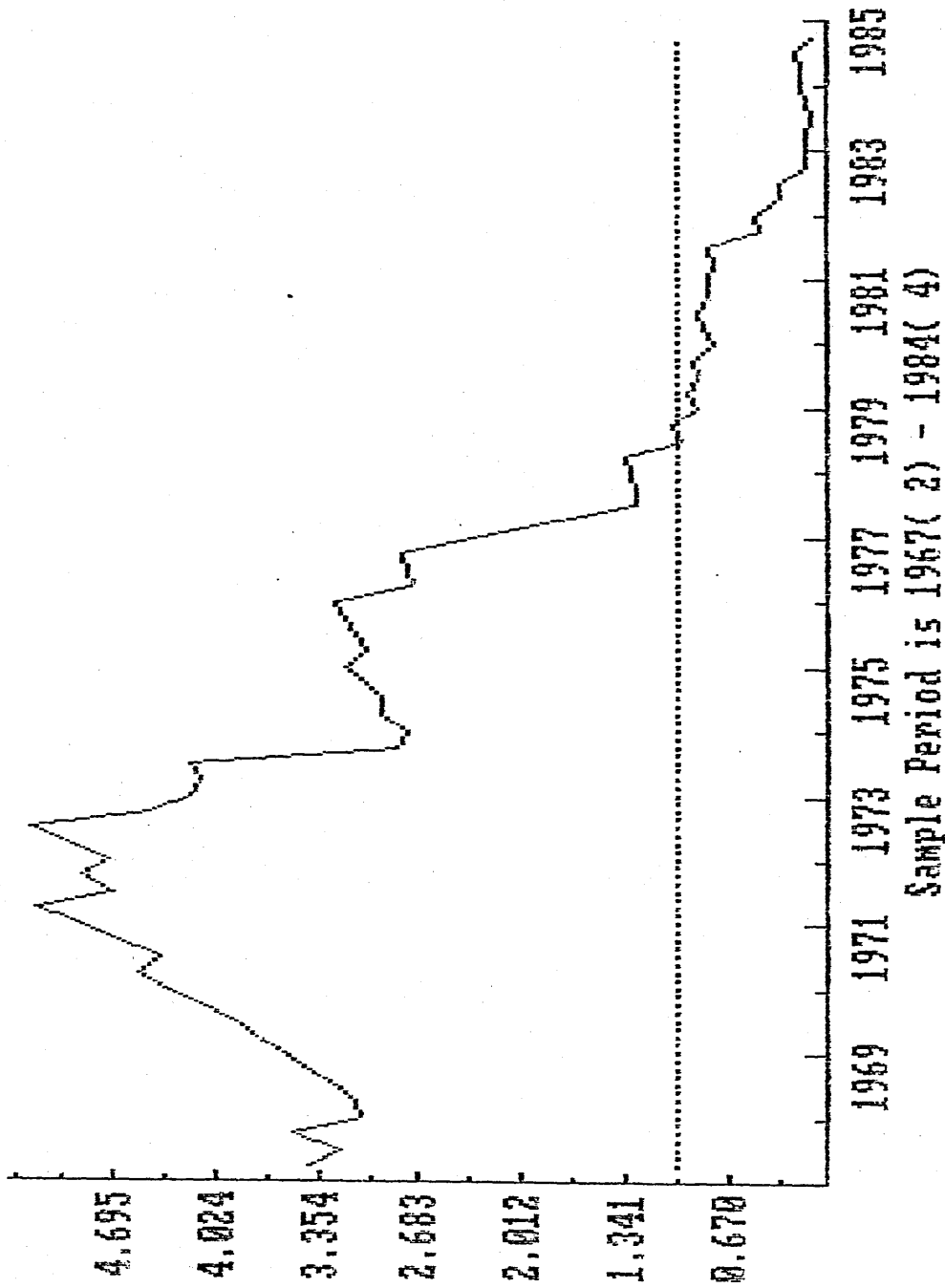


FIGURE 5.14

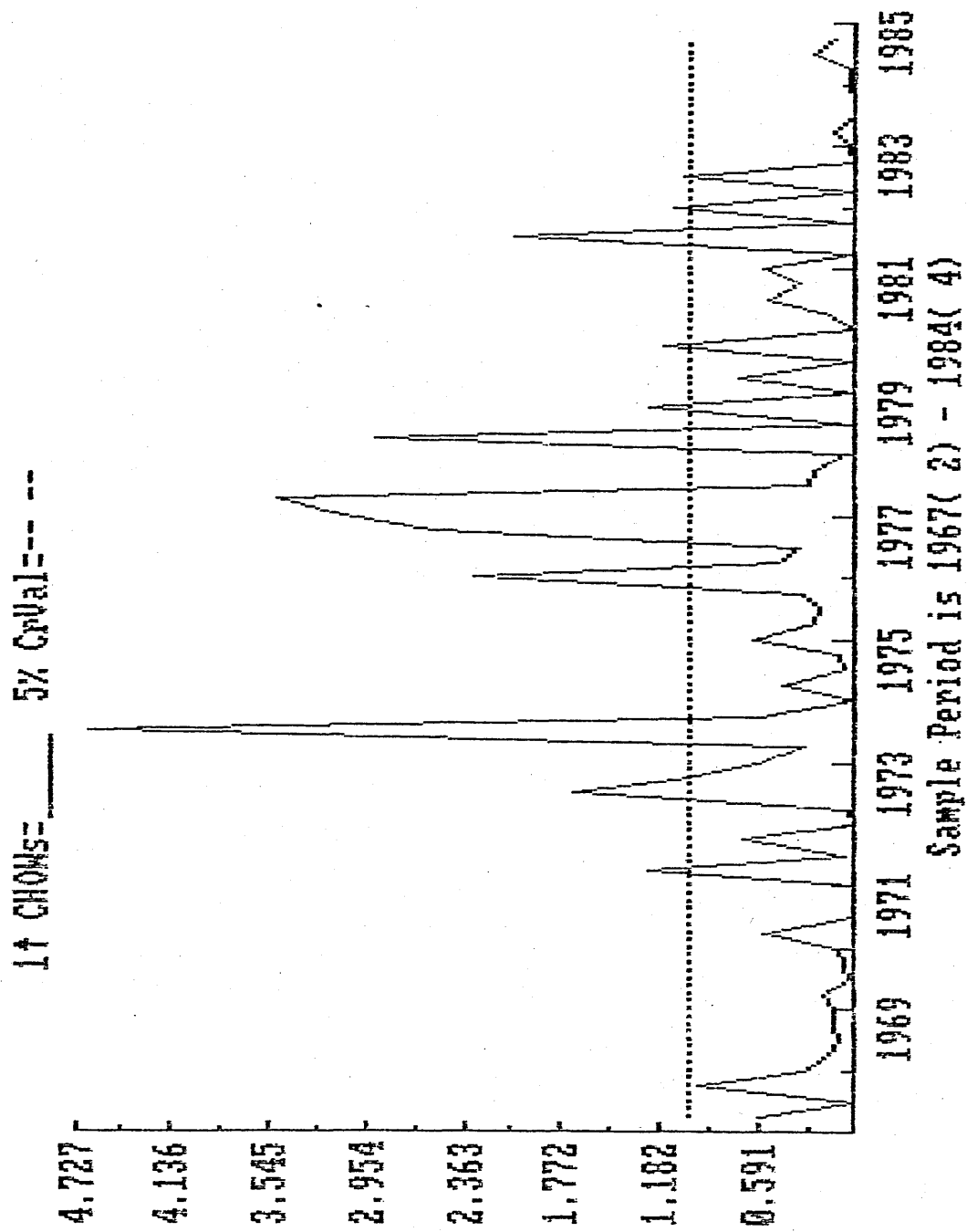


FIGURE 5.15

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satisfactory performance over the whole sample period than we had seen with equation (1). Overall, equation (2) turned out to be the best performer in terms of constancy.

In fact the marginal models for  $y$  and  $p$  constructed with PDI data performed better than those obtained with TFE data. The statistics for the latter are plotted in Figures 5.7-5.9 for the TFE deflator, and in Figures 5.10-5.12 for real TFE. Taking the price level variable first, the inflationary periods are once more a problem from the point of view of constancy of error variance, (see Figure 5.7) with the main outliers appearing in 1969, 1973-74, and 1980. From the point of view of parameter constancy, the 1-step Chow tests (see Figure 5.9) again indicate breaks in 1969-70, 1973-74 and 1980-81, all periods in which discontinuities of one type or another appeared in the UK economy. Figure 5.8 confirms the problems highlighted by Figure 5.9. As far as real TFE is concerned, there are greater problems with equation (4) than with equation (2). The years 1971 and 1980 appear to be problematic (see Figure 5.10), and the 1-step and increasing Chow test sequence confirm this problem from the point of view of forecasting performance.

Lastly, we consider the interest rate equation. This performs worst of all, and indicates that the interest rate is close to being *ex ante* unpredictable. The statistics for this equation are reported in Figures (5.13-5.15). As Hendry (1988) points out, it is difficult to imagine anyone using an equation

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such as (5) in Table 5.1 as a forecasting model for the interest rate. In fact, in his own tests Hendry (1988) finds that the real income variable is also close to being ex ante unpredictable:

"...The weak exogeneity of  $R_t$  and  $y_t$  does not seem implausible in a financial system in which agents are free to determine interest rates (albeit in an effort to control monetary growth)..."

Hendry (1988), p.20

Overall, there seems to be sufficient lack of constancy in the marginal models to suggest that the Lucas critique may well be confirmed or refuted in the case of the demand for M1. If the standard backward-looking models display a sufficient degree of constancy then, in terms of Hendry's analysis described in the previous section, the Lucas critique would to be refuted in this instance.

As we shall see in the next section, we do indeed succeed (not surprisingly) in isolating constant models for the demand for M1 using both forward-looking and feedback-only approaches. However, in addition to Hendry's own method of discrimination, as we anticipated in section two, we shall employ variance encompassing tests to rank the preferred models obtained via the two competing methods.

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### SECTION FOUR: MODEL ESTIMATION

#### 5.4.1 Estimation using TFE data

We begin our modelling process by estimating models using the TFE definitions for P and Y, as described in the previous section. The forecasting equations used for the estimation of the rational expectations models are the ones reported in Table 5.1.

One issue which has to be confronted at the outset is whether seasonal dummies should be included in our two rival models. Recall that, in contrast to some previous studies (see Hendry, 1979, 1985), the data used here is seasonally unadjusted. Seasonal effects in stochastic difference equations may either be captured through the use of seasonal dummies, or by the lag structure of the model itself in the absence of seasonal dummies (see Harvey, 1981a, Davidson *et al.*, 1978). Clearly if seasonal dummies are excluded, the lag structure of the parsimonious model in the case of the 'general-to-specific' strategy will be different than it would have been in the presence of these dummies. On the other hand, forward-looking buffer-stock models of the type considered here do not purport to explain seasonal fluctuations in money holdings, and therefore seasonal dummies should be included in these models from the outset. When presenting our results below, we compare forward-looking and 'general-to-specific' models both where seasonal dummies are present and when they are excluded from the models. As we shall see, the results prove to be unambiguous in both cases.

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Before turning to our estimations, it is useful to summarise the arguments for and against each approach. We have pointed out above that the two model selection strategies are not completely unrelated. Forward-looking dynamic cost-minimisation exercises may lead to models which are observationally equivalent to 'backward-looking' models which incorporate an error-correction mechanism. However, the main differences between the two approaches relate to the restrictive nature of the 'forward-looking' equation. Firstly, only a single lag of the dependent variable is usually allowed for in the estimation equation, whilst in the 'general-to-specific' approach no such untested restrictions are imposed on the data. Secondly, as we have seen so far in the previous sections and in Chapter 4, the forward-looking model obtained from a cost-minimisation process implies certain 'backward-forward' restrictions. These restrictions are usually tested and, if found to be data-acceptable, are imposed on the model. It should also be apparent from our previous discussion that the precise structure of testable restrictions in the forward-looking model depends on the complexity of the cost function adopted.

Thus, the main problem with the 'forward-looking' model is that it appears unduly restrictive from the outset. It is founded on the dubious assumption that economic agents minimise a simple quadratic intertemporal cost function, and this initial problem is compounded by the need to make arbitrary assumptions about the



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discount rate  $\delta$  (or alternatively by the need to use non-linear estimation methods), and the likely problems of aggregation (which are ignored by the usual assumption of the 'single representative agent'<sup>8</sup>). A cost-minimisation exercise may give an initial insight into the type of equation to be estimated, but perhaps at the end of the day the data should provide the main guide to the dynamic structure of the model (see for instance Hendry and Anderson, 1977, Nickell, 1985, for examples of the way in which dynamic cost-minimisation exercises guide but do not constrain estimation).

To counter these criticisms, proponents of forward-looking models have asserted that an RE model may provide a means to circumvent the Lucas critique (cf. the discussion in the previous section). In reply to this assertion one could make two observations.

First, 'general-to-specific' methods have so far been very successful in modelling economic relationships in general, and the demand for M1 in particular (see Hendry, 1985) with no sign of lack of constancy in these models. The significance of the Lucas critique may in fact have been overstated in this context. Furthermore, best-practice econometrics dictates that the applied economist wishing to model an economic relationship using the 'general-to-specific' approach should properly examine the time-series properties of the data he uses to detect any possible pitfalls which may emerge due to major policy changes (see for

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instance Longbottom and Holly, 1984). In any case, in adopting a forward-looking model one is trading off an uncertain advantage in explicitly modelling the processes governing  $p$ ,  $y$ , and  $R$  against a known disadvantage in imposing a dynamic structure derived from a cost function which is unlikely to conform to reality.

Secondly, as suggested in the previous section, the significance of the Lucas critique is potentially verifiable. We have already seen in section three that all but one of the forecasting equations show marked signs of non-constancy. Thus, it now only remains to check whether we can find a constant 'backward-looking' model for this data period to refute the Lucas critique, following Hendry, (1988). Furthermore, we can check whether any one of the two competing approaches produces a model which in a statistical sense provides a better characterisation of the data through the use of variance encompassing tests.

Let us now turn to the estimation of the models using TFE data. We begin with an estimation of the forward-looking model. As pointed out above, we use the forecasting equations from Table 5.1 to construct expected data series. To simplify matters, as in Chapter 4, we follow Artis and Cuthbertson (1985) in assuming that the interest rate follows a random walk, thus removing the need for an interest forecasting equation. Thus, we only use equations (3) and (4) from Table 5.1. There are good reasons for this simplifying assumption: most of our evidence suggests that  $R$

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is essentially ex ante unpredictable. Any model such as equation (5) will surely not provide any useful guide at all to the individual economic agent as to the future path of the interest rate: one might as well assume that it follows a random walk. To further back up our results, one should note that a similar equation to (5) has been estimated by Hendry (1988), and its performance is also abysmal.

The forecasting equations for  $p$  and  $y$  were used to generate the expected data series to be used. Given the number of lags used in the autoregressive equations of Table 5.1, the sample period over which the forward-looking equation was estimated was 1964(2)-1982(4), with the last 8 data periods (1983(1)-1984(4)) used to evaluate the model's ex ante forecasting performance. Note that, as in Chapter 4, we restrict economic agents' time horizon to one year into the future when estimating the solution to the Euler equation.

Using these expected data series we estimated the following conventional unrestricted 'forward-looking' model (see Chapter 4 for a detailed derivation):

$$m_t = k + \text{'seasonal dummies'} + \lambda_1 m_{t-1} + (1-\lambda_1)(1-\lambda_1\delta) \{ \alpha_1 \sum_{i=0}^{\infty} (\lambda_1\delta)^i p_{t+i}^e + \alpha_2 \sum_{i=0}^{\infty} (\lambda_1\delta)^i y_{t+i}^e - \alpha_3 \sum_{i=0}^{\infty} (\lambda_1\delta)^i R_{t+i}^e \} + m_t^u + \epsilon_t \quad (5.27)$$

where, as we can recall from Chapter 4,  $\lambda_1$  is the stable root of the Euler equation, and where

$$m_t^u = \beta_1(p - p^e)_t + \beta_2(y - y^e)_t + \beta_3(R - R^e)_t \quad (5.28)$$

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Equation (i) in Table 5.2 represents the estimated unrestricted (i.e. where the backward-forward restrictions have not yet been imposed) 'forward-looking' equation, and includes seasonal dummies. The diagnostic tests reported in Table 5.2 are the same as the ones we have reported in previous chapters. Note that the unrestricted model passes all the tests at the 5% significance level. However, note also that the price variables have very large standard errors, indicating problems of multicollinearity. These problems had also been detected in our estimations in Chapter 4, although they were less severe in those estimations (which, we should remind ourselves, were carried out using PDI-based definitions for  $p$  and  $y$ ) except where saving was added as an explanatory variable). On the other hand, one should also note that the signs of the summed coefficients for each explanatory variable have the correct sign, indicating that imposing the 'backward-forward' restrictions should yield estimates which look quite sensible. We should also note at the outset that the seasonal dummies appear to be jointly significant, and that the unanticipated shocks seem to have the correct signs, although  $p^u$  seems to be insignificant.

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**TABLE 5.2**

Estimates of the Forward-Looking Model  
TFE data - 1964(2)-1984(4) (less 8 forecast periods)

Regressor	Equation Model 1 Unrestricted	Equation (i) Model 1 Restricted	Equation (ii) Model 1 Restricted	Equation(iii) Model 1 No Seasonals Restricted
$m_{t-1}$	0.847 (0.066)	0.908 (0.037)	0.844 (0.052)	
$p_t^e$	8.826 (5.945)	-	-	
$p_{t+1}^e$	-1.036 (6.959)	-	-	
$p_{t+2}^e$	-25.456 (13.791)	-	-	
$p_{t+3}^e$	18.446 (18.819)	-	-	
$p_{t+4}^e$	-0.599 (10.342)	-	-	
$y_t^e$	0.558 (0.847)	-	-	
$y_{t+1}^e$	-0.321 (0.173)	-	-	
$y_{t+2}^e$	-0.184 (0.176)	-	-	
$y_{t+3}^e$	0.487 (0.269)	-	-	
$y_{t+4}^e$	-0.445 (1.027)	-	-	
$R_t^e$	-0.554 (0.128)	-	-	
$\Sigma(\delta\lambda_1)^i p_{t+i}^e$	-	0.025 (0.009)	0.039 (0.013)	

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TABLE 5.2 (Cont.)

Regressor	Equation Model 1 Unrestricted	Equation (i) Model 1 Restricted	Equation (ii) Model 1 Restricted	Equation (iii) Model 1 No Seasonals
$\Sigma(\delta\lambda_1)^i y_{t+i}^e$	-	0.025 (0.011)	0.035 (0.016)	
$\Sigma(\delta\lambda_1)^i R_{t+i}^e$	-	-0.477 (0.114)	-0.480 (0.168)	
$p^u$	0.076 (0.311)	-0.086 (0.307)	-0.769 (0.423)	
$y^u$	0.151 (0.117)	0.166 (0.104)	0.342 (0.152)	
$R^u$	-0.685 (0.186)	-0.729 (0.176)	-0.287 (0.251)	
Constant	0.638 (0.714)	0.038 (0.584)	0.269 (0.849)	
Q1	-0.049 (0.013)	-0.057 (0.006)	-	
Q2	0.013 (0.011)	-0.027 (0.006)	-	
Q3	-0.006 (0.011)	-0.024 (0.005)	-	
TEST STATISTICS				
$R^2$	0.999	0.999	0.998	
$\hat{\sigma}$	0.0158	0.0166	0.0248	
DW	2.46	2.44	2.67	
$Z_1$	1.66	2.02	0.44	
$E_1$	1.26	1.70	0.36	
LM(4)	2.13	2.18	7.57 *	
LM(5)	1.77	2.85 *	6.46 *	

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TABLE 5.2 (Cont.)

Equation	Equation (i) Model 1 Unrestricted	Equation (ii) Model 1 Restricted	Equation (iii) Model 1 No Seasonals
ARCH(4)	0.23	0.71	0.22
$E_4$	1.111	0.951	1.074
RESET(1)	0.064	0.203	13.59 *
RESET(2)	0.205	0.358	7.97 *

Notes: (a) Test statistics denoted by a \* reject  $H_0$  at the 5% significance level.

(b)  $\sum (\lambda_1 \delta)^i X_{t+i}^e$  denotes  $\sum_{i=0}^4 (\lambda_1 \delta)^i X_{t+i}^e$  for any variable  $X$ . In the case of the interest rate, this is simply equal to  $R_t^e$ .

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Equation (ii) in Table 5.2 estimates the forward-looking model (including seasonals) after the 'backward-forward' restrictions have been imposed (where we make the usual assumption that  $\delta = 0.99$ , following Cuthbertson and Taylor, 1987, to avoid the use of non-linear estimators). The validity of these restrictions were tested using a conventional F-test,  $F(N)$  for  $N$  restrictions on the general model (equation (i)), which is distributed as  $F(N, T-k)$  (where  $T$  is the number of observations and  $k$  is the number of regressors) under the null hypothesis that the restrictions are valid. Testing the backward-forward restrictions yielded a value of  $F(8) = 1.457$ , which is less than the critical value for  $F(8, 53)$  at the 5% significance level.

