ESSAYS IN LONG MEMORY : EVIDENCE FROM AFRICAN STOCK MARKETS

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Essays in Long Memory: Evidence from African Stock Markets

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application for the degree of Doctor of Philosophy.

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Abstract

This thesis explores various aspects of long memory behaviour in African stock markets (ASMs). First, we examine long memory in both equity returns and volatility using the weak-form version of the efficient market hypothesis (EMH) as a criterion. The results show that these markets (largely) display a predictable component in returns; while evidence of long memory in volatility is mixed. In general, these findings contradict the precepts of the EMH and a variety of remedial policies are suggested.

Next, we re-examine evidence of volatility persistence and long memory in light of the potential existence of neglected breaks in the stock return volatility data. Our results indicate that a failure to account for time-variation in the unconditional mean variance can lead to spurious conclusions. Furthermore, a modification of the GARCH model to allow for mean variation is introduced, which, generates improved volatility forecasts for a selection of ASMs.

To further evaluate the quality of volatility forecasts we compare the performance of a number of long memory models against a variety of alternatives. The results generally suggest that over short horizons simple statistical models and the short memory GARCH models provide superior forecasts of volatility; while, at longer horizons, we find some evidence in favour of long memory models. However, the various model rankings are shown to be sensitive to the choice of error statistic used to assess the accuracy of the forecasts.

Finally, a wide range of volatility forecasting models are evaluated in order to ascertain which method delivers the most accurate value-at-risk (VaR) estimates in the context of Basle risk framework. The results show that both asymmetric and long memory attributes are important considerations in delivering accurate VaR measures.

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Acronyms

ARFIMA	Autoregressive Fractionally Integrated Moving Average
ARMA	Autoregressive moving average
APARCH	Asymmetric Power Autoregressive Conditional Heteroskedasticity
ASM	African Stock Markets
CGARCH	Component GARCH
EGARCH	Exponential General Autoregressive Conditional Heteroskedasticity
EMH	Efficient market hypothesis
ES	Exponential smoothing
EWMA	Exponentially weighted moving average
FIEGARCH	Fractionally integrated EGARCH
FIAPARCH	Fractionally integrated APARCH
FIGARCH	Fractionally integrated GARCH
GARCH	General Autoregressive Conditional Heteroskedasticity
HM	Historical mean
HYGARCH	Hyperbolic GARCH
MA	Moving average
MAE	Mean absolute error
MME(*O*)	Mean mixed error (overpredictions)
MME(*U*)	Mean mixed error (underpredictions)
MZ	Mincer-Zarnowitz
RM	RiskMetrics
RMSE	Root mean square error
RW	Random walk
SPA	Superior predictive ability
TGARCH	Threshold GARCH
VaR	Value-at-Risk

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1. Introduction

1.1. Theoretical Background: Long Memory in Time Series

Interest in long memory (or long range dependent) processes can be traced to the examination of data in the physical sciences. Formal models with long memory were introduced by Hurst (1951) and pertained to hydrological studies investigating how to regularise the flow of the Nile river in view of its nonperiodic (flooding) cycles. Mandelbrot and Wallis (1968) described this feature as the “Joseph effect” alluding to the biblical reference in which seven years of plenty where to be followed by seven years of famine. In this sense, long memory process concern observations in the remote past that are highly correlated with observations in the distant future. The implications of long memory in financial markets was first studied by Mandelbrot (1971) who proposed using Hurst’s ‘rescaled range’ statistic to detect long memory behaviour in asset return data. He further observed that if security prices display long memory then the arrival of new market information cannot be arbitraged way, which in turn means that martingale models for security prices cannot be derived through arbitrage. Furthermore, Lo (1991) contends that standard tests of both the capital asset pricing model and the arbitrage pricing theory are redundant if the asset returns display long memory behaviour.

Against this background, long memory (or long-range dependence) describes the correlation structure of a series at long lags; where, the series are characterised by irregular cyclical fluctuations. Mandelbrot (1977) characterises long memory processes as having fractal dimensions, in the form of non-linear behaviour marked by distinct but nonperiodic cyclical patterns and long-term dependence between distant observations.

A variety of measures have been used to detect long memory in time series. For example, in the time domain, long memory is associated with a hyperbolically decaying autocovariance function. Equivalently, the presence of long memory is indicated by a spectral density function that approaches infinity near the zero frequency; in other words, such series display power at low frequencies (Lo, 1991; Di Sario *et al*, 2009). These notions have led several authors to develop stochastic models that capture long memory behaviour, such as the fractionally-integrated I(*d*) time series models introduced to economics and finance by of Granger (1980), Granger and Joyeux (1980), and Hosking (1981). In particular, fractional integration theory asserts that the fractional difference parameter which indicates the order of integration, is not an integer value (0 or 1) but a fractional value (Baillie, 1996). Fractionally integrated processes are distinct from both stationary and unit-root processes in that they are persistent (i.e., they reflect long memory) but are also mean reverting and as a consequence provide a flexible alternative to standard I(1) and I(0) processes. Specifically, the long memory parameter is given by $d \in (0, 0.5)$ while when $d > 0.5$ the series is nonstationary and when $d \in (-0.5, 0)$ the series is antipersistent.

Much of the recent empirical research has focused the role of persistence of shocks, and a large literature has emerged on testing for and estimating fractional processes which have been widely used to describe long memory processes. Indeed, the most familiar model in the analysis of financial time series data is the ARFIMA (*p*, *d*, *q*) model which provide an alternative to ARIMA (*p*, *d*, *q*) processes by not constraining the parameter *d*, to integer values but rather allowing it assume fractional values.

While initial interest in long memory dynamics in financial time series were concentrated on the behaviour of the conditional mean it became widely reported that the autocorrelations of various volatility measures decay at a hyperbolic rate (e.g., Ding *et al*, 1993; Bollerslev and Wright, 2000). In light of these processes, Baillie *et al* (1996) introduced models capturing long memory in the conditional variances. Specifically, they pioneered the FIGARCH class of processes, which capture long memory behaviour in the conditional variance; and, are now extensively used to capture the observed temporal dependencies in financial market volatility.

Since, non-zero values of the fractional differencing parameter imply dependence between distant observations, considerable attention has been directed to the analysis of fractional dynamics in financial time series data. Indeed, long memory behaviour has been reported in the returns of various asset classes, e.g., Granger and Poon (2003) and Poon (2005) for a comprehensive survey. Against this background, a rapidly expanding set of models has been developed to capture long memory dynamics is asset return data (e.g., Poon 2005 and references therein contained).

Using long memory processes as a criterion this research explores various aspects of long memory behaviour in the context of ASMs. In particular, this research aims is to contribute to the extant literature concerning long memory dynamics in equity data that have attracted attention elsewhere. Since evidence of long memory in equity data implies the existence of autocorrelation in the data and as a consequence future stock observations can be predicted on the basis of past realisations of the data. This has topical risk management and policymaking implications that this research will discuss.

For example, our results indicate the rejection of the weak-form version of the EMH in ASMs which in turn suggests strengthening measures designed to enhance the timely disclosure and dissemination of information vis-à-vis the performance of listed companies. In addition, policymakers can consider the implementation of measures to foster the establishment of regional exchanges (e.g., promoting cross-border listings) to enhance liquidity and market efficiency (Irving, 2005 and Adelegan, 2008).

Furthermore, as is well-known, accurate volatility forecasts are an essential component of derivative pricing and risk measurement (Chesney and Scott, 1989; Hull and White, 1987). Against this backdrop we re-examine evidence of volatility persistence and long memory in the presence of structural changes in ASMs. In particular, we focus on the impact of ignored structural breaks and time-variation in the unconditional mean of the variance process. This is relevant because traders (and investors in general) may benefit from a better understanding of how shocks affect volatility over time and the role that structural changes may play in this process.

Indeed, Poterba and Summers (1986) show that the extent to which stock return volatility is persistent, is important, since it affects equity prices (through a time-varying risk premium). This asset pricing paradigm has many other extensions, including the pricing of derivatives. For example, in option markets, traders will be willing to pay higher prices for options if they perceive that shocks are permanent with respect to the life of the option (Malik *et al*, 2005).

In order to evaluate future returns from equity investments or the need for policy intervention it is important to forecast volatility (Loeys and Panigirtzoglou, 2005). As such, we compare and evaluate the forecasting performance of a variety of volatility forecasting techniques. Our results are diverse and show that model performance is sensitive to the choice of evaluation criteria and sample frequency employed. At longer horizons (i.e., at the monthly level) we find some evidence in favour of the outperformance of long memory models reflecting the usefulness of these models in forecasting volatility over longer time spans. Indeed, Bollerslev and Mikkelsen (1996) show that it is necessary to incorporate a long-term volatility structure when pricing derivative contracts with a long maturity.

In terms of VaR estimation our findings may provide guidance on more effective prudential standards for operational risk measurement and, as result, may help ensure adequate capitalisation and reduce the probability of financial distress. Our results highlight the importance of using out-of-sample forecasting techniques and the stipulated probability level for the identification of methods that minimise the occurrence of VaR exceptions. In particular, we find that models incorporating both asymmetric and long memory attributes generally outperform all other methods in estimating VaR across the three percentiles we considered.

In total, our results provide a range of volatility estimates and forecasts which could potentially inform portfolio management strategies and guide policymaking. In particular, while most empirical studies focus on the United States and other developed markets, recent research has begun to look at emerging markets, however, limited evidence exists with respect to ASMs. Against this background, this thesis contributes to the empirical literature by focusing on various aspects of long memory behaviour in African equity data. Finally, the findings from this thesis complement those in previous studies and may provide an interesting comparison to existing studies.

1.2. Synopsis of thesis

After this brief introduction, chapter two details the trends and characteristics of ASMs while chapter three provides a description of the data used in this research. Chapter four starts the empirical analysis by investigating long memory in equity returns and volatility using ARFIMA-FIGARCH and -HYGARCH models in order to assess the informational efficiency of ASMs using the weak-form EMH as a criterion. The results show that these markets (largely) display a predictable component in returns; while evidence of long memory in volatility is very mixed. For the most part these results provide evidence against the EMH. In comparison, results from the benchmark comparators (UK and US) show short memory in returns while evidence of long memory in volatility is mixed. These results show that the behaviour of equity market returns and risks are dissimilar across markets and this may have implications for portfolio diversification and risk management strategies and indeed policymaking. For example, we advocate the introduction of measures designed to enhance the dissemination of information on the performance of listed companies and the cross-listings of stocks and the formation of regional equity markets to bolster liquidity and hence support the price discovery process. Following this, chapter five re-examines evidence of volatility persistence and long memory in light of potential regime shifts (in particular, time-variation in the unconditional (or long run) mean) in the volatility series. In particular, recent evidence has suggested that evidence of long memory may be spurious arising from neglected breaks or time-variation in the unconditional mean. The results obtained suggest that evidence of volatility persistence and long memory are generally overstated when analysed on the assumption that the unconditional variance is constant. Indeed, both breakpoint tests and a moving average application suggest that the unconditional displays substantial time-variation. Furthermore, modification of the GARCH model to allow for mean variation generates improved volatility forecasting performance for some markets. Chapter six evaluates the forecasting performance of a variety of statistical and econometric models at the daily and monthly frequencies under a variety of criteria including both symmetric and asymmetric loss functions. The findings are diverse. In particular, the results show that model performance is sensitive to the choice of evaluation criteria employed. Furthermore, our results indicate that long memory models deliver mixed forecasting performance relative to the other alternatives when both symmetric and asymmetric loss functions are applied at the monthly frequencies (i.e., longer horizons). Nonetheless the evidence indicates the usefulness of these models over longer horizons. Finally, on the basis of the test of superior predictive ability we find that at the daily level the GARCH provides more accurate forecasts while at the monthly frequency evidence in favour of long memory models is mixed.

The seventh chapter extends research concentrating on the evaluation of alternative volatility forecasting models under Value-at-Risk (VaR) estimation in the context of the Basle market risk framework by widening the class of GARCH models used to include more recent extensions of these models (in particular, a variety of long memory models), in addition to the standard RiskMetrics method widely used by financial institutions. These models are then assessed to examine the accuracy of VaR estimates at various confidence levels. The analysis generally reveals that models which capture long memory behaviour (especially multiple volatility components), asymmetric and power effects are important in delivering improved VaR estimates. In addition, we find that all the models considered generally outperform the RiskMetrics and standard GARCH method in estimating the VaR at the three extreme percentiles we consider. In order to verify the adequacy of our results we perform some diagnostic tests which offer some evidence indicating that our selected models are well specified. In sum these VaR results underscore the importance of using the stringent probability level prescribed by the Basle Accord and of using fully out-of-sample methods for the identification and evaluation of volatility models that improve the accuracy of VaR estimates and mitigate the associated regulatory intervention it implies. The thesis concludes with a summary of the work I have done and its potential impact and a brief outline of potential investigations which may evolve from this. Appendix A, gives a list of publications and conference papers produced during the course of my studies.

In summary, this thesis concentrates on the estimation of long memory models in the testing of the weak-form EMH; the examination of structural breaks on measures of volatility persistence and long memory; the forecasting performance of a range of volatility models; and finally the calculation of value-at-risk in the context of the Basle market risk framework. Our results suggest that investment strategies targeting (or linked to) African equities should be based on a complete characterisation of ASM volatility. Indeed, our findings suggest that long memory dynamics are an integral part of that characterisation. Furthermore, our research provides a number of policy conclusions relating to fostering stock market efficiency (e.g., measures to enhance information dissemination and counteract illiquidity) and portfolio management strategies (e.g., derivation of accurate volatility forecasts especially in the presence of structural breaks which could then be used to price options, for example) and risk management (e.g., adoption of appropriate volatility forecasting methods to calculate VaR estimates). This research is therefore of potential interest and value to market participants, policymakers and other researchers; and, fills an important gap in the empirical literature concerning long memory dynamics in ASMs which have hitherto not been comprehensively explored. In particular, a variety of applications of the estimated long memory models are demonstrated and the relevance of these models in the context of ASMs is highlighted.

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2 ASMs: Trends and Characteristics

2.1 Introduction

Prior to 1987, there were only 8 stock markets in Africa, at the end of 2007, there were 22 stock markets ranging from new markets such as those in Cape Verde and Libya (launched in 2005 and 2007, respectively) to the more established markets like those in South Africa and Egypt (founded in 1887 and 1888, respectively). Table 2.1 and 2.2 show that from 1996 to 2007, African stocks markets increased their total market capitalisation from about USD320 billion to approximately USD1,125 billion as these countries opened up to foreign investors. In particular, investors and investment funds have channelled capital into these markets in order to take advantage of high return prospects and concomitant diversification benefits associated with these markets. ASM capitalization had a median value of 19.8 percent of GDP in 1996. By 2007, this proportion had increased to 75.3 percent of GDP amid substantial growth and development of these markets.

2.2 Development of ASMs and Financial Sector Reforms

The establishment or revitalisation of ASMs took place in a context of major reforms, especially during the 1990s. These measures included the liberalisation of their financial sectors, privatisation of state-owned enterprises, the improvement of the investment climate, introduction of a more robust regulatory framework and improvements in the basic infrastructure for capital market operations. (de la Torre and Schmukler, 2005). These reforms set the stage for a significant market expansion, with a trend development in size and liquidity. New equity issues, volume and value of trading, and the number of traded companies all recorded significant progress. As a result, market capitalisation increased from a median of about 19.5 percent of GDP to approximately 57.8 percent of GDP from 1996 to 2007 for ASMs, and the turnover ratio rose from a median of 5.2 percent to 19.3 percent (Table 2.1 and 2.2).

Table 2.1: Stock Market Indicators of ASMs in 1996

	Number of listed domestic companies	Market Capitalisation (USD, billions)	Market Capitalisation of listed companies (% of GDP)	Turnover ratio (%)
Botswana	12	0.3	8.0	9.0
Egypt	646	14.2	18.8	22.2
Ghana	21	1.5	19.8	1.1
Kenya	56	1.8	15.4	3.7
Mauritius	40	1.7	20.1	5.4
Morocco	47	8.7	23.8	5.9
Namibia	12	0.5	10.3	12.1
Nigeria	183	3.6	16.7	2.6
South Africa	626	241.6	218.2	10.9
Tunisia	30	4.3	21.9	6.8
Zimbabwe	64	3.6	38.7	8.8
Median ASM	**47**	**3.6**	**19.8**	**6.8**
UK	100	1,554.6	133.7	78.4
US	500	9,396.2	142.2	92.1

Source: International Financial Statistics; World Development Indicators; Emerging Market Data Base; official web sites of stock exchanges and author’s calculations.

Table 2.2: Stock Market Indicators of ASMs in 2007

	Number of listed domestic companies	Market Capitalisation (USD, billions)	Market Capitalisation of listed companies (% of GDP)	Turnover ratio (%)
Botswana	31	5.9	57.01	2.4
Egypt	591	134.9	102.3	46.3
Ghana	32	2.4	18.4	3.4
Kenya	54	13.4	63.2	15.8
Mauritius	94	6.2	87.9	6.0
Morocco	66	18.5	84.1	27.1
Namibia	28	0.7	11.0	4.6
Nigeria	202	86.3	75.3	13.8
South Africa	401	833.5	327.1	50.0
Tunisia	51	5.0	22.5	19.7
Zimbabwe	82	6.1	106.4	1.5
Median ASM	**66**	**6.2**	**75.3**	**13.8**
UK	100	3,598.2	206.9	83.6
US	500	19,835.7	211.2	104.8

Source: International Financial Statistics; World Development Indicators; Emerging Market Data Base; official web sites of stock exchanges and author’s calculations.

The diversity of ASMs is illustrated in Table 2.1 and 2.2, which gives our calculations for the number of companies listed in each market, market capitalisation in US dollars, and other key market indicators. At the end of 2007, the Johannesburg Securities Exchange (JSE) in South Africa had a market capitalisation of USD855.3 billion and is the largest on the continent. The JSE is an anomaly in several respects. First, it represents almost 75 percent of Africa’s total market capitalisation. Second, while the other ASMs have a low correlation with other global markets (e.g., Smith *et al*, 2002), it is integrated with the major international financial markets (for example, during the financial market crisis of 1998, the JSE overall share index declined by 30 percent in August 1998). Third, it is as a consequence, similar in character to the larger emerging markets found in Latin America and Asia. The second and third largest equity markets are those of Egypt and Nigeria, which have a market capitalisation of USD148.5 billion and USD131.1 billion, respectively. These three markets account for almost 90 percent of the market capitalisation of ASMs. In addition, these markets dominate the number of listed firms in ASMs. The other ASMs we analyse are small and range in market capitalisation from USD18.5 billion (in Morocco) to USD0.7 billion (in Namibia). Beyond these ASMs there are also a number of newer markets in Cameroon and Rwanda which are not analysed because of the unavailability or insufficiency of data.

Although, there are marked differences in the size and number of listed companies, ASMs share a number of attributes. For example in 1996, liquidity as measured by the turnover ratio ranged from 1.1 percent in Ghana to 22.2 percent in Egypt (Table 2.1). By 2007, the turnover ratio ranged from 2.4 percent in Botswana to 50.0 percent in South Africa (Table 2.2). These measures are low relative to comparable figures from other emerging markets. Indeed, the most liquid emerging markets have turnover ratios in excess of 100 percent (Magnusson and Wydick, 2002). Another feature shared by ASMs pertains to the presence of non-synchronous trading or non-trading-effects which in turn reflect small market size and further compound illiquidity (Yartey and Adjasi, 2007). In the case of South Africa, Smith *et al* (2002) suggest that the relative illiquidity of the JSE reflects the domination of share holdings by a small number of very large corporations. This is further reinforced by a pattern of cross-shareholding among these companies which further stifles liquidity. More generally illiquidity implies that the cost of trading remains high and illiquidity begets further illiquidity by limiting the capacity of investors to unwind their positions; which in turn, may potentially stimulate further volatility thereby deterring further market entrants on both buy and sell sides, which, in turn, perpetuates the cycle of illiquidity (see, e.g., de la Torre and Schmukler, 2005).

Another common feature of ASMs is in terms of the composition of listed stocks where market capitalisation is dominated by stocks from the mining and energy, banking and financial services, telecommunications and the tourism sectors. For example, at the end of 2007 stocks in mining accounted for 63 percent and 69 percent of total market capitalisation in Botswana and Ghana respectively; while, in Mauritius banking and financial service stocks represented almost 72 percent of the market value.

Furthermore, ASMs present portfolio diversification benefits given that they are lowly correlated to the major world financial markets. Indeed, Alagidede (2008) presents evidence which shows that average monthly equity return correlation between ASMs and the major international stock markets is 14 percent. In addition, he shows that ASMs are characterised by weak correlations with each other. More precisely, with the exception of South Africa, ASMs are not closely integrated with international capital markets and price action (on local stock exchanges) is driven more by domestic developments than global events (Irving, 2005). Against this background, the potential gains from international portfolio diversification have attracted investors to ASMs.

While key indicators of market development (presented in Table 2.1 and 2.2) point to underdeveloped equity markets, ASMs have been among the fastest growing in the world and have as a result attracted significant investor attention especially in light of the potential portfolio diversification benefits they offer. In addition, ASMs continue to perform well in terms of return on investment relative to other emerging markets and indeed the major international markets. For example, in 2004, the Ghana stock market recorded a growth rate of 144 percent in US dollar terms making it the world's best performing equity market in that year; in comparison, the Morgan Stanley Capital International Global index appreciated by only 30 percent (Databank Group, 2004). Similarly, in Egypt, the Cairo and Alexandria Stock Exchange (CASE) 30, which groups the stocks of the top Egyptian companies in a benchmark index, has risen more than five-fold since Egypt launched its economic reform drive in July 2004. The Zimbabwe stock exchange grew by almost 450,000 percent in 2007 compared to a year ago. Even after adjusting for hyperinflation the ZSE is among the best performing stock market in the world (Irving, 2005).

Table 2.3 illustrates the performance of ASMs expressed in terms in terms of the mean return and the volatility-adjusted return (i.e., Sharpe ratio). The returns are calculated in both local currency terms and on the basis of the US dollar (USD). Returns in Zimbabwe are the highest in both local and USD terms with values of 219.4 and 163.8 respectively. In contrast, Namibia delivers the lowest return in both local currency and US dollar terms at 12.5 and 4.9, respectively. More generally, even after converting these returns into USD terms performance in ASMs remain robust. In local currency terms Egypt records the best Sharpe ratio at 0.48; while, in USD terms Botswana delivers the highest Sharpe ratio at 0.57, indicating superior volatility-adjusted performance compared to the other ASMs. Kenya provides the lowest Sharpe ratio in domestic currency terms (0.11) while Namibia records the lowest Sharpe ratio in US dollar terms (at 0.07). More generally, Senbet (2008) shows that after controlling for risk, USD returns from ASMs are similar to those in other emerging markets (notably in Latin America and Asia) in US dollar terms.

Furthermore, he stresses that there are diversification opportunities in ASMs that derive from their weak correlations with the major international financial markets and their associated risks.

Table 2.3: Risk Adjusted Performance of ASMs

	Performance Based on Local Currency (1996-2007)		Performance Based on S&P-EMDB-USD (1996-2007)	
	Mean Return	Sharp Ratio	Mean Return	Sharp Ratio
Botswana	18.2	0.32	24.4	0.57
Egypt	84.7	0.48	38.1	0.24
Ghana	56.1	0.36	3.49	0.22
Kenya	21.3	0.11	23.1	0.17
Mauritius	13.7	0.42	13.7	0.24
Morocco	16.8	0.39	9.57	0.46
Namibia	12.5	0.17	4.90	0.07
Nigeria	45.4	0.40	16.8	0.10
South Africa	15.4	0.25	9.07	0.12
Tunisia	17.6	0.27	6.92	0.18
Zimbabwe	219.4	0.35	163.8	0.04

Source: Author's calculations. The Sharpe ratio is based on mean stock return and the mean risk free (i.e., treasury bill) rates. Annual data was used for these calculations.

2.3 Overview of National Stock Exchanges

In the section that follows a summary of some of the key characteristics of the eleven ASMs included in this study. These calculations are based on the figures summarised in Table 2.1 and Table 2.2.

i) Botswana Stock Exchange (BSE)

The Botswana Stock Market (BSM) was established in June 1989 with five companies capitalised at USD 40 million, as part of the government's strategy to diversify and broaden the financial sector, and to provide a secondary market for publicly held shares (Jefferis *et al*, 2001). The BSE was formally inaugurated in 1995, following the enactment of the BSE Act in 1994. At the end of 1996, 12 companies were listed on the BSE with a total market capitalisation of USD326 million. In contrast, at the end of 2007, there were 18 listed securities with a total market capitalisation of USD5.2 billion. Despite this increase in market capitalisation, the turnover ratio remains low (relative to global standards). Indeed, at the end of 2007 the turnover ratio was only 2.4 percent compared to 9.0 percent in 1996. The liberalisation of exchange controls announced at the end of 1996 allowing dual listings on to the BSE has resulted in the introduction of two additional indices. The Foreign Companies Index (FCI) which reflects the price movement of the dual listed stocks, while the All Companies Index (ACI) reflects the whole market (i.e., the domestic company index, (DCI) and FCI).

ii) Egyptian Stock Exchange (ESE)

The Egyptian Stock Exchange is second oldest in Africa and comprises two exchanges: the Alexandria Stock Exchange (established 1888) and the Cairo Stock Exchange (set up in 1903). Prior to the introduction of central planning policies and the nationalisation of industries that took place in the 1950s it was the fifth most active stock exchange in the world (Mecagni and Sourial, 1999). However, these policies led to a significant reduction in market activity and as a consequence the market remained dormant until the early 1990s. The revitalisation of the ESE as a market for capital occurred within a context of financial sector reforms that included financial liberalisation, deregulation and privatisation of the economy which began in 1992, with the enactment of the Capital Market Law No. 95, which replaced the plethora of laws previously regulating the securities markets and paved the way for more efficient resource mobilisation and allocation to the corporate sector. Against this background, market capitalisation increased from USD14.1 billion (or 18.8 percent of GDP) in 1996 to USD134.9 billion (or 102.3 percent of GDP) in 2007. Liquidity measured by the turnover ratio more than doubled over this period from 22.2 percent to 46.3 percent.

iii) Ghana Stock Exchange (GSE)

As part of a series of measures geared towards financial reform given the ongoing emphasis on financial liberalisation and deregulation, the Ghana Stock Exchange (GSE) was launched in 1990. Ghana‘s introduction of partial capital account liberalisation in 2006 further opened up participation in domestic capital markets to foreign investors and helped further develop the market. In 1996, 21 companies were listed with a market capitalisation of USD1.5 billion, 11 years later; the GSE has 32 listings, with a market capitalisation of USD2.4 billion. However, liquidity (measured by the turnover ratio) still remains shallow 3.4 percent compared to 1.1 percent ten years earlier. Yartey and Adjasi (2007) highlight the importance of the stock market in financing corporate growth in Ghana. Over the period 1995 to 2002, the stock market financed about 12 percent of total asset growth of listed companies. Although the Ghana Stock Exchange (GSE) has been an important source of financing for corporations, it remains small and illiquid. Despite these structural impediments, the GSE delivers robust growth performance, in terms of investment return, for example, the annual return on the Ghana Stock Exchange, was 144 percent in U.S. dollar terms in 2004 compared with a 30 percent return by MSCI Global Equity Index, making it the best performing stock market in the world.

iv) Nairobi Stock Exchange (NSE) in Kenya

An informal share market began operating in Kenya in the 1920s. However, the Nairobi Stock Exchange was formally constituted in 1954 as a regional exchange for Tanzania, Uganda and Zanzibar. After these countries attained independence the NSE became Kenya's national exchange and accordingly halted regional capital market operations. At the end of December 2007, market capitalisation was USD13.4 billion, an increase of 644.1 percent compared to its value of USD1.8 billion in 1996. As a percentage of GDP market capitalisation has increased from 15.4 percent to 63.2 percent while, the turnover ratio has increased from 3.7 percent to 15.8 percent, in 1996 to 2007, respectively. In contrast, the number of listed companies has barely changed (56 in 1996 compared to 54 in 2007) reflecting among others an elevated pace of mergers and acquisitions. The exchange has three main market tiers dealing with the overall market, alternative investments and fixed income. Plans are currently underway for the establishment of a futures and options market segment.

v) Stock Exchange of Mauritius (SEM)

The Stock Exchange of Mauritius (SEM) opened in 1989 and was liberalised (i.e., opened to foreigner investors) in 1994. Furthermore, the development of the SEM has benefitted from Mauritius's position as an offshore (international) financial service centre. Against this background, the SEM has experienced rapid growth. For instance, at the end of 2007, market capitalisation was USD6.2 billion compared to USD1.7 billion at the end of 1996. As a proportion of GDP market capitalisation has increased from 20.1 percent in 1996 to 87.9 percent in 2007. Over this period, the number of listed securities has more than doubled from 40 in 1996 to 94 at the end of 2007. Despite these developments the level of liquidity as measured by the turnover ratio remains low. In particular, the turnover ratio was 5.4 percent and 6.0 percent in 1996 and 2007, respectively.

vi) Casablanca Stock Exchange (CSE) in Morocco

While the CSE dates back to 1929, its major re-development occurred in the 1990s. At the end of 1996, 47 companies were listed on the exchange; while by the end of 2007, 66 corporations were listed. Over this period market capitalisation has risen 112.6% to USD18.5 billion from USD8.7 billion. Similarly, both market capitalisation as a percentage of GDP and the turnover ratio displayed significant growth, rising from 23.8 percent to 84.1 percent and from 5.9 percent to 27.1 percent, respectively.

vii) Namibia Stock Exchange (NSX)

While, the NSX was formally constituted in 1992, an earlier share market was briefly operational from 1910. At the end of 2007 and on the basis of the number of listed companies, market capitalisation and the ratio of market capitalisation to GDP the NSE is the smallest market in our study of ASMs. From the end of 1996 to the end of 2007, market capitalisation increased 48.9 percent from USD470 million to USD700 million, and the number of companies traded has risen from 12 to 28. As a percentage of GDP, market capitalisation remains little change from 10.3 percent in 1996 to 11.0 percent in 2007. In contrast, liquidity, as measured by the turnover ratio has declined from 12.1 percent to 4.6 percent as stock ownership became more concentrated (following merger and acquisition activity). In addition, close to three-quarters of the stocks traded on the NSX have primary listings on the Johannesburg Securities Exchange (JSE) in South Africa. This in turn also means that the bulk of market activity involves these dual listed equities. Since its inception in 1992, the NSE has been open to foreign investment and has been closely linked with the JSE in terms of the latter's trading system technology.

viii) Nigeria Stock Exchange (NSE)

The Nigeria Stock Exchange (NSE) was set up in 1960. For most of its existence trading activity on the NSE has mainly involved government fixed income securities. However, following the introduction of capital market reforms in the context of overall financial market liberalisation in 1995 the NSE has experienced substantial growth and development in equities trading. For example, market capitalisation has risen by 2297 percent from USD3.6 billion in 1996 to USD86.3 billion in 2007. The growth in the NSE has been driven in large part by the rapid development of the mineral and financial services sector. Over this period, market capitalisation as a percentage of GDP increased from 16.7 percent to 75.3 percent. Although the Nigerian market has a relatively large number of stocks (202 listed stocks in 2007 compared to 183 in 1996) trading levels are low and the market is comparatively illiquid although it has improved (e.g., in 1996 the turnover ratio was 2.6 percent and had risen to 13.8 percent to 2007).

ix) Johannesburg Securities Exchange (JSE) in South Africa

Johannesburg Securities (formerly Stock) Exchange (JSE) in South Africa is the oldest on the continent having being established in 1887. The JSE is the largest and most developed in Africa. Towards the end of 1995, the JSE experienced a comprehensive set of reforms geared to improve operational, institutional and regulatory capacity in line with international best practice. Furthermore, financial reforms where implemented to allow greater foreign participation in the JSE in 1996 the JSE accounted for 86 percent of Africa's total market capitalisation. In 2007, this ratio had declined to 75 percent as other ASMs developed. In 1996, the JSE ranked as the sixteenth largest stock market in the world in terms of market capitalisation (USD242 billion). By the end of 2007, the JSE was the largest emerging equity market in the world, with market capitalisation at USD833.5 billion reflecting significant foreign portfolio inflows (and in turn South Africa's inclusion in major investable global stock market indices).