Although the 'backward-forward' restrictions hold, the results shown in Table 5.2 illustrate that the forward-looking model yields a model (equation (ii)) which exhibits significant serial correlation once the 'backward-forward' restrictions have been imposed. This model therefore fails one of the basic requirements of a properly designed model, even though it passes all the other diagnostic checks reported (in both its restricted and unrestricted forms).

To a large extent this is due to the fact that, as we have pointed out above, forward-looking models attempt to 'shoehorn' what may be a complex dynamic adjustment process into a very simple structure which results from the assumption of a simple quadratic intertemporal cost function. Furthermore, as may be



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gauged from equation (i) in Table 5.2, the estimate of  $\lambda_1$  (given by the estimated coefficient on  $m_{t-1}$ ) is 0.847. It does not take a great deal of mental arithmetic to see that, given the restriction that  $\delta = 0.99$ , the estimated coefficients on the  $p_{t+i}^e$ ,  $y_{t+i}^e$ ,  $R_{t+i}^e$  variables in equation (i) cannot realistically obey the relationship required by the 'backward-forward' restrictions. The only reason why these restrictions prove to be data-acceptable is that equation (i) is overparameterised: a model selection strategy which had not adhered closely to the cost-minimisation exercise (e.g. a 'general-to-specific' model selection strategy based on the forward-looking model) would have undoubtedly led to an equation with a very different structure from that of equation (ii). However, the methodology advanced by proponents of buffer-stock theory seems to have been centred primarily on testing these backward-forward restrictions between estimated parameters which are suggested by 'theory'. In a model which jointly estimates the forecasting equations and the solution to the Euler equation (e.g. Cuthbertson and Taylor, 1987) this would of course also involve the testing of the relevant cross-equation restrictions (see for instance Mishkin, 1983). Proponents of the forward-looking methodology make no attempt to go any further in testing parameter restrictions and to find a parsimonious model, or to take an alternative search route which may yield a model with more satisfactory statistical properties.

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To complete our initial analysis of the forward-looking model, we note that we estimated equation (iii) in Table 5.2 by excluding the seasonal dummies from equation (ii). This is to enable us to compare the forward-looking model with our alternative 'backward-looking' model in the presence and absence of seasonal dummies. We should note that equation (iii) displays an even higher value for the LM(4) and LM(5) statistics and that in addition the model also fails the RESET(1) and RESET(2) tests. These latter failures are another indication of its poor ex ante forecasting performance, and highlights once again the need to take into account a number of tests when evaluating the forecasting performance of a model. In fact, the apparent improved performance of equation (iii) compared to equations (i) and (ii) in terms of the  $Z_1$  and  $E_1$  statistics hides an almost consistent underprediction of the demand for M1. Another problem which is shared by both equations (ii) and (iii) is that the coefficient on the real price unanticipated shock variable ( $p^u$ ) becomes negative (though it remains insignificant). It should be stressed, however, that the zero restrictions on the seasonal dummies are not data-acceptable: the F-statistic for the null against the alternative provided by equation (ii) was found to be  $F(3) = 12.23$ , where the test statistic is distributed as  $F(3, 64)$  under the null: a clear rejection at the 5% significance level. Thus, overall, equation (iii) must be regarded as the 'worse performer' of the forward-looking equations.

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Turning now to our autoregressive distributed lag model, the initial equation we estimated has the following general structure (where seasonal dummies were included from the outset):

$$m_t = k + \sum_{i=1}^5 b_i m_{t-i} + \sum_{i=0}^5 (c_i p_{t-i} + d_i y_{t-i} + e_i R_{t-i}) + \text{'seasonal dummies'} + u_t \quad (5.29)$$

We should recall from our estimation of 'backward-looking' models in Chapter 3 that equations such as (5.29) may be reparameterised at the outset to replace the dependent variable  $m_t$  with  $(m - p)_t$  and the regressors  $m_{t-i}$  by  $(m - p)_{t-i}$ . For economic reasons this alternative form of the model would seem more suitable, but to enable a direct comparison with the forward-looking model, which has  $m_t$  as the dependent variable, especially in terms of non-nested tests, we began our specification search from equation (5.29).

It should be noted that this equation was also estimated over the period 1964(2)-1984(4) minus 8 quarters over the period 1983(1)-1984(4) which were kept aside for ex ante forecasts. The estimated coefficients for this equation are reported in Table 5.3. As expected this equation is overparameterised, and therefore passes all diagnostic checks. It should be noted that the seasonal dummies are significant even at this early stage in the specification process, and hence the exclusion of the dummy variables is likely to increase the value of the estimated variance of our model. This should be borne in mind when examining the final equations.

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Table 5.3

Estimates of the General Autoregressive Distributed Lag Model  
TFE data - 1964(2)-1984(4) (less 8 forecast periods)

Regressor	$m_{t-i}$	$p_{t-i}$	$y_{t-i}$	$R_{t-i}$	$Q_i$	$k$
$i = 0$	-	-0.016 (0.309)	0.079 (0.121)	-0.790 (0.193)	-	0.422 (0.702)
$i = 1$	0.717 (0.142)	0.541 (0.501)	0.255 (0.131)	0.248 (0.279)	-0.062 (0.015)	-
$i = 2$	0.345 (0.177)	-0.010 (0.492)	-0.085 (0.137)	-0.172 (0.288)	-0.060 (0.015)	-
$i = 3$	-0.425 (0.172)	-0.258 (0.443)	-0.316 (0.143)	-0.092 (0.297)	0.012 (0.014)	-
$i = 4$	-0.036 (0.172)	-0.643 (0.445)	0.043 (0.145)	-0.292 (0.293)	-	-
$i = 5$	0.284 (0.143)	0.528 (0.283)	0.147 (0.131)	0.154 (0.219)	-	-

$R^2 = 0.999$   $\hat{\sigma} = 0.0147$   $DW = 2.13$   $Z_1 = 5.52$   $E_1 = 1.89$

$LM(4) = 1.38$   $LM(5) = 1.08$   $ARCH(4) = 0.42$   $E_4 = 0.775$

$RESET(1) = 0.859$   $RESET(2) = 1.09$

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One difficulty faced in undertaking a simplification search in this study was the constraint that, at the end of the day, the forward-looking model and our 'general-to-specific' model had to be comparable. From equation (5.27) and Table 5.2 we see that the dependent variable for the forward-looking model in both its unrestricted and restricted forms is  $m_t$ . Given the presence of  $m_{t-1}$ , we may also reparameterise this model to yield a dependent variable  $\Delta m_t$ . In finding the best 'backward-looking' model we are restricted to a choice between these two dependent variables when reparameterising equation (5.29). As we shall see in the next subsection, this may stack the odds against the 'general-to-specific' approach, in restricting the range of search for the best model.

Within the framework adopted, the most parsimonious model (excluding seasonal dummies) was found to be the following:

$$\begin{aligned} \Delta m_t = & 0.252 - 0.277\Delta m_{t-1} - 0.098\Delta m_{t-3} + 0.323(\Delta p_{t-1} - \Delta p_{t-4}) \\ & (0.605) \quad (0.069) \quad (0.075) \quad (0.179) \\ & -0.221\Delta_2 y_{t-2} - 0.674R_t + 0.370y_t - 0.236y_{t-3} \\ & (0.057) \quad (0.099) \quad (0.054) \quad (0.061) \\ & -0.158(m - p)_{t-1} \\ & (0.036) \end{aligned} \quad (5.30)$$

$$R^2 = 0.746 \quad \hat{\sigma} = 0.0161 \quad DW = 1.87 \quad Z_1(8) = 1.94 \quad E_1 = 1.54$$

$$LM(4) = 1.79 \quad LM(5) = 1.41 \quad ARCH(4) = 0.83 \quad E_4 = 1.31$$

$$RESET(1) = 0.44 \quad RESET(2) = 1.74$$

We also attempted another specification search, this time

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including seasonal dummies. The most parsimonious model which passes all diagnostic checks within the narrow framework adopted is the following:

$$\begin{aligned} \Delta m_t = & 0.744 - 0.031 Q1 - 0.006 Q2 + 0.001 Q3 - 0.177 \Delta m_{t-3} \\ & (0.527) \quad (0.008) \quad (0.008) \quad (0.009) \quad (0.091) \\ & + 0.406(\Delta p_{t-1} - \Delta p_{t-4}) - 0.610 R_t + 0.267 y_t - 0.164 y_{t-3} \\ & (0.162) \quad (0.092) \quad (0.072) \quad (0.075) \\ & - 0.172(m - p)_{t-1} \\ & (0.032) \end{aligned} \quad (5.31)$$

$$R^2 = 0.783 \quad \hat{\sigma} = 0.0150 \quad DW = 2.38 \quad Z_1(8) = 1.52 \quad E_1 = 1.28$$

$$LM(4) = 1.30 \quad LM(5) = 1.65 \quad ARCH(4) = 0.57 \quad E_4 = 1.72$$

$$RESET(1) = 1.93 \quad RESET(2) = 1.37$$

It should be noted that, in sharp contrast to the restricted forward-looking model, both equations (5.30) and (5.31) pass all diagnostic tests, and in particular, there is no significant time dependence in the residuals, unlike equations (ii) and (iii) in Table 5.2. The dependent variable is such that a direct comparison via variance encompassing tests between 'forward'- and 'backward-looking' models is possible, and these will be reported in the next section. However, note that, even on this evidence alone, the general-to-specific modelling strategy appears to have delivered a model which has a more robust design. This is despite the fact that we were restricted in estimating (5.30) and (5.31) in that any reparameterisation had to yield a dependent variable which conformed to the fixed structure of the forward-looking

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model.

In fact, in this particular model it would have made far more sense to transform the model so as to obtain  $\Delta(m - p)_t$  as the dependent variable. This also explains why at first sight, there is no negative inflation effect on the demand for money in equations (5.30) and (5.31) unlike other recent estimates of the demand for M1 (see Hendry, 1979, 1985). If we re-express the dependent variable in terms of growth in real balances (more precisely  $\Delta(m_t - p_{t-1})$ , as in the model for M1 presented in Hendry (1979) and Hendry and Richard (1983)), the coefficient on  $\Delta p_{t-1}$  will become negative, thus producing a significant negative inflation effect in both (5.30) and (5.31). That is, we should expect a negative inflation effect on the demand for real balances, and not on the demand for nominal balances. In contrast, one other major disadvantage of the simple forward-looking model examined here is that it does not explicitly allow for inflation (expectations) in the demand for money.

In section five we attempt a direct comparison between the two models, through the use of conventional variance encompassing tests. Technically the restricted forward-looking models should not be put to the test in this way, as it cannot even pass all the basic diagnostics reported above (see, for example, Hendry, 1983). However, the equations shown in Table 5.2 are the best available for the forward-looking model, and as a result they will be used in our encompassing tests.

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Before we turn to this, however, we briefly provide an account of the results obtained using PDI-based data, to show that our results are not dependent upon the choice of the data set.

### 5.4.2 Estimation using PDI data

We have already carried out the estimation of the forward-looking model using PDI-based data in Chapter 4, when contrasting conventional forward-looking buffer-stock models with our alternative model incorporating saving behaviour. The only difference is that in Chapter 4 these equations were estimated over the period 1965(1)-1984(4) because the forecasting equation for saving was an 8th-order autoregression. We therefore had to re-estimate these equations adjusting the data sample so that it conformed to the one used for the TFE-based models. The estimates obtained for the forward-looking models are reported in Table 5.4.

Note that the results for equation (i) are broadly the same to those we reported in Table 5.2. Again there seems to be a high degree of multicollinearity between the different anticipated price regressors. In contrast to the TFE results, this equation yields a lower estimated value for the stable root  $\lambda_1$ , and the unanticipated price variable is now significant and its coefficient has the correct sign.



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TABLE 5.4

Estimates of the Forward-Looking Model  
 PDI data - 1964(2)-1984(4) (less 8 forecast periods)

Regressor	Equation Model 1 Unrestricted	Equation (i) Model 1 Restricted	Equation (ii) Model 1 Restricted	Equation(iii) Model 1 No Seasonals Restricted
$m_{t-1}$	0.733 (0.079)	0.864 (0.042)	0.745 (0.060)	
$p_t^e$	-2.610 (1.859)	-	-	
$p_{t+1}^e$	0.030 (2.003)	-	-	
$p_{t+2}^e$	2.355 (2.030)	-	-	
$p_{t+3}^e$	7.636 (5.609)	-	-	
$p_{t+4}^e$	-7.255 (4.366)	-	-	
$y_t^e$	-0.256 (1.631)	-	-	
$y_{t+1}^e$	-0.037 (0.604)	-	-	
$y_{t+2}^e$	-0.745 (0.963)	-	-	
$y_{t+3}^e$	-0.600 (1.154)	-	-	
$y_{t+4}^e$	1.844 (3.083)	-	-	
$R_t^e$	-0.652 (0.128)	-	-	

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TABLE 5.4 (Cont.)

Equation	Equation (i)	Equation (ii)	Equation (iii)
	Model 1	Model 1	Model 1
Regressor	Unrestricted	Restricted	No Seasonals
$\Sigma(\delta\lambda_1)^i p_{t+i}^e$	-	0.042 (0.011)	0.082 (0.017)
$\Sigma(\delta\lambda_1)^i y_{t+i}^e$	-	0.043 (0.018)	0.055 (0.443)
$\Sigma(\delta\lambda_1)^i R_{t+i}^e$	-	-0.213 (0.040)	-0.220 (0.061)
$p^u$	0.462 (0.170)	0.435 (0.174)	0.595 (0.245)
$y^u$	0.250 (0.113)	0.131 (0.104)	0.388 (0.152)
$R^u$	-0.604 (0.186)	-0.752 (0.183)	-0.322 (0.261)
Constant	0.755 (0.775)	0.185 (0.600)	1.115 (0.880)
Q1	-0.039 (0.009)	-0.058 (0.007)	-
Q2	-0.023 (0.008)	-0.033 (0.007)	-
Q3	-0.017 (0.008)	-0.027 (0.007)	-
TEST STATISTICS			
$R^2$	0.999	0.999	0.998
$\hat{\sigma}$	0.0156	0.0165	0.0252
DW	2.14	2.34	2.50
$Z_1$	1.72	1.52	0.86

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**TABLE 5.4 (Cont.)**

Equation	Equation (i) Model 1 Unrestricted	Equation (ii) Model 1 Restricted	Equation (iii) Model 1 No Seasonals
E <sub>1</sub>	1.42	1.31	0.43
LM(4)	1.62	2.74 *	10.30 *
LM(5)	1.45	2.88 *	9.55 *
ARCH(4)	0.25	0.60	0.22
E <sub>4</sub>	0.679	0.795	1.293
RESET(1)	0.242	0.225	5.55 *
RESET(2)	0.055	0.435	2.72

Notes: (a) Test statistics denoted by a \* reject H<sub>0</sub> at the 5% significance level.

(b)  $\sum (\lambda_1 \delta)^i x_{t+i}^e$  denotes  $\sum_{i=0}^4 (\lambda_1 \delta)^i x_{t+i}^e$  for any variable X. In the case of the interest rate, this is simply equal to  $R_t^e$ .

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Equation (ii) of Table 5.4 estimates the forward-looking model once the backward-forward restrictions have been imposed. The validity of these restrictions were again tested using an F-test,  $F(N)$  for  $N$  restrictions on the general model (equation (i)). The value of the test statistic was  $F(8) = 1.905$ , which is less than the critical value for  $F(8, 53)$  at the 5% significance level.

Once again, though the backward-forward restrictions are found to be data-acceptable, the results are not too satisfactory: equation (ii) shows clear signs of time dependence in the residuals. Also, excluding seasonal dummies in equation (iii) yields an equation which, as is the case for equation (iii) in Table 5.2 displays significant serial correlation, and fails the RESET(1) test (once again a symptom of consistent underprediction of the demand for M1). Furthermore, the omission of the seasonal dummies was found not to be data acceptable: the F-statistic against the null provided by equation (ii) was found to be  $F(3) = 29.83$ , where the F-statistic is distributed as  $F(3, 61)$  under the null: once more a clear rejection at the 5% significance level.

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Table 5.5

Estimates of the General Autoregressive Distributed Lag Model  
PDI data - 1964(2)-1984(4) (less 8 forecast periods)

i	Regressor	$m_{t-i}$	$P_{t-i}$	$Y_{t-i}$	$R_{t-i}$	$Q_i$	k
i = 0		-	0.381 (0.171)	0.248 (0.119)	-0.750 (0.190)	-	0.608 (0.801)
i = 1		0.619 (0.146)	-0.221 (0.244)	0.004 (0.123)	0.180 (0.280)	-0.042 (0.013)	-
i = 2		0.234 (0.177)	0.132 (0.246)	0.034 (0.144)	0.040 (0.290)	-0.023 (0.013)	-
i = 3		-0.369 (0.175)	-0.042 (0.245)	0.034 (0.142)	-0.320 (0.300)	-0.002 (0.012)	-
i = 4		0.154 (0.172)	-0.405 (0.242)	0.055 (0.136)	-0.220 (0.300)	-	-
i = 5		0.024 (0.144)	0.465 (0.242)	0.113 (0.124)	0.030 (0.220)	-	-

$$R^2 = 0.999 \quad \hat{\sigma} = 0.0148 \quad DW = 1.96 \quad Z_1 = 2.82 \quad E_1 = 1.72$$

$$LM(4) = 1.21 \quad ARCH(4) = 0.38 \quad E_4 = 0.494 \quad RESET(1) = 2.98$$

$$RESET(2) = 4.03$$

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Comparing these results with those obtained from the TFE data and reported in Table 5.2, we note that there are only slight differences in equation standard errors between the two sets of equations. The main differences probably relate to the regressors which seek to capture the unanticipated effects on the demand for money. Note for instance that, whilst  $p^u$  is insignificant and even has the wrong sign in some occasions in the equations of Table 5.2, in Table 5.4 it proves to be positive and significant in all three cases.

Let us now turn to the autoregressive distributed lag model. The general equation which we estimated (corresponding to Table 5.3 for TFE data) is reported in Table 5.5. As in the case of Table 5.3, this equation is overparameterised, and passes all the reported diagnostics.

On the basis of the general equation reported in Table 5.5, we found the following equation after a simplification search which proved to be the best model we could obtain when excluding seasonal dummies:

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$$\begin{aligned} \Delta m_t = & 1.194 + 0.395\Delta m_{t-4} + 0.470(\Delta p_t - \Delta p_{t-4}) + 0.257\Delta y_t - \\ & (0.538) \quad (0.073) \quad (0.129) \quad (0.084) \\ & -0.319\Delta y_{t-3} + 0.340\Delta R_t - 0.108y_{t-1} - 0.600R_{t-1} - \\ & (0.033) \quad (0.170) \quad (0.052) \quad (0.120) \\ & -0.198(m - p - y)_{t-1} \\ & (0.041) \quad (5.32) \end{aligned}$$

$$R^2 = 0.745 \quad \hat{\sigma} = 0.0165 \quad DW = 2.17 \quad Z_1 = 0.90 \quad E_1 = 0.87$$

$$LM(4) = 0.80 \quad LM(5) = 0.76 \quad ARCH(4) = 0.53 \quad E_4 = 0.782$$

$$RESET(1) = 1.30 \quad RESET(2) = 1.22$$

Again, as in the case of the TFE data set, we attempted a search where seasonal dummies were included. The best model which passes all diagnostic tests in this case was found to be the following:

$$\begin{aligned} \Delta m_t = & 0.095 - 0.039 Q1 - 0.017 Q2 - 0.014 Q3 + 0.128\Delta m_{t-4} \\ & (0.012) \quad (0.009) \quad (0.006) \quad (0.006) \quad (0.088) \\ & + 0.385(\Delta p_t - \Delta p_{t-4}) - 0.214\Delta y_{t-3} + 0.650\Delta R_t - 0.214\Delta y_{t-3} \\ & (0.113) \quad (0.090) \quad (0.150) \quad (0.090) \\ & - 0.630R_{t-1} - 0.136(m - p - y)_{t-1} \\ & (0.110) \quad (0.021) \quad (5.33) \end{aligned}$$

$$R^2 = 0.797 \quad \hat{\sigma} = 0.0150 \quad DW = 2.19 \quad Z_1 = 2.30 \quad E_1 = 2.01$$

$$LM(4) = 2.51 \quad LM(5) = 1.80 \quad ARCH(4) = 0.21 \quad E_4 = 0.660$$

$$RESET(1) = 1.07 \quad RESET(2) = 1.58$$

Note that (5.33) has a better goodness-of-fit than (5.32), which suggests that the seasonal dummies should indeed be present. Comparing the results obtained here with the ones we

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reported based on TFE data, we see that the latter seem to lead to a lower standard error in the case where seasonal dummies are excluded, and also lead to 'better' results in terms of time dependence in the residuals in the case where seasonal dummies are included. In general, though, there is very little to choose between the two sets of results: we cannot confirm or deny whether TFE leads to a better model of the demand for M1.

In the next section we shall compare the results obtained from our 'forward'- and 'backward-looking' models more formally. However, before we turn away from the detail of our specification searches, we should recall that all the simplifications in our 'general-to-specific' searches were done so as to yield models with  $m_t$  or  $\Delta m_t$  as the dependent variable, to allow us to calculate variance encompassing tests in section five. We have already pointed out that this might stack the results against the 'backward-looking' model (even though, on the evidence presented so far, the 'general-to-specific' modelling strategy appears to have delivered a model which has a more robust design than the 'forward-looking' model anyway). To show that a better model might have been obtained by not adhering to any arbitrary restrictions, we estimated the following model, using PDI data and including seasonal dummies, which performs marginally better than (5.33):



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$$\begin{aligned} \Delta_4(m - p)_t = & 0.345 - 0.038 Q1 - 0.018 Q2 - 0.013 Q3 + \\ & (0.502) (0.006) \quad (0.006) \quad (0.006) \\ & + 0.856\Delta_3(m - p)_{t-1} - 0.654\Delta p_t - 0.187\Delta y_{t-3} \\ & (0.048) \quad (0.133) \quad (0.084) \\ & + 0.114y_t - 0.610R_t - 0.139(m - p)_{t-5} \\ & (0.027) \quad (0.100) \quad (0.033) \quad (5.34) \end{aligned}$$

$$R^2 = 0.944 \quad \hat{\sigma} = 0.0149 \quad DW = 2.20 \quad Z_1 = 1.66 \quad E_1 = 1.49$$

$$LM(4) = 1.70 \quad ARCH(4) = 0.26 \quad E_4 = 1.20 \quad RESET(1) = 0.26$$

$$RESET(2) = 1.19$$

Although the lag structure differs because of the different reparameterisation used (see Chapters 2 and 3 for a detailed discussion of these issues), the properties of this equation are very similar to those of the other models estimated so far. It has a slightly lower standard error, and performs better than (5.33) in terms of forecasting performance, and yields a lower value for LM(4). However, for the reasons stated above, we shall not use this equation in our formal comparison of the two modelling approaches, to which we now turn.

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### SECTION FIVE: ENCOMPASSING AND LONG-RUN ELASTICITIES

Two test statistics will be used for the variance encompassing tests. We have already provided a detailed discussion of these in Chapter 3, when comparing a number of different approaches to dynamic modelling. First, we employ Pesaran's (1974) formulation of the Cox (1961) approach to testing non-nested models. The test statistic used is distributed normally as a standard normal variate when testing one model against the alternative (non-nested) model. Secondly, we embed both models into a 'general model' which incorporates all the regressors from both models. A standard F-test may then be used to test the first model against the second by testing the validity of the zero restrictions on all the regressors particular to the second model.

We have, however, other non-nested tests at our disposal. Recall from Chapter 3 that in that occasion we also employed the Ericsson Instrumental Variables test, and the Sargan (1964) test. However, the first of these gave identical results in practice to the Cox test, and the Sargan test replicated the results of the F-test. As a result we shall not use these two additional tests here, as the results prove to be conclusive anyway, as we shall see below.

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Table 5.6

## Encompassing Test Statistics

Model 1 = 'General-to-Specific' Model Without Seasonals

Model 2 = 'Forward-Looking' Model Without Seasonals

Model 3 = 'General-to-Specific' Model With Seasonals

Model 4 = 'Forward-Looking' Model With Seasonals

### TFE DATA

	<u>Model 1 vs Model 2</u>	<u>Model 2 vs Model 1</u>
Cox Test	-0.35	-24.89 *
Joint Model	0.25	12.62 *
F-Test		
	<u>Model 3 vs Model 4</u>	<u>Model 4 vs Model 3</u>
Cox Test	0.99	-8.57 *
Joint Model	0.16	2.15 *
F-Test		

### PDI DATA

	<u>Model 1 vs Model 2</u>	<u>Model 2 vs Model 1</u>
Cox Test	-0.68	-22.70 *
Joint Model	2.84 *	19.00 *
F-Test		
	<u>Model 3 vs Model 4</u>	<u>Model 4 vs Model 3</u>
Cox Test	-0.30	-7.67 *
Joint Model	1.24	3.89 *
F-Test		

Note: Test statistics denoted by a \* reject  $H_0$  at the 5% significance level.

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The variance encompassing test statistics comparing the various versions of the 'forward'- and 'backward-looking' models are reported in Table 5.6. We report the values of the test statistics for both the TFE-based and the PDI-based estimations. For both data sets we compare both the case where seasonal dummies are included, and where they are excluded in each model.

Turning first to the models estimated with TFE data, we see that where seasonal dummies are included, the 'forward-looking' model (equation (ii), Table 5.2) is rejected at the 5% significance level against the alternative of the 'general-to-specific' formulation (equation (5.31)) both under the Cox and joint-model F-test. Conversely, the 'general-to-specific' model is not rejected against the alternative of the forward-looking model under either test. Where seasonal dummies are excluded, the Cox and F-tests again reject the forward-looking model (equation (iii), Table 5.2) against the alternative of the 'general-to-specific' model (equation (5.30)), whilst the reverse tests are not significant.

Similar results are obtained from the corresponding models obtained using real PDI and the PDI deflator, with the 'general-to-specific' model encompassing the forward-looking model in all reported cases. The only exception is provided by the joint-model F-test using PDI data where seasonal dummies are excluded (comparing equation (5.32) and equation (iii), Table 5.4). In this case, the test indicates a preference for the joint model

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against each of the two competing models. This is one of the serious problems which arises with the joint model test, because it recommends the acceptance of a joint model with no theoretical basis. However, it should be noted that the 'general-to-specific' model is only narrowly rejected against the joint model at the 5% significance level<sup>9</sup>. Furthermore, we should recall that equation (iii) in Table 5.4 displayed serious serial correlation problems and failed the RESET(1) test. In contrast, equation (5.32) performed much better in terms of the reported diagnostic tests. Overall, it would be fair to say that the 'general-to-specific' models have fared better in the variance encompassing tests compared to the 'forward-looking' models.

However, the variance encompassing tests and the diagnostic tests presented so far only provide us with two sets of criteria on which to judge the suitability of econometric models in accounting for economic behaviour. An important third element is the need to have a model which is consistent with established economic theory. Though only some weight may be given to theoretical priors, they may nevertheless offer a good indication of whether the model is a useful approximation of reality. In the case of the demand for money, a good comparison of the validity of each model may be carried out by comparing the long-run elasticities of the demand for money with respect to the price level, real income and the interest rate.

In Table 5.7 we report the long-run elasticities for the

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'forward-looking' and 'backward-looking' models (where seasonal dummies are included) and in addition, the corresponding results obtained by Cuthbertson and Taylor (1987) in their estimation of a 'forward-looking' model, and by Hendry (1985) in his model of M1 obtained through the 'general-to-specific' modelling strategy. It should be stressed that the sample periods and data definitions used by these authors differ from the ones employed in the present study and that, in contrast to our results, these authors used seasonally adjusted data. Furthermore, Cuthbertson and Taylor (1987) employed a more efficient estimation method in estimating their 'forward-looking' model. As a result of these differences, it is useful to compare our results with those which are derived from what both sets of authors would consider their 'best representative model'. Also to facilitate comparisons, we again report results for both our models which used TFE data, and the models estimated using RPD1 data. Lastly, because the interest rate in our models enters in levels and not logarithms, we report the interest elasticities at a level of interest rates of 10%.

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Table 5.7

### Long-Run Elasticities in the Competing Models

#### TFE DATA

	<u>Price</u> <u>Elast.</u>	<u>Income</u> <u>Elast.</u>	<u>Interest Rate</u> <u>Elast. (at R = 10%)</u>
'General-to-Specific' (equation (5.31))	1	0.602	-0.354
'Forward-Looking' (equation (ii) Table 5.2)	1.030	1.022	-1.932
Transformed Equation (equation (5.35))	1.004	0.492	-0.368

#### RPDI DATA

	<u>Price</u> <u>Elast.</u>	<u>Income</u> <u>Elast.</u>	<u>Interest Rate</u> <u>Elast. (at R = 10%)</u>
'General-to-Specific' (equation (5.33))	1	1	-0.441
'Forward-Looking' (equation (ii) Table 5.4)	2.39	2.29	-1.117
Transformed Equation (equation (5.35))	0.920	0.864	-0.308

#### OTHER MODELS

	<u>Price</u> <u>Elast.</u>	<u>Income</u> <u>Elast.</u>	<u>Interest Rate</u> <u>Elast. (at R = 10%)</u>
Hendry (1985)	1	1	-0.560
Cuthbertson and Taylor (1987)	1.22	2.08	-0.427

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To provide a 'benchmark' against which the estimates of Table 5.7 could be assessed, we attempted to obtain 'direct' estimates of the real income, price and interest rate elasticities from an estimated autoregressive distributed lag equation. The transformation methods to obtain such direct estimates have already been discussed at length in Chapters 2 and 3 (see for instance Bewley, 1979, Wickens and Breusch 1988). By finding an appropriate reparameterisation of equation (5.29), estimates of the long-run elasticities (i.e.  $\Sigma x_i / 1 - \Sigma b_i$ , for  $x = c, d, e$ ) may be obtained. Equation (5.35) below represents our chosen transformed equation, which as we pointed out in Chapter 3, does not require the use of Instrumental Variable Estimation methods, in contrast to the transformations employed by Bewley (1979) and Wickens and Breusch (1988), and is more in the spirit of the vector autoregressive system with an error-correction mechanism proposed by Granger and Weiss (1983) when estimating relationships between cointegrated variables<sup>10</sup>.

$$\begin{aligned} \Delta m_t = & k + \sum_{i=1}^{n-1} \pi_i \Delta m_{t-i} + \sum_{i=0}^{n-1} \tau_i \Delta p_{t-i} + \sum_{i=0}^{n-1} \mu_i \Delta y_{t-i} + \\ & \sum_{i=0}^{n-1} \phi_i \Delta R_{t-i} - (1 - \sum_{i=1}^n b_i) m_{t-1} + (\sum_{i=0}^n c_i) p_{t-1} + \\ & (\sum_{i=0}^n d_i) y_{t-1} + (\sum_{i=0}^n e_i) R_{t-1} \end{aligned} \quad (5.35)$$

Using the estimated parameters of equation (5.35), we can find that the point estimates for the long-run elasticities of the demand for money with respect to the price level, real income and the interest rate (at a level of 10%) are 1.004, 0.492, and -0.368 respectively using TFE data, and 0.920, 0.864, and -



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0.308 respectively using RPDI data. These are also reported in Table 5.7. It is apparent that these estimated elasticities are very close to the values obtained from our 'backward-looking' model (equations (5.31) and (5.33)). The 'forward-looking' model yields implausibly high estimates for both data sets. In particular, in the case of the TFE data the estimated interest elasticity seems strangely at odds with the other findings, whilst the point estimates for the price and real income elasticities using RPDI are also on the high side.

Comparing our results with those obtained in other studies, we see that our RPDI 'backward-looking' model estimates correspond closely with those obtained by Hendry (1985) despite the data discrepancies. On the other hand, our 'forward-looking' model estimates do not match the results obtained by Cuthbertson and Taylor (1987). To some extent this may be due to differences in the data sets used (these authors used a different interest rate and seasonally adjusted data), but it may also be largely due to the difference in estimation methods employed. However, the elasticities reported by Cuthbertson and Taylor (1987) also differ widely from those obtained from all the other models reported in Table 5.7<sup>11</sup>. In particular, their estimate of the real income elasticity seems widely at odds with the other results.

Whatever the merits of the 'forward-looking' equation in terms of 'disentangling' adjustment and expectations parameters,

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there is no reason to believe that equations such as (5.35) which fully specify (in fact overspecify) the model dynamics should not provide reasonable estimates of long-run elasticities as they exploit the properties of cointegrated series which we discussed at length in Chapter 2 (see also Wickens and Breusch 1987, Banerjee *et al.* 1986). It does therefore seem puzzling that both the 'forward-looking' models estimated in this study and the results presented by Cuthbertson and Taylor (1987) yield estimates which differ widely with those obtained from equations such as (5.35).

The last element in our comparison of the two modelling strategies follows from Hendry (1988). As we noted above, Hendry has suggested that an examination of the constancy of both final estimated models and the marginal models generating the forecasts for our 'forward-looking' model may shed light on the whole issue of the Lucas critique.

In section three we have provided some evidence on the lack of constancy in the marginal models for the explanatory variables which has confirmed the results obtained by Hendry (1988) on a narrower data set. Using recursive estimation techniques, we found that the forecasting models reported in Table 5.1 certainly do not exhibit parameter constancy. Given the proven track record of M1 demand models one can only go along with Hendry (1988) and conclude that the Lucas critique is not a problem in that these models may not be given a 'forward-looking' interpretation.

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We have also attempted a different route from that followed by Hendry to present a formal comparison of 'forward-' and 'backward-looking' models by attempting to rank them using variance encompassing tests. This also confirmed the superiority of the 'general-to-specific' approach. It is important to point out, however, that the question of super-exogeneity is quite independent from the question of dynamic structure which has formed the other main part of our analysis, and relates to the whole issue of 'feedback' versus 'feedforward' mechanisms (including those 'forward-looking' models not obtained by dynamic optimisation exercises). The restrictive model structure obtained by slavishly following a cost-minimisation exercise merely puts the final nails in the coffin of the 'forward-looking' model by ensuring that the 'backward-looking' model variance encompasses it.

### SECTION SIX: CONCLUSIONS

In this chapter we have sought to provide a comparison of 'forward-looking' models of the demand for money M1 which had been the main theme of Chapter 4, and corresponding models obtained from the application of a 'general-to-specific' specification search on a general autoregressive distributed lag model. We have shown that 'backward-looking' models of the type analysed in Chapters 2 and 3 are observationally equivalent to 'forward-looking' models, and can therefore be given an interpretation in terms of forward-looking optimising behaviour

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on the part of economic agents. However, it has been generally argued by the proponents of the latter type of approach that 'forward-looking' models enable the applied economist to circumvent the Lucas critique if the demand for money is indeed determined by forward-looking behaviour. On the other hand, we have shown in this chapter that the type of theoretical exercise undertaken by the proponents of forward-looking buffer-stock models is likely to be only a rough approximation of reality given its reliance on quadratic costs of adjustment and static portfolios. The applied economist is therefore asked to trade-off a known benefit by only allowing the data to determine the dynamic structure of the estimating equation to a limited extent against an unknown gain by circumventing the Lucas critique. As Hendry (1985) points out: "...documented empirical evidence where this (the Lucas) critique has been shown to be the main (let alone the sole) explanation for an equation's breakdown are exceedingly rare..." (Hendry, 1985, p.73).

A priori, the attempt to 'shoehorn' a complex dynamic relationship into a rigid structure by adhering strictly to an ad hoc dynamic cost-minimisation exercise does not seem to be a promising avenue of research<sup>12</sup>.

In fact, the results presented in this chapter show that the performance of 'forward-looking' models compared to models obtained using the 'general to specific' model selection strategy in terms of diagnostic tests of model performance is

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disappointing. Furthermore, models of the latter type seem to 'variance encompass' equivalent models of the former type. Even if there may be objections to the simple two-stage OLS method of estimation applied to 'forward-looking' models in this paper, estimates obtained by other authors of 'forward-looking' models using fully efficient methods (notably Cuthbertson and Taylor 1987) also seem to be at odds with the evidence on long-run elasticities presented here. Direct estimates of price, real income and interest elasticities conform more closely to the estimates derived from parsimonious 'backward-looking' equations. Finally, the data raises doubts as to whether the demand for M1 can have a 'forward-looking' interpretation as some authors suggest.

In the absence of strong theoretical priors regarding the nature of economic dynamics it seems that applied economists have little alternative but to rely on the data to guide them towards an appropriate dynamic specification. A dynamic model obtained directly from optimisation exercises may have 'desirable theoretical features' (as in the case of the 'buffer stock' approach to the demand for money) but if it does not fully capture the properties of the data or perform better than competing models it is ultimately destined to fall into the graveyard of empirical models.

One has to point out, however, that, even if the demand for M1 balances may not be given a 'forward-looking' interpretation,

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this does not exclude the possibility that this type of forward-looking model may be applicable to broader definitions of the money stock. In this context, we should recall Milbourne's (1987) criticism of attempts to interpret the demand for transactions balances in terms of a rational expectations model<sup>13</sup>. Our attempts to verify the applicability of these methods to the demand for M1 were instead motivated by the trends followed in the recent literature.

However, some economists may doubt the validity of these models even in a broad money context, as they still involve the usual assumption of a 'representative economic agent' engaging in a multiperiod cost-minimisation exercise (for similar examples in other parts of the macroeconomics literature, see in Lucas and Rapping, 1969, Lucas and Prescott, 1971, and Sargent, 1978, 1979). Aggregation problems when economic agents are heterogeneous are usually ignored, and this may lead to erroneous conclusions<sup>14</sup>. In Chapter 4 we argued that the best way to capture the presence of 'buffering mechanisms' in the portfolio is probably to use a fully-specified multi-asset simultaneous model of the financial system.

Having said this, one cannot totally ignore the possibility that forward-looking models are relevant in the context of the demand for money (possibly for some broader definition of the money stock which may act as a financial 'buffer'). If we follow the tradition of the New Classical school and bypass the problems

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of aggregation, this raises the issue of how the adoption of monetary targets for such an aggregate will affect the process of adjustment in the economy. To answer this question, we need to combine some of the 'forward-looking' models analysed so far Chapters 4 and 5, with the literature on optimal stabilisation policy. This is an issue which we turn to in Chapter 6.

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### Footnotes to Chapter 5

(1) For instance, Nickell (1980, 1985) notes that that a modification of the cost function reported in equation (5.1) to the following expression:

$$C_t = a_0(M_t - M_t^*)^2 + a_1(M_t - M_{t-1})^2 - 2a_2(M_t - M_{t-1})(M_{t-1} - M_{t-1}^*)$$

yields an error-correction rule. Therefore, it follows that a forward-looking interpretation is not the only one which can be given to an error-correction model. Note, however, that with this type of cost function the lag structure is extremely simple.

(2) For some interesting issues of aggregation see Houthakker (1956), Trivedi (1982). As we point out further on, Pesaran (1987) has also criticised the use of a 'representative economic agent' in rational expectations models on the grounds that they may have misleading properties.

(3) The only problem with the inclusion of  $m$  in the marginal models regards the exogeneity of  $m_t$  (see Engle et al., 1983). We return to discuss these issues in the next subsection.

(4) Recall our discussion of aggregation problems in inventory-theoretic models in Chapter 4.

(5) In fact, cases where  $x_t$  and  $z_{t-1}$  have no common elements will be rare, as will be apparent from our previous discussion where in fact  $y_{t-1}$  entered the  $z_{t-1}$  vector.

(6) In particular if IV estimation of the regression of  $y_t$  on  $x_t$  using  $z_{t-1}$  as instruments is employed (see Hendry, 1988).

(7) Here we report the following statistics for each forecasting



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equation. First, we report the one-step residuals, which are graphed together with a line representing  $\pm 2$  times the equation standard error at the given sample size. Secondly, we graph a series of Chow tests for each single 1-step ahead forecast. A dotted line is also plotted to denote the 5% critical value for the appropriate statistic. The statistics themselves are scaled by the PC-GIVE package so as to yield a critical value line which is flat. Thirdly, we plot a Chow test sequence with an increasing horizon (against the final data period), again with a 5% critical value line (i.e. we report the sequence of Chow tests  $(t, \dots, T)$  using the estimated parameters up to  $t-1$  as  $t$  increases. As Hendry (1988) points out, the combination of these test statistics tend to offer a rigid test of the constancy of these forecasting equations.

(8) This assumption is one of the weakest spots of many rational expectations models although it is widely employed (see for example Lucas and Rapping, 1969, Lucas and Prescott, 1971, Sargent, 1978, 1979). See Pesaran (1987) for an example of how aggregation across different economic agents can lead to some surprising results.

(9) Two other variance encompassing tests were carried out in this case (the Sargan test and the Ericsson IV test which were employed in Chapter 3) and were not reported in the main text. These both pointed in favour of the general-to-specific model (i.e. against its rejection), confirming the result of the Cox

test.

(10) The structure of the equation obtained by this procedure is that proposed by Granger and Weiss (1983) and Engle and Granger (1987) to test the significance of the error-correction term (i.e. whether the variables are cointegrated). See Chapters 2 and 3 for further details.

(11) Table 5.7 in the main text reports the long-run elasticities for the forward-looking model which includes seasonal dummies. For the case where they are excluded (equation (iii) in Tables 5.2 and 5.4) the price, income and interest rate elasticities were found to be 1.577, 1.424, -1.943 respectively using TFE data, and 1.230, 0.867, and -0.330 respectively using PDI data. The PDI results are closer to the values obtained from our transformed equation, whilst the TFE results are even further away from the 'benchmark' estimates. One problem with the forward-looking models seems to be the wide range of values which one may obtain by changing any of the elements involved. This statement holds independently from any objections which may be raised against the efficiency of the two-stage OLS method of estimating rational expectations models.

(12) This does not imply that cost-minimisation exercises should be excluded out of hand. See Nickell (1984) for an example of how economic theory and dynamic optimisation may be deployed together to obtain a satisfactory econometric model.