Despite its size, total trade (turnover) represented 10.9 percent of market capitalisation in 1996, reflecting the general illiquidity of the JSE, which in turn reflects a few large listings and the buy-to-hold strategy of domestic institutional investors. At the end of 2007, the turnover ratio had risen to 50.0 percent reflecting a more diverse ownership structure. However, this ratio is still considerably less than that of other large emerging markets in Asia and Latin America (which are in excess of 100 percent). Finally, the JSE is the only ASM that actively trades a variety of derivatives and asset backed securities. For instance, index and single stock futures are actively traded on the JSE; and a variety of currency derivatives are also listed on the JSE.

x) Tunisia Stock Exchange (TSE)

The TSE was established in 1969. In 1995, a variety of financial reforms were introduced in order to promote further stock market activity. While, the number of listed companies rose to 51 in 2007 from 30 in 1996, other market indicators are little changed. For example, market capitalisation was at USD4.2 billion in 1996 and rose only marginally to USD5.0 in 2007. Similarly, as a proportion of GDP, market capitalisation has remained virtually unchanged at 22 percent in both 1996 and 2007.

The turnover ratio meanwhile rose from 6.8 percent to 19.7 percent over this period reflecting increased trading activity.

xi) Zimbabwe Stock Exchange (ZSE)

The Zimbabwe Stock Exchange (ZSE) was founded in 1896. However, its current uninterrupted operations can be traced to 1946. At the end of 1996, the ZSE was the second largest equity market in sub-Saharan Africa, with a total market capitalisation of USD3.6 billion and 64 listed securities. The ZSE was ranked the best performing emerging market from 1999 to 2001 (Irving, 2005). In 2002, the ZSE recorded an increase of more than 100 percent on the S&P global index. This growth has taken place in the context of macroeconomic instability, characterised by a hyperinflationary environment. More specifically, this performance originates from the lack of alternative investment opportunities – notably the relative unattractiveness of the money market in Zimbabwe (given the hyperinflationary environment) – and the use of the official exchange rate (instead of parallel market rates) in calculating equity returns (Irving, 2005). At the end of 2007, market indicators had generally improved – macroeconomic instability notwithstanding – the number of listed securities rose to 82 and market capitalisation rose 69.4 percent to USD6.1 billion. The turnover ratio has fallen from 8.8 percent in 1996 to 1.5 percent in 2007 owing to non-trading effects associated with the lack of alternative investment opportunities.

To summarise, ASMs have grown significantly in size since the early 1990s, driven mostly by strong investor inflows, capital account liberalisation and structural improvements in the respective economies. However, despite the rapid development of these equity markets, key indicators of stock market development show that ASMs are generally characterised by low market capitalisation and few listed companies. There is, however, considerable diversity within this broad outline. For example, at the end of 2007, the JSE in South Africa accounted for almost 75 percent of the continents total market capitalisation. The number of listed companies range from 28 in Namibia to 591 in Egypt. Despite this variability ASMs share a common feature: market liquidity is low (measured by the turnover ratio). At the end of 2007, the turnover ratio ranged from 1.5 percent in Zimbabwe to 50.0 percent in South Africa. In contrast, turnover ratios in the most liquid emerging markets are in excess of 100 percent. Despite the small size of (most) ASMs and low liquidity ASMs are among the fastest growing in the world (in terms of return on investment) and for the most part exhibit low correlation with the major global equity markets. These attributes have motivated investors to take advantage of the opportunity to diversify their portfolios internationally in search of the highest potential returns to their investment. Against this background, this research will explore various aspects of long memory in the volatility of ASMs with potentially useful results for policymakers, risk managers and other market participants.

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3 Data Description

3.1 Introduction

The data used in this study are obtained from Bloomberg and consists of data for eleven African countries. Daily observations on stock returns for the following countries: Egypt, Kenya, Morocco, Namibia, Nigeria, South Africa, Tunisia and Zimbabwe. For Botswana, Ghana, Mauritius and Namibia the data distribution is uneven; hence, the two-day holding return is calculated. The other ASMs are not analysed because of the unavailability of data or the insufficiency of the existing data. In addition, daily data from the UK and US stock indices are included for comparative purposes. These indices are all denominated in local currency and refer to end-of-day quotes. [1]

This study utilises Bloomberg data for several reasons. First, it is convenient from a data collection standpoint. In particular, while there are several possible variants of equity indices for the ASMs (ranging from the synthetic to various national definitions) the use of a single provider for these indices for cross-market comparisons is preferable since it provides a homogenised framework (e.g., Saadik-

[1] As pointed out by Click and Plummer (2005) there are advantages and disadvantages associated with using either the stock market indices measured in local currency terms or in a common currency (e.g., U.S. dollars). In particular, while, indices in local currencies are comparisons of dissimilar units the use of a common currency may be preferable since it negates this limitation by allowing the researcher to control for exchange rate and inflation movements. However, the downside of this approach relates to the concealment of variations in the domestic stock market due to the behaviour of the exchange rate; hence, obscuring the underlying behaviour of the domestic market. In addition, the conversion to a common currency may also mean that the converted indices may uncover some behaviour (i.e., interdependence) emanating from the behaviour of the common currency (e.g., depreciation of the US dollar against all other currencies). Finally, it may be preferable to compare the real returns in domestic currency terms although this creates the additional problem of choosing the appropriate deflator (and the attendant data availability issues in ASMs). Against this background, we prefer the simple and more convenient alternative of using indices denoted in local currency terms.

Sedik and Petri, 2006) in which to conduct the analysis. Second, Bloomberg provides accessible data for a wider coverage of African equity markets, than most data vendors. Third, these indices represent the most actively traded stocks in the respective local markets and also capture a significant portion of market capitalisation.

Throughout this study, the equity return, r_t, is defined as $(p_t - p_{t-1}) \times 100$, where p_t is the log of the equity price at time t. Furthermore, we use absolute returns, $|r_t|$ as a proxy for the volatility of the ASMs (e.g., see chapter 5 and 6). All the time-series include data until December 31, 2007 but the commencement date of the respective time series is variable (owing to data constraints). Table 3.1 provides further details of the data used in this study. In particular, it highlights the name of the stock index used, when the data start, and the number of observations used. Furthermore, the indices used in this study are the benchmark indices in their respective markets.

Table 3.1: Stock Market Data

Country	Index Name	Start Date of Data	Sample size
Botswana	Domestic Companies Index (DCI)	04/04/2001	1237
Egypt	Hermes Financial Index (HFI)	27/07/1995	3243
Ghana	All Share Index (ASI)	20/09/2002	1377
Kenya	Nairobi Stock Exchange (NSE) 20	14/05/1992	4073
Mauritius	All Share Index (SEMDEX)	14/08/1998	2447
Morocco	Casablanca Most Actives Index (CMAI)	22/05/2002	1464
Namibia	Overall Index (NSEOI)	31/01/2003	1275
Nigeria	All Share Index (NSEASI)	21/01/1999	2333
South Africa	FTSE/JSE Africa All Share Index	09/01/1996	3125
Tunisia	All Share Index (TSEASI)	15/03/1999	2296
Zimbabwe	ZSE Industrials Index (ZSEII)	04/10/1994	3455
US	Standard &Poor’s 500 Index (S&P 500)	09/08/1990	4538
UK	FTSE 100	09/08/1990	4538

Table 3.2 below provides details on the construction of the various indices used and the identity of the local currency used in the respective economies.

Table 3.2: Stock Market Index Profile

Country	Index Compilation Method	Currency
Botswana	market capitalisation weighted index	Pula
Egypt	market capitalisation weighted index	Pound
Ghana	market capitalisation weighted index	Cedi
Kenya	price-weighted geometric mean index	Shillings
Mauritius	market capitalisation weighted index	Rupee
Morocco	market capitalisation weighted index	Dirham
Namibia	market capitalisation weighted index	Dollar
Nigeria	market capitalisation weighted index	Naira
South Africa	market capitalisation weighted index	Rand
Tunisia	market capitalisation weighted index	Dinar
Zimbabwe	market capitalisation weighted index	Dollar
US	market capitalisation weighted index	US Dollar
UK	market capitalisation weighted index	UK Pound

3.2 Performance of ASMs

Figure 3.1 and 3.2, present graphic representations of the stock market indices under investigation and their associated percentage rate of daily returns. From Figure 3.1 it is apparent that most of these stock indices follow the same basic pattern. In particular, at the start these markets trade sideways or follow a gentle uptrend. From around mid 2003 these indices experience a rapid acceleration. This pattern is exemplified by the case of Mauritius at the end of 1997 the benchmark index closed at 391.1 and until the end 2002 it traded mostly sideways, ending 2002 at 399.3. From 2002 it rose every year, breaking new records every year. For instance, in 2003 the index closed at 549.6 (up 38 percent year-on-year); in 2004, the index rose 29.3 percent to end at 710.77; in 2005 the market recorded an annual increase of 13.1

percent. In 2006 and 2007, the index closed at 1204.5 and 1852.2, representing annual increases of 49.8 percent and 53.8 percent, respectively. Put differently, the market rose 373.6 percent over the 10 year period ending in December 2007. This evolution has taken place in a context of financial reforms including the liberalisation of domestic capital markets which has led to significant foreign portfolio investments, which in turn have supported the growth of the market. For example, in 2007 net foreign investments into Mauritius reached a record level of net inflows of USD52 million (i.e., the highest annual level reached since the market was opened to foreigners in 1994). Similarly, the benchmark index in Botswana (i.e., the DCI) rose from 2498.7 in 2003 to 8421.6 at the end of 2007, an increase of 237.0 percent, on the back of large portfolio equity flows (from both domestic and foreign institutional investors). This trend also applies in the case of the largest ASMs. For instance, following the implementation of comprehensive financial reforms in 1996 the all-share index in South Africa rose has risen from 4880.5 at the start of 1996 to 28,957.9, an increase of 493.3 percent at the end of 2007. The equity markets of Egypt and Ghana peaked earlier (compared to the other ASMs) and decelerated sharply, though recent data points to their recovery. For example, in 2007, the Ghana Stock Exchange (GSE) all-share index ended the year at 6,599 points, representing an annual increase of 31.8 percent compared to the yearly improvement of 4.3 percent recorded in 2006.

The two major exceptions to this general trajectory are the equity markets of Kenya and Zimbabwe. In Kenya the behaviour of the benchmark index is marked by 3 distinct phases in terms of market evolution. Initially, the index rises rapidly from 914.9 in mid 1995 to 5137.1 in the first quarter of 1995; then, over the next 8 years the index falls to a low of 1004.7 (amid sluggish economic growth and political uncertainty) in April 2003. From here the market rises to 5444.8, its value at the end of 2007. Meanwhile, in the case of Zimbabwe, the stock market follows an almost exponential rise, reflecting the inflow of capital into the stock market amid macroeconomic instability (evidenced most prominently by a hyperinflationary environment), the lack of alternative investment opportunities in the economy, and the imposition of a variety of exchange controls (see Irving, 2005). This notwithstanding, the ZSE registered an increase of more than 100 percent on the S&P 100 global index for 2002 and continues to be adjudged among best-performing markets in the world (after adjusting for inflation).

The robust performance of ASMs over the respective sample periods is attributable, amongst other things, to the robust performance of bellwether stocks in the resource and banking and tourism sectors, increased participation of local retail investors and the growing appetite of foreign institutional investors for ASMs, and a generally improved macroeconomic environment. Indeed, global asset managers have been looking at ASMs with a new interest because during the last 5 years these markets figure among the last truly uncorrelated stock markets. During the last 5 years these ASMs have had on average a 23 percent correlation to the S&P 500 against a correlation of at least 70 percent recorded by other emerging markets in Asia and Latin America (World Bank, 2008). Meanwhile, for the UK and US markets the relevant graphs peak in 2000 and then trough in 2003 following the dot.com bust and thereafter rise. The similar behaviour of these indices suggests a high degree of correlation. Furthermore, as shown in Figure 3.1 the ASMs continued to perform well during the bear market of the early 2000s in the advanced economies suggesting that ASM still offered high return prospects and risk diversification benefits.

Finally, the collapse of the US subprime mortgage market in 2007 and the ensuing global financial instability have led to significant declines in global equity markets. By late 2008 stock markets had dropped (albeit to varying degrees) in all emerging regions (Balakrishnan *et al*, 2009). Furthermore, these developments also brought the risk of a slowing or cessation of portfolio equity inflows to emerging markets. However, recent data suggests that risks to ASMs may currently be reduced reflecting the recovery in commodity prices and the resumption of portfolio inflows (IMF, 2009). In addition, recent gross domestic product (GDP) figures suggests that the pace of decline in economic activity is moderating, although, very unevenly among regions. For example, African economies grew by 5.2 percent in 2008 and are forecast to grow by 1.8 percent and 4.1 percent in 2009 and 2010, respectively; while, advanced economies grew by 3.1 percent in 2008 and are forecast to contract by 1.4 percent in 2009 and grow by 2.5 percent in 2010 (IMF, 2009). This GDP growth differential, in favour of African economies, suggests that growth in ASMs may continue (supported in large measure by rising commodity demand). If this is indeed the case, then it would appear that portfolio equity investments in ASMs may still continue to offer important diversification benefits to investors.

Figure 3.1: African Stock Market Index Levels

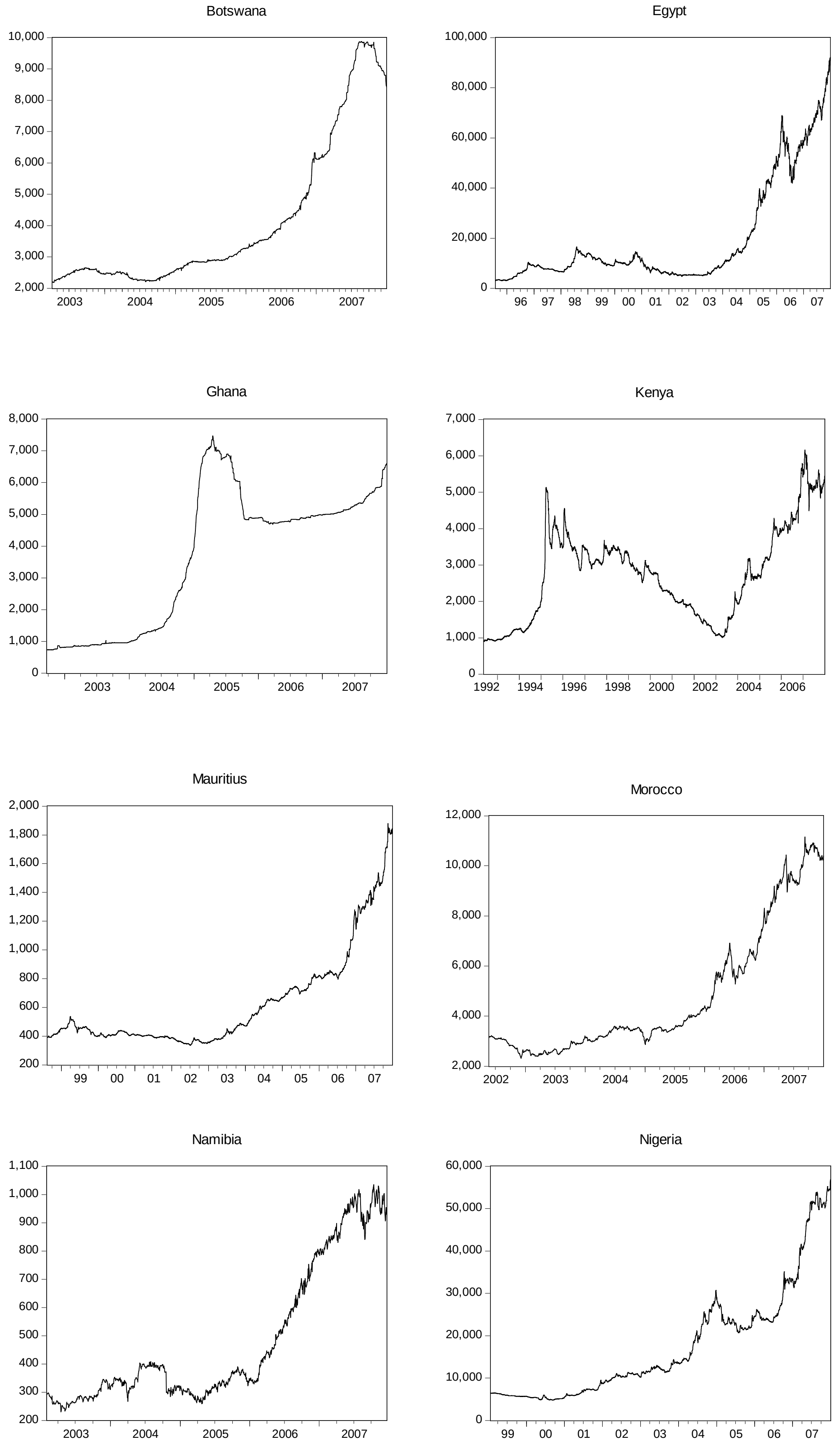


Figure 3.1: African Stock Market Index Levels (continued)

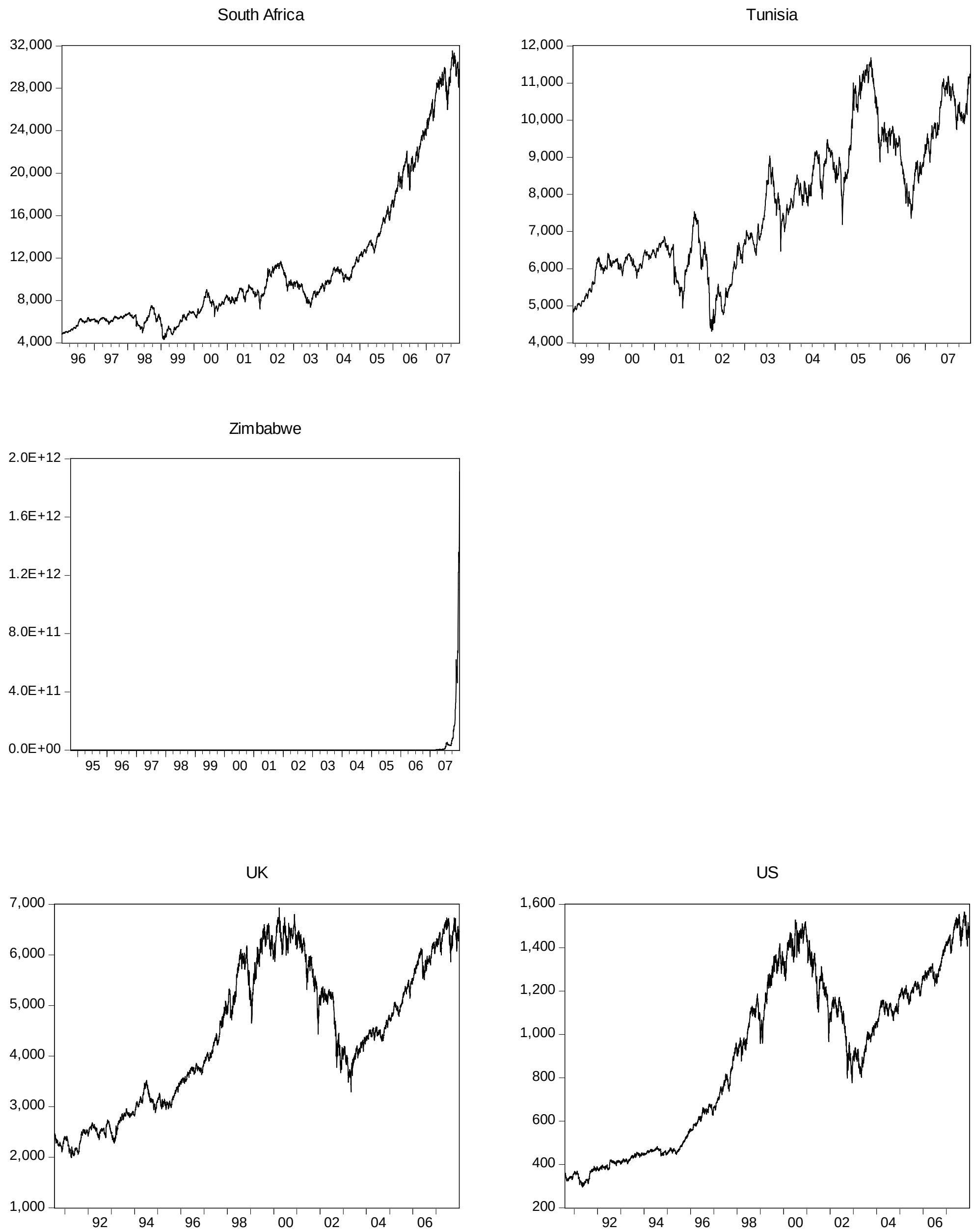


Figure 3.2: African Stock Market Index Returns

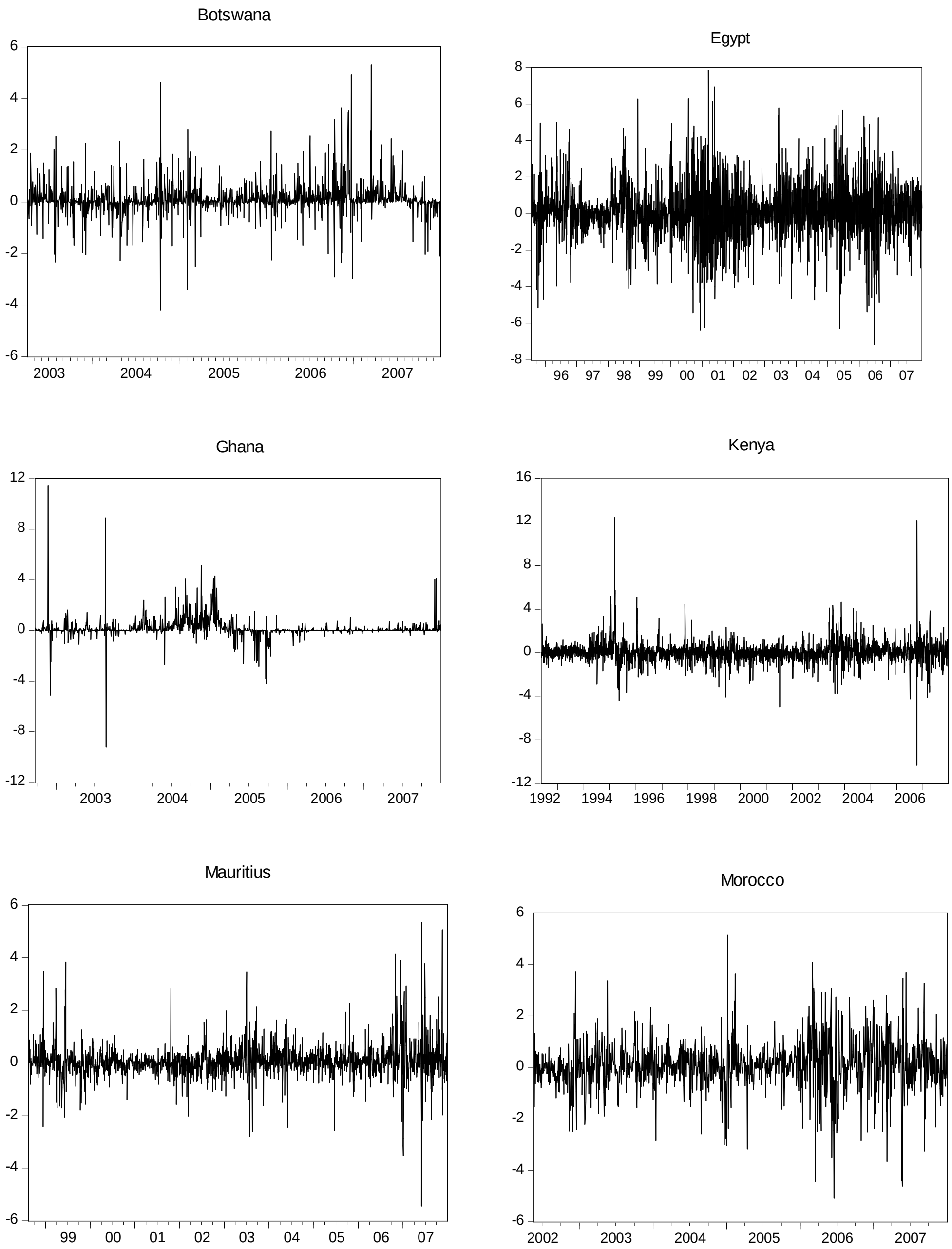
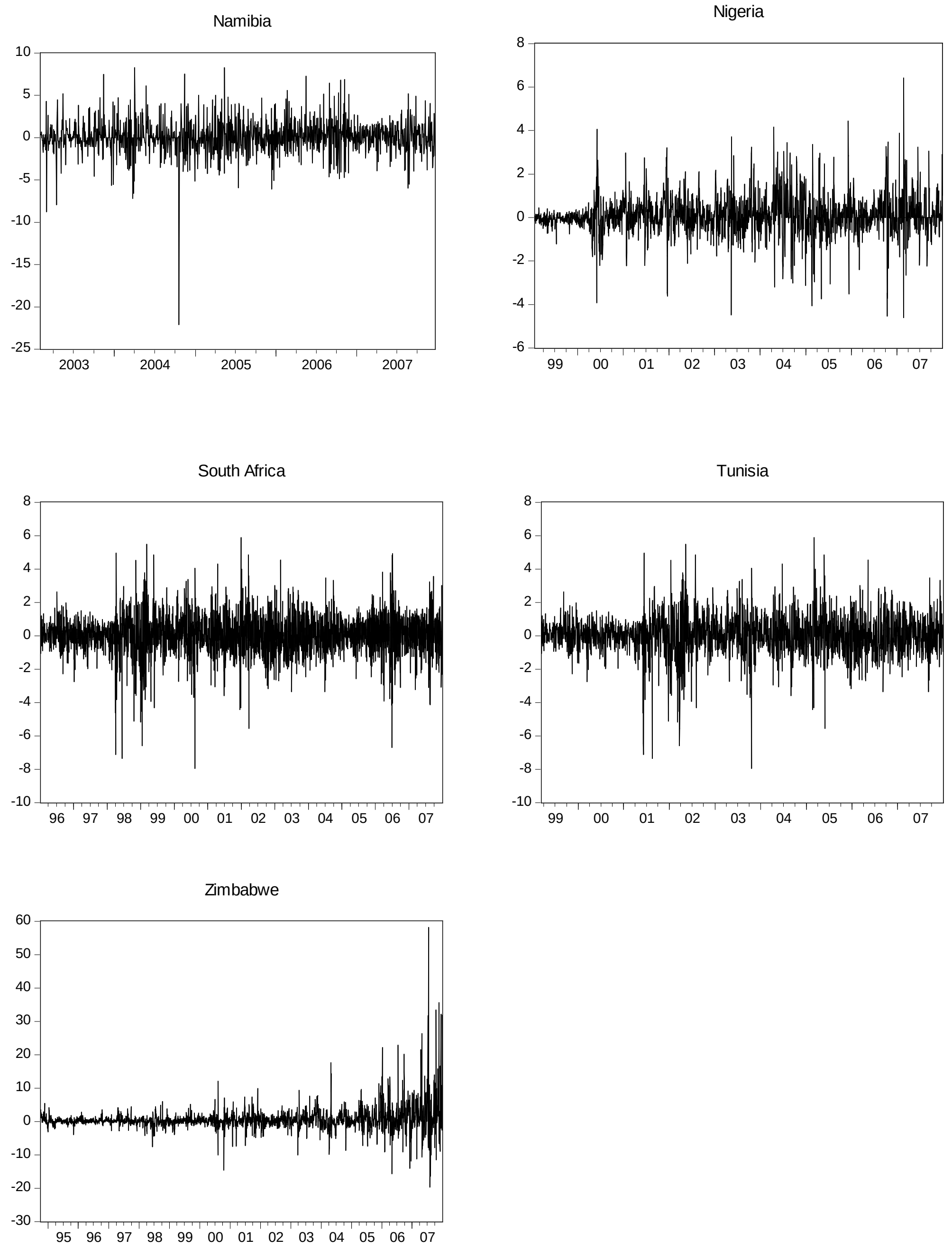
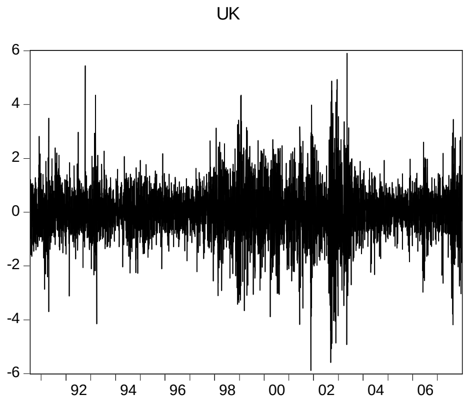
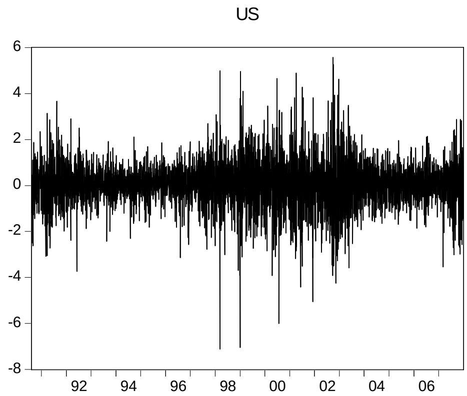


Figure 3.2: African Stock Market Index Returns (continued)







3.3 Examination of Summary Statistics

Table 3.3 presents summary statistics of the daily stock index returns. In particular, key sample statistics for the various stock returns indicate that all ASMs delivered higher mean returns than both UK and US markets. The highest mean returns are recorded in Zimbabwe with a reading of 0.5939% while the lowest mean returns are found in UK with a value of 0.0215%. Put differently mean returns in Zimbabwe are 27.69 times those in the UK. Mean returns from Ghana are the second highest in the selection of ASMs analysed at 0. 1593%. Average returns from Tunisia are at 0.0359% and are the lowest in the sample of ASMs. Median returns mostly, reveal a different ranking, with the exception of Zimbabwe which at 0.1941% records the highest return. South Africa reveals the second highest ranking at 0.0869% and Nigeria and Namibia present the lowest rankings at 0.00% each. On the basis of the median return the rankings of the UK and US are at 0.0427% and 0.0464%, respectively which is higher than most ASMs. Variability as measured by the standard deviation varies considerably among the ASMs. For example, in Zimbabwe, returns are the most volatile at 3.2010, most likely, reflecting the hyperinflationary environment in that country. In comparison returns in Mauritius are the least variable at 0.5830, perhaps reflecting infrequent trading of many listed stocks. At 1.0198 and 0.9955, UK and UK returns are more volatile than half of the ASMs in the sample.