(13) In particular one should recall the aggregation problems in

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an inventory-theoretic model.

(14) See Pesaran (1987) for an example.

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### CHAPTER 6: MONETARY TARGETS AND BUFFER-STOCK MONEY

#### SECTION ONE: INTRODUCTION

In this chapter, we shall outline some of the implications of the buffer stock hypothesis for the performance of money stock targets. To do this, we shall examine the properties of a theoretical buffer stock money model, when it is embedded into a simple model of the economy. The tools used here are the familiar ones of optimal stabilisation policy analysis. The main innovative feature of the material presented in this chapter is the introduction of a forward-looking model of the demand for money into the context of a wider macroeconomic model.

Before we outline the rationale behind the model proposed, we shall briefly examine some of the existing evidence on the performance of monetary targets in theoretical macroeconomic models. This review of the literature is presented in section two. Then in section three we shall present our model, and the main results are reported. A brief conclusion will follow in section four.

#### SECTION TWO: THE PERFORMANCE OF MONETARY TARGETS IN MACROECONOMIC MODELS

##### 6.2.1 Setting the Scene

In Chapter 1 we suggested that the usefulness of monetary targets as the intermediate objectives of monetary policy depends upon the empirical stability of the demand for money. In the succeeding chapters we have shown that there are model selection

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procedures which allow us to estimate models which appear to display a certain degree of parameter constancy. This applies for both M1 and M3. Nevertheless, the resulting empirical models have lag structures which are complex, and data-determined, with little economic rationale behind these short-run dynamics. Forward-looking models may provide one interpretation of these dynamics in terms of some intertemporal optimisation exercise. Although we have shown in Chapter 5 that such an interpretation may not be valid in the case of a 'narrow' definition of the money stock such as M1, this does not imply that forward-looking models of broader aggregates may not perform better. In fact, as we pointed out in Chapter 4, the choice of the appropriate aggregate is likely to be vital in modelling the buffer stock approach in a single-equation context. This is because any such single-equation model is likely to be only an approximation to a more complex multi-asset portfolio adjustment process.

Whatever the outcome of further empirical tests of buffer-stock models, the complex nature of the estimated money demand equations leads us to ask the following questions:

- (i) How is the performance of monetary targets in macroeconomic models affected by the presence of a lagged adjustment process? (In this context, we could also ask whether other intermediate targets are preferable to monetary targets).
- (ii) If forward-looking behaviour is present in the demand for money, how will this affect the economy in the presence of

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monetary targets?

In Chapter 1 we recalled the classic Poole (1970) argument which linked the importance of money supply targets to the 'stability' of the LM schedule in the IS-LM model (and by implication the stability of the demand for money). However, this type of model (like many of its successors) is stochastic but static, and therefore is not a particularly appropriate framework for a discussion of the advantages and disadvantages of monetary targets vis-a-vis other economic variables in a dynamic world. In practice, the application of monetary policy has to contend with the existence of structural lags in the economic system, in addition to the existence of stochastic disturbances (the latter being the main theme of the Poole analysis).

One further disadvantage of the older literature on intermediate targets, indicators and instruments (see for instance Poole, 1970, Pierce and Thomson, 1972, B.Friedman, 1975, Courakis, 1981) is that the terminology adopted is hopelessly muddled, without any trace of a systematic taxonomy, and the models advanced are often rife with strange two-stage approaches which examine separately the links between the instrument and the intermediate objective, and between the intermediate objective and the final objective.

This literature has essentially been overtaken by more recent work which has used control theory in the study of the design of macroeconomic policy and which, if nothing else, has

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helped economists to focus their minds on the structure of the problem under scrutiny. There already exists a vast literature on the performance of monetary targets in stochastic dynamic macromodels, which essentially seeks to answer question (i) above. We survey this material in subsection 6.2.2. Afterwards we shall turn to question (ii) in section three.

Unfortunately, as we shall see, the use of optimal control theory techniques leads to rather complex models which are difficult to solve analytically. Therefore much of the current literature has resorted to simulation methods to obtain some meaningful results from these models. Before turning to these models, however, it is useful to provide a brief account of the optimal control methods which shall form the basis of much of the analysis of this chapter.

### 6.2.2 Optimal Control and Optimal Stabilisation Policy

We may set up an optimal control problem in either continuous or discrete time. Here we shall follow the former course. It is generally assumed that the economic agent (in this context the policymaker) attempts to maximise or minimise an objective function over a (possibly infinite) time horizon. This objective function generally depends on the policymaker's final objectives (targets) and his policy instruments (assuming that the latter may not be varied costlessly over time)<sup>1</sup>. Usually a quadratic cost function is used, of the form:

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$$C = \int_{t_0}^T \frac{1}{2} (x'(\tau) Q x(\tau) + u'(\tau) R u(\tau)) d\tau \quad (6.1)$$

where  $x$  is a vector of state variables (some of which will be the final objectives of interest to the policymaker),  $u$  is a vector of instruments,  $Q$  and  $R$  are weighting matrices, and where the integral is minimised over the period  $t_0, \dots, T$ .

It is generally understood that the economic system may be adequately characterised by a set of differential equations linking the state variables to the policy instruments. This is represented by the following set of equations:

$$dx(t)/dt = Ax + Bu \quad (6.2)$$

where  $A$  and  $B$  are matrices of coefficients. Equation (6.2) may be seen as a reduced form where any non-dynamic endogenous variables have been substituted out.

The problem of minimising (6.1) subject to (6.2) may be solved using Pontryagin's maximum principle (see Intriligator, 1971, Hadley and Kemp, 1971). Usually the solution relates the control variables to the final objectives (see Intriligator, 1971, for details of the solution procedure):

$$u(t) = F(t)x(t) \quad (6.3)$$

where the  $F$  matrix is related to the coefficient matrices  $Q$ ,  $R$ ,  $A$ , and  $B$ . The optimal control solution is therefore to relate the instrument setting to all the state variables of the system. To paraphrase B.Friedman's (1975) view on the conduct of monetary policy, 'all information is valuable' in constructing state-contingent rules for policy instruments.



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One possible answer to the question: 'should we adopt monetary targets?' is therefore: 'yes, but do not rely exclusively on information regarding the money stock in making policy decisions'. This makes questions (i) and (ii) above redundant, as we should not rely on one single 'intermediate objective' for monetary policy. Similar questions such as the 'assignment problem' would also become redundant in such a framework.

However, some economists would argue that this is not a wholly satisfactory approach as it stands. There are a number of reasons for this. First, finding the fully optimal solution requires a considerable amount of computation, especially when these methods are applied to full-blown econometric models of the economy. Secondly, some state variables may prove to be almost irrelevant as a guide to contingent rules (i.e. the feedback coefficients may be negligible). This may occur if two variables contain very similar information about the system dynamics. In this case, a fully optimal rule would seem to be unnecessarily complicated in comparison to simpler rules which excluded such 'almost irrelevant' variables. Thirdly, simple rules where single policy instruments are made to be contingent on single state variables are better understood by the private sector, which explains why they are attractive to governments (see Currie, 1985). For instance, there is evidence that the widespread adoption of monetary targets in the 1970s in many OECD countries

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was in great part due to their simplicity. Fourthly, the above techniques assume that the policymaker knows the structure of the economy with complete certainty. It could be that fully optimal contingent rules are not particularly robust across different economic structures, and that over a number of scenarios they may be dominated by simpler one-target rules. These points make questions (i) and (ii) relevant once more, as in such a context it becomes important to consider whether any given single intermediate objective framework performs better than the others.

The performance of simple rules in stochastic dynamic models (including the use of monetary targets) has been examined by Currie and Levine (1984, 1985). For instance, Currie and Levine (1985) examine the performance in terms of welfare losses of adopting optimal simple rules (i.e. one-target rules where the feedback parameter is chosen so as to minimise the intertemporal cost function) in different scenarios. Currie and Levine (1985) show that no single type of rule performed better than others at all times. The major factor which determined which rule dominated the others seemed to be the source of the disturbances impinging on the economy.

Currie and Levine (1984) also show that for a wider range of parameter values the stability of the system is guaranteed by exchange rate contingent rules than the equivalent money stock rules for the interest rate. This suggests a certain preference for an exchange rate targeting regime, and it should be borne in

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mind that the model allowed dynamics in the money demand equation, so that these experiments can be seen as addressing question (i) above. It is easy to show using the Currie-Levine (1984) model that a more rapid adjustment process in the demand for money equation tends to improve the stability of the model, and hence makes monetary targets for simple rules more feasible.

This literature answers one of our questions regarding the efficiency of monetary targets in the presence of lagged adjustment in the demand for money. However, it does not address the question of what the effect will be of the conjunction of monetary targets and forward-looking behaviour in the demand for money along the lines of the models described in Chapters 4 and 5. We turn to this question in the next section. .PA

### SECTION THREE: FORWARD-LOOKING MODELS AND MONEY SUPPLY TARGETS

#### 6.3.1. A Digression: Solution Methods in Models with Forward-Looking Variables

Before we examine the performance of monetary targets in a simple macroeconomic model with forward-looking behaviour in the demand for money, we need to survey some results relating to the solution methods of dynamic stochastic rational expectations models which shall turn out to be useful further on in this section. The solution procedures considered below follow from the work of Blanchard and Kahn (1980), Dixit (1980), Chow (1979), and Currie and Levine (1982).

Consider a model which can be described by a series of  $n$

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differential equations<sup>2</sup>:

$$\begin{bmatrix} dy \\ dx^e \end{bmatrix} = A \begin{bmatrix} y \\ x \end{bmatrix} dt + \begin{bmatrix} du_1 \\ du_2 \end{bmatrix} \quad (6.4)$$

where  $y(t)$  is a vector of  $(n - k)$  variables which are predetermined at time  $t$  (i.e. these variables are not 'free' to take up any value at time  $t$ ), and where  $x^e(t)$  is a vector of  $k$  expectational, 'free', or non-predetermined variables. Matrix  $A$  is a  $(n * n)$  matrix of coefficients, and the  $du_i$  are Wiener processes with a covariance matrix  $\theta$ .

To solve a rational expectations model of this type we first have to examine the solution for the mean trajectories. We shall turn to the stochastic properties further on. In order for a rational expectations model of this type to be stable, we require there to be the same number of positive eigenvalues as there are 'free' or 'jump' variables, i.e.  $k$ . If we then define  $M$  as the  $(n * n)$  matrix of  $n$  eigenvectors of  $A$ , and  $\Lambda$  as the  $(n * n)$  diagonal matrix with the eigenvalues of  $A$  on the main diagonal, <sup>x</sup> then it follows from the definition of characteristic roots that:

$$MA = \Lambda M \quad (6.5)$$

Assuming that there are the same number of positive eigenvalues as there are 'free' variables, we may then partition these matrices conformably as follows:

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$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad (6.6)$$

where  $\Lambda_1$  is the  $((n - k) * (n - k))$  diagonal matrix of negative (stable) eigenvalues, and  $\Lambda_2$  is the  $(k * k)$  diagonal matrix of positive unstable eigenvalues. The submatrices  $M_{ij}$  and  $A_{ij}$  are partitioned conformably such that  $M_{11}$  and  $A_{11}$  are  $((n-k)*(n-k))$ ,  $M_{12}$  and  $A_{12}$  are  $((n - k) * k)$ ,  $M_{21}$  and  $A_{21}$  are  $(k * (n - k))$ , and  $M_{22}$  and  $A_{22}$  are  $(k * k)$ .

The general solution to a system of differential equations is found by first transforming the dynamic variables by pre-multiplying them by the matrix of eigenvectors, to yield a vector of variables  $c$ :

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = M \begin{bmatrix} y \\ x \end{bmatrix} \quad (6.7)$$

From (6.4), (6.5) and (6.7), it follows that the  $c$  vector follows the path given by (6.8) below:

$$dc = \Lambda c dt \quad (6.8)$$

In the case of the last  $k$  variables, i.e.  $c_2$ , the eigenvalues are positive, and hence applying an ordinary differential equation solution to these  $k$  equations will produce unstable paths. The problem is that rational expectations models have  $k$  unstable characteristic roots, and hence to solve them we have to set the initial values of the 'free' variables to the appropriate value such that the system is put on the stable manifold. That is, the expectational variables are assumed to 'jump' so as to place the

dynamic system on its unique stable path towards equilibrium. The mathematical counterpart of this 'transversality assumption' is that we set the initial values for  $c_2$ , i.e.  $c_2(0)$  (and hence implicitly  $x^e(0)$ ) so as to eliminate the unstable roots  $\Lambda_2$  from the solution in (6.8). This involves setting  $c_2(0) = 0$ , and hence:

$$c_2 = M_{21}y + M_{22}x = 0$$

This implies the following relationship between the non-predetermined and the predetermined variables which holds at time  $t = 0$ , and at all other times on the stable manifold:

$$x = M_{22}^{-1}M_{21}y \quad (6.9)$$

Given the initial values of the 'jump variables' in (6.9), from (6.4) it then follows that the predetermined variables follow a mean trajectory given by:

$$dy = Bydt$$

$$\text{where } B = A_{11} - A_{12}M_{22}^{-1}M_{21}$$

That is, the mean path of the predetermined variables is given by:

$$y = \exp(Bt)y(0) \quad (6.10)$$

and the mean path of the 'jump variables' then follows from (6.10) and (6.9).

The above analysis only refers to the mean path of the dynamic variables, and we may be interested in the stochastic properties of the system (i.e. the stochastic properties of the dynamic variables around their mean paths). To examine these

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properties, we now derive the asymptotic covariance matrix of the  $\{y \ x\}'$  vector. It is relatively trivial to express the covariance properties of the 'jump' variables in terms of the covariance properties of the predetermined variables, given (6.9). Thus, for instance:

$$\text{cov}(yx') = E((y - y^e)(x - x^e)')$$

and therefore

$$\begin{aligned} \text{cov}(yx') &= -E((y - y^e)(y - y^e)')(M_{22}^{-1}M_{21})' \\ &= \text{cov}(yy')(M_{22}^{-1}M_{21})' \end{aligned} \quad (6.11)$$

Similarly, it can be shown that:

$$\text{cov}(xx') = (M_{22}^{-1}M_{21})\text{cov}(yy')(M_{22}^{-1}M_{21})' \quad (6.12)$$

It now remains for us to find  $\text{cov}(yy')$ . It can be shown (see Chow, 1979, Currie and Levine, 1984), that  $\text{cov}(yy')$  evolves as follows (where  $Y$  denotes  $\text{cov}(yy')$ ):

$$dY/dt = YB' + BY + \theta_1$$

where  $\theta_1$  contains the first  $n-k$  elements of  $\theta$ . In examining the stochastic properties of these dynamic models we focus on the asymptotic covariance properties, and hence where  $dY/dt = 0$ . We may then solve the above equation to find the elements of  $Y$  according to the following expression:

$$Y^* = (B \otimes I + I \otimes B)^{-1} \theta^* \quad (6.13)$$

where the  $\otimes$  denote Kronecker products, the  $I$  are conformable identity matrices, and the  $Y^*$  and  $\theta^*$  are vectors defined as:

$$Y^* = (y_{11}, y_{12}, \dots, y_{1(n-k)}, y_{21}, \dots, y_{2(n-k)}, \dots, y_{(n-k)1}, \dots, y_{(n-k)(n-k)})$$

and in a similar fashion  $\theta^* = (\theta_{11}, \theta_{12}, \dots, \theta_{(n-k)(n-k)})$ .

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Having established some results relating to the solutions of dynamic rational expectations models, we now turn to the model which shall form the main object of our analysis in this chapter.

### 6.3.2 A Model of Buffer Stock Money in Continuous Time.

We now examine the effects of adopting monetary targets in a money market in which economic agents treat the money stock as a 'buffer asset'. As we have argued in Chapter 1, the adoption of strict targets for monetary aggregates in the UK as intermediate objectives of monetary policy has been judged to be a failed experiment. The apparent instability of the demand for broad monetary aggregates led some economists to point to the uselessness of targeting a nominal quantity which seemed to be a poor indicator of the behaviour of final objectives. Furthermore, the consistent overshooting of monetary targets was greeted by both calls to abandon a strict monetarist strategy for a more eclectic policy of looking at a number of economic indicators<sup>3</sup>, and by an incitement to pursue a stricter policy of monetary control<sup>4</sup> so as to achieve monetary targets.

To some extent, as we have seen in Chapters 2 and 3, some of the problems in the estimation of the demand for money function have evoked a response of a purely econometric character, and the adoption of modern techniques of dynamic modelling (see, for instance Hendry and Mizon, 1978, Hendry, 1979, 1985) has led to a marked improvement in the quality of estimated demand for money functions. Furthermore, as we have seen in Chapters 4 and 5, it



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has been suggested by a number of authors that one explanation for the success of dynamic models of the demand for money which incorporate 'error-correction mechanisms' might lie in the role of money as a 'buffer asset'. As we have seen in Chapters 4 and 5, the buffer stock approach stresses the forward-looking nature of the demand for money (see Cuthbertson and Taylor, 1986a), and we have shown that under certain circumstances such a forward-looking model is 'observationally equivalent' to backward-looking models which embody an error-correction mechanism (see, for instance, Hendry et al, 1984, Nickell, 1985, Cuthbertson, 1985a). Furthermore, the buffer stock approach can, in this context, also explain why the dynamic lag structure of backward-looking models of the demand for money may alter over time in the presence of policy changes (the Lucas critique).

We have seen that empirical tests of the buffer stock approach have met with mixed success. Although some authors claim a degree of success in the use of forward-looking models in modelling the demand for money (in particular see Cuthbertson and Taylor, 1987), we have seen in Chapter 5 that this success may to some extent be attributable to the observational equivalence property rather than to a degree of 'forward-lookingness' in the demand for money. Nevertheless, we have also suggested that there may be potential for an improvement of forward-looking models by extending their application to broader monetary aggregates, by adopting less restrictive cost function formulations, and/or by

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using multi-asset models (see Chapters 4 and 5).

However, despite the fact that there exists a vast empirical literature on the buffer stock approach to the demand for money, there remains the need for a theoretical analysis of the behaviour of the money market in the presence of buffer stock money. We have seen in section two that whilst there exists a literature on the performance of monetary targets in the presence of lagged adjustment in the demand for money (our question (i) in Section 2 above), there is the need for an evaluation of the performance of such targets when the demand for money conforms to the buffer-stock hypothesis (our question (ii) in section 2 above).

In what follows we model the pursuit of monetary targets by the monetary authorities in a theoretical 'buffer stock' model, and show that even in the context of a relatively simple model we can shed some light in the behaviour of the money market if money indeed acts as a 'buffer asset'. In section 6.3.3 we shall then see if some of the predictions of our simple model may be related to the actual dynamics experienced by money market variables in the UK in the last two decades.

The model of buffer stock behaviour presented in this section is a continuous time variant of the discrete time model which is estimated in the empirical literature (see Chapters 4 and 5). The reason for the use of a continuous time variant is that it simplifies our analysis somewhat, enabling us to derive

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some analytic solutions. As we pointed out in Chapter 4, these types of model bear a striking resemblance to problems describing the firm's investment decision (see for instance Lucas, 1967, Sargent, 1979). It is assumed that the representative economic agent wishes to minimise a quadratic intertemporal cost function,  $V$ , which penalises deviations of actual money balances,  $M$  from desired levels,  $M^*$ , and also penalises adjustments in money balances,  $DM$  (where the operator  $D$  is such that  $DX = dX/dt$ , and  $D^2X = d^2X/dt^2$ ):

$$V = (1/2) \int_0^{\infty} \{a(M(\tau) - M^*(\tau))^2 + b(DM(\tau))^2\} \exp(-\delta\tau) d\tau \quad (6.14)$$

where  $a$  and  $b$  are the relative weights assigned by the economic agent to the costs of being away from  $M^*$  and the costs of adjustment, and  $\delta$  is a subjective discount rate. Note that, as in the case of the class of models considered in Chapter 4 and 5, the desired money stock is considered to be a function of real income, the price level, and the rates of return on alternative assets. Again, economic agents are assumed to undertake a two-stage process, with a microeconomic optimisation exercise yielding the desired demand for money function  $M^*$ , and the cost-minimisation exercise in (6.14) yielding the optimal adjustment path of  $M$  towards the 'target',  $M^*$ . The drawbacks of this type of analysis has been discussed in Chapter 4. Once again it is worthwhile to mention that its appeal lies in its simplicity.

Let us also recall that, although the  $DM$  component of the

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cost function may seem somewhat perplexing given the argument that the individual is supposed to consider money as a 'buffer asset' and hence less costly to adjust than alternative components of his financial wealth, in this simple model it is assumed that the individual's portfolio choice is merely between money and bonds,  $B$ , so that it follows that  $DB = -DM$ , and that we need not enter  $DB$  in our quadratic cost function. This, however, need not be the case if we allow for a more varied portfolio, or net saving so that wealth is not constant over time (see Muscatelli, 1988a, and Chapters 4 and 5).