Table 3.3: Summary Statistics of ASM Returns

	Mean	Median	Max	Min	Standard Deviation	Skewness	Sharpe Ratio
Botswana	0.109	0.033	5.307	-4.190	0.693	0.964	0.157
Egypt	0.104	0.064	7.856	-7.173	1.414	0.024	0.074
Ghana	0.159	0.018	11.44	-9.227	0.787	2.451	0.202
Kenya	0.044	0.010	12.39	-10.34	0.851	1.481	0.052
Mauritius	0.063	0.027	5.344	-5.452	0.583	0.854	0.108
Morocco	0.081	0.060	5.148	-5.094	0.935	-0.207	0.086
Namibia	0.091	0.000	8.270	-22.08	1.995	-0.955	0.046
Nigeria	0.093	0.000	6.422	-4.604	0.886	0.205	0.105
South Africa	0.057	0.087	5.890	-7.948	1.175	-0.531	0.049
Tunisia	0.036	0.056	5.072	-6.061	1.191	-0.519	0.030
Zimbabwe	0.594	0.194	58.18	-19.70	3.201	4.258	0.186
UK	0.022	0.043	5.903	-5.885	1.019	-0.133	0.021
US	0.031	0.046	5.573	-7.113	0.995	-0.124	0.031

Note: Both mean and median are expressed in terms of percent. Max and min, refer to the maximum and minimum value, respectively

In terms of the risk and returns relationship, the data show mixed evidence of the risk-return hypothesis (i.e., high risks imply high returns). For example, Zimbabwe registers the highest mean (0.5939) among the ASMs and also has the highest variability, measured in terms of Zimbabwe's average standard deviation of returns (3.2010). Similarly, Egypt has a relatively high mean return of 0.1041 and a high standard deviation of 1.4136. In relation to the Sharpe ratio, Ghana has the highest Sharpe ratio (0.2024) while among ASMs Tunisia records the lowest Sharpe ratio (0.0302) indicating that these markets have the best and worst volatility-adjusted performance, respectively.

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4. The Efficiency of African Stock Markets

4.1. Introduction

Long memory (or long-range dependence) in stock market data has important implications for the efficiency of the market in pricing securities. The efficient market hypothesis (EMH) provides the standard framework to analyse and interpret the dynamics of equity data. While, a number of definitions of market efficiency are available, the random walk version of the EMH proposed by Bachelier (1900) and formalised by Osborne (1959) and Fama (1965, 1970) asserts that for an equity market to be efficient, future prices cannot be predicted from currently available information (or alternatively the best forecast of the equity price in the next period is the price in the current period).

If equity data exhibit long memory then it displays significant autocorrelation between distant observations. This in turn implies that the series realisations have a predictable component; and hence, past trends in the data can be used to predict future returns. Therefore, long memory provides evidence against the weak-form version of the efficient market hypothesis (EMH) [1]

The extant literature indicates that there has been little analysis of the time series properties of ASMs despite their increasing importance in terms of portfolio diversification, notably as the markets of South-East Asia and Latin America have

[1] The weak form of the EMH asserts that the current price incorporates all relevant historical information about share prices. As such, changes in equity prices cannot, therefore, be predicted from past trends in prices.

become increasingly aligned with those of the US and Europe. In addition, while ASMs have been among the fastest growing markets in the world, most African markets – excluding South African remain very small by world standards (Yartey and Adjasi, 2007). Small size and associated low levels of liquidity, raise questions regarding the efficiency of these markets and the process of price determination. Against this background, this paper attempts to fill this gap by investigating the long memory properties of equity returns and volatility using data from eleven ASMs in order to evaluate the informational efficiency of these markets, using the EMH as a criterion.

This is relevant because the ability of an equity market to efficiently process information affects its allocative capacity, and, therefore, its contribution to economic growth (El-Erian and Kumar, 1995; Errunza, 2001). Indeed, in a competitive market with little informational impediments equity prices are expected to adjust very rapidly to new information relating to investment opportunities and business prospects. In contrast, in markets where information on the performance and policies of corporations are not readily available to investors, the resulting uncertainty may distort the decision making process. For example, investors, may shorten their investment horizons, or choose to invest elsewhere where the business environment is more stable. Similarly, investor exposure may be curtailed if investors perceive they are being penalised for bearing risk, or if excessive volatility weakens confidence and deters neutral or risk averse investors (Mecagni and Sourial, 1999). In sum, the efficiency of an equity market in processing information affects its allocative capacity and therefore its contribution to output growth, which, in itself, is a key motivation behind the establishment or revitalisation of ASMs.

This study makes a contribution to the existing literature in the following ways. First, by determining if long memory exists in ASMs, since there does not appear to be any previous tests of long memory in these markets. Second, we simultaneously model long memory in equity returns and volatility using the ARFIMA-FIGARCH and ARFIMA-HYGARCH models which represent a relatively new innovation in time series analysis. These two approaches also provide a means to examine the sensitivity of the findings to the choice of method used. Third, the allowance of possible long memory in stock volatility may provide useful information to market participants given that volatility is key for measuring risk, pricing derivatives and hedging strategies (e.g., Hull and White, 1987; Chesney and Scott, 1989). Indeed, Bollerslev and Mikkelsen (1996) demonstrate the importance of including a long memory volatility structure when pricing long-term derivative contracts. Fourth, this study will also perform the same tests on US and UK data in order to assess if the findings are sensitive to the degree of market development, and hence, the implications this may have on investment strategies.

Based upon the empirical results we conclude that it is important to model long memory in African equity data. Furthermore, we argue that ASMs (largely) display a predictable component in returns (and hence do not conform to weak form efficiency); while evidence of long memory in volatility is very mixed. In comparison, results from the US and UK indicate short memory in returns; while, evidence of long memory in volatility is mixed. These results show that the behaviour of equity market returns and risks are dissimilar across markets and this may have implications for portfolio diversification and risk management strategies.

4.2. Review of Relevant Literature

To investigate long memory in equity data previous studies have used different estimation procedures and data of varying frequencies. For example, to detect long memory in asset markets Mandelbrot (1971) proposed the ‘range over standard deviation’, which was modified by Lo (1991). A standard long memory model is the Auto-Regressive Fractionally Integrated Moving Average (ARFIMA) (p, d, q) model introduced by Granger and Joyeux (1980) and Hosking (1981). These models provide an alternative to ARIMA (p, d, q) process by not restricting the parameter d to an integer value (0 or 1) but allowing it to assume any real value. Because the fractional differencing parameter (d) implies dependence between distant observations, recent empirical research has focused on the analysis of fractional dynamics in equity market data. Furthermore, these methods have been applied to international equity data with mixed results.

There are a number of empirical studies that report evidence of long memory in equity data. For example, Greene and Fielitz (1977) examine the daily returns of a plethora of securities listed on the New York Stock Exchange using the classical ‘rescaled range’ (or “R/S” statistic) and present evidence attesting to the presence of long memory behaviour. Cheung and Lai (1995) analyse data from Austria, Italy, Japan and Spain and detect long memory in these markets. In addition, this finding was invariant to the choice of estimation methods employed. In particular, results from both the modified ‘rescaled range’ and the spectral regression method, which was used to model an ARFIMA process indicated the presence of long memory dynamics in the data. Using weekly data Barkoulas *et al* (2000) report evidence of long memory in the Greek stock market for data spanning ten years. They estimate the fractional differencing parameter through application of the spectral regression technique. In addition, they report that the ARFIMA model provides better out-of-sample forecasting accuracy in comparison to the benchmark linear (random walk) models. Investigating Australian stock market returns over a period spanning 120 years (i.e., from 1876-1996), McKenzie (2001) using a time series composed of monthly observations finds evidence consistent with long memory in Australia. Similarly, Lee *et al* (2001) using data for China report evidence which suggests the existence of long-range dependence in stock price changes. In a seven country study of mostly Asian countries, over the period 1983 to 1998, Sadique and Silvapulle (2001) also document evidence of long memory in these stock markets. Wright (2001) finds evidence of long memory behaviour in a selection of emerging markets. Nagayasu (2003) finds evidence of long memory in the equity returns and volatility of the Nikkei before and after the implementation of financial reforms in Japan, suggesting that the Japanese stock market remains inefficient despite the enactment of comprehensive financial market reforms. Assaf and Cavalcante (2005) report evidence of long memory in a variety of stock return volatility measures in Brazil, while little evidence of long-range dependence is found in the stock returns. DiSario *et al* (2008) use methods based on wavelets and aggregate series to show the existence of long memory in stock return volatility in Turkey. McMillan and Thupayagale (2008) examine the long memory properties of South Africa's equity market using the ARFIMA-FIGARCH model and conclude that financial market reforms had a no effect with respect to improving the efficiency of the market.

In contrast, evidence against long memory has also been reported in a number of empirical studies. For example, Lo (1991) does not find evidence consistent with long memory in US daily and monthly equity returns over several time periods including various sub-periods from 1962 to 1987. Mills (1993) investigates monthly UK stock returns from 1965-1990 and his results indicate that the data is not long-range dependent. Chow *et al* (1995) are also unable to corroborate evidence of long memory in US equity returns from 1962 to 1991, even after splitting the data into two sub-periods and after controlling for seasonalities in equity returns. In a study comprising monthly stock market indices obtained from the Morgan Stanley Capital International (MSCI) stock index data for eighteen industrialised countries from 1970 to 1992, Cheung and Lai (1995) report that their empirical results in general provide little evidence of long memory in these stock returns. Huang and Yang (1995) test for the presence of long memory in nine Asian equity markets together with two benchmark comparators (the US and UK) using the modified rescaled range statistic and data of various frequency and find that in most cases the existence of long memory can be rejected, with the exception of data from the UK. Finally, Resende and Teixeira (2002) do not find evidence of long memory in Brazil for periods before and after the introduction of the Real Stabilisation Plan.

Most of the research to date on long memory behaviour has been concentrated on the major international stock markets (e.g., US, Japan and UK) and some applied empirical work has been conducted on the more prominent emerging markets (e.g., Brazil, China and Turkey). However, comparatively little is known about the long memory behaviour of the returns and volatility of the smaller emerging markets, especially in Africa. These markets are typically much smaller, less liquid and more volatile than developed country equity markets (Domowitz *et al*, 1998). In addition, Assaf and Cavalcante (2005) argue that the industrial structure found in these markets is often quite different from that found in developed countries. In relation to ASMs, both Irving (2005) and Yartey and Adjasi (2007) report that ASMs are characterised by a small number of listed companies, low liquidity levels and a large number of nonactively traded shares. These underlying conditions may produce different stock return and volatility behaviour than that obtaining in the larger international equity markets. These differences, in turn, may have implications on domestic financial policy and investment strategies. Indeed, Nagayasu (2003) argues that stock returns in developing countries can be expected to display a long memory in light of the shallowness of their markets and their less mature institutional and regulatory frameworks.

Against this background, this paper examines the long memory properties of equity returns and volatility using African data in order to evaluate the efficiency of these markets. This is relevant because the ability of an equity market to efficiently process information affects its allocative capacity, and therefore its contribution to economic growth (El-Erian and Kumar, 1995). In addition, the ability of stock markets to convert information (on economic fundamentals) into accurate stock prices is important, in the overall economic development context; in which, the establishment (or in some cases revitalisation) of ASMs is being pursued by the authorities as part of a broader set of financial and economic reforms.

4.3. ASMs: Institutions and Information flows

A voluminous literature on ASMs suggests that macroeconomic stability, a well-developed banking system, robust accounting and disclosure standards, a broad investor base, and effective protection of shareholders' rights are important prerequisites for the efficient functioning of these equity markets (e.g., Marone, 2003; Irving, 2005; Yartey and Adjasi, 2007).[2] Indeed, a well-functioning stock market is expected to influence growth through increased capital accumulation and by influencing the efficiency of capital allocation (Levine, 2001).

ASMs vary considerably in terms of their history, size and degree of development, both generally and in terms of the financial sector in particular. Differences in institutional and infrastructural characteristics may potentially have a bearing on how accurately information is processed (and therefore on market efficiency).

First, infrastructural bottlenecks in ASMs may impede how well these markets process information. For example, the trading, settlement and clearing procedures in Egypt, Mauritius, Nigeria, South Africa and Tunisia are electronic; while, in the other ASMs these procedures are mostly manually-driven. In addition, all the ASMs have a central depository system with the exception of Botswana, Ghana, Kenya, Namibia and Zimbabwe which may have a bearing on how well information is disseminated. These differences may help contribute to deficiencies in the transmission and processing of information, especially on a real-time basis. In particular, the benefit of

[2] For a more general discussion refer to Bekaert *et al* (2001, 2005).

automation derives from its elimination of the risks and costs associated with paper-based transactions and hence can more robustly support trading activity. In addition, automation provides a platform on which information may be more rapidly and accurately transmitted. Second, the JSE in South Africa is the most sophisticated market on the continent and has more developed disclosure requirements aimed at the timely provision and dissemination of information on the performance of listed companies. For example, the South African Stock Exchange News Services (SENS) stipulates strict disclosure requirements for listed companies and enhances investor confidence and market transparency. Such a mechanism does not appear to currently exist for the other ASMs. In addition, reporting systems in ASMs vary widely. For instance, Botswana, Ghana, Namibia, South Africa and Tunisia employ locally devised accounting and auditing reporting systems while the other ASMs use a standard international reporting system. These differences imply that information in these markets may be processed at different speeds given different settlement and clearing procedures and operational settings. More precisely, these differences may hamper the real-time availability of market information. Third, regulatory oversight with respect to disclosure requirements, accounting standards and contract enforcement varying substantially among ASMs (Yartey and Adjasi, 2007; Irving, 2005). These differences may constrain the dissemination of timely and accurate information on the performance of listed companies. These differences may also therefore impact on the extent to which security prices adequately reflect available information and hence on market efficiency.

Despite the institutional and infrastructural differences, there are several structural similarities between the ASMs. First, they are generally illiquid when compared to the most liquid markets in the world which have turnover ratios in excess of 100 percent. Lack of liquidity in these markets is widely attributed to the absence of an active and well developed investor base (Magnusson and Wydick, 2002). ASMs in general, including the JSE, are illiquid (i.e., measured by the turnover ratio is low compared to other emerging markets and indeed) by international standards, reflecting a few large listings and the buy-and-hold strategy of domestic institutional investors, a characteristic that may well have negative implications for market efficiency. Indeed, liquid markets are generally perceived as desirable because of the multiple benefits they offer, including improved allocation and information efficiency (Sarr and Lybek, 2002). Furthermore, Jefferis *et al* (2001) suggest that illiquidity in ASMs may help explain why the emergence of stock markets in Africa has had little broader economic impact

Second, there exists a pattern of cross-shareholdings among many corporations in Africa. The largest conglomerates have traditionally attempted to establish and maintain strong and stable business relationships by holding stocks of partner institutions or companies linked to the conglomerates. For example, Jefferis *et al* (2001) and Irving (2005) point out that equity ownership in ASMs is dominated by a small number of large conglomerates. This means that the ownership of stocks remains highly concentrated, with large shareholdings held by a few dominant companies. This in turn implies that trading activity and the information it provides are effectively limited (however, this effect may be diminishing in importance as the turnover ratio in ASMs has increased but it is still significantly lower than that in the most liquid markets in the world). Third, the existence of a 'buy and hold' investment strategy in many ASMs reinforces nontrading effects. Furthermore, nontrading effects may be reinforced by delisting rules that stipulate a minimal number of transactions a year. For example, Mecagni and Sourial (1999) describe how delisting rules may have exacerbated illiquidity in Egypt. In the smaller ASMs, the limited number of stocks traded implicitly results in a 'captive market' with little scope for investors to trade, given few buyers and sellers. In addition, illiquidity fosters even more illiquidity by limiting the capacity of investors to unwind or re-establish their positions without promoting greater volatility in the market and hence discourage the entry of new players, which, in turn limits liquidity. The implications are far reaching since illiquidity hinders the price revelation process (e.g., de la Torre and Schmukler, 2005). In addition, illiquid markets are associated with high costs of trading. This further reduces the benefits of equity markets, deterring further market entrants on both buy and sell sides (e.g., see Bekaert *et al*, 2005).

While there is no presumption that the list expressed above is exhaustive (since it is beyond the purview of this study to examine the exact causes of market inefficiencies in ASMs) they may nonetheless reinforce expectations of long memory behaviour in ASMs.

4.4 Long Memory in Time Series

Long memory describes the correlation structure of a series at long lags. In the time domain, long memory is characterised by a hyperbolically decaying autocovariance function. Indeed, this slow decay of the autocorrelation function is considered to be the defining characteristic of long memory process (Lo, 1991; Campbell *et al*, 1997).

To define a long memory model formally, a stationary stochastic process X_t is called a long memory process if its autocovariance function $\rho(\tau)$ is such that the autocorrelations are positive and decay monotonically and hyperbolically to zero. This asymptotic property can be expressed as:

$$\rho_\tau \approx |\tau|^{2d-1} \quad \text{as } |\tau| \to \infty \tag{4.1}$$

when $d \in (0, 0.5)$ the series is stationary and said to have long-memory, while if $d >$ 0.5, the series is nonstationary and hence unpredictable. For $d \in (-0.5, 0)$, the series is described as having short memory, which is a measure of the decline in statistical significance between distant observations. [3]

In this paper we follow Campbell *et al* (1997) and Nagayasu (2003) by considering both long and short memory processes to be reflective of market inefficiency (since they present evidence against the EMH) given the speed of the convergent process $d \in (-0.5, 0.5)$ is slower than that of a stationary ARMA process.

[3] The autocorrelation functions for the stocks returns (for 100 lags) of the sample countries are presented in Chapter 5 (see Figure 5.1), together with the 5 percent critical value, which examines this behaviour in detail. From these graphs only Ghana present evidence consistent with a of a hyperbolic decay pattern; however, a few autocorrelations between lags 70 and 80 are insignificant. All other markets present an erratic decay structure from which no inference with respect to the presence of long memory can be made.

4.5. Empirical Methodology

In the ensuing empirical analysis, the informational efficiency of ASMs is examined by using the martingale model. The stochastic process of a stock price (or stock index) P_t follows a martingale process when

$$E\left[P_{t+1} - P_t \middle| P_t, P_{t-1}, \ldots\right] = 0 \tag{4.2}$$

This means that stock price changes are unpredictable and hence future prices changes cannot be predicted from currently available information. Furthermore, when the time-series property of equity returns are expressed as an ARIMA (m, d, n) process, the hypothesis of market efficiency can be tested by analysing the size of its differencing parameter, d. Specifically, the martingale process implies that equity returns are stationary and can be expressed as an ARIMA (m, d, n) where $d = 0$. In contrast, $d \neq 0$ implies a departure from the EMH; hence, future price (or return) movements can be predicted on the basis of past information.

In addition, the martingale criterion has the advantage that it allows for the conditional volatility of (stock) returns to be predictable on the basis of past volatility (see Cuthbertson, 1996, for a more detailed exposition). This property of the martingale model is less restrictive than the condition embodied by the random walk hypothesis which stipulates that variance process should be time-invariant. Therefore, in order to estimate long memory in stock market data (specifically, in both stock returns and stock return volatility) the ARFIMA-FIGARCH and ARFIMA-HYGARCH models are employed. The following section outlines the construction of these models. In particular, the AFRIMA model represents an extension of the standard ARIMA process; while both the FIGARCH and HYGARCH models have been formulated to capture long memory in stock return volatility. More precisely, using a conditioning information set that is composed of a sequence of past equity returns, this model tests for the weak form version of the EMH. This involves determining whether there is a pattern of time dependence in stock returns that may allow for past return realisations to be used to improve the predictability of future returns. In the context of the ARFIMA-FIGARCH and ARFIMA-HYGARCH models the fulfilment of weak-form EMH is therefore associated with the absence of long memory in both stock returns and stock return volatility.

4.5.1. ARFIMA Model

In order to model long memory in equity returns the ARFIMA (*m, d, n*) model developed by Granger and Joyeux (1979) and Hosking (1981) is used. As previously discussed, this technique has been extensively used to analyse the behaviour of financial time series. This process can be expressed as:

$$\phi(L)(1-L)^d y_t = \theta(L)\varepsilon_t \tag{4.3}$$

Where the stock return series is denoted by y_t and d refers to the fractional differencing parameter. The L designates a lag operator, and, $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator of orders m and n respectively. Further,

$\phi(L)=1-\sum_{j=1}^{m}\phi_j L^j$ and $\theta(L)=1+\sum_{j=1}^{n}\theta_j L^j$. All the roots of $\phi(L)$ and $\theta(L)$ lie outside the unit circle. The innovation, ε_t, follows a white noise process with variance, σ^2, i.e., $\varepsilon_t \sim \text{IID } N(0,\sigma^2)$.

Granger and Joyeux (1980) and Hosking (1981) show that when the lag operator $(1 - L)^d$ is extended to noninteger powers of *d*, the result is a well-defined time series that is said to be *fractionally differenced* of order *d* (or equivalently, *fractionally integrated* of order –*d*). The fractional differencing parameter, *d*, measures the level of integration of the time series, i.e., $y_t \sim I(d)$, and the fractional differencing operator $(1 - L)^d$ has a binomial expansion (see Lo, 1991) which can more conveniently be shown in terms of the hypergeometric function (Baillie *et al*, 1996):

$$
\begin{aligned}
(1-L)^d &= F(-d,1,1;L) \\
&= \sum_{j=0}^{\infty}\Gamma(j-d)\Gamma(j+1)^{-1}\Gamma(-d)^{-1}L^j \\
&= \sum_{j=0}^{\infty}\frac{\Gamma(j-d)L^j}{\Gamma(-d)\Gamma(j+1)} \\
&= \sum_{j=0}^{\infty}\pi_j L^j \qquad (4.4)
\end{aligned}
$$

where $\Gamma(\cdot)$ represents the gamma function. The restriction of *d* to integer values in equation (4.3) results in the basic autoregressive integrated moving average (ARIMA) While, the long memory processes arises because the fractional differencing parameter, *d* is allowed to assume any real value. This in turn is shown by Granger and Joyeux (1980) and Hosking (1981) to capture persistence in such a way that the extent to which shocks remain important for long periods into the future is significantly larger in relation to the case of a stationary ARIMA (i.e., $d = 0$ and y_t is white noise) whose autocorrelation function exponentially converges to zero ($\rho_j = c\theta^j$ with $|\theta| < 0$).

In the case of an ARFIMA *(0, d, 0)* and $d \in (-0.5, 0.5)$, the process y_t is weakly stationary and invertible (Hosking 1981). For $d < 0.5$ and $d \neq 0$, the autocorrelation function of the time series data is proportional to $\rho_j \approx cj^{2d-1}$ as $j \rightarrow \infty$ where c is the ratio of gamma functions, which shows that the autocorrelation function of the ARFIMA process hyperbolically converges to zero at a pace controlled by the size d (Granger and Joyeux 1979, Hosking 1981). Consequently, the speed of the convergent process for $d \in (-0.5, 0.5)$ is slower than that of the geometric decay of a stationary ARMA (or white noise) process.

In particular, when $d \in (0, 0.5)$, the autocorrelations are positive and decay monotonically and hyperbolically to zero. Since the processes with $0 < d < 0.5$ displays a slower convergence than the stationary ARMA case they are defined as long-memory processes (Nayagasu, 2003). These series are described as long-memory processes given that their autocorrelations functions decay considerably slower than those of more conventional time series. Therefore the finding of long memory in equity data raises evidence the weak-form EMH because it indicates that the autocorrelation function decays at a slow rate, implying that the correlation between a stock's price movements and any shock to that price will have a lasting impact.

4.5.2. FIGARCH Model

The FIGARCH (p, d, q) model was introduced by Baillie *et al* (1996) and is used to capture long memory in volatility. The general representation for this model can be derived from the standard GARCH process, which is given by:

$$h_t = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)h_t \tag{4.5}$$

where h_t and ε_t^2 are conditional and unconditional variances of ε_t respectively, $\omega = \varepsilon^2[1 - \beta(1) - \alpha(1)]$, and $\phi(L) = 1 - \sum_{j=1}^{q} \phi_j L^j$ and $\beta(L) = 1 + \sum_{j=1}^{p} \beta_j L^j$. The GARCH ($p$, q) process in Equation (4.5) can be rewritten as an ARMA (m, p) process in ε_t^2 such that we have:

$$[1 - \alpha(L) - \beta(L)]\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t \tag{4.6}$$

where $v_t \equiv \varepsilon_t^2 - \sigma_t^2$. To ensure covariance stationarity the roots $[1 - \alpha(L) - \beta(L)]$ and $[1 - \beta(L)]$ are constrained to lie outside the unit circle. When the autoregressive lag polynomial, $1 - \alpha(L) - \beta(L)$, contains a unit root, the model is referred to as an Integrated GARCH process (Engle and Bollerslev, 1986) and is specified by:

$$\phi(L)(1-L)\varepsilon_t^2 = \omega + [1-\beta(L)]v_t \tag{4.7}$$

From this model, the FIGARCH model is obtained by introducing the fractional differencing operator, $(1-L)^{\bar{d}}$, such that:

$$\phi(L)(1-L)^{\bar{d}}\varepsilon_t^2 = \omega + [1-\beta(L)]v_t \tag{4.8}$$

Like the ARFIMA (*m, d, n*) process for the mean, the fractional differencing operator, $(1-L)^{\bar{d}}$, can also be given by the gamma function as in equation (4.4). In addition, $\bar{d} \in$ (0, 1) and all the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit circle. The FIGARCH (*p, d, q*) model nests a variety of other GARCH models, and is equivalent to the standard GARCH model and the IGARCH process, when $\bar{d} = 0$ and $\bar{d} = 1$, respectively.

While $\bar{d}$ captures long memory in the FIGARCH model, it's interpretation is not identical to that reflected by the ARFIMA because the FIGARCH process may not be covariance stationary but strictly stationary and ergodic for $0 \le \bar{d} \le 1$ and hence the unconditional variance of ε_t does not exist (Baillie, *et al*, 1996). Furthermore, equation (4.8) can be represented as $[1-\beta(L)]h_t = \omega + [1-\beta(L)-\phi(L)(1-L)^{\bar{d}}]\varepsilon_t^2$ which for the FIGARCH $(1,\bar{d},1)$ process is equivalent to:

$$h_t = \omega + [1-(1-\beta_1 L)^{-1} + \left\{1-\left[1-\beta(L)^{-1}\right]\phi(L)(1-L)^d\right\}\varepsilon_t^2 \tag{4.9}$$

Indeed, Baillie, *et al* (1996) contend that ω > 0, $0 \le \bar{d} \le 1-2\phi_1$, and $0 \le \beta_1 \le \phi_1 + \bar{d}$ which they demonstrate are sufficient conditions to ensure a positive conditional variance of the FIGARCH $(1,\bar{d},1)$ process for all *t*.

4.5.3. HYGARCH Model

The Hyperbolic GARCH (HYGARCH) model was developed by Davidson (2004) in order to address theoretical limitations associated with the FIGARCH process. In particular, Davidson shows that in the FIGARCH model, the long memory parameter *d* behaves counterintuitively given that *d* approaches zero as the memory of the relevant stochastic process rises. In view of this anomaly, Davidson argues that the FIGARCH process is more akin to the 'knife-edge nonstationary' class of models exemplified by the IGARCH model. Therefore, Davidson proposed the HYGARCH model to overcome this deficiency. This model generalises the FIGARCH process so that it behaves in a more intuitive way, such that an increase in *d* reflects greater long memory. More precisely, the HYGARCH is obtained by modifying equation (4.8) to

$$\phi(L)\left((1-\tau)+\tau(1-L)^{\bar{d}}\right)\varepsilon_t^2 = \omega + \left[1-\beta(L)\right]v_t \tag{4.10}$$

by incorporating the additional parameter $\tau \geq 0$. The HYGARCH model nests the GARCH model under the restriction $\tau = 0 \quad (or\ \bar{d} = 0)$ and the FIGARCH model under the restriction $\tau = 1$. When $\bar{d} = 1$ the parameter τ becomes an autoregressive root and the HYGARCH reduces to either a stationary GARCH (τ < 1), an IGARCH (τ = 1) or an explosive GARCH (τ >1). The conditional variance of the HYGARCH model is given by:

$$h_t = \omega + [1 - \beta(L)]^{-1} + \{1 - [1 - \beta(L)]^{-1} \phi(L)[1 + \alpha((1 - L)^d - 1)]\}\varepsilon_t^2 \qquad (4.11)$$

Davidson (2004) provides more details on the construction and application of the HYGARCH process.

Previous studies have used a variety of techniques to estimate the fractional differencing parameter, *d*. For example, the size of $\bar{d}$ can be estimated by using a semi-parametric approach in the frequency domain like Geweke and Porter-Hudak (GPH, 1983) and Robinson (1994). While these methods are widely used they have vital weaknesses. For instance, while the semi-parametric estimator of GPH is potentially robust to non-normality, the results are overly sensitive to serial correlation (Agiakloglou *et al*, 1992). Similarly, Robinson's method, is essentially a discretely averaged periodogram and is compromised by discontinuity in the asymptotic distribution theory (see Baillie, 1996) hence the derived conclusions may be biased. To address these shortcomings, this study utilises maximum likelihood methods which are both consistent and asymptotically efficient.

4.6. Empirical Results

4.6.1. Preliminary Observations

Before performing formal long memory tests, the time series properties of the data are examined using standard diagnostic methods. To test for nonstationarity of the data series, each market index is tested for the presence of unit roots using the Augmented Dickey Fuller (ADF) test. Assuming the series have a non-zero mean, a constant is included in the regression. The null hypothesis of a unit root is tested against the alternative hypothesis of a stationary autoregressive process and a stationary autoregressive process with a trend. To ensure that the ordinary least squares (OLS) regression will give an unbiased estimate of the lag coefficients, the number of lags included in the ADF is optimised by minimising the Schwarz Information Criteria (SIC). Relaxing the independent and identically distributed $\left(0, \sigma^2\right)$ assumption and allowing errors to be dependent with heteroscedastic variance, a Phillips-Perron (PP) test is conducted to verify the ADF results. The results of the unit root tests are reported in Table 4.1, below. To complement, the ADF and PP tests we also perform the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test which evaluates the stationarity hypothesis (compared to the unit root hypothesis tested by both the ADF and PP).

Table 4.1: ADF and PP Unit Root Tests

	Augmented Dickey-Fuller		Phillips-Perron	
	Constant	Constant and Trend	Constant	Constant and Trend
Botswana	-49.60^{**}	-46.64^{**}	-43.90^{**}	-43.46^{**}
Egypt	-46.03^{**}	-46.02^{**}	-27.20^{**}	-27.21^{**}
Ghana	-51.41^{**}	-51.33^{**}	-33.76^{**}	-33.82^{**}
Kenya	-57.15^{**}	-57.14^{**}	-20.59^{**}	-20.59^{**}
Mauritius	-35.59^{**}	-35.46^{**}	-24.77^{**}	-24.98^{*}
Morocco	-24.93^{**}	-25.12^{**}	-24.91^{**}	-25.12^{**}
Namibia	-63.50^{**}	-63.65^{**}	-36.35^{**}	-36.40^{**}
Nigeria	-28.94^{**}	-28.95^{**}	-23.52^{**}	-23.58^{**}
South Africa	-48.94^{**}	-49.35^{**}	-51.65^{**}	-50.83^{**}
Tunisia	-33.64^{**}	-33.65^{**}	-25.17^{**}	-25.28^{**}
Zimbabwe	-44.02^{**}	-43.57^{**}	-36.04^{**}	-35.63^{**}
US	-68.42^{**}	-68.47^{**}	-66.87^{**}	-66.89^{**}
UK	-65.52^{**}	-65.50^{**}	-40.81^{**}	-40.80^{**}

Note: '**' and '*' indicate statistical significant at the 1% and 5% levels, respectively.

Both the ADF and PP unit root tests offer evidence in favour of stationary equity returns. Furthermore, the KPSS test presented in Table 4.2 provides additional support to the conclusions of the ADF and PP tests. While these tests may be deficient in terms of their ability to capture an order of integration that may not be an integer (see Baillie, 1996), the results herewith obtained are consistent with those of previous research, for example, Lo (1991), Mills (1993), Resende and Teixeira (2002) and Nagayasu (2004).

Table 4.2: KPSS Tests

	KPSS	
	Constant	Trend and Constant
Botswana	-49.60**	-46.64**
Egypt	-46.03**	-46.02**
Ghana	-51.41**	-51.33**
Kenya	-57.15**	-57.14**
Mauritius	-35.59**	-35.46**
Morocco	-24.93**	-25.12**
Namibia	-63.50**	-63.65**
Nigeria	-28.94**	-28.95**
South Africa	-48.94**	-49.35**
Tunisia	-33.64**	-33.65**
Zimbabwe	-44.02**	-43.57**
US	-68.42**	-68.47**
UK	-65.52**	-65.50**

Note: '**' and '*' indicate statistical significant at the 1% and 5% levels, respectively.

However, based on the Jarque-Bera normality (1987) and Engle's Lagrange Multiplier ARCH (1982) test reported in Table 4.3, equity data exhibit non-normality and ARCH effects. These results provide evidence against the market efficiency specified in the random walk version of the EMH.

Table 4.3: Jarque-Bera Normality and LM ARCH Tests

	Normality test	ARCH(5) test	ARCH(10) test
	Chi^(2)	F(5, *n*)	F(10, *n*-10)
Botswana	6914.66**	39.16**	27.13**
Egypt	958.68**	51.73**	34.77**
Ghana	14087.53**	62.13**	39.08**
Kenya	197920.50**	86.10**	70.22**
Mauritius	12631.17**	42.22**	27.55**
Morocco	863.07**	31.58**	17.00**
Namibia	19555.61**	117.06**	83.57**
Nigeria	2638.09**	92.12**	50.98**
South Africa	2688.68**	67.72**	47.15**
Tunisia	13061.38**	64.97**	41.96**
Zimbabwe	411956.30**	89.58**	58.42**
US	2700.18**	79.26**	54.23**
UK	1827.22**	157.75**	103.14**

Note: '**' and '*' indicate statistical significant at the 1% and 5% levels, respectively.

4.6.2. Results from GARCH Model

The GARCH (*p*,*q*) model allows for the modelling of volatility persistence based on some stylised facts usually observed in high-frequency financial time series data, among them, the presence of thick tails, time-varying correlations and volatility clustering. Indeed, this framework is consistent with the characteristics of leptokurtosis and volatility clustering observed in the time series of ASM returns. To this end, we have estimated a variety of GARCH (1, 1) models.[4] Table 4.6 to Table 4.8 (shown at the end of the chapter) presents our models highlights the importance GARCH effects by showing that GARCH and ARCH terms, are statistically significant for most markets, with the exception Namibia and Tunisia where the

[4] The mean equation is estimated through the best fitting model for the daily returns. The implementation procedure followed is described by Brooks (2002) and involves selecting the lag orders (where the maximum *p* and *q* is set at five) that minimise the information criteria. While a number of information criteria exist, this study utilises the Schwarz Information Criteria (SIC, 1978) on the basis that it will asymptotically deliver the correct model (see Brooks, 2002). The selected ARMA (p,q) models are as follows: Botswana (1, 2); Egypt (1, 1); Ghana (0, 2); Kenya (1, 1); Mauritius (2, 0); Morocco (1, 0); Namibia (1, 0); Nigeria (1, 1); South Africa (1, 0); Tunisia (0, 2); Zimbabwe (3,3); UK (0, 0) and US (0,0).

ARCH terms are not statistically significant. Furthermore evidence of persistence in variance as measured by the GARCH model is reflected in the magnitude and significance of the ARCH and GARCH terms (indeed, as this sum approaches unity the greater the degree of persistence). Therefore, in order to have an indication of long memory in stock return volatility we assess the level of volatility persistence (this concept is examined in greater detail in the next chapter; see Table 5.1). The results obtained are mixed. In particular, evidence of volatility persistence is various considerably among the ASMs. For example, this measures ranges from 0.7964 in Botswana (which presents comparatively mild evidence in favour of long memory) to 0.9905 in Kenya (which strongly suggests long memory in stock return volatility). Meanwhile, volatility persistence in Nigeria and Zimbabwe is explosive, which does not suggest the existence of long memory in equity return volatility in these markets. In contrast, the UK and US exhibit strong persistence, since the level of persistence is 0.9844 and 0.9940, respectively, which is indicative of long memory. In addition, we note that the parameters of the conditional variance equations are all positive and statistically significant. Furthermore, they satisfy the positivity constraint for the GARCH (1, 1). This confirms the existence of a time-varying (conditional) variance of ASM returns, which may be interpreted as time-varying uncertainty among investors with respect to equity price fluctuations.

To further evaluate the statistical properties of the GARCH models (reported in Table 4.6 to 4.8), some diagnostic tests were performed by applying the Box-Pierce Q statistic test to standardised and squared standardised residuals. These diagnostics show that the estimated models are generally appropriate for the stock indices considered in this study.

4.6.3. Results ARFIMA-FIGARCH Model

We now turn to estimation of the fractionally differencing parameter in ASMs using the ARFIMA-FIGARCH model which is the main interest of this paper. To model persistence in equity returns and volatility simultaneously, maximum likelihood methods are used to specify the ARFIMA-FIGARCH model. The ARFIMA part of the equation provides a basis to test for market efficiency by examining the size of the fractional differencing term, d, in the mean equation. In particular, d measures the adjustment speed (relative to a stationary ARIMA case where $d = 0$) and hence permits conclusions based on the EMH (i.e., adjustment speed measured by the fractionally differencing term) as a criterion. On the other hand, the FIGARCH part of the model captures long memory in the conditional variance (or volatility) of the data. The next step is to estimate the appropriate ARFIMA (m, d, n)-FIGARCH (1, $\bar{d}$, 1) models. Table 4.4 presents the findings on the size and sign of d and $\bar{d}$, respectively. Tables 4.9 to 4.13 (shown at the end of the chapter) provide a more detailed presentation of the corresponding ARFIMA-FIGARCH models.

Table 4.4: Fractional Differencing Parameters from ARFIMA-FIGARCH Model

Country	d-ARFIMA (d)	d-FIGARCH ($\bar{d}$)
Botswana	0.1116 [0.0254]*	0.7476 [0.1495]**
Egypt	0.1244 [0.0349]*	0.4878 [0.0558]**
Ghana	0.0399 [0.0275]	0.2864 [0.1217]*
Kenya	0.2388 [0.0222]**	0.2265 [0.0559]*
Mauritius	0.1583 [0.0321]**	0.5826 [0.1768]*
Morocco	0.1182 [0.0443]*	1.4315 [0.1163]**
Namibia	-0.0357 [0.0306]	0.3301 [0.1023]*
Nigeria	0.1770 [0.0650]*	0.6926 [0.1061]**
South Africa	0.0001 [0.0278]	0.4717 [0.0826]**
Tunisia	0.0723 [0.0268]*	0.3586 [0.0708]**
Zimbabwe	-0.0027 [0.0700]	0.4173 [0.0431]**
US	-0.0315 [0.0120]*	0.4225 [0.0529]**
UK	-0.0158 [0.0125]	0.5221 [0.0717]**

Note: '**' and '*' indicate statistical significant at the 1% and 5% levels, respectively.

4.6.3.1 ARFIMA MODEL

Market efficiency is considered by examining the size of the fractional differencing parameter, *d*, in the mean equation. Table 4.3 highlights the prevalence of long memory in returns in ASMs. Most of the ASMs considered in this study display evidence of long memory in stock returns, with the exception of Namibia and Zimbabwe, where the results indicate the existence of short memory in stock returns. The fractional differencing parameter estimates are concentrated between 0.00 and 0.25. Furthermore, this parameter is statistically significant in all these markets except for Ghana and South Africa. The exceptions to this pattern are Namibia (d = -0.0357) and Zimbabwe (d = -0.0114) where returns follow a short memory process; however, these results are not statistically different from zero. Meanwhile, results from the UK and the US also suggest that equity returns follow a short memory process; however, for the UK the results are not statistically significant.

4.6.3.2 FIGARCH Model

Evidence of long memory in volatility is mixed both across ASMs and the benchmark comparators (i.e., US and UK). In the ASMs, the results largely corroborate the existence of long memory in volatility with the exception of Botswana $(\bar{d} = 0.7476)$, Mauritius $(\bar{d} = 0.5826)$, Morocco $(\bar{d} = 1.4315)$ and Nigeria $(\bar{d} = 0.6926)$, where the results obtained indicate that volatility does not have a predictable component. The US has a fractional differencing value of 0.422495 respectively which suggests a long memory component in volatility. On the other hand, the UK has $\bar{d} = 0.5221$ which implies that volatility is a nonstationary process and hence unpredictable. Finally, the long memory parameters in the conditional variance equations are significantly different from zero across all markets examined in this study.

Statistically, the FIGARCH model embodies a positivity constraint which impacts on the validity of the estimated results. The benchmark comparators all satisfy the positivity constraint of the FIGARCH model. However, the results from Ghana, Kenya, Mauritius and Namibia do not satisfy the positivity constraint of the FIGARCH model and, therefore, results from these markets must be interpreted cautiously.

The significant size of d and $\bar{d}$ obtained from this model illustrates the importance of modelling long memory in ASMs. Furthermore, the result of $d \neq 0$ from these models is in contrast to our findings from the unit root tests that led to a conclusion of $d = 0$.

In sum, the results of the ARFIMA-FIGARCH model suggest that stock returns in ASMs are characterised by stochastic processes which have a potentially predictable component, this in turn implies a departure from the EMH suggesting that relevant market information was only partially or gradually reflected in stock price changes. This pattern of time dependence in stock returns may allow for past information to be used to improve the predictability of future returns. Evidence of long memory in equity return volatility in ASMs is mixed, with evidence for and against long-memory.

4.6.4. Results from ARFIMA-HYGARCH Model

By construction, the FIGARCH does not specify a covariance stationary process. Consequently, Davidson (2004) proposed the Hyperbolic GARCH (HYGARCH) model, which nests both GARCH and FIGARCH as special cases. The HYGARCH shares with the GARCH model the desired property of covariance stationary, while at the same time it obeys hyperbolically decaying impulse response coefficients as does the FIGARCH (see section 4.5.3 for model specification).

4.6.4.1 ARFIMA Model

From here we estimate the appropriate ARFIMA (m, d, n)-HYGARCH (p $\bar{d}$, q) models. Table 4.5 presents our results on the size and sign of the ARFIMA d and the HYGARCH $\bar{d}$, respectively. The fractional differencing terms in the returns equation for most ASMs are within the range $d \in (0, 0.5)$. The exception is Namibia d = -0.0343. Our results are statistically significant except for Ghana and South Africa. Both US and UK data show short memory in returns; and, this effect is statistically significant. Again, these findings suggest deviations from the EMH suggesting bottlenecks in the processing of new information. In addition, our results reinforce the different pattern of time dependence characterising ASMs compared to more developed markets. In particular, ASMs generally follow long memory processes; while short memory dynamics distinguish both the UK and US. Furthermore, the *d*-ARFIMA estimates in this model (ARFIMA-HYGARCH) are very similar (in magnitude and statistical significance) to those obtained using the ARFIMA-FIGARCH model. The only inconsistency with respect to ARFIMA estimates between these models concerns the long memory parameter d for Zimbabwe. In particular, $d = 0.1036$ and is statistically significant in the ARFIMA-HYGARCH model, while, $d =$ -0.0027 and is not statistically important in the ARFIMA-FIGARCH model. These discrepancies may reflect parameter instability owing to the complexities of modelling long memory in equity returns and volatility simultaneously in a context of macroeconomic instability (marked by hyperinflation) as exists in Zimbabwe.

Table 4.5: Long Memory Results from ARFIMA-HYGARCH Model

Country	*d*-ARFIMA (d)	*d*-HYGACH ($\bar{d}$)
Botswana	0.1425 [0.0282]**	1.5137 [0.2896]**
Egypt	0.1197 [0.0338]*	0.2637 [0.0316]*
Ghana	0.3980 [0.0276]	0.3269 [0.3124]
Kenya	0.2394 [0.0221]**	0.3186 [0.0574]*
Mauritius	0.1865 [0.0255]**	0.7844 [0.3005]*
Morocco	0.1169 [0.0444]*	0.6124 [0.2077]*
Namibia	-0.0343 [0.0305]	0.4486 [0.2114]*
Nigeria	0.1747 [0.0641]*	0.6779 [0.1229]*
South Africa	0.0001 [0.0278]	0.4741 [0.1026]*
Tunisia	0.0726 [0.0269]*	0.5259 [0.2071]*
Zimbabwe	0.1036 [0.0437]*	0.2034 [0.9528]*
US (S&P 500)	-0.0316 [0.0120]*	0.3696 [0.0666]*
UK	-0.0157 [0.0125]*	0.5857 [0.0892]**

Note: '**' and '*' indicate statistical significant at the 1% and 5% levels, respectively.

4.6.4.2 HYGARCH Model

Parameter estimates for long memory in volatility in all markets considered are statistically significant, except for Ghana. In particular, under the FIGARCH model the long memory parameter, $\bar{d}$, is statistically significant at the 5 percent level while in the HYGARCH model the long memory parameter is not statistically significant. For Botswana, Mauritius, Morocco, Nigeria and Tunisia $\bar{d} > 0.5$, which implies that the volatility process is nonstationary and therefore unpredictable. For the other ASMs we find evidence of a pattern of time dependence in volatility that may allow for past information to be used to improve the predictability of future volatility. For the US our results indicate evidence of long memory in volatility; while in the UK volatility is shown not to have any association with its distant realisations. Indeed, the long memory estimates for the benchmark comparators are also very similar to those obtained in the FIGARCH model. More generally, we find that the results from the HYGARCH model are very similar (in magnitude and statistical significance) to those obtained from the FIGARCH model previously described. The noticeable exception relates to the case of Tunisia where the fractional differencing parameter, $\bar{d} = 0.5259$ in the HYGARCH model compared to 0.3586 in the FIGARCH model. In other words, in the HYGARCH model Tunisia's volatility process is identified as a non-stationary process while in the context of the FIGARCH model Tunisia's volatility structure displays long memory in volatility. The few discrepancies between the FIGARCH and HYGARCH model results that we find underscore the importance of using recently developed models in time series econometrics which ensure that the estimation techniques used appropriately reflect the time series characteristics of the data. Therefore, in view of the limitations of the FIGARCH model, results from the HYGARCH process can be considered more robust. Indeed, from this perspective market participants and policymakers are advised to utilise models which are more robust in order to derive more accurate measures of long memory in volatility and hence avoid misleading inferences. Finally, the results obtained from this model are largely consistent with those of the ARFIMA-FIGARCH model suggesting that these models are close substitutes.

4.7 Summary and Conclusion

Using the weak-form version of the EMH as a criterion this study examines the long memory properties of ASMs. This is important because the efficiency of a market in processing information affects its allocative capacity, and therefore its contribution to economic growth. Furthermore, our results show that the behaviour of equity market returns and the associated volatility are dissimilar across the various countries analysed and this may have implications for portfolio diversification and risk management strategies. In particular, these results may be useful to investors given that price volatility is an important driver of active investment returns; and, volatility is also a key determinant of risk premia in equity markets.

The results from this study show that ASMs generally have a long memory component associated with their stock returns. In contrast, the developed market countries display short memory in their returns process. This result may suggest differences in how information is processed in these markets. For example, the ASMs are generally shallow and have less mature institutional and regulatory frameworks (Yartey and Adjasi, 2007); whereas the reverse holds in the US and UK. In particular, the preponderance of long memory in the equity returns of ASMs may reflect a variety of factors that influence the processing of new information, such as illiquid trading conditions and the still largely limited role of mutual funds and professionally managed intermediaries in many ASMs.

This study finds that evidence of long memory in volatility is mixed across countries – both ASMs and benchmark comparators. For Botswana, Mauritius, Morocco, Nigeria, Tunisia (only in the HYGARCH model) and the UK volatility is unpredictable. For all other markets, volatility is characterised by long memory indicating that shocks to the stock return volatility decay slowly and distant observations are associated with each other and therefore potentially predictable.

The results obtained from the ARFIMA-FIGARCH and ARFIMA-HYGARCH are generally similar. The ARFIMA-FIGARCH results for Ghana, Kenya, Mauritius and Namibia do not satisfy the positivity constraint specified by the FIGARCH model hence their statistical validity and associated interpretations cannot be considered robust or reliable. In contrast, the HYGARCH model is shown to be more encompassing as it adequately explains all the data. However three differences in these models are evident. First, the ARFIMA-FIGARCH results show that Zimbabwe returns follow a short memory process which is also statistically insignificant; while, the HYGARCH model suggest returns in Zimbabwe follow a long memory process that is statistically different from zero. The FIGARCH model indicates that volatility in Tunisia has a statistically significant long memory component whereas the HYGARCH model indicates that volatility in Tunisia is unpredictable. In addition, the FIGARCH model reports long memory in Ghana’s volatility is statistically different from zero; while, results from the HYGARCH version suggest that it is not. Apart from this difference the results obtained from these models are similar.

In terms of policy implications, the rejection of the market efficiency hypothesis implies that addressing trading frictions; promoting timely disclosure and dissemination of information to investors on the performance of listed companies; and strengthening regulatory oversight are key elements of a strategy aimed at improving the efficiency of ASMs (Barkoulas *et al*, 2000; Yartey and Adjasi, 2007).

Further policies to resolve the informational inefficiencies in ASMs also relate to upgrading the operational infrastructure. For example, Malone (2001) and Irving (2005) report that many of the trading and settlement modalities in ASMs are paper-based and therefore prone to a variety of delays (e.g., time lags in reporting) and errors (relating to the accuracy of the provided information). It follows that the adoption of an electronic trading platform and adoption of a central depository system can be anticipated to help improve the speed and accuracy of informational flows in ASMs (Yartey and Adjasi, 2007).

In addition, to the extent that market inefficiency in ASMs may be linked to the lack of breadth and depth in these markets, it may be important to counteract these effects by promoting the establishment of regional stock markets. Indeed, Irving (2005) suggests that cooperation and integration of ASMs may help improve the liquidity and efficiency of these markets. This may, in turn, improve the mobilisation and allocation of capital and in so doing foster greater output growth. Further evidence of the beneficial effects of stock market integration are provided by Adelegan (2008) who finds that regionally integrated stock markets (measured by the number of cross-listings) generally tend to develop and grow faster than their nonregionally integrated counterparts.

Finally there are several implications of this study for further research. First, the accuracy of long memory estimates can improved if regime shifts are explicitly accounted for in the modelling framework. For example, Diebold and Inoue (2001), Granger and Hyung (2004) and McMillan and Ruiz (2009) show that the long memory parameter is overestimated if such breaks are not incorporated. Second, this inclusion may result in an improvement in both stock return and volatility forecasts, especially over long horizons. Indeed, such forecasts are important in a variety of settings, including the formulation of risk and portfolio management strategies.

GARCH Estimates

Table 4.6: GARCH Results (Botswana, Egypt, Ghana and Kenya)

Botswana

Constant (mean)	0.0449 [0.0142]*
AR(1)	0.9345 [0.0191]**
MA(1)	-1.0405 [0.0340]**
MA(2)	0.1334 [0.0268]*
Constant (variance)	0.1043 [0.0047]**
ARCH (α1)	0.4941 [0.0325]**
GARCH (β1)	0.3023 [0.0214]**
Q(5) 1/	11.590
Q(10) 1/	13.721
Q(5) 2/	2.6833
Q(10) 2/	13.350

Egypt

Constant (mean)	0.0386 [0.0186]*
AR(1)	0.2092 [0.0179]**
MA(1)	0.0412 [0.0182]*
Constant (variance)	0.0217 [0.0026]**
ARCH (α1)	0.0753 [0.0048]**
GARCH (β1)	0.9059 [0.0052]**
Q(5) 1/	2.0584
Q(10) 1/	3.9161
Q(5) 2/	2.6332
Q(10) 2/	4.2954

Ghana

Constant (mean)	0.0047 [0.0035]
MA(1)	-0.5918 [0.2892]*
MA(2)	0.2731 [0.1241]*
Constant (variance)	0.0024 [0.0002]*
ARCH (α1)	0.3292 [0.0219]**
GARCH (β1)	0.6517 [0.0108]**
Q(5) 1/	3.3831
Q(10) 1/	1.3456
Q(5) 2/	3.8864
Q(10) 2/	3.0295

Kenya

Constant (mean)	0.0150 [0.0157]
AR(1)	0.8431 [0.0183]**
MA(1)	-0.6788 [0.0269]**
Constant (variance)	0.0480 [0.0015]*
ARCH (α1)	0.4283 [0.0101]**
GARCH (β1)	0.5622 [0.0069]**
Q(5) 1/	0.8838
Q(10) 1/	1.3716
Q(5) 2/	0.6505
Q(10) 2/	0.8742

Note:
1/ The Ljung-Box Q test applied to standardised residuals.
2/ The Ljung-Box Q test applied to squared standardised residuals.
The numbers in ( ) and [ ] refer to lag lengths and standard deviations
‘**’ and ‘*’ indicate statistical significant at the 1% and 5% levels, respectively.

Table 4.7: GARCH Results (Mauritius, Morocco, Namibia and Nigeria)

Mauritius		*Morocco*	
Constant (mean)	0.0299 [0.0095]*	Constant (mean)	0.0654 [0.0224]*
AR(1)	0.2239 [0.0183]**	AR(1)	0.2917 [0.0283]**
AR(2)	0.1254 [0.0199]**	Constant (variance)	0.0323 [0.0029]*
Constant (variance)	0.0473 [0.0021]*	ARCH (α1)	0.2238 [0.0170]**
ARCH (α1)	0.4869 [0.0193]**	GARCH (β1)	0.7305 [0.0137]**
GARCH (β1)	0.3702 [0.0178]**	Q(5) 1/	4.7507
Q(5) 1/	1.0180	Q(10) 1/	1.6653
Q(10) 1/	2.1912	Q(5) 2/	3.6944
Q(5) 2/	0.7185	Q(10) 2/	6.6888
Q(10) 2/	1.6511		
Namibia		*Nigeria*	
Constant (mean)	0.0639 [0.0316]*	Constant (mean)	0.0019 [0.0152]
AR(1)	0.3395 [0.0253]	AR(1)	0.6384 [1.0361]
Constant (variance)	0.2336 [0.0940]	MA(1)	-0.0181 [0.3466]
ARCH (α1)	0.0792 [0.0257]*	Constant (variance)	0.0164 [0.0017]*
GARCH (β1)	0.8872 [0.0307]	ARCH (α1)	0.3448 [0.0157]**
Q(5) 1/	6.7376	GARCH (β1)	0.7003 [0.0083]**
Q(10) 1/	9.8531	Q(5) 1/	6.0485
Q(5) 2/	2.2373	Q(10) 1/	4.3667
Q(10) 2/	6.8114	Q(5) 2/	4.7294
		Q(10) 2/	3.1392

Note: See Table 4.6.

Table 4.8: GARCH Results (South Africa, Tunisia, Zimbabwe, UK and US)

South Africa

Constant (mean)	0.0880 [0.0181]*
AR(1)	0.1083 [0.0191]**
Constant (variance)	0.1066 [0.0211]**
ARCH (α1)	0.1134 [0.0057]**
GARCH (β1)	0.8436 [0.0049]**
Q(5) 1/	7.2045
Q(10) 1/	6.4889
Q(5) 2/	5.6485
Q(10) 2/	4.2857

Tunisia

Constant (mean)	0.0182 [0.0105]
MA(1)	0.2295 [0.0240]**
MA(2)	0.1309 [0.0239]**
Constant (variance)	0.0174 [0.0016]
ARCH (α1)	0.0905 [0.0048]
GARCH (β1)	0.8911 [0.0039]**
Q(5) 1/	1.9609
Q(10) 1/	4.5123
Q(5) 2/	2.9400
Q(10) 2/	6.4021

Zimbabwe

Constant (mean)	0.1398 [0.0371]*
AR(1)	0.5171 [0.1785]*
AR(2)	0.4788 [0.2012]*
AR(3)	0.0354 [0.1599]
MA(1)	0.0989 [0.1861]
MA(2)	-0.4298 [0.1522]*
MA(3)	-0.2315 [0.1284]
Constant (variance)	0.0736 [0.0029] **
ARCH (α1)	0.4038 [0.0117]**
GARCH (β1)	0.7052 [0.0047]**
Q(5) 1/	25.770
Q(10) 1/	29.531
Q(5) 2/	9.6661
Q(10) 2/	8.1274

UK

Constant (mean)	0.0096[0.0026]*
Constant (variance)	0.0091 [0.0010] **
ARCH (α1)	0.1011 [0.0049]**
GARCH (β1)	0.8833 [0.0054] **
Q(5) 1/	17.048
Q(10) 1/	15.572
Q(5) 2/	8.1892
Q(10) 2/	8.0058

US

Constant (mean)	0.0549 [0.0112]**
Constant (variance)	0.0037 [0.0006] *
ARCH (α1)	0.0670 [0.0035]**
GARCH (β1)	0.9275 [0.0036] **
Q(5) 1/	7.7790
Q(10) 1/	11.193
Q(5) 2/	6.2086
Q(10) 2/	9.9768

Note: See Table 4.6.