In our simple model we assume that the desired demand for money,  $M^*$  is a simple log-linear demand for money function:

$$M^* = \alpha_0 Y - \alpha_1 r + P \quad (6.15)$$

where  $Y$ ,  $r$  and  $P$  represent (the logarithm of) real income, the rate of interest on bonds and the price level respectively.

The problem facing the economic agent in (6.14) is a classical calculus of variations problem (see Intriligator, 1971) as the economic agent has a single objective,  $M$ , and uses its first time derivative  $DM$  as a single controller. This avoids a full application of the Pontryagin maximum principle outlined in section two above, although the two techniques may be shown to be equivalent (see Intriligator, 1971, Hadley and Kemp, 1971). To solve a calculus of variations problem, we must first of all define the so-called intermediate function,  $I$ :

$$I = (1/2)\{a(M(\tau) - M^*(\tau))^2 + b(DM(\tau))^2\}\exp(-\delta\tau) \quad (6.16)$$

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The optimal path for the controller DM must then satisfy the Euler-Lagrange condition. Applying the Euler-Lagrange condition to I:

$$(\partial I / \partial M) - (d/dt)(\partial I / \partial \dot{M}) = 0 \quad (6.17)$$

The sense behind this condition should be apparent: the appropriate path for DM will be such that the contribution to instantaneous cost function, I, of an infinitesimal change in M equals the time derivative of the contribution to I of an infinitesimal variation in DM. The application of the Euler-Lagrange condition in (6.17) yields a second-order differential equation describing the optimal dynamics of the controller DM (and consequently of the target, actual money stock holdings, M):

$$D^2 M(t) - \delta M(t) = (a/b)(M(t) - M^*(t)) \quad (6.18)$$

Equation (6.18) is of some interest in itself. First of all, we should note the close similarity to the Euler equations whose solutions were estimated in Chapters 4 and 5: the optimal path for money balances follows a second-order differential equation whose properties depend crucially on the relative costs of being away from equilibrium and of adjusting money balances ( $a/b$ ), and on the subjective discount rate,  $\delta$  (see Chapter 4). Secondly, as can be readily inferred from (6.18), the characteristic roots of this equation have opposite signs, thus suggesting that this model has a saddlepoint equilibrium, with a unique convergent path towards it (the saddlepath). The signs of the eigenvalues can readily be found as follows. The characteristic equation is:

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$$\lambda^2 - \delta\lambda - (a/b) = 0 \quad (6.19)$$

yielding the following values for the characteristic roots:

$$\lambda_i = \{\delta \pm (\delta^2 + 4(a/b))^{1/2}\}/2 \quad i = 1, 2 \quad (6.20)$$

From (6.20) it is apparent that one of the eigenvalues must be positive, and the other negative as  $(\delta^2 + 4(a/b))^{1/2} > \delta$ .

Although the presence of a saddlepoint equilibrium is usually a problem in a dynamic system with an arbitrary departure point, this problem does not arise here because we are dealing with a forward-looking optimisation problem. It is assumed, in line with similar calculus of variations problems in investment theory or Ramsay-type growth problems (see Sargent, 1979, Hadley and Kemp, 1971, Nagatani, 1981) that the economic agent rules out all divergent paths from equilibrium, as these lead to large positive costs. The individual is instead assumed to set the initial value of DM so as to ensure convergence to equilibrium where  $M = M^*$ , as  $t \rightarrow \infty$ . This initial condition for DM may be seen as a type of 'transversality' condition imposed on the system with DM as the non-predetermined variable. There is a straightforward analogy between the method of solution to this type of optimal control problem and the method of solution applied to dynamic rational expectations models, which we examined in an earlier subsection. In fact, this is easily seen when the differential equation (6.18) is rewritten in the form:

$$DM(t) = G(t)$$

$$DG(t) = \delta G(t) + (a/b)(M(t) - M^*(t)) \quad (6.21)$$

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where we have rewritten the second-order differential equation as a two-equation system of first-order differential equations, defining  $G$  as  $DM$ . In solving (6.21) when the system is saddlepoint stable we then treat  $G$  as the jump variable, and  $M$  as the predetermined variable. Whilst the stock of money holdings is predetermined at any given time, its rate of change is clearly not, enabling us to solve the calculus of variations problem satisfactorily<sup>5</sup>.

In passing it is worthwhile to note that the solution to (6.18), just like its discrete time counterparts, emphasises the forward-looking nature of the demand for money. To obtain the particular integral for (6.18) requires knowledge of the exact past and future time path of  $M^*$  and hence of real income, the price level, and the interest rate. If we for the moment ignore equation (6.15) and assume that  $M^*$  is exogenously fixed, the solution to (6.18), having imposed the transversality condition can be shown to be:

$$M(t) = (M(0) - M^*)\exp(\lambda_2 t) + M^* \quad (6.22)$$

where  $M(0)$  is the initial value of the money stock, and  $\lambda_2$  is the stable (negative) characteristic root.

However, the above analysis does little more than confirm some of the results obtained in discrete time cost-of-adjustment models of the demand for money of the type analysed in Chapters 4 and 5. A more interesting model of the dynamics in the money market would have to include an endogenous desired demand for

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money, and a role for government policy. We now turn to the task of building such a model.

### 6.3.3 A Small Economic Model in Continuous Time

To simplify the order of dynamics involved, we inevitably have to trade-off realism against simplicity in constructing an appropriate macroeconomic model. As a stylised fact, we assume that the monetary authorities have full control over the rate of interest which is their instrument of monetary policy. The supply of money is then assumed to be demand-determined at any given level of the interest rate. As we have already seen in Chapters 4 and 5, this makes sense if we are proposing a costs-of-adjustment model of the demand for money where the private sector is able to plan its money holdings on the basis of some forward-looking cost-minimisation exercise. In this sense it lies at the opposite pole from the usual textbook assumption of an exogenous money stock.

Although some economists would probably regard this as an extreme assumption, it nevertheless provides a closer approximation of actual methods of monetary control by the monetary authorities<sup>6</sup> than the assumption that the monetary authorities exogenously fix the money supply, and policy rules based on this assumption have now become commonplace in the literature on small macromodels (see for instance Currie and Levine, 1984, 1985, Taylor, 1988). Furthermore the identification of the interest rate as the instrument of monetary policy and the



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money stock as an intermediate objective linked to the former via some economic relationship is also common in the literature on intermediate targets (see B. Friedman, 1975). Effectively, we are assuming that the monetary authorities exert their control over the economy by treating the money stock as an intermediate objective (or an information variable in the optimal control sense), and manipulating interest rates to achieve their monetary targets via their effect on the desired demand for money. In practice the actual money stock is likely to be demand-determined due to the existence of a banking system (see Tobin, 1963), and our model of buffer stock money highlights the possibility that the actual money stock may not coincide at all times with the desired demand for money<sup>7</sup>.

In what follows we have assumed that the level of real income,  $Y$ , contains both an exogenous component,  $\beta_0$ , and is a negative function of the interest rate:

$$Y = \beta_0 - \beta_1 r \quad \beta_0, \beta_1 > 0 \quad (6.23)$$

In addition to this static IS relationship, we also retain our simple desired demand for money function (6.15) except that, for simplicity, we will restrict our analysis to a fixed price model. By an appropriate choice of units we can then set  $P = 0$  in equation (6.15). We assume that the authorities implement a proportional policy rule for their monetary instrument (see, for instance Phillips, 1954, 1957, Turnovsky, 1977) so as to achieve a particular targeted level of real income,  $\hat{Y}$ :

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$$r - \hat{r} = \mu(M - \hat{M}) \quad \mu > 0 \quad (6.24)$$

where  $\hat{r}$  and  $\hat{M}$  are appropriately chosen so as to achieve the desired target for real income. (This restriction is necessary, as a simple proportional rule will not generally restore the system to the desired equilibrium unless it is properly 'calibrated'. See Phillips, 1954, 1957, Turnovsky, 1977). The policy rule may be interpreted as follows:  $\hat{r}$  is the steady state level of the interest rate which enables the authorities to achieve  $\hat{Y}$ . If we, for simplicity set  $\hat{Y} = 0$ , by an appropriate choice of units, from equation (6.23) it is then apparent that  $\hat{r} = \beta_0/\beta_1$ . Equation (6.24) then merely states that the authorities raise the interest rate above their desired steady state level whenever the money stock rises above its target,  $\hat{M}$ . By setting  $\hat{Y} = 0$  in equation (6.15) it is furthermore apparent that an appropriate choice of target for the money stock is  $\hat{M} = -(\alpha_1\beta_0/\beta_1)$ . Thus, (6.24) can be written as:

$$r = \mu(M + (\alpha_1\beta_0/\beta_1)) + \beta_0/\beta_1 \quad (6.25)$$

Equation (6.25) describes a rational policy for the monetary authorities to follow, if they treat the money stock as their intermediate objective of monetary policy, and they have a final objective for real income  $\hat{Y} = 0$ .

We may ask why the monetary authorities should wish to adopt a money stock target instead of focusing directly on their final objective  $Y$  in a situation where the 'LM side' of the model is dynamic, whilst the 'IS side' (equation (6.23)), contains no such

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lags. We have refrained from introducing expenditure-income lags in this model to keep its dynamics as simple as possible and to focus on the operation and dynamics in the money market when money stock targets are combined with a forward-looking model of the money stock. The comparison between (nominal) income and money stock targets is a matter dealt at length with elsewhere (see Currie and Levine, 1984, 1985, Vines et al., 1983), and lies outwith the scope of this chapter, where we focus our attention on the consequences of implementing money stock targets in the presence of buffer stock behaviour on the part of economic agents. We shall return to discuss the issue of other intermediate objective/information variables further on. However, it is worth noting in passing that, in practice, money stock targets are often preferred to income targets given the availability of accurate money stock data over short time intervals. Such 'information lags' may be important, and are ignored in most of the literature on simple rules. In our simple model we therefore assume that the authorities cannot focus directly on their final objective,  $Y$ .

Whilst we have made the realistic assumption that the authorities implement their monetary policy via the manipulation of interest rates, it would be foolish not to consider the possibility of policy lags, given that the monetary authorities do not usually set their policy instruments instantaneously to their desired levels (see Phillips, 1954, 1957, Turnovsky, 1977).

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We therefore modify equation (6.25) so as to take account of such lags:

$$Dr = \phi[\mu(M + (\alpha_1\beta_0/\beta_1)) + \beta_0/\beta_1 - r] \quad (6.26)$$

where  $1/\phi$  may be interpreted as the mean lag for the interest rate for given values of the other endogenous variables (see Currie and Levine, 1984).

This simple dynamic model of the economy is completed by the inclusion of our buffer stock adjustment equation (6.18), which describes the adjustment of the actual to the desired money stock in the model on the assumption that economic agents minimise a quadratic intertemporal cost function. The adoption of equations (6.18) and (6.26) to model the dynamics of the money stock involves a number of implicit assumptions which must be made clear at the start.

First, the policy rule adopted has been assumed a priori, and not derived from an explicit intertemporal optimisation process on the part of the monetary authorities. A full optimal control exercise could be carried out here by using the maximum principle, along the lines described in an earlier subsection. Though it would certainly be interesting to model the authorities' preferences explicitly, this would result in a dynamic system of a higher order, which could be analysed exclusively using numerical simulation techniques. Furthermore, it should be noted that the whole point of our analysis here is to examine the performance of monetary targets, i.e. the

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performance of a simple rule in the presence of forward-looking behaviour by economic agents. (The incorporation of optimal simple rules would add little to our analysis.) More importantly, the emphasis in the recent literature has been more on the relative performance of simple and decoupled policy rules (see Vines et al. 1983, Currie and Levine, 1985), and less on that of fully optimal rules, and this chapter lays more emphasis on the behaviour of the money market given the adoption of simple monetary targets, and on the performance of monetary targets in a 'buffer stock environment'. A further extension of the analysis in this chapter would obviously be the introduction of lags in the income-expenditure relationship and a comparison of simple rules where the authorities alternatively target the money stock and real income.

Secondly, economic agents are assumed to be atomistic and hence cannot be assumed to act strategically in deciding the path of their money holdings. Thus, the adoption of equations (6.18) and (6.26) as our 'buffer stock' and policy rule equations makes sense in the context of this model, though a natural extension of our analysis would be to consider the authorities' preferences explicitly by modelling the outcome in the money market as a Stackelberg differential game, with the authorities as the leader (for an early attempt at such a model, see Molana and Muscatelli, 1986).

Thus, the full macroeconomic model can be characterised by

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a third order dynamic system containing equations (6.18) and (6.26). We first of all substitute out the non-dynamic endogenous variables  $M^*$  and  $Y$  out of equation (6.18) by using equations (6.15) and (6.23):

$$D^2M = \delta(DM) + (a/b)M - (a/b)\alpha_0(\beta_0 - \beta_1 r) + (a/b)\alpha_1 r \quad (6.27)$$

As discussed above, the dynamic system is best illustrated by representing  $DM$  by the variable  $G$ :

$$DM = G$$

Equations (6.26) and (6.27) may then be written in matrix form as:

$$\begin{bmatrix} DM \\ Dr \\ DG \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \phi\mu & -\phi & 0 \\ (a/b) & (a/b)(\alpha_1 + \alpha_0\beta_1) & \delta \end{bmatrix} \begin{bmatrix} M \\ r \\ G \end{bmatrix} + \begin{bmatrix} 0 \\ \phi(\beta_0/\beta_1)(1+\mu\alpha_1) \\ -(a/b)\alpha_0\beta_0 \end{bmatrix} \quad (6.28)$$

$A$ 
 $G$

In solving our model we must recall that our economic agents are assumed to solve their intertemporal optimisation problem by imposing the 'transversality' condition referred above. Thus, we solve (6.28) by treating  $G$  (the growth in money holdings) as a forward-looking non-predetermined variable, whilst  $M$  and  $r$  are predetermined in the usual rational expectations sense. Thus, following an exogenous shock to the system, it is assumed that  $G$  'jumps' so as to take the system onto the stable manifold. For

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this 'transversality condition' to be imposed, however, the matrix  $A$  must satisfy the conventional 'saddlepoint property' discussed in the previous subsection. It can be confirmed that our model satisfies this property by examining the characteristic polynomial of matrix  $A$ :

$$\lambda^3 - (\phi - \delta)\lambda^2 - (\phi\delta + (a/b))\lambda - (a/b)(\phi + \phi\delta(\alpha_1 + \alpha_0\beta_1)) = 0 \quad (6.29)$$

We may apply the algebra of polynomials to (6.29) in order to identify the signs of the eigenvalues  $\lambda_i$ . We know that, (see for instance Turnbull, 1957):

$$\lambda_1\lambda_2\lambda_3 = (a/b)(\phi + \phi\delta(\alpha_1 + \alpha_0\beta_1)) > 0$$

which implies that either two eigenvalues are negative and one positive, or all three are positive. To rule out the latter possibility, we may also note that:

$$\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = -(\phi\delta + (a/b)) < 0$$

which implies that at least one of the eigenvalues must be negative, so that the system definitely has one positive eigenvalue. Thus the number of positive eigenvalues equals the number of 'free' or non-predetermined variables, and the system is stable in the usual rational expectations sense.

To examine the full dynamic properties of the system, we next obtain the left-eigenvector associated with the unstable eigenvalue, say  $\lambda_1$ :

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$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \end{bmatrix} \begin{bmatrix} -\lambda_1 & 0 & 1 \\ \phi\mu & -(\phi + \lambda_1) & 0 \\ (a/b) & (a/b)(\alpha_1 + \alpha_0\beta_1) & (\delta - \lambda_1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (6.30)$$

Normalising by setting one of the elements of the left-eigenvector, say  $c_{12}$ , equal to one, it then follows from (6.30) that:

$$c_{13} = (\phi + \lambda_1)/((a/b)(\alpha_1 + \alpha_0\beta_1)) = \theta_1$$

$$c_{11} = (\phi\mu + (a/b)\theta_1)/\lambda_1 = \theta_2$$

Given that  $\lambda_1 > 0$ , it follows that  $\theta_i > 0$ ,  $i=1,2$ .

The dynamics of the system can now be fully described, in the light of these results. We know that the non-predetermined variable,  $G$ , will be related to the predetermined variables as follows (see the solution methods reported in the previous subsection):

$$\begin{aligned} G(t) &= -(c_{11}/c_{13})M(t) - (c_{12}/c_{13})r(t) \\ &= -(\theta_2/\theta_1)M(t) - (1/\theta_1)r(t) \end{aligned} \quad (6.31)$$

and the dynamics of the predetermined variables follow the following (2 x 2) subsystem of differential equations:

$$\begin{bmatrix} \frac{dM}{dt} \\ \frac{dr}{dt} \end{bmatrix} = (A_{11} - A_{12} \begin{bmatrix} -(\theta_2/\theta_1) & -(1/\theta_1) \end{bmatrix}) \begin{bmatrix} M \\ r \end{bmatrix} + G' \quad (6.32)$$

where  $A_{11}$  is the top left-hand (2 x 2) submatrix of matrix  $A$ ,  $A_{12}$  is the top right-hand (2 x 1) submatrix of  $A$ , and  $G'$  is a subvector of the vector of exogenous variables,  $G$ .

We may use (6.31) and (6.32) to answer some questions of how



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the money market will react in response to an exogenous shock to the system. In the present model, we only allow for changes in autonomous expenditure,  $\beta_0$ . We first of all examine the comparative static properties of the system, which are relatively straightforward. It is easy to show that the total differentials are:

$$dM/d\beta_0 = (1/\Delta) [ -(a/b)(\alpha_1 + \alpha_0\beta_1)(\phi/\beta_1)(1 + \mu\alpha_1) + \phi(a/b)\alpha_0 ]$$

and

$$dr/d\beta_0 = (1/\Delta) [ (a/b)(\phi/\beta_1)(1 + \mu\alpha_1) + (a/b)(\phi\mu\alpha_0) ]$$

where  $\Delta$  is the determinant of matrix A. Given that from the characteristic polynomial (6.29), we know that  $\Delta > 0$ , it should be apparent that  $dM/d\beta_0 < 0$ , and  $dr/\beta_0 > 0$ . The economics of this result is very simple and may be interpreted in IS-LM terms: given that a positive expenditure (IS curve) shock increases income above its target level,  $\hat{Y} = 0$ , the authorities raise interest rates so as to cause a fall in the money stock, causing the LM curve to shift back to the left so that in full equilibrium,  $Y = \hat{Y} = 0$ . It naturally follows that in full equilibrium,  $G = 0$ . Given this result, we may examine whether  $G$  overshoots or undershoots its long-run equilibrium value in the short run. That is, it is interesting to know if in response to an increase in expenditure monetary growth responds positively or negatively. From (6.31), it follows that:

$$G(0+) - \bar{G} = -(\theta_2/\theta_1)(M(0+) - M) - (1/\theta_1)(r(0+) - r) \quad (6.31')$$

where  $X(0+)$  represents the value of a variable  $X$  the instant

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after the exogenous shock, and  $\bar{X}$  its long-run equilibrium value. It is difficult to say much about the sign of  $G(0+)$  unless we assume that the economic system was initially in full equilibrium at  $Y=0$ , so that we may conveniently set  $M(0+) = M^* = -\alpha_1 r(0+)$ . Equation (6.31') then simplifies to (6.31''):

$$G(0+) = (r(0+) - r)(1/\theta_1)(\theta_2\alpha_1 - 1) \quad (6.31'')$$

If we then consider an exogenous positive expenditure shock to the system, then from our comparative statics,  $(r(0+) - r)$  is obviously negative, but we do not know whether this will call forth a rise or fall in money supply growth in the short run, unless we can sign  $(\theta_2\alpha_1 - 1)$ . If  $G(0+)$  is positive it follows that there will be at least some time period over which both interest rates and the money stock are increasing, following an expenditure shock.

Though this result shows that, in the presence of buffer stock money, the money stock and interest rates may move in the same direction, thus explaining why the money stock may seemingly react perversely to a deflationary policy of higher interest rates, it still offers an incomplete picture of the dynamics of the economic model. From equation (6.32), and matrix A we know that the transition matrix, B, where

$$B = (A_{11} - A_{12}[-(\theta_2/\theta_1) \quad -(1/\theta_1)])$$

is given by:

$$B = \begin{bmatrix} -(1/\theta_1) & -(\theta_2/\theta_1) \\ \phi\mu & -\phi \end{bmatrix} \quad (6.33)$$

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which implies that the stable eigenvalues  $\lambda_2$  and  $\lambda_3$  will be complex numbers provided  $((\theta_2/\theta_1) - \phi)^2 < 4\phi\mu/\theta_1$ . If the eigenvalues are complex, the system will display cycles in which the money market will move to equilibrium whilst the money stock and the interest rate move in phase and out of phase over different time periods. In other words, it is perfectly plausible to expect periods over which the money stock and interest rates are both rising. In itself, this is not a very surprising result, as it is well known from Phillips' study of stabilisation policies that in the presence of policy lags, adjustment to full equilibrium of an economic system may not be monotonic. However, given the complicated expressions for the  $\theta_i$ , it is difficult to interpret this expression in terms of the structural parameters of our model. Nevertheless, with the aid of phase diagrams for the model we may reach some conclusions about how the behaviour of the system depends upon the government policy parameter and the economic agents' preferences.