ARFIMA-FIGARCH Estimates

Table 4.9: ARFIMA-FIGARCH Results (Botswana, Egypt, Ghana and Kenya)

Botswana		*Egypt*	
Constant (mean)	0.0389 [0.0147]*	Constant (mean)	0.0447 [0.0216]*
d-ARFIMA	0.1116 [0.0254]*	*d*-ARFIMA	0.1244 [0.0349]*
AR(1)	-0.3532 [0.2716]	AR(1)	0.0650 [0.0467]
MA(1)	0.3776 [1.0422]	MA(1)	0.0428 [0.1771]
MA(2)	-0.0442 [0.0721]	Constant (variance)	0.9795 [0.0051]**
Constant (variance)	0.2104 [0.1417]	*d*-FIGARCH	0.4878 [0.0558]**
d-FIGARCH	0.7476 [0.1495]**	ARCH (α1)	0.3073 [0.0933]*
ARCH (α1)	0.1818 [0.5482]	GARCH (β1)	0.4177 [0.3357]**
GARCH (β1)	0.1454 [0.5363]	Q(5) 1/	9.1087
Q(5) 1/	20.907	Q(10) 1/	15.305
Q(10) 1/	24.330	Q(5) 2/	4.7535
Q(5) 2/	2.3833	Q(10) 2/	3.2281
Q(10) 2/	4.7667		
Ghana		*Kenya*	
Constant (mean)	0.0071 [0.0059]	Constant (mean)	0.0237 [0.0155]
d-ARFIMA	0.0399 [0.0275]	*d*-ARFIMA	0.2388 [0.0221]**
MA(1)	0.7868 [0.5531]	AR(1)	0.8425 [0.1796]**
MA(2)	-0.0210 [0.0342]	MA(1)	-0.6945 [0.0258]**
Constant (variance)	-0.1118 [0.0394]*	Constant (variance)	0.0611 [0.0174]*
d-FIGARCH	0.2864 [0.1217]*	*d*-FIGARCH	0.2265 [0.0559]*
ARCH (α1)	0.1306 [0.1606]	ARCH (α1)	0.3182 [0.0281]**
GARCH (β1)	0.6371 [0.2835]*	GARCH (β1)	0.9120 [0.3181]*
Q(5) 1/	4.6097	Q(5) 1/	35.500
Q(10) 1/	8.1305	Q(10) 1/	21.599
Q(5) 2/	1.6529	Q(5) 2/	12.052
Q(10) 2/	1.9631	Q(10) 2/	9.2337

Note: See Table 4.6.

Table 4.10: ARFIMA-FIGARCH Results (Mauritius, Morocco, Namibia and Nigeria)

Mauritius	
Constant (mean)	0.0615 [0.0224]*
d-ARFIMA	0.1583 [0.0321]**
AR(1)	0.2901 [0.0277]**
AR(2)	0.3306 [0.0416]**
Constant (variance)	0.0191 [0.0290]
d-FIGARCH	0.5826 [0.0177]*
ARCH (α1)	0.4186 [0.0559]**
GARCH (β1)	0.2221 [0.1215]
Q(5) 1/	2.7870
Q(10) 1/	1.3937
Q(5) 2/	1.3974
Q(10) 2/	0.5331

Morocco	
Constant (mean)	0.0579 [0.0319]
d-ARFIMA	0.1182 [0.0443]*
AR(1)	0.5031 [0.2553]
Constant (variance)	0.1235 [0.0561]*
d-FIGARCH	1.4315 [0.1162]**
ARCH (α1)	0.3482 [0.0994]*
GARCH (β1)	0.1303 [0.0422]*
Q(5) 1/	14.266
Q(10) 1/	20.391
Q(5) 2/	8.0056
Q(10) 2/	11.254

Namibia	
Constant (mean)	0.0778 [0.0180]*
d-ARFIMA	-0.0357 [0.0306]
AR(1)	0.1052 [0.0186]
Constant (variance)	-0.1408 [0.0154]**
d-FIGARCH	0.3301 [0.1023]
ARCH (α1)	0.0538 [0.0162]*
GARCH (β1)	0.1829 [0.0199]**
Q(5) 1/	5.4449
Q(10) 1/	2.9531
Q(5) 2/	1.3118
Q(10) 2/	0.9695

Nigeria	
Constant (mean)	0.0189 [0.0106]
d-ARFIMA	0.1770 [0.0649]*
AR(1)	0.2397 [0.0855]*
MA(1)	0.1274 [0.1601]
Constant (variance)	0.0298 [0.0248]
d-FIGARCH	0.6926 [0.1061]**
ARCH (α1)	0.2041 [0.0273]**
GARCH (β1)	0.6198 [0.3141]*
Q(5) 1/	27.271
Q(10) 1/	22.680
Q(5) 2/	10.337
Q(10) 2/	6.3389

Note: See Table 4.6.

Table 4.11: ARFIMA-FIGARCH Results (South Africa, Tunisia and Zimbabwe)

South Africa

Constant (mean)	0.0876 [0.0181]*
d-ARFIMA	0.0001 [0.0278]
AR(1)	0.1082 [0.0347]*
Constant (variance)	0.0375 [0.0154]*
d-FIGARCH	0.4717 [0.0826]**
ARCH (α1)	0.1098 [0.0664]
GARCH (β1)	0.5022 [0.1096]*
Q(5) 1/	6.6296
Q(10) 1/	9.5504
Q(5) 2/	3.1053
Q(10) 2/	5.7628

Tunisia

Constant (mean)	0.1508 [0.0387]
d-ARFIMA	0.0723 [0.0268]*
AR(1)	0.0438 [0.1380]
MA(1)	0.4709 [0.1527]*
Constant (variance)	0.5121 [0.1407]**
d-FIGARCH	0.3586 [0.0708]**
ARCH (α1)	0.0920 [0.1863]
GARCH (β1)	0.2933 [0.0975]*
Q(5) 1/	3.3048
Q(10) 1/	0.4039
Q(5) 2/	0.0573
Q(10) 2/	0.0064

Zimbabwe

Constant (mean)	0.1383 [0.0345]*
d-ARFIMA	-0.0027 [0.0700]
AR(1)	1.7091 [0.1761]**
AR(2)	-0.9605 [0.2788]*
AR(3)	0.2087 [0.1085]
MA(1)	-1.3136 [0.2017]**
MA(2)	0.5460 [0.2365]*
MA(3)	-0.1229 [0.0555]*
Constant (variance)	0.1649 [0.0712]*
d-FIGARCH	0.4173 [0.0431]**
ARCH (α1)	-0.2488 [0.4147]
GARCH (β1)	-0.2247 [0.4238]
Q(5) 1/	22.157
Q(10) 1/	32.157
Q(5) 2/	1.7871
Q(10) 2/	3.8832

Note: See Table 4.6.

Table 4.13: ARFIMA-FIGARCH Results (UK and US)

UK		*US*	
Constant (mean)	0.0248 [0.0118]*	Constant (mean)	0.0536 [0.0085]*
d-ARFIMA	-0.0158 [0.0125]	d-ARFIMA	-0.0315 [0.0120]*
Constant (variance)	0.0112 [0.0137] *	Constant (variance)	0.0181 [0.0068] *
d-FIGARCH	0.5221 [0.0717]**	d-FIGARCH	0.4225 [0.0528]**
ARCH (α1)	0.3361 [0.0853]**	ARCH (α1)	0.2115 [0.0402]*
GARCH (β1)	0.5187 [0.0729]**	GARCH (β1)	0.6087 [0.0606]**
Q(5) 1/	12.994	Q(5) 1/	17.858
Q(10) 1/	17.466	Q(10) 1/	21.463
Q(5) 2/	8.4603	Q(5) 2/	11.472
Q(10) 2/	10.006	Q(10) 2/	13.950

Note: See Table 4.6.

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5 Measuring Volatility Persistence and Long Memory in the Presence of Structural Breaks: Evidence from African Stock Markets

5.1 Introduction

A great deal of research modelling financial time series has focused on estimating time-varying volatility. Indeed, an extensive literature has established the presence of non-constant and time dependent volatility in high frequency asset returns data.[1] The main representatives of this class of models are the Autoregressive Conditional Heteroscedasticity (ARCH) models (Engle, 1982) and its extensions including the Generalised ARCH (Bollerslev, 1986) and the Fractionally Integrated GARCH (Baillie *et al*, 1996). These models explicitly recognise the difference between the conditional and unconditional (or long run) variance, where the former is allowed to change over time and the latter remains constant.

Against this background, this paper examines the properties of stock return volatility in ASMs with a view to characterising the behaviour of the conditional variance. This is important since the potential gains from international portfolio diversification have attracted investors to these markets yet little is known about the volatility profile in ASMs. In particular, this paper investigates the effect of structural breaks on volatility persistence, long memory and the forecasting performance of equity data from African economies.

[1] Bollerslev *et al* (1992) provide an extensive survey of the literature.

These questions may provide investors with a better understanding of how shocks affect volatility over time and the role that regime changes may play in this process. Indeed, Poterba and Summers (1986) show that the extent to which stock-return volatility is persistent, is important, since it affects stock prices (through a time-varying risk premium). More specifically, in this paradigm, an increase in (expected) volatility persistence would imply a decline in the current stock price. Second, Andersen and Bollerslev (1998) demonstrate that GARCH models provide accurate volatility forecasts when volatility persistence is accurately estimated. In addition, Stărică and Granger (2005) examine US data and show that the incorporation of occasional breaks in the unconditional variance generates better out-of-sample forecast in comparison to the standard GARCH (1, 1).

Since the most familiar example of an observation-driven volatility model is the GARCH model, it is appropriate to consider it at the outset. In this model, the speed of mean reversion is determined by the persistence of the volatility process. That is, these models are based on the assumption that although volatility is persistent it ultimately reverts to a constant mean volatility.[2] However, the degree of persistence may be biased due to ignored breaks in the data. For example, Mikosch and Stărică (2004a), and Malik *et al* (2005) show that volatility persistence is overestimated if regime shifts are not accounted for in the standard GARCH model. Further, Mikosch and Stărică (2004b) present evidence indicating that the unconditional volatility of a GARCH model may not be constant; hence, leading to spurious evidence of persistence. In a more formal long memory context, Diebold and Inoue (2001),

[2] This is because GARCH models are effectively moving average models that calculate that volatility will eventually pull back to some form of long-term average.

Granger and Hyung (2004), and McMillan and Ruiz (2009) show that the failure to account for structural breaks can lead to spurious evidence of long memory.

These issues have not been examined so far for ASMs, and this paper attempts to fill the gap by addressing the following questions. First, this paper examines the extent of both volatility persistence and long memory in ASMs. Second, the possibility of structural breaks; in particular, occasional structural breaks and time-varying unconditional volatility will be explored insofar as they may trigger spurious long memory results. Third, the properties of a time-varying unconditional variance will be explored. Specifically, its attributes will be examined, with a view, to assess if they can be exploited to provide for improved volatility forecasts. These findings may have potential value for market participants in asset pricing and risk management especially given that accurate volatility forecasts are important in a wide range of applications from measuring risk (e.g. Value-at-Risk management) to pricing derivatives (e.g., option pricing).

To summarise our findings at the outset, we find that accounting for neglected breaks and time-variation in the unconditional mean of the volatility series leads to lower estimates of volatility persistence and long memory. In particular, we find that the degree of volatility persistence generally decreases after incorporating structural breaks; while, the long memory estimate is substantially reduced. This suggests that the long memory parameter may be more sensitive to the impact of neglected structural breaks in the equity data. The results indicate that previous studies on the volatility behaviour of ASMs may have overstated the degree of volatility persistence and long memory. Specifically, the results of this analysis indicate that the long memory property is amplified when analysed on the premise that the unconditional (or long-run) variance is constant. Furthermore, it is shown by means of both breakpoint tests and a moving average application that the unconditional volatility of the stock-return volatility series displays time-variation. Finally, the modification of the GARCH model, through the introduction of a rolling window in order to update the unconditional variance and hence reflect mean variation produces improved volatility forecasting performance for a selection of the ASMs. In sum, the findings on the impact of structural breaks on volatility in this paper complement those in previous studies and may provide an interesting comparison to existing studies.

5.2 Review of Relevant Literature Review

While a wide range of methods have been employed to model volatility, the GARCH model (developed by Engle, 1982 and generalised by Bollerslev, 1986) provides a widely accepted basis with which to analyse high-frequency financial time series data. In the analysis of various asset returns, a common finding that has emerged from empirical work using the GARCH (1,1) model is that shocks to the conditional variance process are highly persistent (i.e., the impact of these shocks endure for long periods into the future).[3]

[3] Volatility persistence is reflected by the magnitude and significance of the coefficients of the ARCH and GARCH parameters in the conditional variance equation. Furthermore, the relevant time series displays more persistence when the sum of these coefficients is closer to one.

However, Lastrapes (1989) reports that an important disadvantage of the autoregressive structure of the ARCH model is that it produces a high degree of volatility persistence; which may not be consistent with the distinct changes in the mean level of volatility associated with a regime shift, as might occur during a financial crisis or in a context of economic reform. In this case, the ARCH model overestimates the true magnitude of volatility persistence. In a related paper Lamoureux and Lastrapes (1990) demonstrate through both analysis of US daily stock return data and a Monte Carlo simulation that evidence of persistence in variance in GARCH models can be overestimated by the failure to account for deterministic structural shifts in the volatility process. Indeed, Engle and Mustafa (1992) analyse the stock market crash of October 1987 and showed that the ARCH specification implied greater volatility persistence than actually was the case over this period. Hamilton and Susmel (1994) employ a specification of the GARCH model that uses an application of the Markov-switching methodology to modelling a conditional volatility process subject to regime shifts. Applying this method to stock return data they present evidence showing that shocks to the variance process are less persistent when structural changes in the data are accounted for. In addition, the episodes of regime shifts are determined endogenously by the data and not exogenously determined as in the paper by Lastrapes (1989).[4] Malik *et al* (2005) use the iterated cumulated sum of squares (ICSS) algorithm to detect regime shifts in Canadian stock

[4] Traditionally, regimes are described by changes in the constant of the process:
$$y_t = \gamma_1 + (\gamma_2 - \gamma_1)D_t + \vartheta y_{t-1} + \varepsilon_t,$$
where y_t is the stock return series, D_t denotes a dummy variable which assumes the value of zero before the structural change and one thereafter, ε_t represents the innovation process, γ_1 and γ_2, pertain to the mean of the return series before and after the regime shift. The primary limitation of this approach relates to the implicit assumption that the regime shift is produced by a single event, which is not going to repeat itself, and can be exogenously determined.

market data. This methodology also allows for the endogenous detection of sudden changes in volatility levels at certain points in time (Inclan and Tiao, 1994). Their results further highlight the finding that after taking account of volatility shifts in the GARCH model, the estimated persistence of volatility shocks is substantially reduced. Furthermore, they emphasise the potential implications their findings may have on financial market participants.

The GARCH model explicitly recognises the difference between the conditional (or time-varying) and unconditional (or non-varying) variance; where, the former may depend upon random variables in the conditioning set such as past disturbances, while the latter is assumed to be constant (as is traditionally the case in econometrics). In light of these assumptions, Mikosch and Stărică (2004b) show that evidence of volatility persistence could be an artefact of structural change in the data; in particular, nonstationarity of the unconditional variance. In total, the extant literature provides significant evidence to substantiate the hypothesis that GARCH measures of volatility persistence are subject to model misspecification arising from the failure to take account of regime shifts in the data being analysed (e.g., McMillan and Ruiz, 2009, and references therein).

Similarly, in the case of long memory estimates, a growing body of econometric research suggests that evidence of long memory may be a spurious experimental result. For example, Lobato and Savin (1998) test for the presence of long memory in daily US stock return data and show that evidence of long memory may in fact be spurious owing to nonstationarity and aggregation effects within the data.[5] In order to address these possibilities they split their data into stationary subperiods and employ individual stock return data (as opposed to stock index data). Their results indicate no evidence of long memory in the stock returns; while, ample evidence suggests the existence of long memory in the squared returns process. To examine the possibility that the observed evidence of long memory is, potentially spurious, Gourieroux and Jasiak (2001) show how stochastic processes with infrequent regime switching may precipitate a long memory effect in the autocorrelation function, thereby, giving the appearance of long memory behaviour. In order to investigate the relation between stochastic regime switching and the appearance of long memory in stock return volatility Diebold and Inoue (2001) use both theoretical and empirical methods to show that only a minute regime switching effect is necessary to generate a long memory effect. In other words, the authors demonstrate that regime switching confounds evidence of fractional integration. In another study, Mikosch and Stărică (2004a) argue that the long memory effect may be spurious owing to the failure of statistical techniques to distinguish between a stationary long memory process and a nonstationary financial time series. This line of analysis is further advanced by Granger and Hyung (2004) who show that an occasional break in time series data can produce a long memory effect in the autocorrelation function. More precisely, the authors show that if the long memory parameter is estimated from data generated by a process marked by occasional level shifts in the mean level of volatility then this can precipitate the false inference that the data generating process is a genuine long memory process and not a short memory process with breaks in the mean. Bisaglia

[5] Aggregation (of data) as a source of long memory behaviour derives from the notion that the aggregation of a weakly dependent series may generate a strong dependent series and hence display long memory behaviour.

and Gerolimetto (2008) extend the existing literature by considering the forecasting performance of long memory models and (long memory) models with stochastic regime switching by using Monte Carlo simulation. Their results are ambiguous, but long memory models generally underperform their counterparts with regime switching. Furthermore, McMillan and Ruiz (2009) analyse the equity markets of ten industrialised countries that show that a modification of the GARCH model to allow for mean variation generates improved volatility forecasting performance over long horizons. Against this background, the following analysis seeks to contribute to this literature by examining data from ASMs, for which there appears to be limited or no discussion of this subject matter.

5.3 Empirical Analysis

5.3.1 GARCH Estimates

A GARCH model is defined by its first and second moments, which, are typically referred to as the mean and variance equation, respectively. The mean equation captures the return process (r_t) which is composed of a conditional mean, μ, which may include autoregressive and moving average terms and an error term ε_t, which follows a conditional normal density with a zero mean and a variance h_t. In addition, the information set available to investors up to time $t-1$ is given by I_{t-1}, hence:

$$r_t = \mu + \varepsilon_t, \quad \text{where } \varepsilon_t | I_{t-1} \sim N(0, h_t) \tag{5.1}$$

The specification of the conditional volatility is consistent with a forecast of the variance at time t (h_t) on the basis of a long-term average (the constant unconditional mean value, ω), the volatility forecast from the previous period (h_t) and information about volatility in the last period (ε_{t-1}^2):

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_t \quad (5.2)$$

Furthermore, the inequality restrictions $\omega > 0$ and $\alpha, \beta \geq 0$ are imposed to ensure that the conditional variance is strictly positive. The GARCH (1, 1) specification has the useful property that shocks to volatility decay at a constant rate and the speed of decay is measured by the estimate of $\alpha + \beta$. The sum of α and β also measures volatility persistence (i.e., the extent to which shocks to current volatility remain important for long periods into the future). As this sum approaches unity, the persistence of shocks to volatility becomes greater. However, when $\alpha + \beta = 1$ then any shock to volatility is permanent and the unconditional variance is infinite. In this case, the process is denoted an IGARCH (Integrated GARCH, Engle and Bollerslev, 1986). This process implies that volatility persistence is permanent; hence, past volatility is significant in predicting future volatility over all finite horizons. When the sum of α and β is greater than unity, then volatility is explosive, i.e., a shock to volatility in one period will result in even greater volatility in the next period (Chou, 1988).

As a simplification the standard GARCH model restricts the dynamics of the unconditional variance to an arbitrary constant in order to focus on the conditional variance. This paper will relax this assumption in order to examine the implications on the behaviour of the conditional variance; especially, the ARCH and GARCH parameters. In particular, the unconditional variance $\left(\sigma^2\right)$ in this model is equal to $\omega/(1-\alpha-\beta)$. This specification allows us to investigate the possibility of structural breaks in the variance constant, ω, which in turn imply shifts in the unconditional variance; and hence, may lead to spurious evidence of volatility persistence in a standard GARCH model.

To provide another perspective on the estimates of volatility persistence, the half-life of volatility shocks are also presented. The half-life measures the number of days over which a shock to volatility decays to half its original size and is calculated as $\ell = \log(0.5)/\log(\alpha+\beta)$. The GARCH model estimates are reported in Table 5.1.

Table 5.1. GARCH and Fractional Integration Estimates.						
	GARCH(1,1)					GPH
	ω	α	β	α+β	ℓ	*d*
Botswana	0.104 (22.03)	0.494 (15.16)	0.302 (14.10)	0.796 (0.00)	3.04	0.25 (1.96)
Egypt	0.022 (8.37)	0.075 (15.48)	0.906 (172.84)	0.980 (0.00)	34.57	0.44 (4.58)
Ghana	0.002 (8.58)	0.329 (15.05)	0.652 (60.45)	0.981 (0.22)	36.27	0.43 (3.51)
Kenya	0.048 (32.05)	0.428 (42.49)	0.562 (81.55)	0.991 (0.23)	74.10	0.40 (4.41)
Mauritius	0.047 (22.27)	0.487 (25.24)	0.370 (20.78)	0.858 (0.00)	4.51	0.35 (3.34)
Morocco	0.032 (11.04	0.224 (13.17)	0.731 (53.55)	0.955 (0.00)	15.03	0.44 (3.62)
Namibia	0.234 (2.49)	0.079 (3.07)	0.887 (28.92)	0.966 (0.16)	20.33	0.27 (2.09)
Nigeria	0.016 (9.36)	0.345 (22.00)	0.700 (84.63)	1.045 (0.00)	N/A	0.32 (3.07)
South Africa	0.020 (11.39)	0.113 (19.84)	0.864 (175.23)	0.977 (0.00)	29.27	0.40 (4.08)
Tunisia	0.017 (10.37)	0.091 (18.98)	0.891 (228.08)	0.982 (0.00)	37.69	0.38 (3.59)
Zimbabwe	0.074 (25.19)	0.404 (34.59)	0.705 (149.79)	1.109 (0.00)	N/A	0.54 (5.73)
UK	0.009 (8.89)	0.101 (20.77)	0.883 (163.48)	0.984 (0.00)	42.87	0.51 (5.87)
US	0.004 (6.37)	0.067 (18.96)	0.928 (255.47)	0.994 (0.00)	122.03	0.60 (6.91)

Notes: Equation specification and discussion in Section 2. Numbers in parentheses are *t*-statistics, except under column α+β where entries are *p*-values from a Wald test that α+β=1. Half-lives, ℓ are calculated as log(0.5)/log(α+β). For, Nigeria and Zimbabwe, the half-life cannot be interpreted (since the half-life approaches infinity as α+β →1) and are denoted N/A to indicate that the method used is not applicable.

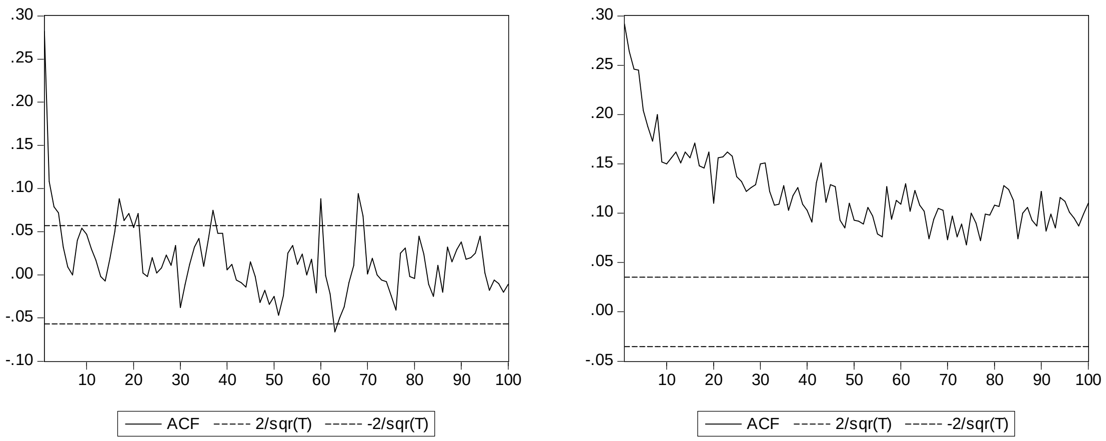
The estimates of volatility persistence for the eleven ASMs vary considerably. In Nigeria and Zimbabwe, volatility persistence is explosive, and is equal to 1.045, and 1.109, respectively. For all other markets $\alpha + \beta < 1$. For example, among ASMs, Kenya exhibits the greatest persistence at 0.991; while, Botswana, registers the lowest level of volatility persistence at 0.796. Meanwhile, evidence of high volatility persistence is also found in the UK and US, at 0.984 and 0.994, respectively. Kenya's high volatility persistence translates into a half life of 74 days; while Botswana's relatively lower degree of volatility persistence is equivalent to a half life of only 3 days. South Africa and Egypt, are Africa's largest equity markets, and have half-lives of 29 and 35 days respectively. This means that shocks to volatility will taper off with a half life of about 4 and 5 weeks, respectively. In comparison, it takes 122 days for a shock to volatility to diminish to half its original size in the US. However, in the case where volatility is explosive, as in the case of both Nigeria and Zimbabwe, the half-life cannot be interpreted (since the half-life approaches infinity as $\alpha + \beta \rightarrow 1$; hence, persistence should be considered large for these indices). The generally smaller degree of volatility persistence (and hence half life) of ASMs in comparison to the developed markets in our sample, may reflect differences in the structural composition of these markets. For example, ASMs are characterised by nonsynchronous trading (or nontrading) effects which may lead volatility shocks to dissipate more quickly given the fragmented nature of trading in these markets. This in turn may also be a manifestation of the illiquid trading conditions prevalent in ASMs.

5.3.2 ACF and Fractional Integration

In the time domain, long memory is characterised by a very slow mean-reverting hyperbolically decaying autocorrelation function (ACF). To investigate this asymptotic property in ASMs it is necessary to examine the sample ACF. The ACF for the absolute index returns and up for up to 100 lags for each respective time series are presented in Figure 5.1, together with the 5 percent critical value based upon $2/\sqrt{(T)}$.

Figure 5.1: ACF for Absolute Returns[11]

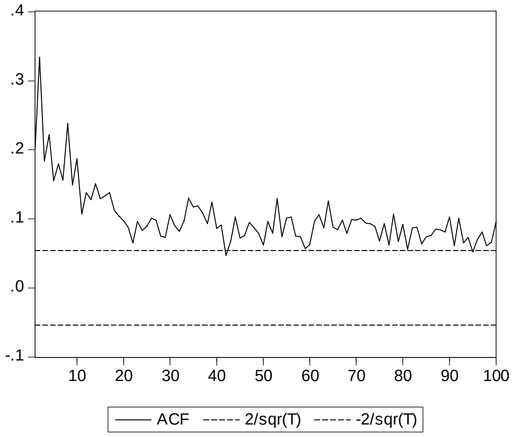
Botswana Egypt



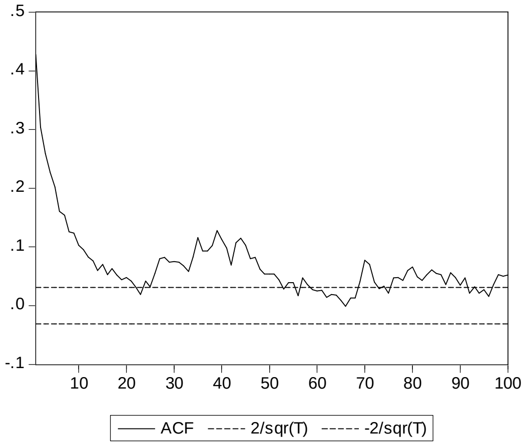
[11] In all the graphs presented in Figure 5.1, the horizontal axis captures the lag length up to the 100th lag while the vertical axis captures the size of both the critical value of the test statistic and the ACF for absolute returns.

Figure 5.1: ACF for Absolute Returns (continued)

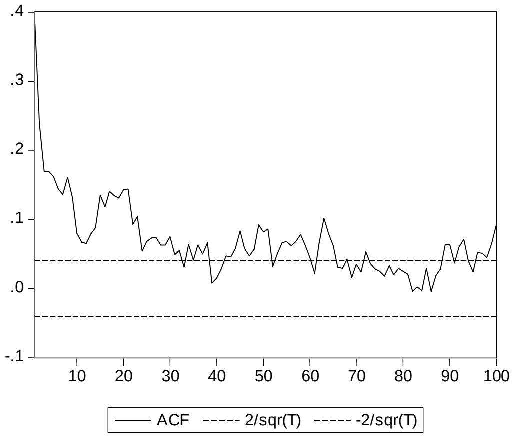
Ghana



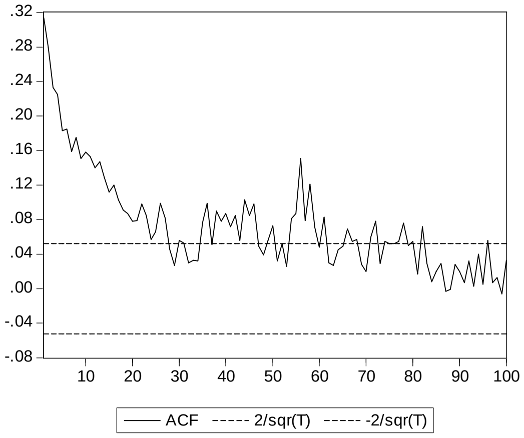
Kenya



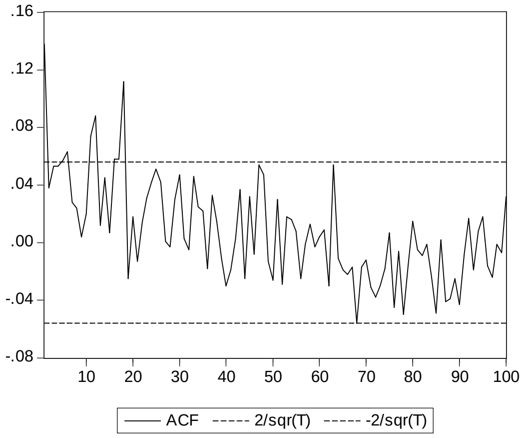
Mauritius



Morocco



Namibia



Nigeria

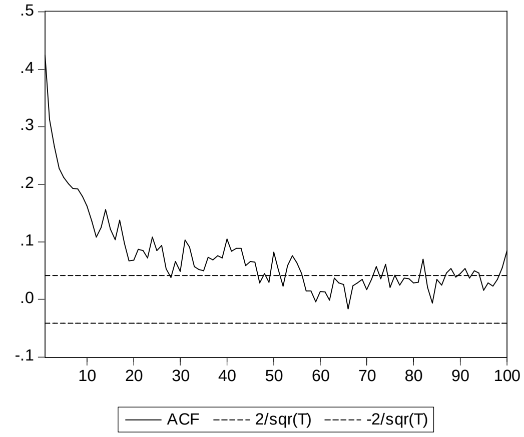
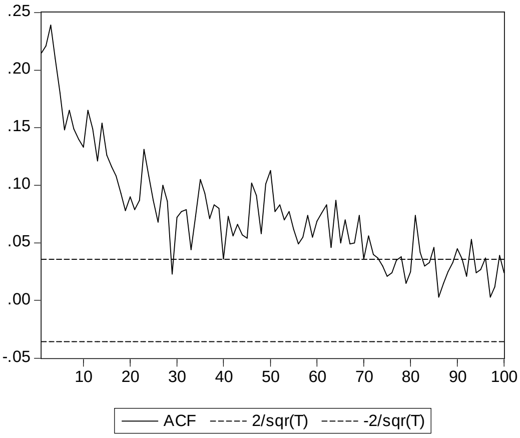
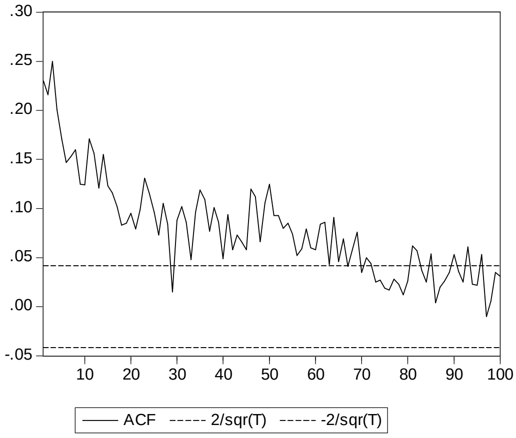


Figure 5.1: ACF for Absolute Returns (continued)

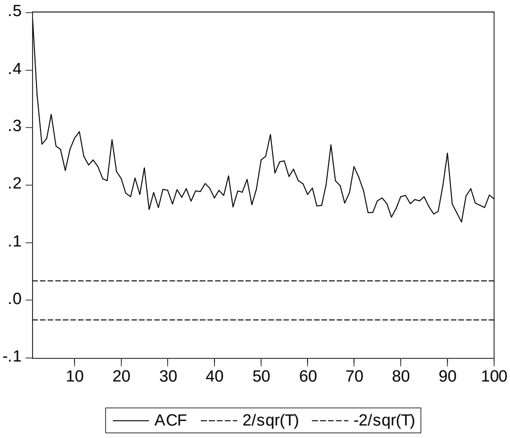
South Africa



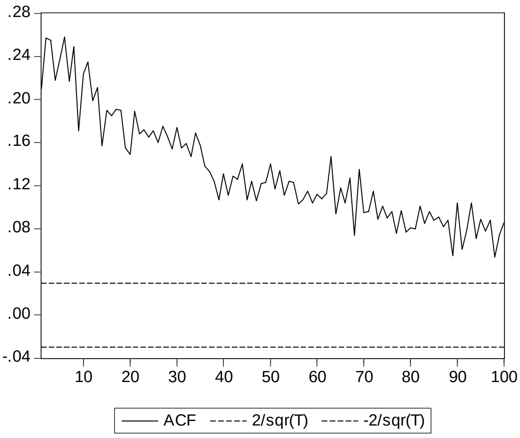
Tunisia



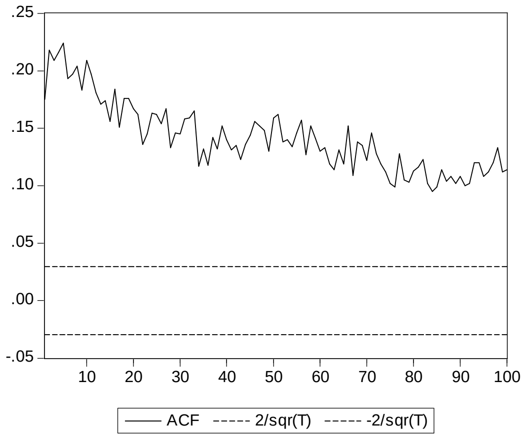
Zimbabwe



UK



US



The graphical evidence shows that in ASMs most autocorrelations are not significant at all lags except for Egypt. Ghana comes close to having all autocorrelations being significant at all lags, however, at around lag 45 the ACF dips into insignificant territory but rebounds into significance thereafter. Botswana, Namibia and Zimbabwe are the extreme examples, in that, their ACFs are mostly insignificant at all lags, a pattern inconsistent, with the existence of long memory. In total, the ACFs of the majority of the ASMs do not suggest the presence of long memory. In comparison, the ACFs from the US and UK suggest the existence of long memory in volatility.

In the frequency domain, a long memory process is revealed by the behaviour of its spectral density function $f(\lambda_j)$ estimated at the harmonic frequencies $\lambda_j = 2\pi\, j/T$ **,** where j = 0, 1, 2,…,*m* defines the set of harmonic frequencies. Indeed, a stationary process is defined to have long memory when

$$f(\lambda_j) \approx c\lambda_j^{-2d} \text{ as } \lambda_j \to \infty \qquad (5.3)$$

where $c > 0$ and $d \in (0, 0.5)$ is the long memory parameter.

Geweke and Porter-Hudak (1983) proposed a semi-parametric procedure to obtain an estimate of the fractional differencing parameter *d* based on the slope of the spectral density function around the angular frequency $\lambda_j = 0$. In particular, let $I(\lambda_j)$ denote the sample periodogram at the *j*th Fourier frequency, $\lambda_j = 2\pi j / T$, *j* = 1, 2, …, [*T*/2]. The estimator of the parameter of fractional integration, *d*, is then based on the least-squares regression

$$\log\left(I\left(\lambda_j\right)\right) = \beta_0 + \beta_1 \log\left(\lambda_j\right) + \varepsilon_j \qquad (5.4)$$

where *j* = 1, 2, …,*m*, and $\hat{d} = -1/2\hat{\beta}_1$ provides the estimated long memory parameter using the conventional truncation $m = T^{0.5}$, for the equity returns, presented in the final column of Table 5.1. The results highlight the prevalence of long memory in ASMs. This result may reflect the structural features of ASMs – notably small size and illiquidity. Nagayasu (2004) argues that stock markets in developing countries are likely to display long memory component because of the shallowness of their markets coupled with their less mature institutional and regulatory environment. In particular, The size of *d* ranges from 0.25 for Botswana to 0.54 in Zimbabwe. For the UK and US, *d* equals 0.51 and 0.56, respectively. Further, all these results are statistically different from zero. However, we observe that the ACF provides generally ambiguous evidence with respect to the existence of long memory compared to the GPH estimates. This may be because our choice of volatility proxy $|r_t|$ is a potentially 'noisy' proxy for the true volatility process (Poon and Granger, 2003; Poon, 2005).

5.4 Evidence of Structural Change in Volatility

In order to test for potential breaks within the conditional mean of the volatility process (proxied by absolute returns), we perform the 'breakpoint' tests of Bai and Perron (1998, 2003a,b).[12] This method allows us to identify shifts in volatility endogenously in contrast to methods where regime shifts are imposed on *a priori* grounds. The break tests involves regressing absolute returns (or more generally the variable of interest) on a constant and testing for breaks within that constant. In particular, this test aims to identify the number of breaks *m* (i.e., *m*+1 regimes) in a given time series. The procedure estimates the following equation:

$$x_t = \beta_j + \varepsilon_t ; \quad t = T_{j-1} + 1, \ldots, T_j \tag{5.5}$$

For j=1,…,m+1, where x_t is the variable of interest and β_j (j=1,…,m+1) is the mean level in the *jth* regime. The *m*-partition represents the breakpoints for the different regimes. Each partition is estimated by OLS with the estimate of β_j (j=1,…,m+1) generated by the minimisation of the sum of squared residuals. Further information on this procedure can be found in a series of papers, Bai and Perron (1998, 2003a, b). The null hypothesis of this test assumes that there is no break within the (equity) data. In contrast, the alternative hypothesis stipulates that there are a pre-specified number of breaks in the data. In addition, this method allows the user to specify a minimum distance between break points. In performing these tests, a maximum of up to five break points are specified (following McMillan and Ruiz, 2009). However, for the

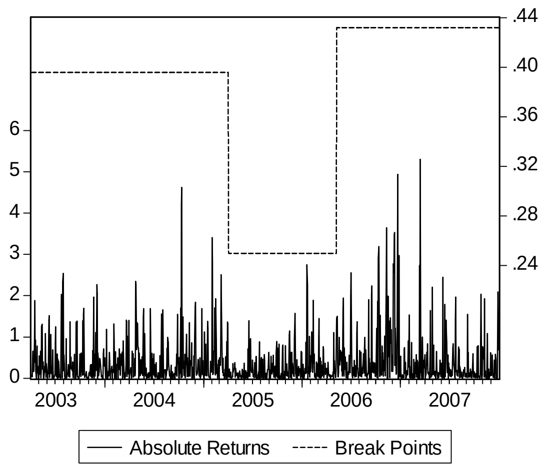
[12] The breakpoint estimators correspond to the global minimum sum of squared residuals. Further details of the Bai and Perron procedure can be found in a sequence of papers, Bai and Perron (1998, 2003a, b).

shorter samples, an even smaller number of breakpoints are specified, in order to avoid congestion of breakpoints. More precisely, breakpoints should occur infrequently, in a manner similar to structural changes, rather than a large number of changes, which would make the level shift an integrated process.

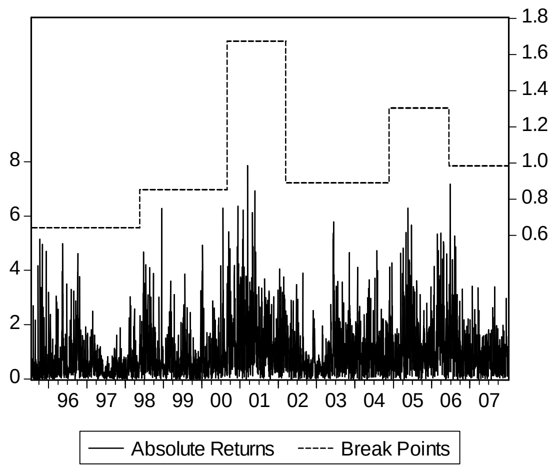
The results of the breakpoint tests are plotted in Figure 5.2, and indicate different patterns and levels of volatility in the ASMs. More specifically, these graphs show that the variance process is indeed time-varying and characterised by distinct regimes. The number of regime changes in ASMs are generally more numerous than those found in the UK and US reflecting a variety of country-specific developments.

Figure 5.2: Absolute Returns Mean Break Points[13]

Botswana



Egypt



[13] In Figure 5.2, the horizontal axis reflects the time period (expressed in years). The right-hand vertical axis measures the absolute return; while, the left-hand vertical captures the magnitude of the breakpoints as identified by the Bai-Perron multiple break point test.

Figure 5.2: Absolute Returns Mean Break Points (continued)

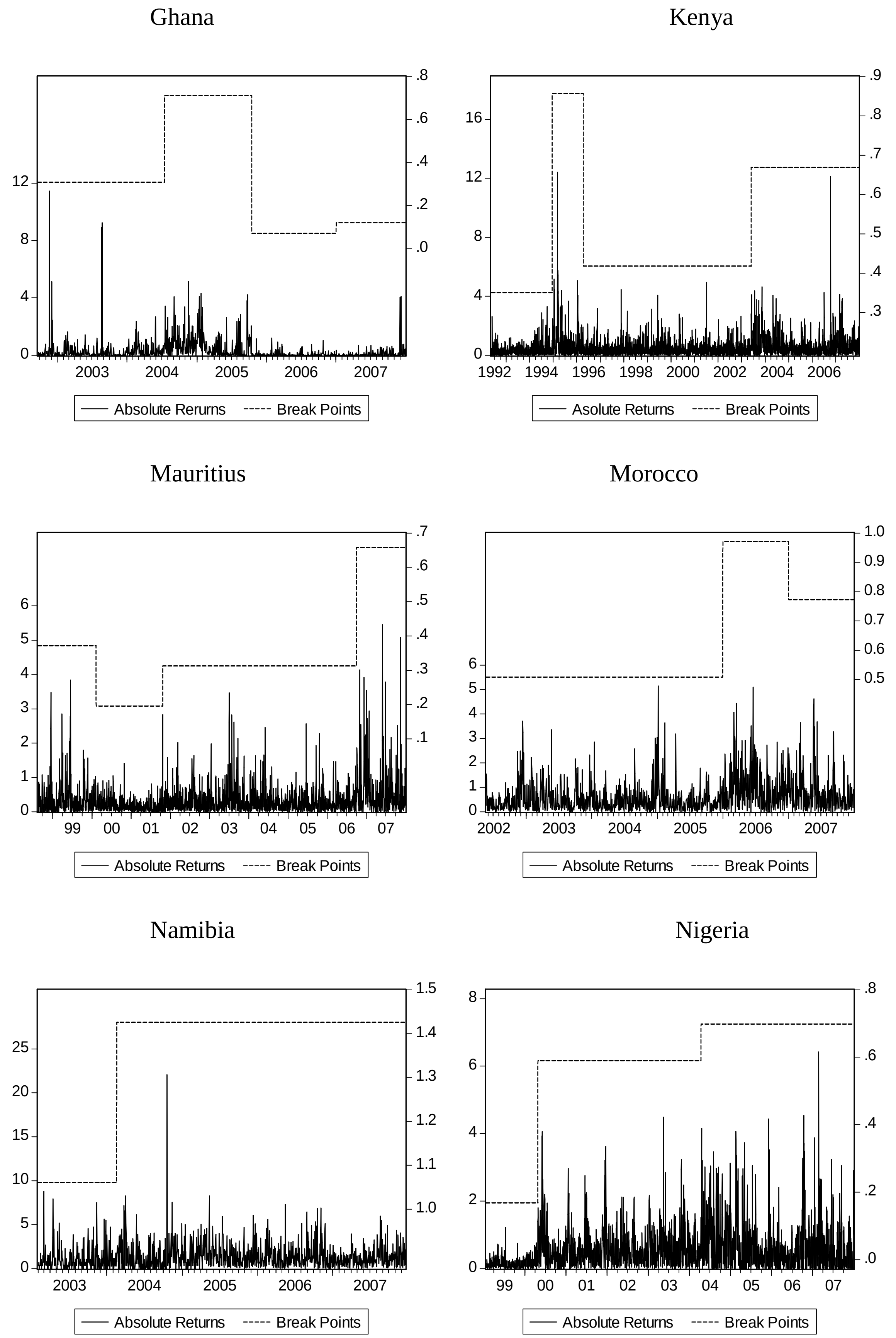
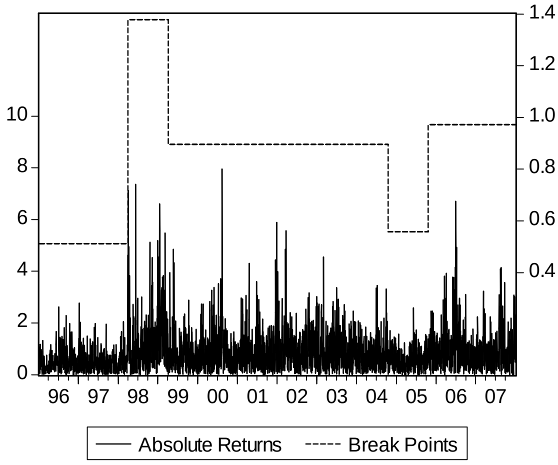
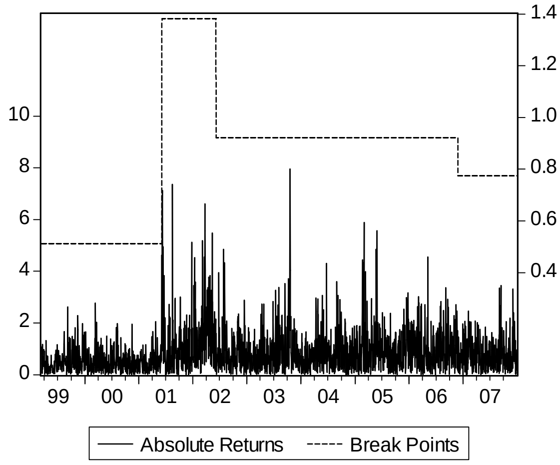


Figure 5.2: Absolute Returns Mean Break Points (continued)

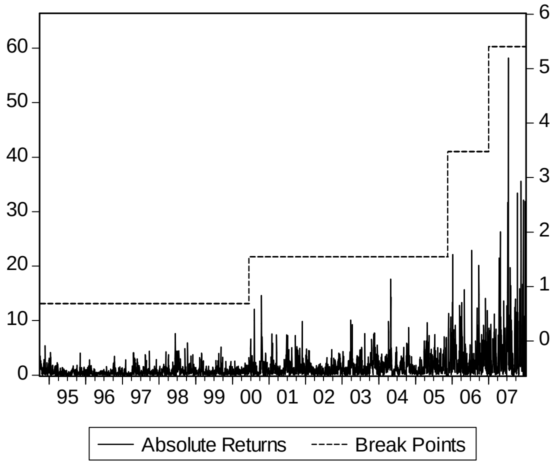
South Africa



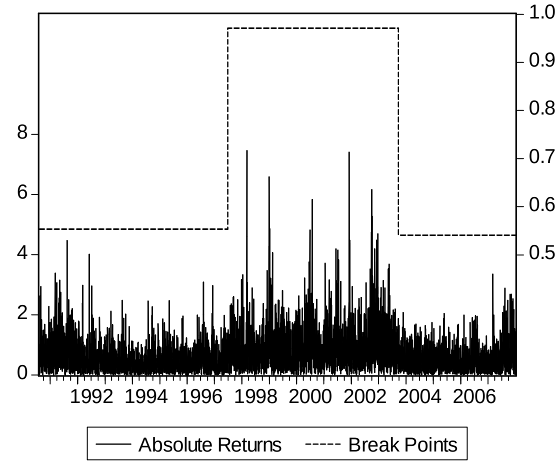
Tunisia



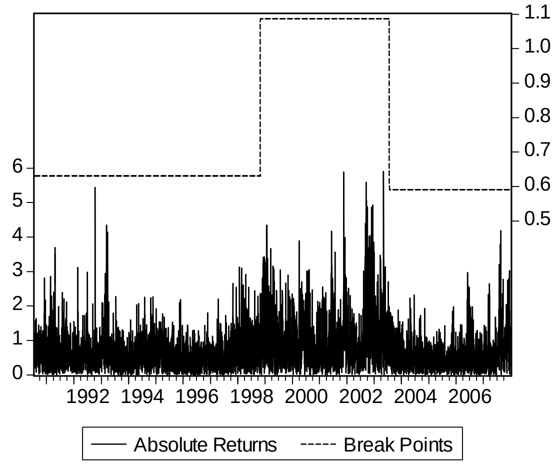
Zimbabwe



US



UK



For example, break point tests show that Namibia is characterised by one point of sudden change in volatility and therefore two volatility regimes; while, Egypt and South Africa both exhibit six well-defined episodes of variability in the mean level of their respective unconditional variance processes.

To illustrate this point further, Kenya displays four distinct regimes: the first is from the start of the sample until 1995; the second is from 1995 to 1996 where volatility spikes substantially; the third is from 1996 to 2003 where the level of unconditional volatility is considerably lowered; and finally, volatility rises to a higher regime in 2003 to the end of 2007. The first period encompassed the period of stock exchange modernisation; the second period, coincides with a relaxation of exchange control; the third period was characterised by the adoption of international accounting standards; and, the fourth coincided with economic and political uncertainty.[14]

South Africa has five distinct regimes. First, a period from the start of the sample period until early 1998, volatility was at its lowest. Second, a period, from early1998 to late 1999, the unconditional mean volatility rose significantly, following financial crisis in emerging markets. For example, during the financial crisis in Brazil and Russia in 1998, the JSE’s overall share index fell by 30 percent in the month of August 1998 alone. The third and fourth periods, from late 1998 until late 2004 and late 2004 to the start of 2006, respectively, are marked by significant declines in the level of unconditional mean volatility, associated with steady economic growth and

[14] For a more detailed overview visit www.nse.co.ke/newsite/pdf/factbook_07.pdf

various measures aimed at modernising the operation of the JSE. The last period is from early 2006 until end of the sample and is characterised by a rise in the level of volatility (though less than volatility during the emerging market financial crisis) due to uncertainties relating to economic growth given the backdrop of rising oil prices.

Zimbabwe has four distinct regimes. The first, is from the start of the sample until mid-2000. The second, is from mid-2000 to the end of 2005. The third, is from end of 2005 until the end of 2006. The final regime is from the end of 2006 until the end of the sample. These four regimes are characterised by a progressively rising level of the unconditional mean volatility in Zimbabwe. These regimes also correspond to the deteriorating economic and political environment in Zimbabwe. More precisely, the level of macroeconomic instability has progressively deteriorated over the sample period. For example, inflation in Zimbabwe is the highest in the world, at more than 66,000 percent at the end of the sample period. For the most part of the last 5 years, Zimbabwe's inflation roughly doubles once every three or four months.

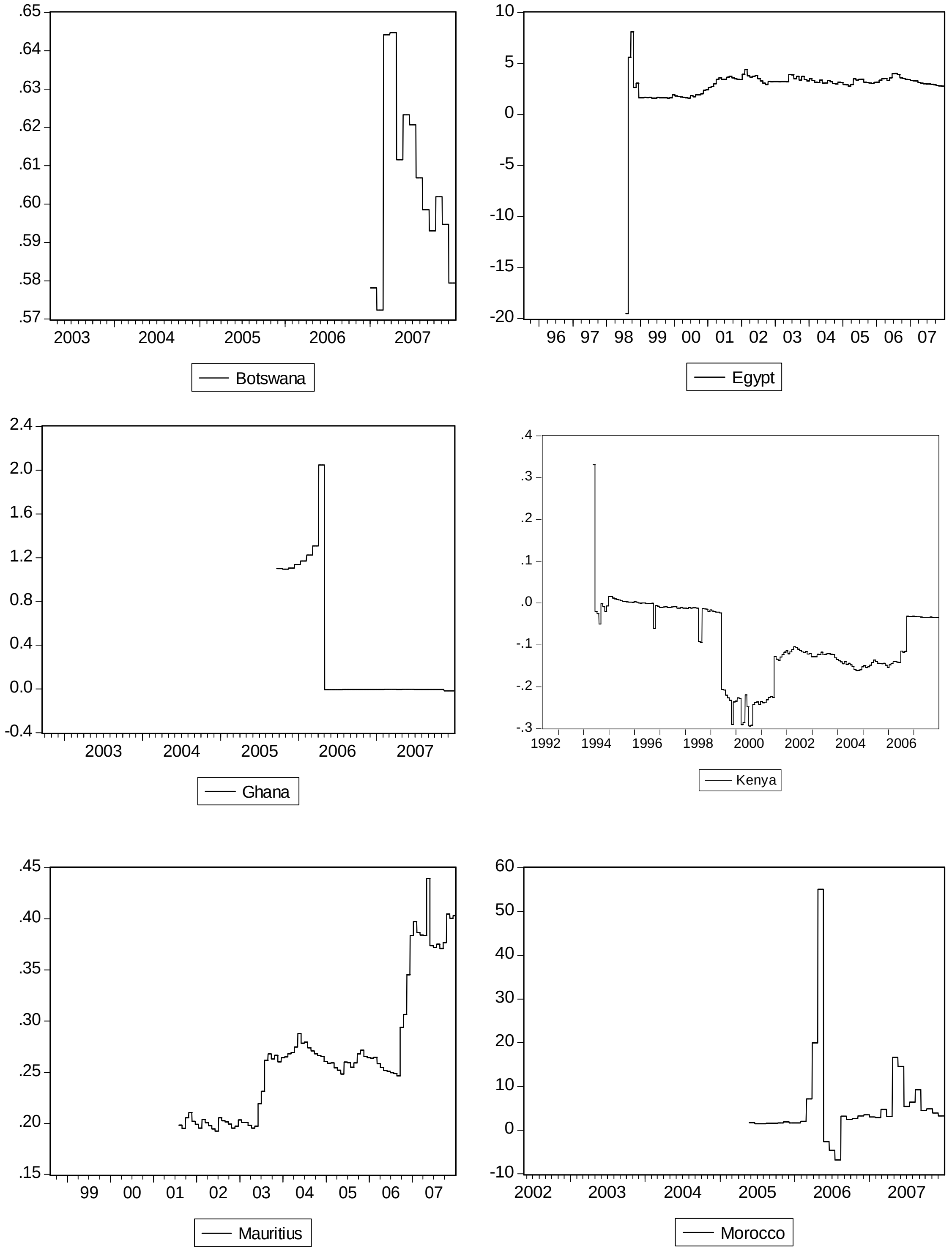
The preceding captured some of the key developments in the behaviour of the level of the unconditional mean of the variance process in some ASMs. These can be generalised to other ASMs. In particular, the level of the unconditional volatility in ASMs is marked by a number of distinct regimes. These shifts are different from country to country and underscore that these markets are mostly unconnected from each other; and, largely insulated from the major global stock markets. Also, as pointed out in Chapter 4, these markets are also largely small and illiquid and are characterised by nonactively traded stocks.

In comparison, the US and UK exhibit three volatility regimes these appear to be synchronised and similar in timing and duration. In particular, there appear to be fewer breaks and the regimes shifts appear to be synchronised. In particular, the behaviour of the unconditional mean volatility of the S&P 500 and the FTSE 500 are essentially identical. In these markets, there is an initial period from the start of the sample until (late) 1997; second, a period between (late) 1997 and (late) 2003 where the level of unconditional volatility rises sharply; finally, from (late) 2003 until the end of the sample where the mean of the volatility process falls substantially, but to a different level to regime one (i.e., volatility is only slightly more elevated in regime three than in regime one). The spikes in volatility in these markets coincide with periods marked my turbulence in international financial markets. In particular, the Asian financial crisis (which started in July 1997), the Russian 'Ruble' financial crisis (began in August 1998) and the Brazil financial crisis (January 1999). Also the technology stock bubble in early 2000 occurred in this period. In short, heightened volatility from 1997-2003 coincide with a period of global financial crisis. After 2003 volatility moderated significantly following the pick-up in economic growth and the lowering of GDP growth volatility.

While, the breakpoints in the US and UK correlate with major international financial events, those in the ASMs coincide with country-specific economic or political developments. This in turn may reflect that ASMs are not integrated into the wider international financial markets. This may be because these markets are illiquid which may deter entrants on both buy and sell sides. In addition, restrictions on foreign participation in some ASMs (e.g., Zimbabwe) may also explain the disconnection with the major international equity markets.

Breakpoint tests imply that the volatility processes are marked by discrete shifts in the level of fluctuations. However, Mikosch and Stărică (2004b) demonstrate that the unconditional volatility may exhibit a more gradual pattern of time-variation. In order to investigate this aspect and following a procedure used by McMillan and Ruiz (2009) which allows us to extract the time-variation in the absolute returns of each series. In particular, this procedure involves the derivation of the unconditional variance through recursive estimation of the GARCH (1, 1) model. The time-varying nature of the unconditional volatility is illustrated in Figure 5.3 below.

Figure 5.3: Evidence of Time-Varying Unconditional Variance[15]



15 Note: All graphs presented in Figure 5.3 are such that the horizontal axis captures the relevant time period (expressed in years) and the vertical axis measures the size of the time-varying unconditional variance.