We may represent our full dynamic system (equation system (6.29)) on a pseudo-three-dimensional diagram, and our subsystem of predetermined variables (equation system (6.33)) on a two-dimensional diagram of the stable manifold, respectively Figures 6.1 and 6.2. Given our dynamic system, we know that  $G = 0$ , ( $DM = 0$ ) is given by the desired demand for money function (i.e. the locus  $M = M^*$ ). The  $M^*$  function effectively represents the locus in Figure 6.1 where the  $DM = 0$  locus (the horizontal plane)

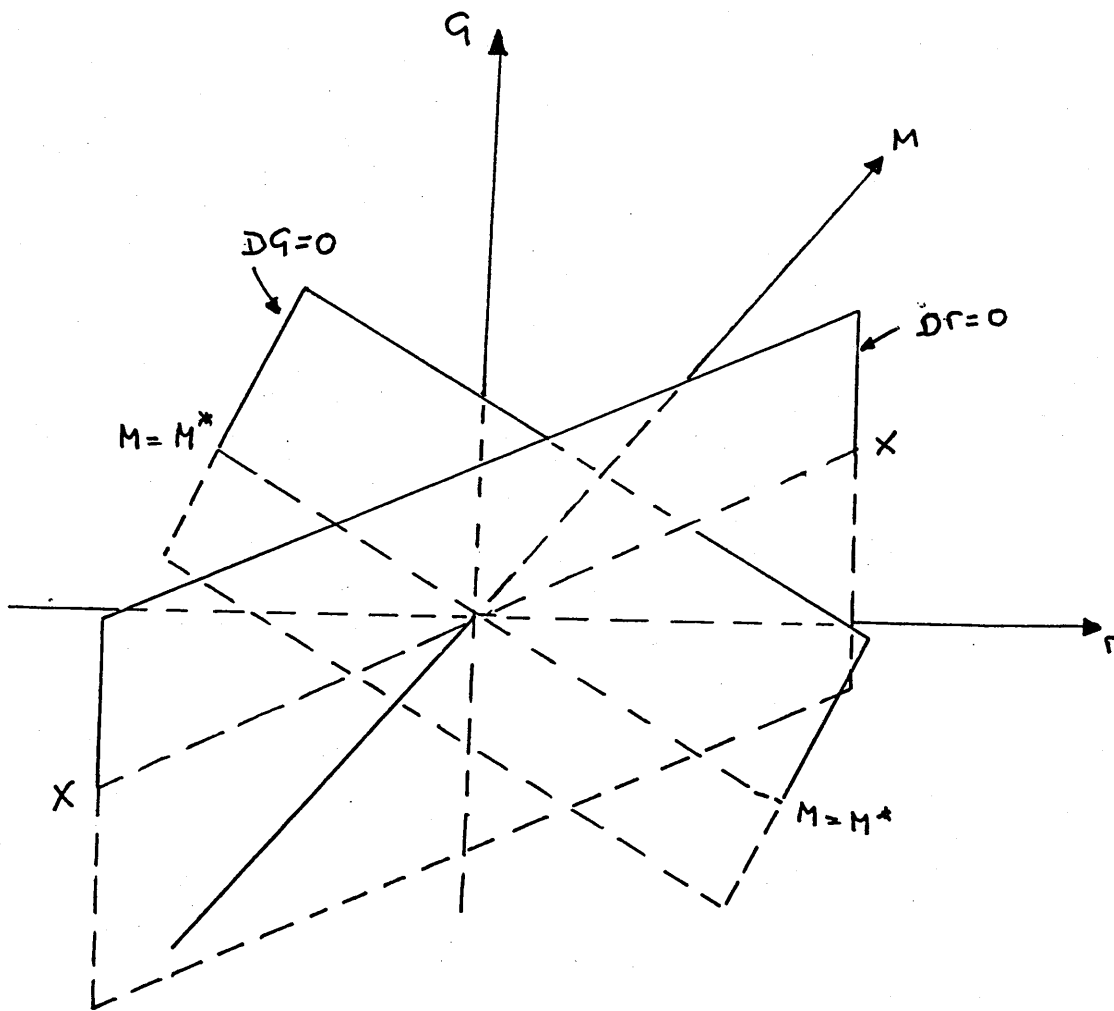


Figure 6.1

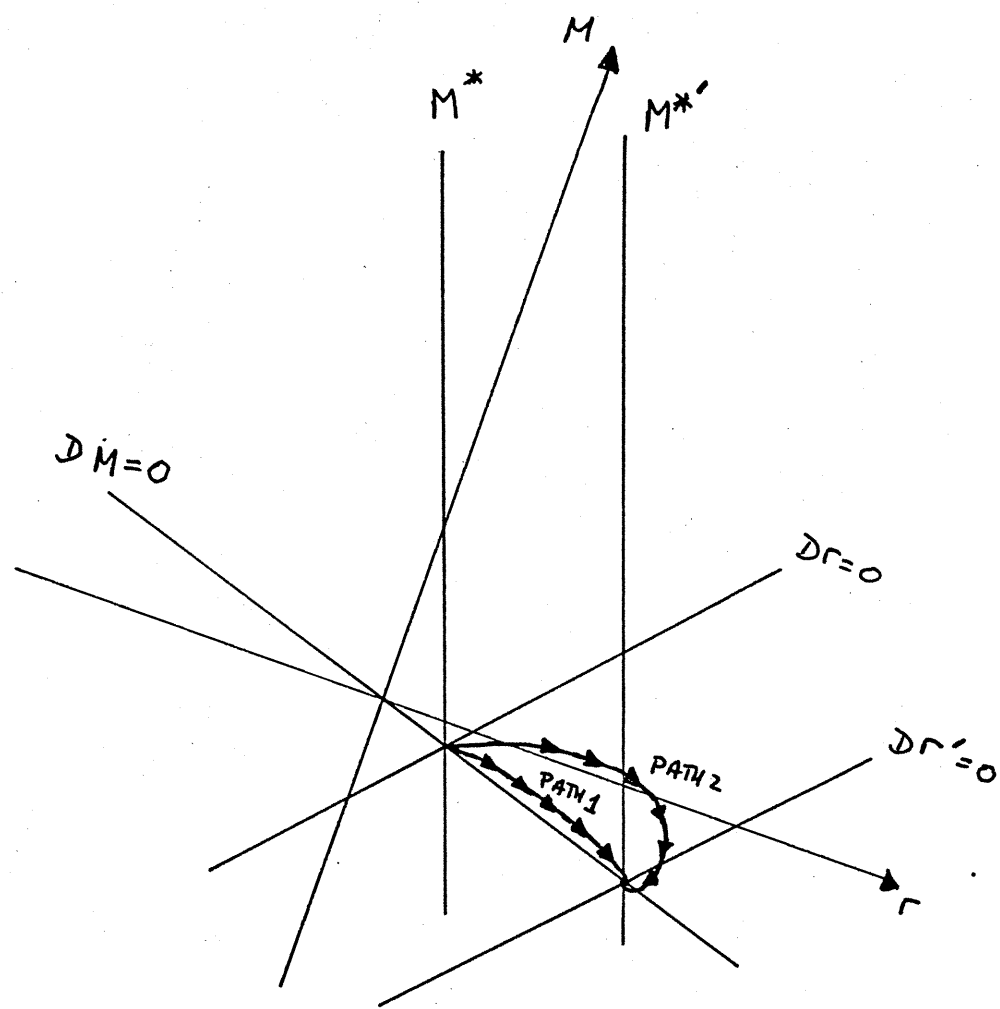


Figure 6.2

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intersects with the  $\dot{G} = 0$  plane which slopes upwards with respect to both the two horizontal axes. Furthermore, the  $Dr = 0$  plane intersects the horizontal plane perpendicularly in Figure 6.1, and the XX line represents the line of its intersection with the horizontal plane. However, a clearer picture of the model emerges by focusing on the stable manifold. From (6.33), we know that the  $DM = 0$  schedule has an equation:

$$M = -(1/\theta_2)r \quad (6.34a)$$

and the  $Dr = 0$  schedule has an equation given by:

$$M = (1/\mu)r - (\beta_0/\beta_1\mu)(1 + \mu\alpha_1) \quad (6.34b)$$

Furthermore, we may plot the desired demand for money equation, which, after substituting for  $Y$  from expenditure function (6.24) yields the following equation:

$$M^* = -(a_1 + \alpha_0\beta_1) + \alpha_0\beta_0 \quad (6.34c)$$

The schedules given by (6.34a)-(6.34c) have been drawn in Figure 6.2. Note that after an expenditure shock (a rise in  $\beta_0$ ), the  $Dr = 0$  and  $M^*$  curves shift to  $Dr' = 0$ , and  $M^{*'} respectively. yielding a higher interest rate and lower money balances in equilibrium in accordance with the multipliers reported above. However we cannot, a priori, draw the exact path of motion in Figure 6.2, without knowing more about the relative signs of the parameters. From the discriminant of the characteristic equation of matrix B, we know that the two stable roots of the system may be real or complex, and hence both path 1 and path 2 are feasible in Figure 6.2. Note that if the system had real stable roots, the$

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system would follow a path like path 1 to its new equilibrium, giving rise to a case of 'missing money' with  $M$  consistently below  $M^*$  until the system returns to equilibrium. On the other hand, if  $DM > 0$  initially, then the system may follow a path such as path 2 to equilibrium, where velocity initially falls, and the money market displays cycles. Thus, whether the system is a stable focus or a stable node i.e. whether paths 2 or 1 are relevant depends entirely on the overshooting condition.

However, this description of events merely tells us that 'almost anything can happen' in a money market where money performs a buffer role. A more interesting question is how the type of path of adjustment depends upon the value of the parameters of the model. In particular, we shall focus on the relative weights attached by the economic agents to the instantaneous costs of being away from equilibrium and the instantaneous costs of adjustment ( $a/b$ ), and on the policy parameter  $\mu$ , which reflects the strenght of the authorities desired response to deviations from monetary targets<sup>8</sup>.

Turning first the role of the government policy parameter  $\mu$ , we may examine the result of variations of its value on the curves on the stable manifold illustrated in Figure 6.2. In Figure 6.3(a) we illustrate the extreme case where  $\mu = 0$ , i.e. where monetary policy is not very responsive to deviations of the money stock from its prescribed target. From (6.34b) we know that the  $Dr = 0$  schedule becomes vertical. To know the effect on

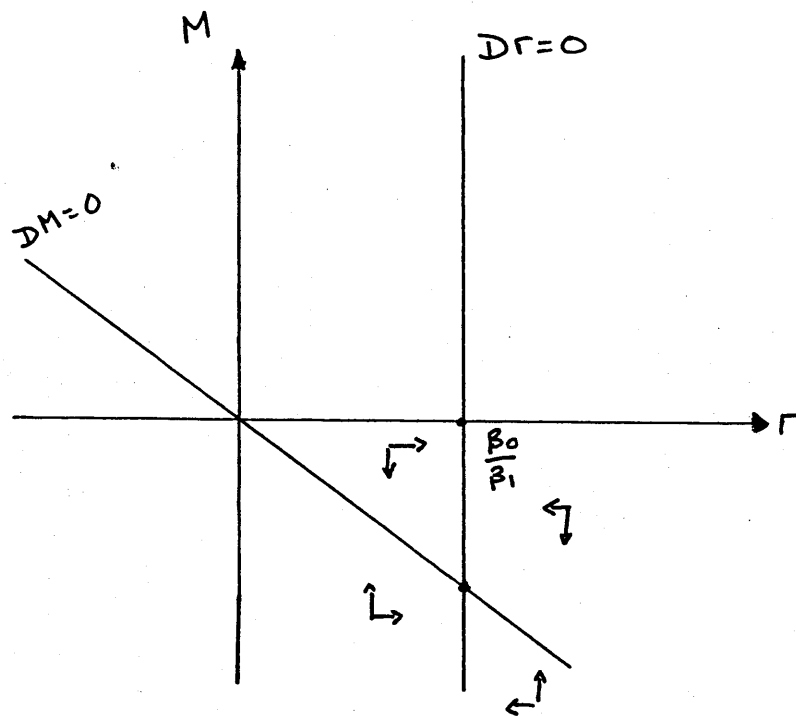


Figure 6.3 (a)

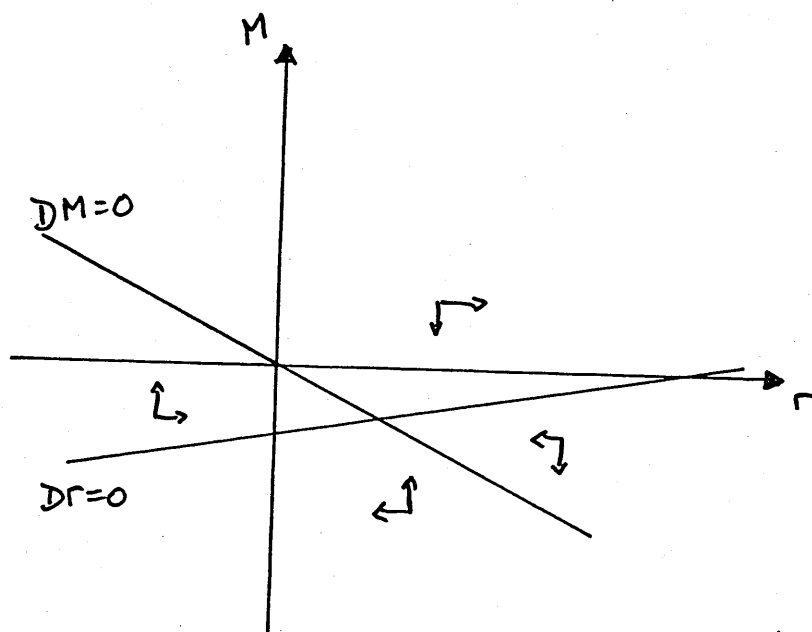


Figure 6.3 (b)



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the  $DM = 0$  schedule, we need to find out the effect of setting  $\mu = 0$  on  $\theta_2$ . From the definitions of the  $\theta_i$ , we can find that:

$$\theta_2 = \{\phi\mu + [(\phi + \lambda_1)/(\alpha_1 + \alpha_0\beta_1)]/\lambda_1\}$$

From this expression it would seem that setting  $\mu = 0$  does not drastically affect the slope of the  $DM = 0$  line, provided, of course that the value of  $\lambda_1$  is not drastically affected. From our knowledge of  $\lambda_1\lambda_2\lambda_3$  we know that setting  $\mu = 0$  does not lead any eigenvalue to take on a zero value. Thus, it is safe to assume that  $DM = 0$  retains its downward slope in Figure 6.3(a). As we can see from the arrows of motion in this figure, the equilibrium is a stable node. Thus, a smaller response of desired policy to a deviation of policy from target is less likely to cause cycles in the money market. This result should not be seen as surprising, as it is well known from Phillips' exercises with simple policy rules that a large policy response may produce cycles in the economy due to overadjustment (see Phillips 1954, 1957, Turnovsky, 1977). Thus, a strong policy response may not immediately produce the desired effect in the targeted monetary aggregate, whilst a small policy response may cause monotonic adjustment, as in this case.

We may also quickly dispose of the case where  $\mu$  is large and positive, that is where  $Dr = 0$  is shallow in slope (see Figure 6.3(b)). In this case, the arrows of motion suggest that the equilibrium is a stable focus, thus confirming our assertion in the previous paragraph that a large policy response may initially

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raise monetary growth.

We next turn to the behaviour of the system for different values of the relative cost of adjustment parameter,  $(a/b)$ . This parameter is notable for its absence in equations (6.34a)-(6.34c), thus suggesting that it plays no role in determining the dynamics of the system. However, this would be a wrong conclusion, since, though  $\theta_1$  appears to be independent of  $(a/b)$ ,  $\lambda_1$  depends on it.

From our knowledge of  $\lambda_1\lambda_2\lambda_3$ , we know that the product of the three eigenvalues depends positively on  $(a/b)$ . Given that the stable subsystem is independent of the value of  $(a/b)$ , we may conclude that  $\lambda_1$  depends positively on  $(a/b)$ : the larger the relative costs of being away from equilibrium, the larger the unstable root of the system. That this is the role played by  $(a/b)$  may be verified by its presence in the last equation of the full dynamic system (equations (6.29)). Thus, the larger  $(a/b)$ , the smaller the value of  $\theta_2$ , and the more likely that  $G(0+)$  will initially be positive from (6.31'''). In terms of our phase diagram, this implies that the system is more likely to be a stable focus. Whether this is indeed the case in practice is impossible to say without assigning arbitrary values to the other parameters of the model, an exercise which we undertake in part below.

At this stage, however, we may note a similarity with the results obtained for the policy parameter  $\mu$ . The greater economic

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agents' costs of deviating from their desired money balances, the more likely that the system will display cycles, just as a greater policy response leads to cycles. Again, this result should not be seen as paradoxical, as it is merely another example of the Phillips' (1954, 1957) result mentioned above. The difference here is that the cycles in the economy are not caused by a strong policy response, but by a strong desire on the part of individuals to keep their actual money balances close to their long-run desired levels, thus causing 'overadjustment'.

The argument so far has been to show that the implementation of monetary targets in the presence of buffer stock behaviour in the money market may lead to targets not being met through an apparent perverse response of money holdings to interest rate changes where the money market displays cycles. In other words, we can explain both 'missing money' and 'excess money' episodes and relate these to policy parameters or agents' preferences. Whilst this explains some of the recent episodes in monetary targeting in OECD countries, it does not in itself represent an indictment of monetary targets. We cannot say that a strong desired policy response (a large  $\mu$ ) is necessarily counterproductive purely on the basis that it causes monetary targets to be overshoot rather than undershot. A proper evaluation of the policy response requires some evidence on the effects of policy on the authorities' desired objectives. We now attempt such an exercise, very much in the spirit of the analysis of

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Buiter and Miller (1981, 1983) in the case of a small open economy.

### 6.3.4 Evaluating the Performance of Monetary Targets

In what follows we evaluate the performance of monetary targets in the presence of buffer stock behaviour, by assuming that the authorities' aim is to minimise deviations of the level of real income from its target, which in this case is  $\hat{Y} = 0$ . A small problem arises in cases where adjustment is non-monotonic, where a measure of welfare loss such as:

$$\int_0^{\infty} Y(t)dt$$

will seriously understate the true welfare loss for the authorities (see, for instance Buiter and Miller, 1983). We therefore choose a quadratic loss measure of the type:

$$\int_0^{\infty} [Y(t)]^2 dt$$

This is in contrast to Buiter and Miller (1983) who prefer to measure absolute deviations from the target value. We prefer the above measure because it seems to be easier to evaluate than the one proposed by Buiter and Miller for both the monotonic and non-monotonic adjustment cases so that we may apply the same measure of welfare loss to both cases. From the IS curve, we know that income changes are directly matched by interest rate changes, as the income-expenditure relationship is static. Thus, our measure of welfare loss (i.e. total (square) deviation of output from its target value) may be approximated by:

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$$\int_0^{\infty} [r(t) - \bar{r}]^2 dt$$

where, as we can recall from the previous section,  $\bar{r} = (B_0/B_1)$ . We know that the form of the dynamic path for  $r(t)$  depends on whether the eigenvalues of  $A$  are real or complex. If the two stable roots  $\lambda_2$  and  $\lambda_3$  are real numbers then the solution has the form:

$$r(t) = \bar{r} + E_1 \exp(\lambda_2 t) + E_2 \exp(\lambda_3 t) \quad (6.35)$$

where the  $E_i$  are coefficients dependent upon the initial conditions of the system. Alternatively, if the  $\lambda_i$  are complex such that  $\lambda_i = s \pm wi$  where  $s < 0$ , and  $i = \sqrt{-1}$ , then the solution has the form:

$$r(t) = \bar{r} + \exp(st)[F_1 \sin(wt) + F_2 \cos(wt)] \quad (6.36)$$

where  $s$  is known as the damping factor, and  $w$  as the frequency of the cyclical solution, and the  $F_i$  are coefficients dependent upon the initial conditions of the system. Thus, in the case of monotonic adjustment, the welfare loss may be expressed as:

$$\int_0^{\infty} [E_1 \exp(\lambda_2 t) + E_2 \exp(\lambda_3 t)] dt \quad (6.37)$$

and, similarly, in the case of non-monotonic adjustment:

$$\int_0^{\infty} \exp(st)[F_1 \sin(wt) + F_2 \cos(wt)] dt \quad (6.38)$$

Whilst it is relatively straightforward to evaluate the integral in equation (6.37), given that it converges, the integral in (6.38) appears more problematic. Nevertheless, the presence of the exponential factor in (6.38) enables us to use a standard table of Laplace transforms to evaluate this integral.

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The Laplace transform of a function  $f(t)$  is defined by the equation:

$$L(f) = \int_0^{\infty} f(t)\exp(-pt)dt = F(p) \quad (6.39)$$

for some  $p$  (some restrictions on  $p$  exist in the cases of particular functionals  $f(t)$ , see for instance Boas (1966)). The convenience of using Laplace transforms is that  $F(p)$  is tabulated and hence what seems like a complicated integral or differential equation may often be turned into a simple algebraic problem by reference to such transforms. One other advantage of the Laplace transform method is that it would allow us, if we so wished, to evaluate the welfare loss formula for finite time horizons or for the case where the equilibrium suddenly switches from being a stable node to a stable focus or vice-versa.