Figure 5.3: Evidence of Time-Varying Unconditional Variance (continued)

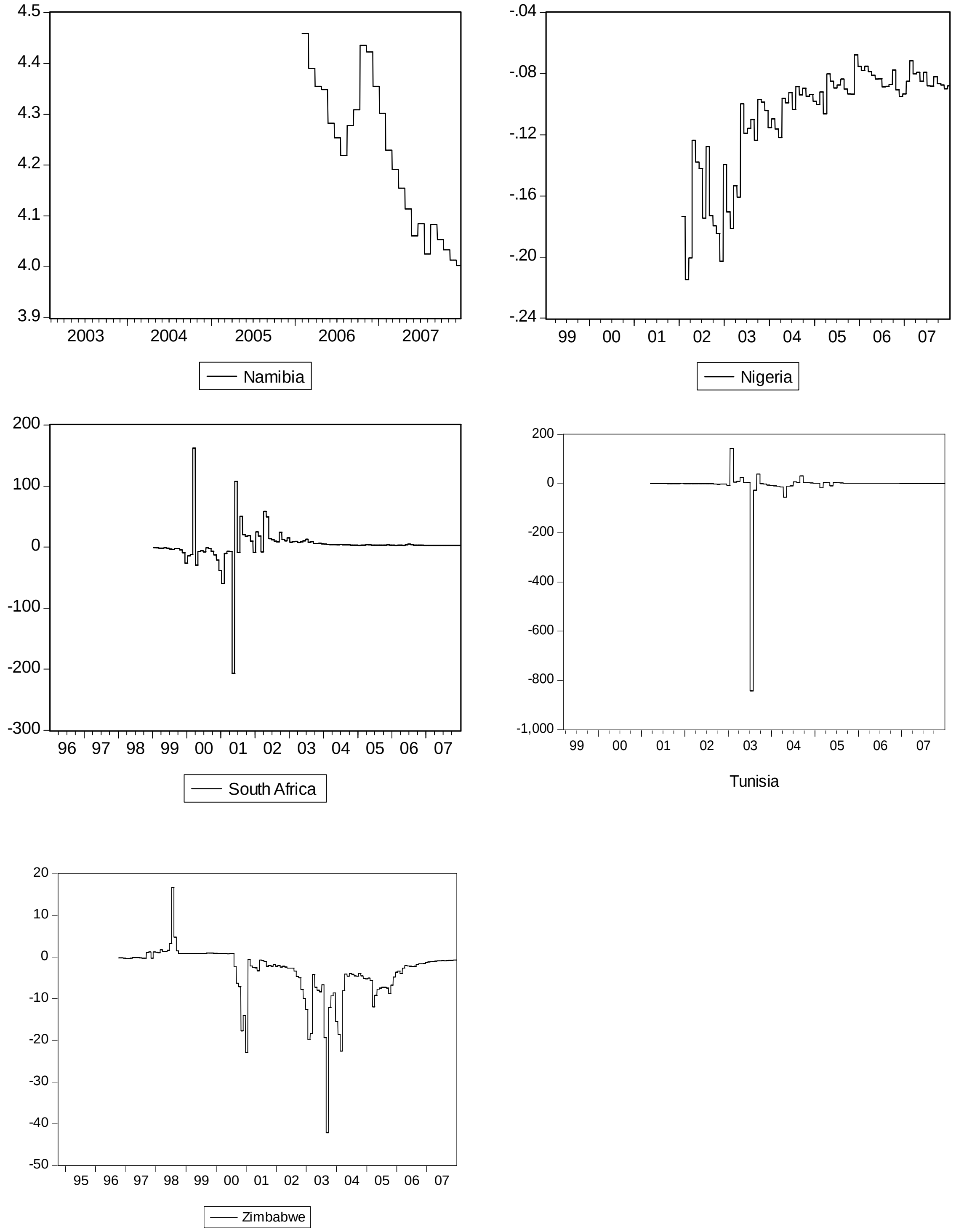
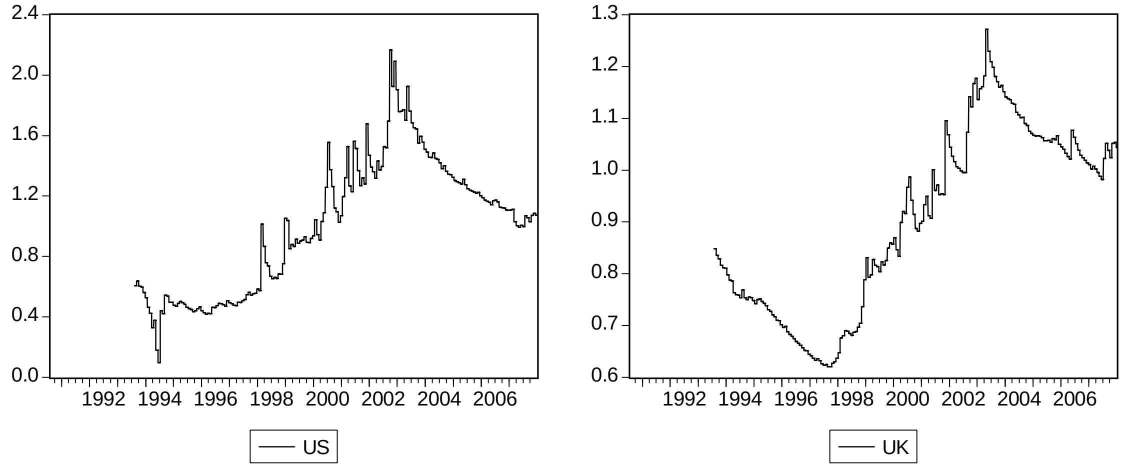


Figure 5.3: Evidence of Time-Varying Unconditional Variance (continued).



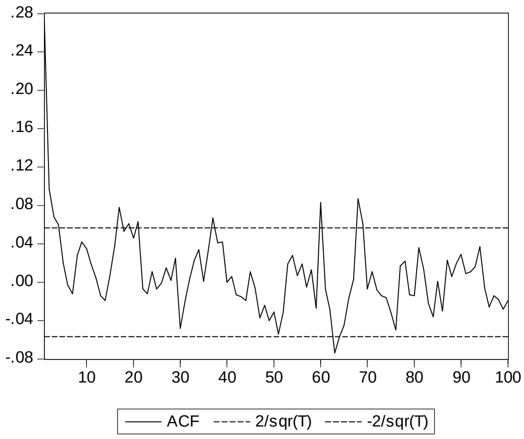
These results indicate that the unconditional variance process is not constant as the GARCH model assumes but rather exhibits wide fluctuations and in some cases very abrupt changes. Furthermore, the form and timing of time-variation of the ASMs volatility series differ considerably and appear to reflect country-specific developments. In addition, ASMs with the exception of South Africa, are isolated from the major global equity markets and are therefore driven more by domestic economic fundamentals (Smith *et al*, 2001; Irving, 2005).[16] In comparison, the broad thrust of time-variation in the unconditional variance observed in the US and UK appears synchronised perhaps reflecting that these markets are driven by common events (e.g., international financial crisis).

[16] However, for ASMs, this paper makes no attempt to absolutely identify the causes of regime shifts and instead focus on identifying the time periods of sudden changes themselves since information regarding the sources of these shifts cannot readily be imputed.

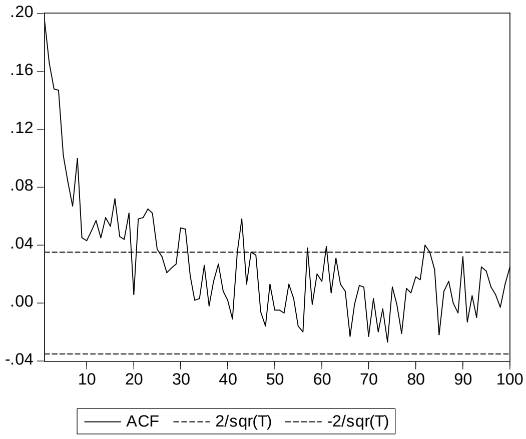
In order to examine the impact of removing the time-varying unconditional variance component on the long memory property of the equity volatility we examine the behaviour of the ACF. The derivation of this adjusted ACF is based on the analysis of the multiple structural break test of Bai and Perron (2003a) which is expressed in equation (5.5). The results from this procedure indicate the existence of sharp level shifts in the evolution of the volatility series (Figure 5.2). However, the unconditional variance may display a gradual evolution. Accordingly, we implement the rolling GARCH model presented by equation (5.6). This model is able to capture time-variation in the unconditional mean of the volatility series (shown Figure 5.3) by filtering any slow moving trend within the volatility series. Against this background, Figure 5.4 presents the ACF for 100 lags after accounting for structural breaks in the volatility data.

Figure 5.4: ACF for Adjusted Absolute Returns[17]

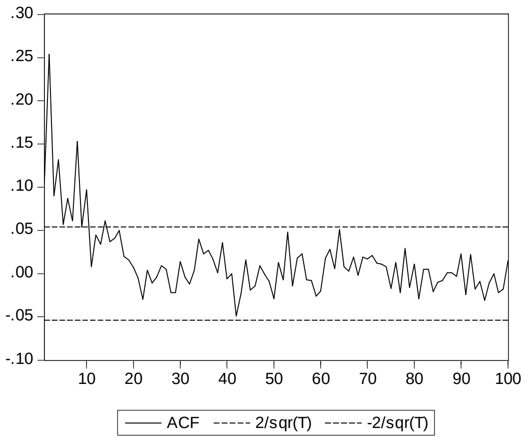
Botswana



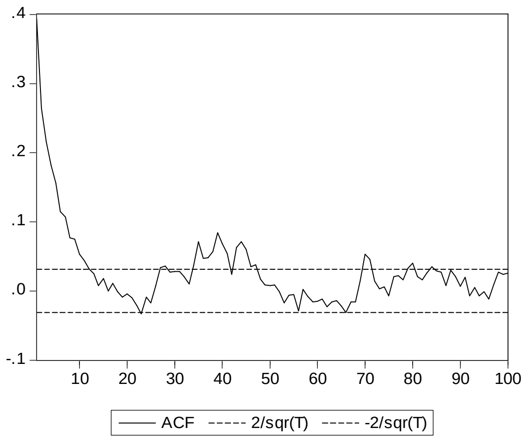
Egypt



Ghana



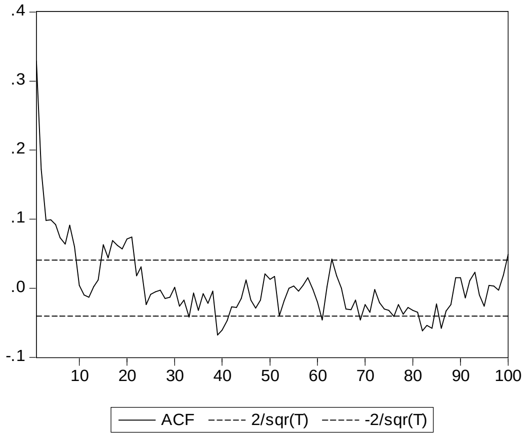
Kenya



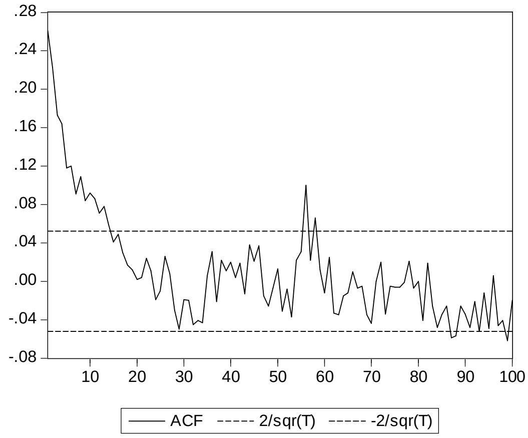
[17] Note: In all the graphs presented below the horizontal axis captures the lag length up to the 100th lag while the vertical axis captures the size of both the critical value of the test statistic and the ACF for adjusted absolute returns.

Figure 5.4: ACF for Adjusted Absolute Returns (continued)

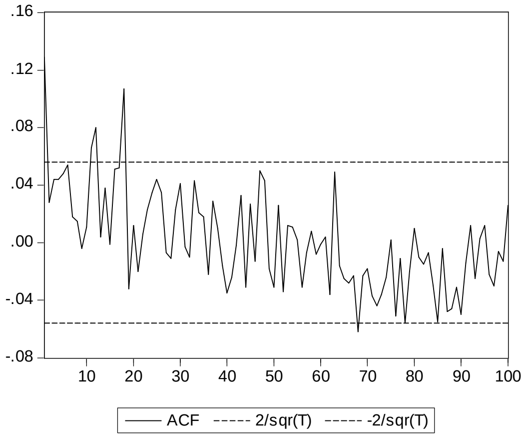
Mauritius



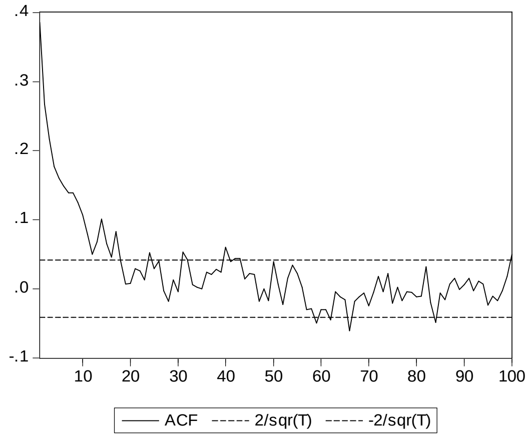
Morocco



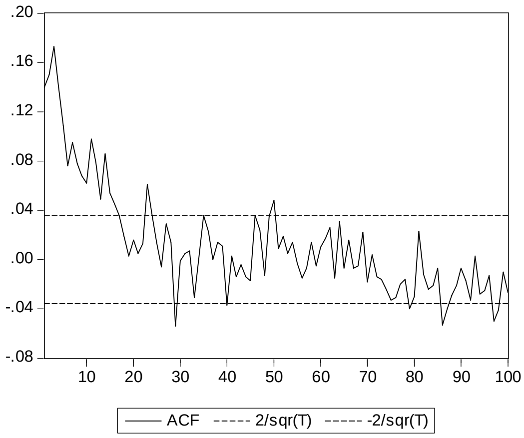
Namibia



Nigeria



South Africa



Tunisia

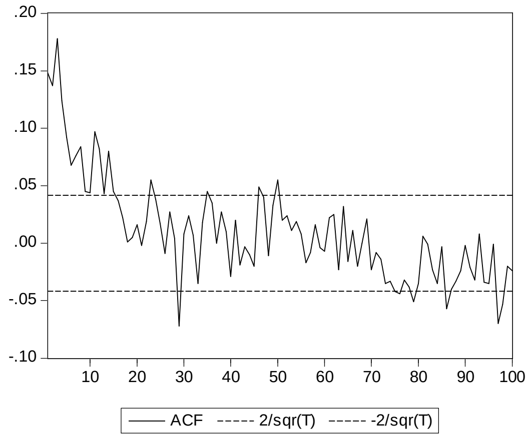
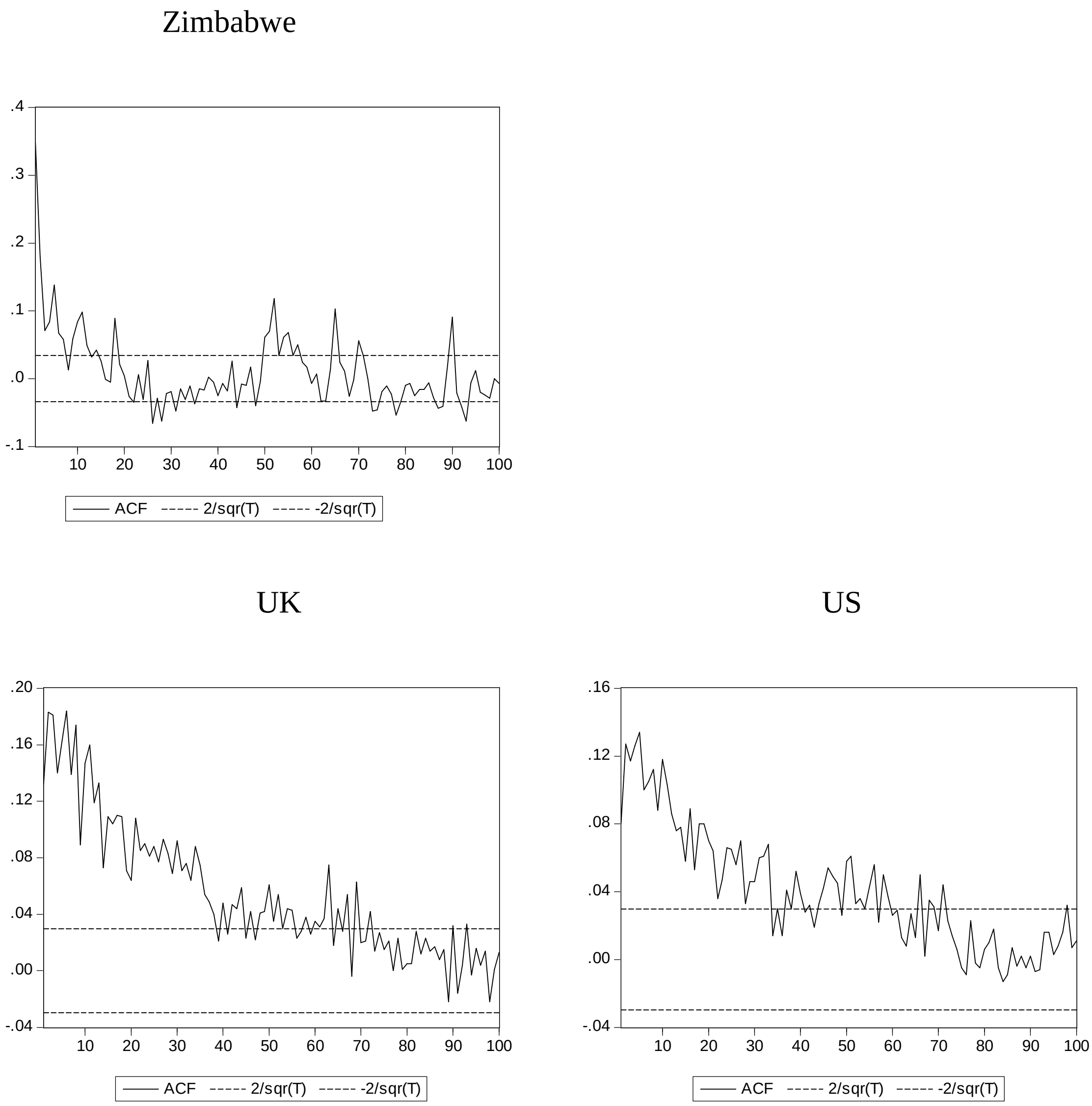


Figure 5.4: ACF for Adjusted Absolute Returns (continued)



In particular, from these graphs it is apparent that after adjusting for time-variation in the unconditional volatility process then the ACF decays quickly and erratically. For example, Egypt's ACF decays quickly and becomes insignificant in at around lag 20 and crosses zero at lag 35. Similarly, Morocco's ACF decays quickly and fluctuates in a choppy manner below zero. In sum, these results show that after accounting for time-variation in the unconditional mean variance the ACF decays quickly and for the most part suggest that the long-memory component in the data is diminished. For some markets like Botswana and Tunisia, the ACF remains erratic (before and after accounting for structural change in the data) and does not offer any support in favour of a hyperbolic decay structure.

To further evaluate the extent to which structural change has a bearing on the degree of long memory we re-estimate the FIGARCH model and the findings are presented in (the final column of) Table 5.2. The results show that the extent of long memory in volatility is reduced for all the markets considered in this study. For example, the long memory parameter for Kenya and Mauritius decline to 0.24 and 0.08 from 0.40 and 0.35, respectively. These results suggest that the long memory parameter is overstated if structural breaks are not accounted for in the FIGARCH model.

Table 5.2. Adjusted GARCH and Fractional Integration Estimates.						
	Adjusted GARCH(1,1)					GPH
	ω	α	β	α+β	ℓ	*d*
Botswana	0.083 (6.40)	0.403 (12.81)	0.359 (14.83)	0.762 (0.00)	2.55	0.22 (0.69)
Egypt	0.027 (9.45)	0.073 (14.20)	0.903 (149.23)	0.976 (0.00)	28.28	0.13 (1.27)
Ghana	0.021 (10.15)	0.522 (70.71)	0.442 (87.51)	0.965 (0.00)	19.29	0.17 (0.85)
Kenya	0.061 (18.31)	0.315 (39.04)	0.637 (71.39)	0.953 (0.00)	14.36	0.24 (1.80)
Mauritius	0.034 (4.41)	0.469 (25.33)	0.323 (15.04)	0.792 (0.00)	2.97	0.08 (1.92)
Morocco	0.063 (3.64)	0.182 (12.73)	0.778 (47.88)	0.960 (0.00)	17.12	0.08 (1.50)
Namibia	0.017 (1.68)	0.088 (3.25)	0.752 (1.88)	0.840 (0.00)	3.96	0.24 (2.14)
Nigeria	0.003 (5.10)	0.177 (26.37)	0.851 (3.97)	1.028 (0.00)	N/A	0.21 (1.95)
South Africa	0.107 (5.07)	0.006 (8.64)	0.928 (62.19)	0.935 (0.00)	10.26	0.04 (1.97)
Tunisia	0.022 (0.052)	0.138 (1.33)	1.022 (31.30)	1.160 (0.01)	N/A	0.09 (2.09)
Zimbabwe	0.015 (10.61)	0.326 (36.49)	0.751 (183.92)	1.078 (0.00)	N/A	0.21 (1.84)
UK	0.004 (8.33)	0.080 (12.05)	0.902 (106.39)	0.982 (0.00)	37.41	0.33 (1.97)
US	0.007 (5.15)	0.056 (13.25)	0.937 (197.59)	0.994 (0.00)	108.84	0.25 (1.93)

Notes: Equation specification and discussion in Section 2. Numbers in parentheses are *t*-statistics, except under column α+β where entries are *p*-values from a Wald test that α+β=1. Half-lives, ℓ are calculated as log(0.5)/log(α+β). For, Nigeria, Tunisia and Zimbabwe, the half-life cannot be interpreted (since the half-life approaches infinity as α+β →1) and are denoted N/A to indicate that the method used is not applicable.

While the results presented so far highlight the bias introduced by structural breaks, Perron and Qu (2006) argue that a short-memory process with breaks will bias upward the persistence estimate of a short-memory process; while, the persistence of a long memory process is biased downward after filtering structural break tests. Since these two processes may be confused Perron and Qu propose a test which distinguishes a long memory process from a short-memory process with breaks.[18] This test stipulates that a genuine long memory process is such that the estimates of the fractional integration parameter *d* should be invariant to the choice of the bandwidth *m*, while the test statistic should not be statistically different from zero. This test is performed and the results are presented in Table 5.3. The results across all markets show an inverse relationship between *d* and *m*. These test results suggest that volatility in these markets is not intrinsically long memory but rather short-memory processes subject to level shift in the unconditional variance.

[18] The test equation is given by: $t_d(a,b) = \sqrt{\frac{24(T^a)}{\pi^2}} \times (\hat{d}_a - \hat{d}_b)$, where 0<a<b<1 (a=1/3; b=4/5) and $\hat{d}$ is the long memory parameter. For more details see Perron and Qu (2006).

Table 5.3. Fractional *d* Estimates for Across Different *m*					
	Truncation Value, *m*				
	$T^{1/3}$	$T^{1/2}$	$T^{2/3}$	$T^{4/5}$	Perron-Qu
Botswana	0.23	0.25	0.15	0.17	6.43
Egypt	0.52	0.44	0.29	0.28	8.80
Ghana	0.60	0.46	0.36	0.32	11.62
Kenya	0.34	0.40	0.25	0.31	4.99
Mauritius	0.58	0.35	0.25	0.25	21.59
Morocco	0.44	0.44	0.35	0.30	8.81
Namibia	0.29	0.27	0.15	0.10	4.02
Nigeria	0.71	0.32	0.31	0.30	16.63
South Africa	0.47	0.40	0.37	0.31	9.05
Tunisia	0.45	0.38	0.41	0.30	8.24
Zimbabwe	0.80	0.54	0.40	0.29	4.80
UK	0.53	0.51	0.52	0.27	14.27
US	0.61	0.56	0.43	0.25	16.80
Notes: Entries are estimates of the fractional integration parameter over different truncation values, see equation (5.4). The final column is the Perron-Qu test (2006), which is normally distributed.					

5.4.1 Time-Varying Mean Adjusted GARCH Model

The empirical results of the previous section show that the standard GARCH assumption of a constant unconditional variance is not tenable. Accordingly, this assumption is relaxed, in order to account for time-variation in the mean process of the unconditional variance. In particular, we follow Mikosch and Stărică (2004b) and McMillan and Ruiz (2009) and estimate a rolling GARCH model to capture time-variation in the unconditional mean variance. In particular, the rolling GARCH is a modification of the GARCH model given in equation (5.2) that allows the unconditional mean to vary.

This model is presented below:

$$h_t = \omega\mu^{-1}\sum_{w=1}^{260}\left|r_{t-1-w}\right| + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \qquad (5.6)$$

where μ is the window length of the moving average.[19] The results of this model are presented in Table 5.2. The results generally point to a lower degree of volatility persistence in the rolling GARCH model in relation to the standard GARCH model. For example, the level of volatility persistence in Egypt and Kenya decrease to 0.976 and 0.953 from 0.980 and 0.991, respectively. This is further illustrated by the half-life decay periods which are generally lower. For instance, before allowing for structural breaks the half-lives, in ASMs ranged from just over 3 days (in Botswana) to 74 days (in Kenya); while the UK and US registered 43 and 122 days, respectively. In contrast, after incorporating breaks into the model, the half-lives, ranged from just under 3 days (in Botswana) to 28 days (in Egypt). Similarly, half-lives were reduced to 37 and 109 days in the UK and US, respectively. Further, where volatility persistence was explosive as in the case of Nigeria and Zimbabwe; the use of the rolling GARCH reduced (but did not change) the fact that volatility persistence in these markets was still explosive. However, there are two exceptions to this trend which are represented by the cases of Morocco and Tunisia where the extent of volatility persistence rises. This means that a moving average application may not be the appropriate method to address the phenomena of structural change in these markets.

[19] A window length of 260 corresponds with a 5-day trading year and also seems reasonable since investment managers commonly rebalance their portfolios on an annual basis (e.g., McMillan and Ruiz, 2009).

Finally, it is important to assess if the inclusion of a time-varying mean component to the GARCH model yields superior forecast performance, relative to the GARCH and FIGARCH models. This is relevant since it would confer a benefit to investors in applications where volatility forecasts are necessary.

5.5. Forecast Performance

To gauge forecasting power we simply start by splitting our respective data sets in half and we estimate each model for all series from the sample spanning the first half; then, we use those estimates to forecast volatility over the second part of the sample. This strategy is motivated by the simple fact that our respective samples are of differing time span; hence, by splitting our data sets in half allows for consistency in terms of enhancing comparability of results. Against this background, forecasting power is then evaluated using the following regression:

$$r_t^2 = \alpha + \beta h_t^{2f} + \varepsilon_t \qquad (5.7)$$

where r_t^2 denotes squared daily returns and we use this measure as a proxy for the 'actual' volatility observed in the various markets examined. In addition, h_t^{2f} represents the volatility forecast obtained from estimating the first half of the sample. We then use the coefficient of determination, R^2, after estimating equation (5.6) above, as a criteria against which to evaluate the forecast performance of the three models examined. The results of this exercise are presented in Tables 5.4 and 5.5. At the daily level, the results indicate that the standard GARCH model generally delivers the best forecasting results for most ASMs, with the exception of Egypt, Namibia, Nigeria and Tunisia. For these markets the rolling GARCH is superior. For Morocco and the US the results of the GARCH and rolling GARCH are almost the same. In contrast the FIGARCH model delivers the weakest results. However, this is not entirely surprising because this model explicitly captures long term volatility.

Table 5.4: Forecast R^2

Daily			
	GARCH	Rolling GARCH	FIGARCH
Botswana	0.068	0.044	0.056
Egypt	0.133	0.147	0.125
Ghana	0.075	0.070	0.058
Kenya	0.191	0.114	0.097
Mauritius	0.063	0.028	0.011
Morocco	0.088	0.069	0.051
Namibia	0.037	0.053	0.007
Nigeria	0.211	0.246	0.177
South Africa	0.129	0.115	0.081
Tunisia	0.044	0.127	0.109
Zimbabwe	0.242	0.168	0.176
UK	0.141	0.144	0.119
US	0.072	0.073	0.066

Notes: R^2 values are from Equation (5.7)

Since volatility forecasts are additive it is straightforward to derive the monthly forecasts. Over longer horizons the results are mixed. The standard GARCH generally delivers superior forecasting performance for ASMs. The rolling GARCH model produces better results for Tunisia and Namibia. Meanwhile, in the case of Zimbabwe, South Africa, Nigeria and Egypt the long memory model performs best. Meanwhile, in the US and UK the standard GARCH and rolling GARCH produce almost similar results at the daily level. At the monthly frequency the FIGARCH models deliver the best performance forecast for the UK; while, the rolling GARCH performs best for the results for the US.

Table 5.5: Forecast R^2			
Monthly			
	GARCH	Rolling GARCH	FIGARCH
Botswana	0.156	0.127	0.122
Egypt	0.195	0.217	0.246
Ghana	0.240	0.156	0.212
Kenya	0.343	0.294	0.227
Mauritius	0.195	0.140	0.113
Morocco	0.253	0.146	0.177
Namibia	0.096	0.163	0.145
Nigeria	0.275	0.260	0.354
South Africa	0.319	0.374	0.417
Tunisia	0.188	0.213	0.164
Zimbabwe	0.309	0.347	0.452
UK	0.285	0.288	0.316
US	0.207	0.213	0.204
Notes: R^2 values are from Equation (5.7)			

In order to further evaluate the forecasting performance of the GARCH, modified GARCH and FIGARCH models, forecasting encompassing tests are performed. These tests are used to determine if a competing forecast carries additional information over a base model forecast (Chong and Hendry, 1986). If the former contains no useful marginal information then the latter model is said to encompass it.

To formally test for forecasting encompassing this study considers two regression models:

$$r_t^2 = \alpha + \beta_1 h_{1,t}^f + \beta_2 h_{2,t}^f + \varepsilon_t \tag{5.8}$$

$$\xi_{1,t} = \beta_2 \left(\xi_{1,t} - \xi_{2,t} \right) + \varepsilon_t \tag{5.9}$$

Equation (5.8) is an extension of equation (5.7) and equation (5.9) was proposed by Ericsson (1992) to further ascertain forecast performance by comparing data of a specified model with the data of alternative models. The subscripts 1, 2 denote the forecast models (1 refers to the GARCH/FIGARCH and 2 refers to the rolling GARCH) and ξ denotes the forecast error $\left(r_t^2 - h_t^f\right)$. In both equations the null hypothesis is that Model 1 (GARCH/FIGARCH) encompasses Model 2 (the rolling GARCH model) in which case β_2 is equal to zero. In contrast, if β_2 is greater than zero then Model 2 embodies additional information over Model 1 which implies that Model 2 is not encompassed by Model 1.[20] The estimated results from these models are shown in Table 5.6 and 5.7, respectively, where the parameter and *t*-values for equation (5.8) are presented along with the *p*-value associated with β_2 from equation (5.9).