Using such Laplace transform tables (see Boas, 1966) we found the following expressions for the welfare loss in the monotonic and non-monotonic case respectively:

$$-[(2E_1E_2)/(\lambda_2 + \lambda_3) + (E_1)^2/2\lambda_2 + (E_2)^2/2\lambda_3] \quad (6.40)$$

$$-(F_1)^2/2s + ((F_2^2 - F_1^2)/2)(-s/2(s^2 + w^2)) + F_1F_2(w/2(s^2 + w^2)) \quad (6.41)$$

Expression (6.40) tells us, quite naturally, that in general the larger the size of the negative real roots, the less the total deviation of output from its target, whilst expression (6.41) tells us that the larger the damping factor and the frequency of the cycles, the smaller the deviation of output from

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target. However, these expressions do not allow us to say much a priori about the relative advantages and disadvantages of adopting different values of the policy parameter,  $\mu$ . To compare the effects of policies which imply a strong desired policy response to deviations of the actual money stock from monetary targets (a high  $\mu$ ), and policies which imply a weak desired response to deviations of the money stock from monetary targets (a low  $\mu$ ), it is best to impose a set of arbitrary, but plausible, parameter values on the economic model. This will then enable us to obtain numerical values for the total output loss under the two different policy regimes. Furthermore, it will enable us to consider the effect of these policies when the system is shocked whilst in an initial position of disequilibrium, and when the system starts from full equilibrium.

In what follows, we have fixed the values of the structural parameters of the model as shown in Table 6.1:

Table 6.1: Structural Parameter Values

$$\alpha_0 = 1 \quad \alpha_1 = 1 \quad \delta = 0.5 \quad \beta_1 = 0.1 \quad \phi = 0.5$$

In addition we consider two scenarios: first, one in which agents' preferences are such that  $(a/b) = 0.001$ , and second, one in which  $(a/b) = 1$ . Economic agents are assumed to attach a relatively smaller penalty to being away from equilibrium than to the speed of adjustment in the first case. As we shall see below,

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the first scenario produces a model in which adjustment is monotonic, whilst the second produces cyclical adjustment (as we predicted from the analytical solution presented in the previous subsection). Within each scenario, we then consider the implementation of a monetary policy where  $\mu = 5$ , and of a relatively stronger policy response strategy, where  $\mu = 15$ . Given these parameter values, we evaluate the welfare loss using the formulae shown in (6.40) and (6.41) above, after a unit shock in exogenous expenditure  $\beta_0$ . In addition, we are of course compelled to make some assumption regarding the initial state of the system. Again, we consider various possibilities within each of our two scenarios. First, the dynamic system may start from a position of full equilibrium, where  $M(0) = M^*(0)$ . Given our comparative static results, and the values of the parameters shown in Table 6.1, this would imply that the initial conditions are such that  $(r(0+) - \bar{r})$  and  $(M(0+) - \bar{M})$  the instant following the shock are equal to 0.862 and -0.862 respectively. However, we also consider alternative situations, where actual money stock holdings initially exceed or are below their desired values. Whilst keeping  $(r(0+) - \bar{r})$  at its 0.862 value, we consider two possible initial conditions for actual money holdings. We first set  $(M(0+) - \bar{M}) = -2$ , where actual money holdings initially lie below their desired value, and then  $(M(0+) - \bar{M}) = 1$ , where actual holdings initially lie above their desired value. This will enable us to examine the relative performance of a strong and a



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weak response to monetary targets given a variety of possible scenarios.

First, let us examine the scenario where  $(a/b) = 0.001$ , the results for which are reported in Table 6.2 below. As predictable from (6.40), these results tell us that the greater the policy response to deviations from target, the smaller the welfare loss. Furthermore, it seems to matter little whether or not the initial conditions were such that the money market was in equilibrium to begin with. In all three settings, a greater policy response reduces the welfare loss considerably.

Table 6.2: Welfare Loss Given  $(a/b) = 0.001$

Initial Conditions	$M < M^*$	$M = M^*$	$M > M^*$
Value of Policy Parameter	$M(0+) - \bar{M} = -2$	$M(0+) - \bar{M} = -0.862$	$M(0+) - \bar{M} = 1$
$\mu = 5$	29.53	29.38	29.12
$\mu = 15$	11.42	11.37	11.26

We may contrast this with the situation where  $(a/b) = 1$ , i.e. where the adjustment to equilibrium is non-monotonic. The results for this particular case are reported in Table 6.3. One must note that the net gains from a larger value for  $\mu$  are smaller than in the case of the previous scenario. Furthermore, if the initial condition is such that  $M > M^*$ , the gains from a greater policy response become smaller. If the government's welfare function were to penalise deviations of the policy

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instrument from its final equilibrium, in addition to deviations from target income levels, it is doubtful whether the monetary authorities would prefer a larger value of  $\mu$ .

Table 6.3: Welfare Loss Given  $(a/b) = 1$

Initial Conditions	$M < M^*$	$M = M^*$	$M > M^*$
Value of	$M(0+) - \bar{M} = -2$	$M(0+) - \bar{M} = -0.862$	$M(0+) - \bar{M} = 1$
Policy Parameter			
$\mu = 5$	0.531	0.356	0.217
$\mu = 15$	0.304	0.253	0.199

We may draw two main conclusions from these results. First, although in the previous section we noted that a large value of  $\mu$ , for a given value of  $(a/b)$ , is more likely to produce cycles in the money market, this in itself does not mean that monetary targets should not be pursued vigorously if the monetary authorities wish to minimise a loss function of the type described above. However, in the case where adjustment is non-monotonic due to a large value of  $(a/b)$ , the gains from a more vigorous policy may be less, and if the monetary authorities penalise deviations of the monetary instrument around its equilibrium value, the net effect of a more vigorous policy may be a comparative loss in welfare. In this regard, it should be stressed that the authorities may well wish to avoid large and sudden increases in interest rates.

Second, in deciding its best policy, the monetary authorities cannot ignore the initial conditions in the money

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market. In particular, if the actual money supply initially exceeds its long run value, the monetary authorities may do better by adopting a gradualist policy, given that the money stock overstates the degree of expansion in the economy. Thus whether a gradual or a rapid approach to meeting monetary targets is adopted very much depends on the particular dynamic structure of the economic system when 'buffer stock money' is present.

In general, though, we can say that when monetary targets are quickly overshoot after the implementation of policy, or where there seems to be an initial slump in the velocity of money at the moment when the targets are implemented, a gradualist approach may be better in terms of our economic welfare measure. The opposite is of course true if the economy enters 'missing money' episodes (i.e. where the adjustment is monotonic), where a larger value of  $\mu$  may be appropriate.

Overall, the problem the authorities face in this model is one where the money stock provides a poor signal about the underlying real conditions in the economy. Our model would therefore perhaps suggest that a more eclectic policy is appropriate, where the authorities look at money velocity or the long-run demand for money as well as money stock data (in addition, of course, to real income data when this is available). One feature of the above analysis which is often ignored in optimal policy analysis is the initial conditions of the system. We have demonstrated that the welfare losses vary considerably

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depending on the initial conditions of disequilibrium in the money market. Unless the monetary authorities are able to perceive this state of disequilibrium, they will generally adopt an incorrect policy strategy.

The problems are compounded for the monetary authorities if  $(a/b)$  or  $\delta$  changes over time, as this will cause the dynamics to alter. There is therefore no guarantee that a policy stance which is more appropriate over a given time period will always remain the most appropriate over time.

In passing one should also note that any change in the value of the policy parameters will cause the dynamics of the system to alter, thus providing an obvious example of the Lucas (1976) critique, and explaining why the short run dynamics of estimated money demand functions will not remain invariant over time. Furthermore, it is unlikely that  $(a/b)$  and  $\delta$  will remain invariant over time in the presence of innovations in financial markets and a certain degree of variability in the riskiness of operating in asset markets.

We conclude the analysis of our simple deterministic model by noting that the performance of monetary targets in the presence of buffer stock behaviour in the money market is highly variable. Having analysed the dynamic properties of the deterministic model presented above, it would be interesting to analyse its stochastic properties once we introduce random disturbances in some of its constituent parts. It is to this task

that we now turn.

### 6.3.5 Stochastic Properties of the Model

To render the above model two stochastic, let us first define  $dz_i$  to be Wiener processes which are normally and independently distributed with a zero mean and constant variance:

$$dz_i \sim N(0, \sigma_i^2 dt) \quad \forall i \quad (6.42)$$

We define two Wiener processes, where the first may be interpreted as a government policy error,  $dz_2$ , where the authorities unintentionally (and randomly) deviate from their announced policy path. The second,  $dz_1$ , may be seen as an unanticipated error by the economic agents in their holdings of money balances. It is quite natural to expect that individuals will face unexpected random accumulations or decumulations of money balances. This random error plays a similar role to the unanticipated elements of money demand in empirical studies of buffer stock money (see for instance Cuthbertson, 1988, Cuthbertson and Taylor, 1987).

We may now rewrite our dynamic model (6.29) in its stochastic form, though it should be noted (see Currie and Levine, 1982, 1984) that the  $z_i$  are not differentiable with respect to time (i.e.  $dz_i/dt$  does not exist) given the definition of the continuous time Wiener process, and hence the model must be rewritten in a slightly different form:

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$$\begin{bmatrix} dM \\ dr \\ dG \end{bmatrix} = A \begin{bmatrix} M \\ r \\ G \end{bmatrix} dt + \begin{bmatrix} dz_1 \\ dz_2 \\ 0 \end{bmatrix} \quad (6.43)$$

As we saw in the previous subsection, we may find the asymptotic covariance matrix for the money stock and the interest rate via the following expression (for a detailed derivation see Currie and Levine, 1982):

$$[\text{var}(M) \quad \text{cov}(Mr) \quad \text{cov}(Mr) \quad \text{var}(r)]' = -((B \otimes I) + (I \otimes B))^{-1} [\sigma_{11} \quad \sigma_{12} \quad \sigma_{21} \quad \sigma_{22}]'$$

where the  $\otimes$  are Kronecker products and where  $\sigma_{ij}$  is the  $(ij)$ th element of the covariance matrix of the vector  $[dz_1 \ dz_2]$ . If we assume that the two random disturbances in this vector are independently distributed, so that the  $(ij)$ th element of the vector  $i \neq j$  is equal to zero, then it makes the task of finding an expression for  $\text{cov}(Mr)$  simpler.

In what follows, we denote the matrix  $((B \times I) + (I \times B))$  by  $R$ . It can be shown, from our above definition of  $B$  that the determinant of  $R$  is:

$$\begin{aligned} \det(R) = & 2(\theta_2/\theta_1)\{(\phi + (\theta_2/\theta_1))[2\phi(\phi + (\theta_2/\theta_1)) + (\phi\mu/\theta_1)] - \\ & (1/\theta_1)[\phi\mu(\phi + (\theta_2/\theta_1))]\} + (2/\theta_1)[\phi\mu(2\phi(\phi + (\theta_2/\theta_1)))] \end{aligned} \quad (6.44)$$

which, from our knowledge of the parameters of the system, has a positive sign. Thus, as is apparent from the definition of the asymptotic covariance matrix, we can identify the sign of  $\text{cov}(Mr)$

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by examining the sign of the minors of the (1, 2)th and (4, 2)th element of the R matrix,  $R_{12}$  and  $R_{42}$ . In practice  $\text{cov}(\text{Mr})$  is given by:

$$\text{cov}(\text{Mr}) = -[(R_{12})\sigma_1^2 + (R_{42})\sigma_2^2]/\det(R) \quad (6.45)$$

It can be shown that the relevant minors of R are:

$$R_{12} = -[\phi\mu(2\phi(\phi + (\theta_2/\theta_1)))] < 0$$

$$R_{42} = 2(\theta_2/\theta_1)[(\phi + (\theta_2/\theta_1))/\theta_1] > 0 \quad (6.46)$$

Substituting these terms into (6.45), it is apparent that  $\text{cov}(\text{Mr})$  may have either a positive or negative sign. Given the complexity of this expression, we may only conjecture on the possible effects on the sign of  $\text{cov}(\text{Mr})$  of manipulating various structural parameters. Nevertheless, it should be apparent from (6.46) that, the larger the value of  $\mu$ , the more likely that  $\text{cov}(\text{Mr})$  will be positive as the  $R_{12}$  minor will dominate expression (6.45), whilst the larger  $(a/b)$  (and hence the smaller the value of  $\theta_1$  and the larger  $(\theta_2/\theta_1)$ ), the more likely that  $\text{cov}(\text{Mr})$  will be negative, as the  $R_{42}$  minor will dominate (6.45). The rationale for this result is simply that a greater responsiveness of interest rates to deviations from monetary targets will induce a positive covariance between M and r, whilst a greater convergent force towards the long run demand for money will induce a more conventional negative correlation between these two variables. Again, we should not therefore be surprised to find a positive asymptotic covariance between the interest rate and money stock movements (and also between G and r) in

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periods where monetary policy places a strong emphasis on meeting monetary targets.

These results may be confirmed by again substituting some plausible values for the parameters of the model to evaluate the values of the covariances and cross-correlations of the elements of the vector  $[M \ r \ G]$ . We again use the same structural parameter values reported in Table 6.1, and evaluate these covariances and correlations for different alternative values for  $\mu$  and  $(a/b)$ , on the assumption that the random shocks have unit variances. The results are reported in Tables 6.4 and 6.5 below.

Table 6.4: Covariances of the Dynamic Variables

	(a/b) = 1 $\mu = 0.0001$	(a/b) = 1 $\mu = 15$	(a/b) = 0.001 $\mu = 0.0001$	(a/b) = 0.001 $\mu = 15$
cov(Mr)	-0.48	0.170	-0.002	210
cov(Gr)	-0.24	-0.17	-0.001	-7.2
cov(MG)	-0.50	-0.50	-0.50	-0.50

Table 6.5: Correlations of the Dynamic Variables

	(a/b) = 1 $\mu = 0.0001$	(a/b) = 1 $\mu = 15$	(a/b) = 0.001 $\mu = 0.0001$	(a/b) = 0.001 $\mu = 15$
corr(Mr)	-0.477	0.185	0	0.968
corr(Gr)	-0.328	-0.732	-0.035	-0.991
corr(MG)	-0.673	-0.806	-0.999	-0.993

This confirms our previous argument that a larger value of  $(a/b)$  tends to make the covariance and correlation coefficient of



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M and r more negative, whilst a larger value of  $\mu$  tends to make it more positive. Again, this may be used to provide an interpretation of the casual empirical evidence regarding the short-run movements of the money stock vis-a-vis the interest rate during the 1970s in the UK, given the greater emphasis on meeting money stock objectives during this period compared to the 1960s<sup>10</sup>. In the next subsection we examine how this model may be matched with some casual empirical evidence on the UK experience of monetary targets in the 1970s and early 1980s, and we attempt to place the results of this model in the context of other accounts of the performance of monetary targets.

### 6.3.6. Monetary Targets and Actual UK Experience

It is always rather hazardous to relate purely theoretical models such as the model proposed in this section to actual experience of monetary targeting via casual empiricism. However, our model does suggest that in a regime of strict adherence to monetary targets, one would expect a positive relationship between movements in interest rates and movements in the money stock, both in the mean path of these variables and in their asymptotic covariances (i.e. in their stochastic movements around these mean paths).

To some extent this conforms with the pattern in the growth of broad monetary aggregates in the UK in the late 1970s and early 1980s, where attempts to bring the rate of growth of  $\text{£M3}$  under control via sudden large interest rate increases brought

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forth a short-term acceleration of monetary growth. Thus, for instance:

"Rapid monetary growth continued throughout the late summer and early autumn (of 1979)...The authorities' response came on 15th November (1979), when MLR was raised from 14% to a record 17% in order to encourage funding and discourage bank borrowing...even allowing for distortions (i.e. the removal of the corset)...(In 1980) monetary growth remained above target, although the substantially lower growth in M1, the rise in real interest rates,...,the strength of sterling and the perceived weakness in the real economy were accepted as evidence of the tightness of policy and hence the need to resist forces that might generate further increases in nominal interest rates..." (Hall, 1984, p.71, and p.103)

Furthermore at that period in time, the Governor of the Bank of England stated that:

"The lesson, perhaps, is the need to avoid attaching undue importance to short-term developments in any single monetary aggregate; it is sounder to take into account, as we in fact do, the underlying developments in both the aggregates as a whole and in the real economy. Taken overall, this evidence suggests that policy has been restrictive rather than otherwise..." (Bank of England Quarterly Bulletin, December 1980, p.458).

This is exactly the point highlighted by our model. If indeed one accepts the existence of forward-looking buffer stock

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behaviour then such short-term increases in the money stock can be expected when a strict monetary policy is imposed. Furthermore, our model suggests that the actual dynamics will depend upon the costs of adjustment (the  $a/b$  and  $\delta$  parameters in our model), and hence one may not expect the same type of dynamic adjustment in the money market in, say, the 1980s, compared to the 1960s.

The point to remember, though, is that whilst the information value of the current money stock appears to be reduced by such dynamic adjustment, in practice a fully rational 'theoretical model-type' policymaker and a fully rational private sector will not worry about the fluctuations in the money stock provided that they have perfect information about the underlying model of the economy. The problem with the practical application of monetary targets, as we can gauge from the above quotations, is that in practice we do not live in a world of perfect information, and failure to meet such targets in the short-term may lead to a lack of confidence in policymakers. Furthermore, as we saw in our model, an optimal setting of the policy feedback parameter, as we saw in the previous subsection, requires precise knowledge both about the current state of the money market (i.e. if it is in 'disequilibrium' at the outset) and about the value of the 'behavioural' parameters ( $a/b$ ) and  $\delta$ . This makes the implementation of monetary targets difficult, and may on occasion lead to overcontractionary or overexpansionary policies.

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### 6.3.7 Criticisms of the Model and Possible Alternative Frameworks

We now turn to ask whether the results obtained with our simple forward-looking model are in any way different from those which could have been obtained through the use of a simple backward-looking model. For instance, it would have been possible to analyse a model with the following structure:

$$DM(t) = -Y(M(t) - M^*(t)) \quad (6.47)$$

$$Dr(t) = \phi(\mu(M(t) + (\alpha_1 \beta_0 / \beta_1)) + (\beta_0 / \beta_1) - r) \quad (6.48)$$

where money holdings adjust gradually towards their desired value  $M^*$  according to a backward-looking adjustment process (the continuous-time equivalent of the simple partial adjustment mechanism). It can be shown that this dynamic system can also produce cyclical behaviour in the money stock, as it has two negative (stable) eigenvalues, which may or may not have an imaginary component. The eigenvalues of the system defined by (6.47) and (6.48) are given by:

$$\lambda_i = -(Y + \phi) \pm (Y^2 + \phi^2 - 2Y\phi(1 + 2\mu(\alpha_0 \beta_1 - \alpha_1)))^{1/2} \quad (6.49)$$

where  $i = 1, 2$ .

At first sight, this would seem to imply that the dynamic properties of our model which embodies forward-looking behaviour do not differ markedly from those of a model where money holdings adjust according to a simple partial adjustment process. However, there are a number of ways in which our model offers additional insights compared to the model given in (6.47)-(6.48).

Firstly, it should be pointed out that the dynamics of our

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forward-looking model are substantially different. The structure of our model imparts a higher order of dynamics on the system than the simple partial adjustment mechanism. This is easily seen by turning to Figure 6.2 above. In our forward-looking model, the  $DM = 0$  and  $M^*$  schedules are not the same, whilst in the model given by (6.47) and (6.48) it is apparent that  $DM = 0$  where  $M = M^*$ .

Furthermore, it should be apparent that, due to the presence of a forward-looking jump variable in our model, namely  $DM$ , the dynamic properties following a policy announcement will be substantially different. This will have not been apparent from our above analysis since we have focused mainly on current unanticipated shocks. However, announcement effects may have an important role in the dynamics of the money market and hence on the performance of monetary targets. Thus, the effectiveness of the money stock as an information variable is reduced if there is forward-looking behaviour and there is a degree of uncertainty regarding the authorities' future interest rate policy<sup>11</sup>. The same is not true of backward-looking partial adjustment mechanisms, where announcements have no effects. Furthermore, as we pointed out above, the effectiveness of the money stock as an information variable requires the authorities to know exactly what the current state of disequilibrium in the money market in order to implement an efficient interest rate policy. From the above subsection we have seen that in the UK in 1979-1980 there

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seemed to be considerable disagreement about the actual restrictiveness of monetary policy in the light of contrasting signals from the real economy and monetary growth statistics.

Secondly, in contrast to a partial adjustment model, which is essentially ad hoc, our model enables us to relate the dynamic and stochastic properties of the model (and the resulting efficiency of different interest rate policies) to the 'fundamental parameters' of the cost-minimisation process, namely  $a, b$  and  $\delta$ . As we pointed out above, this also enables us to speculate about the changes which may drive these parameters. Thus, if we believe that the costs of adjustment in the money market change over time, this may account to changes in the dynamics of the demand for money over time (the Lucas critique). (See also footnote 10).

These are some of the differences introduced by our model compared to a more conventional 'lagged-adjustment' model. On the other hand, there are a number of drawbacks in using this type of model to analyse the performance of monetary targets. The main problem, of course, is that the simple model used does not allow us to contrast the performance of monetary targets with the performance of alternative simple rules. In our model, given that there are no lags in the expenditure sector, it is obvious that it makes far more sense to target interest rate policy on real income itself. We chose to assume that this option was not available to policymakers thus ruling out any comparison between

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different simple rules. In a more realistic model we should have allowed for some price flexibility, included some open-economy aspects, and introduced lags in the wage-price and income-expenditure adjustment processes. This would have allowed us to contrast the performance of monetary targets with those of simple rules where monetary policy is targeted on (say) exchange rates, or nominal income. However, as we pointed out above there are already numerous studies which evaluate the performance of simple rules (see for instance Currie and Levine 1984, 1985, Edison et al., 1987, Taylor, 1988), albeit none of these studies allow for forward-looking behaviour in the money market. Furthermore, such a study would be a laborious one in itself, as it would involve a whole host of numerical simulation exercises. Analytical insights are impossible in larger models.

In addition to considering a more realistic model structure, there are other extensions which may be made to our analysis. First of all, there is the possibility of moving away from non-optimal simple monetary target rules, and actually introduce an optimising government, to find the optimal feedback parameter for the policy rule. One could then analyse the dynamics of the money market in terms of game theory with two separate optimisers, the private sector and the government, each taking account of each other's strategies. In particular such an extended model would allow us to analyse both Nash solutions and solutions which take the government as a Stackelberg leader. (It would on the other

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hand be unrealistic to assume that the private sector acts as a Stackelberg leader given its atomistic nature, see Backus and Driffill, 1985, Borio, 1986.) The nature of the government's commitment to policy and its reputation would then affect the behaviour of the money market. Our model offers the first base towards such an exercise.

### SECTION FOUR: CONCLUSIONS

In this chapter we have argued that the inclusion of policy rules in a buffer stock model of the demand for money reveals certain features about the dynamics of the money stock which may offer one explanation of the apparently peculiar behaviour of the money stock once monetary targets have been implemented. Thus, in periods where little emphasis is put on meeting money stock targets (i.e. a low value for  $\mu$  in our model), it is more likely that a strong negative correlation between movements in the real money stock and the interest rate will be observed. The opposite will be true when a greater emphasis is placed on meeting such targets through a 'vigorous' interest rate policy. This may provide us with a simple explanation for the casual empirical evidence on the behaviour of the UK money market during the 1970s and 1980s.

Furthermore, we have shown that implementing a more rigorous interest rate policy may or may not be an appropriate response to the apparent failure to meet monetary targets. Overall, it may be more sensible for the monetary authorities to adopt an eclectic



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policy when faced with such difficulties, focusing on other information variables in addition to the money stock. (However it should also be remembered that lags in other sectors of the economy will reduce the efficiency of other information variables.) Our model shows that whether a 'large' or a 'small' interest rate response is appropriate is entirely dependent upon the 'initial conditions' in the money market and the parameters of the forward-looking demand for money model. In each case the authorities may do far better if (in the absence of more reliable intermediate objectives, e.g. the exchange rate, nominal income) they were to make the short-term pursuit of monetary targets conditional on the behaviour of the short-run velocity of money, instead of sticking rigidly to money stock contingent targets.

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### Footnotes to Chapter 6

(1) This avoids the use of extreme policy settings. In practice there are real economic and political costs involved in varying policy instruments.

(2) Note that this system has no constant terms, and hence the equilibrium (if it is attainable) is at  $\bar{y} = 0$ ,  $\bar{x} = 0$ . This is not a special case, as any model in the form:

$$\begin{bmatrix} dy \\ dx \end{bmatrix} = A \begin{bmatrix} y \\ x \end{bmatrix} dt + Gdt$$

can be re-expressed in deviation from equilibrium form, where

$$\begin{bmatrix} \bar{y} \\ \bar{x} \end{bmatrix} = -A^{-1}G$$

(3) As we pointed out above, fully optimal policy rules of this type exploit 'all the available information' in the dynamic system.

(4) One proposal being put forward in the early 1980s was the introduction of monetary base control to exert a closer control over monetary aggregates (see Bank of England Quarterly Bulletin, 1979)

(5) A similar problem arises when computing the optimal investment decision. In that context, investment is a 'free' variable, whilst the capital stock is clearly a predetermined variable.

(6) This offers a closer approximation of the system of monetary control currently in use in the UK, given that the Bank of

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England never refuses to provide additional liquidity to the banking system if this is requested, but it will do so through eligible bill purchases at rates which it deems suitable for the purposes of exerting monetary control.

(7) Actual and desired money holdings may also not coincide in models where the monetary authorities are assumed to control the money stock exogenously, but where there is stickiness in the price level, real income, or the interest rate (see Artis and Lewis, 1976, Laidler, 1983). In this chapter we are focusing on a buffer-stock model which emphasises demand-side shocks, along the lines of the Cuthbertson and Taylor model analysed in Chapters 4 and 5.

(8) We should remember from our assumptions that the actual response of the interest rate depends in part upon the policy lag effect, controlled by the parameter  $\phi$ , over which the authorities may not necessarily have any control.

(9) There is an analogy here with the way in which one would use logarithmic tables to solve complex arithmetic problems, in the sense that these transforms allow us to solve calculus problems.

(10) It should also be pointed out that  $(a/b)$  is unlikely to remain invariant over time if the economic environment changes. In more uncertain times it would be reasonable to assume that economic agents would attach less importance to being away from equilibrium compared to the costs of adjustment. As we saw in the main text, one would expect such a fall in  $(a/b)$  to lead to a

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more positive correlation between  $M$  and  $r$ , thus offering an indication for what happened in the late 1970s, when firms tried to increase their borrowing from banks in the face of deteriorating conditions in the real economy. Overall, it is by no means sure that  $(a/b)$  will be independent of the stance of monetary policy, especially given the possibility that any financial innovation triggered off by monetary policy may affect the value of  $(a/b)$ .

(11) This problem of uncertainty regarding future policy is sometimes referred to (quite appropriately) as the 'finance minister problem'.

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### CHAPTER 7: CONCLUSION

In this chapter we summarise some of the results presented in this study, and examine some of the implications for the empirical modelling of the money market in general and the demand for money in particular. We begin, in Section one, by bringing together some of the themes covered in the first six chapters. In Section two we then consider an agenda for future research in the light of current work on asset demands. This will help us to consider the results obtained in this thesis in the context of the wider literature on asset demand models and the demand for money.

#### SECTION ONE: RESULTS AND CONCLUSIONS OF THE PRESENT STUDY

The objective which we set ourselves at the outset of this thesis was that of investigating some of the aspects underlying the empirical modelling of the demand for money. We chose the particular context of the United Kingdom to conduct our empirical studies, but the study has focused more particularly on the underlying empirical methodology of the models under scrutiny than on the institutional features of the UK economy<sup>1</sup>. This narrow focus is apparent from our choice of a limited set of explanatory variables in all the models examined.

We basically examined two different approaches to the empirical modelling of the demand for money, which we now summarise in turn.

##### 7.1.1 'Feedback-Only' Models

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First, we considered so-called 'feedback-only' models which may be seen as generalisations of the simple 'partial-adjustment' and 'adaptive-expectations' which gained such popularity in the 1960s and early 1970s. The feedback-only approach does not rely on a deep theoretical basis for its suggested specifications<sup>2</sup>, and, as we saw in Chapter 2, this very lack of a theoretical base tends to make the process of estimating such models rather ad hoc.

In fact, a number of different approaches to constructing feedback models have been advanced in the literature. The differences between these approaches consist in the order in which different steps are taken in the specification process. There are certainly common themes between them: in each case the search for an 'appropriate' model (in the statistical sense) proceeds in such a way as to lead us from a model which has a very 'general' specification, to one which is more parsimonious in character. However, there are also important differences between these various approaches (which we surveyed in Chapter 2). The main ones concern the following two points: first, whether one investigates the long-run properties of an economic relationship and imposes these on the model at the outset (i.e. before simplifying the dynamic structure of the model). Second, whether one should transform (reparameterise) the model at the outset, which will in fact influence the simplification process by which one reduces the parameter space in obtaining a

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parsimonious model.

As we stated in Chapter 2, these distinctions between different modelling strategies are by no means rendered trivial by the observation that all the different proposed models essentially originate from the same initial general Autoregressive Distributed Lag (ADL) model. Although a number of approaches in constructing feedback-only models have been advanced in the recent econometrics literature, there have so far been few attempts at exploring the consequences of taking these alternative paths when building an econometric model in practice<sup>3</sup>.

We therefore presented a survey in Chapter 2 of several alternative methods of constructing feedback-only models, and also suggested a different transformation to that usually advanced in the econometrics literature (see Wickens and Breusch, 1988) which had several advantages in estimation. These alternative modelling strategies were then compared in Chapter 3 and we have shown that they do not lead to identical results. In fact, it is quite possible, using non-nested testing techniques, to discriminate between them in terms of explanatory power.

These results are intriguing, and have implications which stretch far beyond the realm of the demand for money. In fact, the techniques scrutinised here have become the basis of a large proportion of recent single-equation econometric studies in economics. This, in our view, requires us to consider more

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carefully the differences between these approaches both at the level of econometric theory, and through further studies along the lines of the work presented in Chapter 3. We have no way of knowing whether our 'ranking' of different modelling approaches at the end of Chapter 3 is in any way robust: hopefully any further comparative studies will help resolve this question. At the very least our results in Chapters 2 and 3 should act as a warning to applied economists in how the choice of method is not a trivial one in modelling dynamic relationships. This should be seen as the main conclusion of Chapters 2 and 3.

Another conclusion which was reached in examining feedback-only models relates to the direct estimation of long-run model properties. As we pointed out above, some techniques (most notably the Engle-Granger two-stage procedure) rely on first of all establishing the existence of (and subsequently quantifying) a long-run relationship between the economic time-series under scrutiny. In Chapter 2 we demonstrated that the cointegration tests proposed to establish the existence of a long-run economic relationship tend to have a number of drawbacks, primarily due to their low power. Furthermore, we examined some of the difficulties encountered in obtaining reliable point estimates of long-run elasticities using cointegration equations. Given the way in which these techniques have infiltrated the applied economics literature, these results must be seen as rather worrying.



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The results also highlight a rather important aspect regarding cointegration tests. Because they purport to test for the 'existence' of an economic relationship, there is a temptation to regard the results obtained from these tests as being rather unambiguous: after all, something either exists or does not exist. However, at this point we should remember that these cointegration tests are nothing more than statistical tests. Thus, one may still make type I and type II errors, and 'existence' must be viewed in probabilistic terms. Perhaps one should view the issue of cointegration as somewhat analogous to Heisenberg's famous uncertainty principle in quantum mechanics<sup>4</sup>.

### 7.1.2 'Forward-Looking' Models

The second type of model which we have considered in this thesis are so-called 'forward-looking' models. In Chapter 4 we surveyed various theories which seek to emphasise the role of money as a buffer asset in economic agents' portfolios. In particular we focused on the multiperiod quadratic costs of adjustment model which has been proposed by Cuthbertson and Taylor. This model shows clearly how the buffer role played by money implies that the demand for money has a forward-looking nature. This approach therefore suggests the estimation of rational expectations models of the demand for money.

In Chapter 4 we considered two possible criticisms of this approach. We demonstrated that the cost-of-adjustment function used by Cuthbertson and Taylor in constructing their estimation

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equations is fundamentally flawed, in that they assume a simple dichotomy between money and non-money assets, ignoring both the role of saving which affects the total wealth stock and possible complications which arise when their simple framework is extended to a multi-asset framework (i.e. to a model with 3 or more assets in the portfolio). We therefore modified this model in two different ways.

First, whilst remaining in the context of the simple two-asset model, we allowed for the possibility of saving. We demonstrated that the resulting estimation equation for the forward-looking model differs considerably from that proposed by Cuthbertson and Taylor. We estimated both the simple Cuthbertson-Taylor model and our saving-based model, and the evidence obtained tends to indicate that the saving variable is significant in explaining the demand for money. Although the saving model obtained is less than totally satisfactory, we concluded that it seemed unadvisable to totally ignore an explicit role for saving and wealth in these forward-looking models.

Second, we extended the simple Cuthbertson-Taylor model to the three asset case, with the inclusion of saving. The resulting model is very similar to a forward-looking version of the 'interdependent adjustment' portfolio model suggested by Brainard and Tobin (1968). Although we did not estimate this model on the grounds that it seemed rather difficult to implement, we

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demonstrated (through the use of numerical simulation methods) that the real nature of money as a financial 'buffer' can really only be captured in a context where there are more than two assets. This again suggests that focusing on a single-equation model to illustrate the buffer stock nature of the demand for money is not likely to yield fruitful results.

### 7.1.3 Comparing Forward-Looking and Feedback-Only Models

Having examined two apparently diverse approaches to constructing single-equation models of the demand for money, the following two questions follow naturally:

- (a) What is the relationship between the two approaches at the theoretical level?
- (b) Under which circumstances is either model likely to outperform the other at the empirical level?

In Chapter 5 we demonstrated that in fact there is a degree of observational equivalence between the two types of model. This is not a problem unique to demand for money studies, but applies also to other areas of applied economics where the rational expectations hypothesis is applied (see Sargent, 1976). This observational equivalence in fact explains the reason why both approaches have been successful to some degree in constructing empirical models of the demands for money.

However, the second question is more difficult to resolve, and has been the subject of much debate in the recent literature. In Chapter 5 we have demonstrated, following Hendry (1988) and

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Muscatelli (1988b) that the two approaches can be tested against each other at the empirical level. Thus, whether the forward-looking model dominates the feedback-only model or not depends entirely on whether the 'Lucas critique' is relevant in the case of the demand for money. In Chapter 5 we examine different ways of testing the relevance of the 'Lucas critique', and conclude from the evidence that the feedback-only model dominates the forward-looking model in the case of the demand for M1 in the UK<sup>5</sup>.

We concluded our study with an analysis in Chapter 6 of the methods which could be applied to examine the implications of given specifications for the demand for money for the theoretical literature on monetary targets. In particular, we examined the way in which a forward-looking specification of money demand can distort the use of a monetary aggregate as an indicator of the stance of monetary policy.

### 7.1.4 Implications for the Empirical Verification of Single-Equation Demand for Money Studies

Having summarised the results obtained in the previous chapters we now turn to a discussion of the main implications which result for the modelling of the demand for money in the context of single-equation studies.

From the previous subsection it should be apparent that we believe that there are methods to discriminate between the forward-looking and backward-looking approach of the demand for

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money at the empirical level. In this thesis we have demonstrated that the Lucas critique does not appear to have much relevance for the demand for M1 in the UK. This does not of course exclude the possibility that it may prove relevant for other aggregates and in the context of other countries<sup>6</sup>. However, there are reasons to believe that, even in cases where the Lucas critique proves to be relevant, the 'forward-looking' model will still be handicapped by its rather restrictive dynamic structure (see Muscatelli 1988b, 1988c). As we pointed out in Chapters 4 and 5, the theoretical basis for the forward-looking model are not, in our view, sufficiently sound.

In most cases, therefore, we would advocate the use of a feedback-only model. In the context of feedback-only models, our results in Chapters 2 and 3 indicate that, whilst cointegration tests may prove an interesting check of whether some long-run relationship is likely to exist between integrated series, ultimately the results are likely to prove inconclusive, with a tendency to accept the null hypothesis of no cointegration. Furthermore, one cannot conduct any exercises in statistical inference with the standard errors obtained from cointegration (static) equations given the bias present. In these circumstances, it may be better to use transformed model to judge the appropriateness of explanatory variables in demand for money studies.

Thus, the results of Chapters 2 and 3 seem to have the

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following implications for the estimation of feedback-only models. First, cointegration tests provide some insights into the appropriateness of some variables in explaining the demand for money, but the significance of long-run multipliers (which may be gauged instantaneously from transformed models) may prove a better guide to the applied economist in deciding on a set of appropriate explanatory variables. Second, regarding the specification of the short-run dynamics, a number of routes may be taken. Our study has suggested that transformed models may have the benefit of ensuring that the long-run properties of the estimated model do not alter as a result of the simplification search. There is no guarantee, however, that transformed models will perform unambiguously better than other approaches in building a feedback-only model in other contexts. Third, it may be worth pursuing a number of different specification routes by starting the specification search with both a transformed model and a conventional unrestricted ADL model.

These conclusions relate to the material covered in this thesis. However, there are a number of issues relating to the modelling of the demand for money which we were unable to cover here. We now examine some possible avenues for future research which may help to shed further light on the issues discussed in the first six Chapters of this thesis.

### SECTION TWO: AN AGENDA FOR FURTHER WORK

There are a number of issues which, in our view, demand

further investigation in the context of the demand for money. We now examine these in turn.

### 7.2.1 Areas of Possible Further Research

(a) The role of additional variables. This has not received any attention in this thesis, but is a matter of great importance. Already some studies in the demand for money literature have indicated that variables such as wealth, the variance of asset returns, etc. are relevant in explaining some definitions of the money stock (see Grice and Bennett, 1984, Baba *et al.*, 1988). In the case of the UK, we suggest that it may be appropriate to investigate the role of these two variables, given the difficulties in modelling the demand for broad monetary aggregates with a limited set of explanatory variables. Furthermore, it may be appropriate to analyse in detail the open economy influences on the demand for money, which have been almost completely ignored in the current literature. If one seeks to model the demand for money across a time period when the UK experienced both fixed and floating exchange rates, it seems reasonable to argue that this should be allowed for somewhere in the model.

(b) The role of theory in determining the model used. As should be apparent from the models estimated in the past chapters, theory plays a secondary role in the estimation of asset demands. Recently Courakis (1988) has suggested that theory should play a greater role in the estimation of asset demands by, e.g.,

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determining the functional form used.

(c) The role of the supply of money. Another worrying aspect of the single-equation models surveyed in this thesis is that the role of the variables determining the supply of money is totally ignored. Typically, we seek to estimate a single-equation demand for money model without reference to a theory of money supply determination. In practice we should recognise that, given that we can only observe money holdings, our so-called 'demand for money studies' are probably better seen as semi-reduced forms including elements of both supply and demand. Recently, Foster (1988) has attempted to include supply elements in a model of the money stock, with some degree of success. Given the tenuous link to theory which demand for money studies currently have, it may make some sense to abandon any pretence of theoretical coherence, and concentrate on obtaining a model which may include elements of the supply-side of the money market.

The difficulty in abandoning theory entirely is, of course, that the policy implications of any model obtained in that manner are not at all obvious. A more appropriate way to allow for the behaviour of money supply is that of abandoning the use of single-equation models, and instead building a complete model of the financial sector which may lead to a better understanding of the interactions of money demand, money supply, and the real economy.

(d) The theory of short-run dynamic adjustment. Further research



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is required on the theory underlying the short-run dynamics of adjustment which were analysed in this thesis. Thus, for instance, in Chapter 4 and 5 we criticised the use of the single representative agent in the context of the forward-looking demand for money models. This suggests that more work needs to be done to develop models which allow for a degree of disaggregation to see if they can be rendered operational at the empirical level, and if they can give some insights into the nature of short-run dynamic adjustment.

### 7.2.2 Concluding Remarks

To conclude, in this thesis we have been able to offer some indications regarding the appropriateness of a number of approaches to the estimation of single-equation models of the demand for money. Many of the results obtained have important implications for the applied economist in the light of recent developments in the field of applied econometrics. Although the conclusions obtained are only strictly valid in the context of the demand for money data used here, some of the results may have important implications for the empirical modelling of other single-equation time series econometric models.

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### Footnotes to Chapter 7

(1) Some observers may argue that this may not be a satisfactory approach. We shall return to this issue in Section two.

(2) Although, as we pointed out in Chapters 2 and 3, some of the error-correction mechanisms which feature in these models are linked with the feedback control mechanisms which have been advanced in control theory for the control of dynamic systems (see Salmon, 1982).

(3) I am not currently aware of any attempts to compare these different approaches in the existing applied econometrics literature.

(4) The two issues are in fact similar: just as in physics one would imagine that physical quantities are not a matter for dispute (cf. Einstein's famous dictum that 'God does not play dice with the world'), so one would imagine that the 'existence' of an economic relationship should be verifiable. However, measurement difficulties in both physics and economics lead to the presence of uncertainty.

(5) However, the results presented in Chapter 5 have recently come under further scrutiny and their apparent conclusive nature has been questioned by Cuthbertson and Taylor (1988). Muscatelli (1988c) has argued that in fact the balance of the evidence points against the appropriateness of the forward-looking model and ascribes its poor performance vis-a-vis feedback-only models to the restrictive nature of the dynamic adjustment allowed for

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in Cuthbertson and Taylor's model.

(6) Although Muscatelli and Papi (1989) demonstrate that the forward-looking approach also seems to perform relatively badly when compared to a feedback-only model in the context of the demand for M2 in Italy.

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