[20] Clements and Harvey (2006) observe that in equation (5.8) both β parameters are unrestricted while in equation (5.9) it is implicitly assumed that that the β parameters sum to one. In other words, equation (5.8) can be rewritten as $r_t^2 = \alpha + \left(1 - \beta_2\right)h_{1,t}^{2f} + \beta_2 h_{2,t}^{2f} + \varepsilon_t$ from which equation (5.9) can then be derived. Finally, the authors show that where the forecasts are correlated equation (5.8) is preferred.

Table 5.6. Forecast Encompassing Test Results - GARCH				
	Daily		Monthly	
	GARCH	Rolling GARCH	GARCH	Rolling GARCH
Botswana	12.20 (0.86)	-1.55 (-3.93) [0.60]	-1.52 (-3.81)	2.11 (4.07) [0.00]
Egypt	2.26 (5.95)	0.017 (0.36) [0.042]	3.02 (4.73)	-2.09 (-2.56) [0.00]
Ghana	0.48 (1.32)	-0.143 (-0.255) [3.606]	-4.16 (-5.07)	4.23 (5.82) [0.00]
Kenya	-1.72 (-4.60)	0.39 (1.07) [0.01]	-1.88 (-0.91)	0.79 (0.56) [0.00]
Mauritius	5.67 (3.06)	0.06 (0.19) [0.008]	0.13 (0.27)	1.94 (2.05) [0.32]
Morocco	0.56 (0.11)	3.05 (1.28) [0.00]	-0.40 (-0.42)	0.55 (0.65) [0.41]
Namibia	0.966 (0.03)	-7.90 (-3.13) [1.12]	0.56 (0.60)	0.79 (0.88) [0.37]
Nigeria	3.37 (0.36)	0.86 (1.09) [0.031]	0.82 (0.87)	0.93 (1.09) [0.16]
South Africa	0.024 (0.11)	0.546 (0.317) [0.085]	-0.56 (-0.78)	0. 95 (1.08) [0.02]
Tunisia	-0.135 (-0.27)	3.08 (4.66) [0.20]	-1.15 (-1.72)	1.79 (2.59) [0.01]
Zimbabwe	15.73 (4.06)	-4.60 (-5.267) [0.546]	0.06 (0.21)	0.33 (0.50) [0.00]
UK	0.112 (0.47)	1.157 (3.81) [0.06]	3.83 (4.07)	-2.09 (-2.52) [0.00]
US	1.12 (0.24)	0.66 (1.53) [0.02]	0.92 (1.03)	2.52 (2.82) [0.00]

Notes: Entries are estimated coefficients from equation (5.8), with *t*-values in parentheses and *p*-values from equation (5.9) in brackets.

The results from this analysis suggests that at the daily level the GARCH model does indeed encompass the rolling GARCH model for most ASMs with the exception of Egypt, Namibia, Nigeria and Tunisia. In other words, apart from these four markets there is no additional information contained within the rolling GARCH model over the standard GARCH model. This finding is mostly consistent with the results from Table 5 where the R^2 from the rolling GARCH (for these Egypt, Namibia, Nigeria and Tunisia) are higher than those from the GARCH model. Meanwhile, for the UK and US, the results indicate that the rolling GARCH encompasses the GARCH model.

At the monthly level, the results are more mixed. For example, test results from Botswana and Ghana indicate that the encompassing tests are rejected. In contrast, the encompassing test results are not rejected for Egypt and the UK. However, for the remaining markets the results are less conclusive. For instance, the encompassing test in equation (5.8) is rejected for Kenya and marginally so for both South Africa and Tunisia. On the other hand, the encompassing test represented by equation (5.9) is rejected for Zimbabwe along Kenya, South Africa and Zimbabwe.

Table 5.7. Forecast Encompassing Test Results - FIGARCH

	Daily		Monthly	
	FIGARCH	Rolling GARCH	FIGARCH	Rolling GARCH
Botswana	-1.46 (2.36)	1.83 (3.39) [0.02]	2.00 (2.29)	0.09 (0.56) [0.32]
Egypt	-0.64 (2.33)	1.60 (3.36) [0.00]	-0.08 (0.12)	1.16 (2.05) [0.00]
Ghana	-1.73 (1.98)	2.01e-04 (7.36e-03) [0.00]	1.22 (0.62)	0.03 (1.56e-03) [0.23]
Kenya	2.34 (3.22)	-0.66 (-1.90) [0.35]	1.36e-05 (0.44)	-0.308 (-0.06) [0.75]
Mauritius	0.78 (1.34)	0.187 (0.392) [0.02]	2.01 (2.57)	-1.17 (-2.40) [0.54]
Morocco	0.17 (0.89)	-2.19 (-2.77) [0.30]	6.58e-03 (0.125)	0.29 (0.06) [0.11]
Namibia	1.26 (2.18)	-5.61 (-7.74) [0.19]	3.28 (5.63)	5.32 (2.53) [0.01]
Nigeria	-1.08 (-1.59)	2.66 (4.20) [0.08]	3.09 (4.17)	-2.44 (-3.90) [0.03]
South Africa	-1.29 (-1.42)	0.93 (3.50) [0.04]	0.68 (0.36)	0.51 (0.18) [0.82]
Tunisia	-0.28 (0.45)	1.23 (3.92) [0.27]	1.02 (1.41)	-0.07 (-0.03) [0.06]
Zimbabwe	2.42 (3.19)	-0.02 (-2.19) [0.05]	4.60 (7.12)	0.01 (5.71e-05) [0.43]
UK	-0.39 (-1.58)	0.71 (1.77) [0.00]	2.23e-04 (0.08)	0.14 (0.19) [0.77]
US	-0.53 (-1.11)	1.07 (2.03) [0.00]	1.41 (1.03)	1.05 (0.83) [0.00]

Notes: Entries are estimated coefficients from equation (5.8), with *t*-values in parentheses and *p*-values from equation (5.9) in brackets.

With respect to the FIGARCH encompassing results, the findings indicate that at the daily frequency the FIGARCH encompasses rolling GARCH model for Kenya, Morocco, Namibia and Zimbabwe. In comparison, at the monthly frequency the results indicate that the hypothesis that the FIGARCH model encompasses the rolling GARCH is not rejected for Mauritius and Nigeria. Meanwhile, for all other markets the evidence is more mixed across the two tests exemplified by equations (5.8) and (5.9). In particular, additional information, is contained within the rolling GARCH forecasts for Egypt, Namibia, Nigeria, Tunisia and the US.

In total, the encompassing tests provide some support to the relevance of a rolling GARCH model to some ASMs at both the daily and monthly frequency; however, this support is narrow in the sense that it applies to a small selection of ASMs and it is revealed to be sensitive to the specification of the time frequency (i.e., whether a daily or monthly frequency is specified).

5.6 Conclusion

Since analysis of the impact of regime changes on measures of volatility persistence and long memory are relatively sparse in the context of ASMs this essay extends the empirical work to examine the impact of structural breaks on volatility behaviour. In particular, this essay models stock-return volatility using an estimation technique for the variance of returns that accounts for the regime shifts evident in the data. The aim of this paper was to examine the behaviour of stock return volatility in ASMs, especially since volatility is an important driver of active investment returns and risk premia in the market (Loeys and Panigirtzoglou, 2005). Furthermore, accurate volatility estimates have become a basic input for shaping hedging strategies and managing risks. As such, these findings may have potential value for market participants in portfolio management.

Against this background, this empirical analysis found that both persistence and long-memory estimates are biased higher when structural breaks are not accounted for in standard GARCH and FIGARCH models. In particular, and with respect to the GARCH model, the results indicate time-variation in the unconditional mean of the volatility series, which in turn, is shown to bias upward the finding of volatility persistence. Moreover, these results are consistent with the notion that misspecification of the GARCH model due to ignored structural breaks in the unconditional variance can lead to an overstatement of the extent of volatility persistence in equity data (Mikosch and Stărică, 2004b; McMillan and Ruiz, 2009). Furthermore, the analysis supports the notion that long memory in volatility is reduced by incorporating regime shifts in the model. More generally, failure to account for these breaks can lead to incorrect inference. With respect to forecast performance this study finds that accounting for time-variation in the unconditional volatility provides useful information which improves forecasting performance in some cases. However, the standard GARCH model is generally found to deliver superior forecasting performance at the both the daily and monthly level on the basis of the R^2 from a Mincer-Zarnowitz regression. Similarly, encompassing tests offer some support to the rolling GARCH model. However, evidence from these tests is mostly mixed and consistent results apply to only a small selection of ASMs. In total therefore, the forecasting results appear to generally support the superiority of the standard GARCH in relation to the other alternatives considered in generating accurate forecasts.

Finally, there are several implications of our study. While we implemented a rolling GARCH model to account for time-variation in the unconditional variance, future research may wish to explore more subtle ways to capture breaks in the unconditional mean process, especially since in some markets (e.g., Morocco and Tunisia) the implementation of a rolling GARCH model actually resulted in an increase in the level of volatility persistence. Another possibility would be to use nonlinear models to allow for an asymmetric reaction of volatility to good and bad innovations (e.g., Bollerslev and Mikkelsen, 1996). This is especially relevant since many of the break points identified in ASMs coincide with shifts in policies or changes in the regulatory environment, which in turn may affect information flows in these markets and hence the behaviour of investors and policymakers. In addition, the capture of asymmetric movements in equity data may enhance the quality of volatility forecasts.

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6. Forecasting African Stock Market Volatility

6.1 Introduction

Episodes of volatility in emerging equity markets have been the subject of much research and policy attention. This focus reflects a number of considerations. First, volatility is a key for measuring risk, pricing derivatives and shaping portfolio management strategies (e.g., Hull and White, 1987; Chesney and Scott, 1989).[1] In addition, volatility provides investors with information on market sentiment and the changing attitude toward risk from bearish to bullish and vice-versa. Second, (higher) volatility in equity markets raises important public policy issues about the stability of financial markets and the pace of economic growth. For example, at the macroeconomic level, equity market volatility may affect economic performance through changes in consumer and business spending. In particular, economic theory suggests that fluctuations in stock prices may have an impact on private consumption through a wealth effect (Case *et al*, 2001; Davis and Palumbo, 2001). Indeed, Garner (1990) finds that the stock market crash of 1987 reduced consumer spending in the US, a conclusion consistent with the suggestions of economic theory that fluctuations in stock prices (or asset prices more generally) have an impact on private consumption through a wealth effect. In addition, Funke (2002) examines a sample of sixteen emerging markets and finds evidence of a small but statistically significant relationship between stock price volatility and private consumer spending in these

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[1] A standard formula for pricing derivatives is the Black-Scholes formula (1973). For example, the price of a European call option is given by
$C(S,K,T,t,r,\sigma)=S.N.(d_1)-K.N(d_2)$, where S is the current price of the underlying asset, K is the strike price, T is the maturity time, t is the current time, r is the risk free return, and σ is the volatility of S. $N(.)$ denotes the cumulative normal distribution:
$d_1=\log(S)-\log(K)+\left(r+0.5\sigma^2\right)(T-t)/\sigma\sqrt{T-t}$ and $d_2=d_1-\sigma\sqrt{T-t}$. In this model, all the variables, except σ, are known. Therefore, pricing an option is equivalent to valuing volatility, which implies estimating σ. Indeed, traders quote options' prices in terms of volatilities (Hull, 2002).

economies. Third, in its surveillance role, the International Monetary Fund (IMF) monitors volatilities of asset prices and related derivatives' prices in both mature and emerging markets with a view to promoting macroeconomic policies conducive to financial stability (Krichene, 2003)[2]. Fourth, the increased interdependence between financial markets leads to a more rapid and larger transmission of national financial disturbances – through contagion effects – to other markets. For example, during the financial crises in Russia and Brazil in 1998, the South Africa's JSE all-share index fell by 30 percent in August 1998. In addition, South Africa is the dominant economy is southern Africa. As a consequence of this position developments on the JSE may affect regional equity markets, especially, since many South African companies own or control numerous listings on regional equity markets. In total, in order, to assess future returns from both active and passive risk taking (the alpha and the beta) or the need for policy intervention, it is important to forecast volatility (Loeys and Panigirtzoglou, 2005).

Most of the research to date on the out-of-sample forecasting performance of various models applied to stock return volatility has been concentrated on the major international financial markets, comparatively little is known about the performance of volatility models in the context of ASMs. In particular, the development and growth of stock markets in emerging markets have provided important portfolio diversifications benefits to investors - with respect to risk reduction and opportunities to earn high returns. As a consequence, accurate volatility forecasts are important in a broad range of portfolio management activities including derivative pricing, the

[2] More generally the experiences of 1929 and 1987 and more recent episodes of stock market volatility highlight the impact of stock-return volatility on economic activity.

formulation of trading strategies and risk management (e.g., in the calculation of hedge ratios and Value-at-Risk measures).

Against this backdrop, this paper compares and evaluates the forecasting performance of a spectrum of models – from the simple to the relatively complex – using data from ASMs in order to determine which model(s) are of greatest relevance in these markets. This study is motivated by recognition of the benefit that accurate volatility forecasts confer to financial market participants and the limited empirical evidence available to date in ASMs. Therefore, this study provides an opportunity to augment the existing evidence.

Accordingly, this paper extends the empirical literature in a number of ways. First, it focuses on ASMs, where there appears to be little or no previous work on establishing the forecasting ability of a broad range of models. We also include the UK and US market for comparative purposes. Second, we utilise a wide variety of models including a number of long memory models, which may be useful for forecasting volatility over longer time horizons. Indeed, Bollerslev and Mikkelsen (1996) show that it is important to model the long-term volatility structure when pricing derivative contracts with long maturity. Third, in addition to standard (symmetric) assessment measures used in the literature such as the forecast RMSE and forecast MAE, we also examine the utility of an asymmetric loss function; in particular, the mean mixed error (MME) static introduced by Brailsford and Faff (1996) in order to evaluate forecast accuracy, when under (or over)-predictions of volatility are penalised differently. Fourth, we implement Hansen's (2001) test of superior predictive ability (SPA), in order to observe if our preferred models indeed outperform the (specified) benchmark models in terms of forecast accuracy. Finally, all forecast evaluations at performed at both the daily and monthly frequencies in order that the sensitivity of our results to the choice of sample frequency may be examined. A total of fifteen volatility forecasting models are considered. These are the random walk (RW), historical average (HA), moving average (MA), exponential smoothing (ES), exponentially weighted moving average (EWMA), and simple regression (SR) models. We also evaluate the performance of the following conditional variance models. Specifically, the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) (Bollerslev, 1986), integrated GARCH (IGARCH) (Engle and Bollerslev, 1986) exponential GARCH (EGARCH) (Nelson, 1991), threshold GARCH (TGARCH) (Glosten *et al*, 1993), asymmetric power ARCH (APARCH) (Ding *et al*, 1993), fractionally integrated GARCH (FIGARCH) (Baillie *et al*, 1996), FIEGARCH (Bollerslev and Mikkelsen, 1996), FIAPARCH (Tse, 1998) and the component GARCH (CGARCH) (Engle and Lee, 1999).

To summarise our results from the outset, we find that (for the most part) the various model rankings are sensitive to the choice of forecast error statistic used, the specification of the loss function (i.e., symmetric versus asymmetric) and the frequency of the forecasts (i.e., daily forecasts versus monthly forecasts). As such it is difficult to make broad generalisations. However, at the daily frequency, and using the forecast MAE statistic as a criterion we find that the random walk model provides the most accurate forecasts for the most number of ASMs. These markets include Egypt, Mauritius, Zimbabwe and the benchmark comparators (i.e., UK and US). On the basis of the forecast RMSE our results show that the random walk process yields superior forecasts in Botswana, Egypt, Zimbabwe and the US. Furthermore for Egypt (RW), Kenya (CGARCH), Tunisia (GARCH), Zimbabwe (RW) and the US (RW) our results indicate consistency in terms of model selection across both the forecast MAE and RMSE. It is also worth pointing out that in some cases our results were almost indistinguishable or constrained within a narrow range, indicating that model performance was very close.

In terms of the application of an asymmetric loss function, we also find that the results are also very diverse (at the daily level). However, the results indicate that for most ASMs a variety of statistical methods deliver the best volatility forecasting performance if under-predictions are penalised more heavily. On the other hand, if over-predictions are penalised more heavily then GARCH-type models are preferred. These results may be of particular interest to option traders, for example, an under-prediction of equity volatility implies a downward biased estimate of the call option price and vice-versa. This situation may influence market entrants on both the buy and supply sides and hence volumes and values traded.

Meanwhile, at the monthly frequency we generally find greater support for the conditional variance models, in terms of producing more accurate forecasts. In particular, we find some evidence of the viability of long memory models, although their outperformance is not ubiquitous. At the monthly frequency we find that the forecast MAE and RMSE statistics yield consistent results for Botswana, Egypt, South Africa and Zimbabwe where the CGARCH, FIGARCH, FIGARCH and random walk models are preferred, respectively. On the basis of the forecast

MME(*U*) and MME(*O*) statistics our results are once again very mixed; with long memory models not dominating. Finally, we also implement a test of superior predictive ability (Hansen, 2005). On the basis of this test our results suggest that the GARCH model generally delivers superior forecast accuracy at the daily level. In contrast, at the monthly frequency our results provide mixed evidence in favour of long memory models delivering better forecast performance.

6.2 Review of Relevant Literature

Tse (1991) and Tse and Tung (1992) show that the exponentially weighted moving average model (EWMA) provides more accurate forecasts than those of the GARCH model, in the equity markets of Japan and Singapore, respectively. Brailsford and Faff (1996) use a number of models to forecast volatility in Australia. In addition, they employ a variety of loss functions to capture both symmetric and asymmetric behaviour. Their results are ambiguous insofar as no model consistently outperforms its alternatives. As such, the authors suggest that the selection of an appropriate error measure and model must be consistent with the application for which the forecasts are required.

Brooks (1998) find that the GARCH model generates superior forecasting when analysing US stock return volatility. McMillan *et al* (2000) analyse the forecasting performance of a host of models focused on the UK stock index volatility. As a general result the authors report that the moving average and GARCH models deliver the most consistent forecasting performance. Yu (2002) tests the forecasting performance of an array of models with respect to daily volatility data from New

Zealand. The author finds that the stochastic volatility model delivers the most accurate volatility forecasts. Taylor (2004) using data from equity indices of seven industrialised countries presents evidence indicating the superiority of forecasts obtained from smooth transition exponential smoothing (STES) models relative to those from GARCH-family of models and moving average models. Hansen and Lunde (2005) utilise the test of superior predictive ability (SPA) and find that the GARCH (1,1) is outperformed by alternative models. In particular, their results suggest that in order to arrive at better out-of-sample forecasts then models that include a leverage effect would be more appropriate.[3]

While the existence of long memory is well-documented comparatively less is known about the forecasting performance of long memory models (especially in the context of emerging markets). Barkoulas *et al* (2000) find that long memory models provide superior out-of-sample forecasting accuracy over longer horizons in the Greek Stock Market compared to the random walk process and a short memory model. Degiannakis (2004) examines the stock return volatility of the leading European equity markets and finds that the FIAPARCH (1,1) with skewed-*t* conditional distributed innovations generates better one-day ahead volatility forecasts than a variety of conditional heteroskedastic models. Lux and Kaizoji (2007) consider a long out of sample period and show that long memory models generally produce more accurate forecasts than naïve volatility models and the standard GARCH model. McMillan and Speight (2007) compare and evaluate the forecasting performance of a

[3] These results pertain to the analysis of IBM stock return volatility (see, Hansen and Lunde, 2005).

variety of models in the context of calculating Value-at-Risk in eight emerging stock markets in Asia. They find that long memory models that incorporate an asymmetric (or leverage) effect deliver superior forecasts compared to simple models (e.g., RiskMetrics) and standard (short-memory) GARCH models.

While there is a growing literature on forecasting equity return volatility, see for example, Granger and Poon (2003) and Poon (2005) and references therein. Four important conclusions emerge. First, the extant literature presents mixed evidence regarding the superiority of relatively complex models (e.g., GARCH-type models) relative to more simple alternatives, in terms of delivering accurate volatility forecasts. Second, the model rankings are revealed to be sensitive to the specification of the forecast error statistic used to measure their accuracy. Third, the testing of the forecasting performance of long memory models is still at its infancy. Fourth, evidence from ASMs appears non-existent (or very limited) owing perhaps to data availability constraints or the historically smaller stock market size in these countries (and hence the limited interest of major institutional investors).

6.3 Data Analysis

We focus on squared daily returns, r_t^2, as the volatility proxy of ASMs. In addition, to calculating daily forecasts we also calculate monthly forecasts using the additive property of volatility forecasts.[4] In order to examine forecasting performance we simply split the respective time series in half and we estimate each model over the

[4] It is well-known that volatility forecasts are additive, i.e., the sum of five daily volatility forecasts produces the weekly forecast. Similarly, the sum of the weekly forecasts produces the monthly forecast (Brooks, 2002).

first part of the sample and then use those results to forecast volatility over the second part of the sample for the out-of-sample evaluation and comparison. In other words, the sample data is split between the in-sample period, $t = 1,\dots,T$ and the out-of-sample period $t = T,\dots,\tau$. This modelling strategy is motivated by convenience owing to the differing time spans of the available ASM data.

6.4 Volatility Modelling and Forecasting

As noted in the Introduction, we consider a wide spectrum of volatility models, ranging from simple statistical methods (so called naive models) to an array of GARCH models and their fractionally integrated extensions.

6.4.1 Simple Statistical Methods

i) Historical mean model

The extrapolation of the historical mean in volatility represents a basic method of forecasting future volatility. Furthermore, if the volatility process has a stationary mean, it follows that variation in estimated volatility is due to measurement error; hence, the historical mean calculated as the unweighted average of volatility observed in-sample provides a basis for the derivation of optimal forecasts of volatility, h_{t+1}, for all future periods. In other words, assuming that the conditional expectation of volatility is constant then the best forecast of future volatility is the historical average of past observed volatilities.

$$h_t = \bar{\sigma}^2 = \frac{1}{\tau - T}\sum_{t=1}^{\tau}\sigma_t^2 \qquad (6.1)$$

Throughout the forecasting chapter, and in line with previous research, (e.g., McMillan *et al*, 2000) forecasts based on the historical mean will also serve as a benchmark for the comparative evaluation of the alternative forecasting models considered in this chapter.

ii) *Moving average*

Under the moving average model volatility is forecast by an unweighted average of past observed volatilities over a stipulated time interval. The choice of the moving average estimation period or 'rolling window' (Z) is arbitrary and in this chapter we adopt a rolling window of one trading year.[5]

$$h_t = \bar{\sigma}^2_{j,Z} = \frac{1}{Z}\sum_{j=1}^{Z}\sigma_j^2 \; ; \qquad (6.2)$$

iii) *Random walk*

The preceding models presume that volatility reverts to a stable or gradually evolving trend in volatility. Under the random walk model the best predictor of the stock return volatility in the next period is the volatility in the previous period

$$h_t = \sigma^2_{t-1} \qquad (6.3)$$

The random walk hypothesis suggests that the optimal forecast of volatility is for no change since the last true observation. This model also provides an alternative

[5] We choose the moving average period to be 260 days which is consistent with the annual portfolio rebalancing exercising (e.g., McMillan and Ruiz, 2009).

benchmark for evaluating the relative forecasting performance of methods employed in the literature, being a standard comparative method in econometric forecast appraisal.

iv) Simple regression

This model employs an ordinary least squares (OLS) regression of observed volatilities on immediate past observed volatility. The one-step ahead forecast based on the simple linear regression of the volatility at period *t*+1 on the volatility at period *t*. This procedure entails the application of ordinary least squares regression to in sample data in order to estimate out-of-sample volatility.

$$h_t = \psi_1 + \psi_2 \sigma_{t-1}^2 \tag{6.4}$$

v) Exponential smoothing

Exponential smoothing represents a basic form of adaptive forecasting. The forecast of volatility in this model is a weighted function of the immediately preceding volatility forecast and actual volatility, where the weights decline exponentially.

$$h_t = \Theta_T h_{t-1} + (1 - \Theta_T)\sigma_{t-1}^2 \tag{6.5}$$

More precisely, forecasts from the exponential smoothing model adjust based upon past forecast errors. Furthermore, the smoothing parameter, Θ, is such that $0 \le \Theta \le 1$ and is chosen to produce the best fit by minimising the sum of the squared in-sample forecasts errors.

vi) Exponentially weighted moving average (EWMA) or RiskMetrics (RM) Model

The exponentially weighted moving average is a blend of both the exponential smoothing and moving average models. In particular, the past observed volatility in equation (6.5) is replaced with a moving average forecast as specified in equation (6.2).

$$h_t = \lambda_T h_{t-1} + (1-\lambda_T)\frac{1}{T}\sum_{j=T-T}^{t}\sigma_{j-1}^2 \qquad (6.6)$$

where $0 \leq \lambda \leq 1$ is the smoothing parameter. When $\lambda = 0$ the model reduces to a random walk process and when $\lambda = 1$ the model is equivalent to the prior period forecast of volatility. In this study we set λ to 0.94 following standard market practice, which is also consistent with previous research which indicates that this value produces accurate forecasts (e.g., RiskMetrics, 1996; Fleming et al, 2001).

6.4.2 Symmetric (or first-generation) GARCH Model

The most familiar example of observation-driven volatility models are represented by the GARCH family of models developed by Engle (1982) and Bollerslev (1986) which can account for the can account for the difference between the unconditional and conditional variance of a stochastic process. [6]

[6] The GARCH model was developed to capture the empirical evidence of non-constant variance of shocks observed in many financial time-series. In particular, the GARCH class of models are capable of capturing leptokurtosis, skewness and volatility clustering, which are the three stylised features most often observed in high frequency financial time series data. Bollerslev *et al* (1992) and Bera and Higgins (1993) provide an extensive survey of time-variation in conditional volatility of asset returns.

i) *GARCH (p, q) Model*

The general specification of the GARCH model is given by

$$h_t = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)h_t \tag{6.7}$$

where h_t and ε_t are the conditional and unconditional variances of ε_t respectively; in addition, the long-run variance is $\omega = \varepsilon^2[1-\beta(1)-\alpha(1)]$, and $\phi(L) = 1 - \sum_{j=1}^{q}\phi_j L^j$ and $\beta(L) = 1 + \sum_{j=1}^{p}\beta_j L^j$. The model is well defined if the coefficients of the infinite autoregressive representation are all non-negative and the roots of the moving average polynomial squared innovations lie outside the unit circle. In the basic GARCH (1, 1) model, the effect of a shock on volatility declines geometrically over time.

ii) *IGARCH (p, q) Model*

The GARCH model above can also be expressed as $[1-\alpha(L)-\beta(L)]\varepsilon_t^2 = \omega + [1-\beta(L)]\vartheta_t$ where $\vartheta_t \equiv \varepsilon_t^2 - h_t$ with zero and serially uncorrelated and $E_{t-1}(\vartheta_t) = 0$. In order to be concordant with the covariance stationary process of ε_t, all the roots of $[1-\alpha(L)-\beta(L)]$ and $[1-\beta(L)]$ lie outside the unit circle. Accordingly, an IGARCH process can be written as $\alpha(L)(1-L)\varepsilon_t = \omega + [1-\beta(L)]\vartheta_t$. An IGARCH model refers to an integrated (nonstationary) GARCH. This process implies that any shock to volatility is permanent and the unconditional variance is infinite. Put differently, integrated processes have theoretically infinite variances, exhibit long stochastic swings and are not mean-reverting. IGARCH implies infinite persistence of a volatility shock. The

IGARCH can also be presented in terms of the conditional variance through its infinite ARCH representation

$$h_t = \omega[1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\alpha(L)(1-L)\}\varepsilon_t^2 \qquad (6.8)$$

6.4.3 Asymmetric (or 'second-generation') GARCH Models

The models examined so far are symmetric in that negative and positive shocks have the same effect on volatility. However, negative innovations to stock returns have been found to increase volatility more than positive disturbances of the same magnitude (e.g., volatility is higher in a down- than an upmarket). In other words, returns are thus said to have an asymmetric impact on volatility. Black (1976) and Christie (1982) suggest that stock price fluctuations are negatively correlated with volatility; in particular, falling equity prices imply increased leverage of firms, which entails more uncertainty and hence generates more volatility. As such, asymmetric behaviour is also referred to as the leverage effect.[7] Loeys and Panigirtzoglou (2005) suggest that the 'directional' nature of market volatility reflect three factors, namely: the asymmetry of growth volatility to growth shocks (which reflects that economic recessions tend to be associated with higher growth volatility), the prevalence of stop losses (which imply the dissipation of risk capital during bear markets which makes forced selling more likely in bear markets), and the third factor is return correlation, which is known to rise in down-markets, thus amplifying the impact of individual stock volatility (Loeys and Panigirtzoglou, 2005). In order to account for this

[7] Campbell and Hentschel (1992) propose the 'volatility feedback hypothesis' as an alternative to the 'leverage effect'. In particular, their theory asserts that if expected returns rise when equity price volatility increases as a reward to investors for assuming greater risk; then, equity prices should fall when volatility rises. For further details on the leverage effect see Engle and Ng (1993) and Henry (1998).

asymmetric response of volatility to such shocks we evaluate three widely used asymmetric GARCH models. We begin with the TGARCH (p,q) model introduced by Glosten *et al* (1993).[8]

i) TGARCH (p,q) Model

The TGARCH process is specified as:

$$h_t = \omega + \sum_{i=1}^{q} \left(\alpha_i \varepsilon_{t-1}^2 + \pi_i I_{t-i} \varepsilon_{t-i}^2 \right) + \sum_{i=1}^{p} \beta_i h_{t-i} \qquad (6.9)$$

where the leverage effect is represented by the indicator (or dummy) variable I_{t-1}, whose behaviour is such that $I_{t-1} = 0$ in the event of a positive shocks (i.e., ε_{t-1}>0) and $I_{t-1} = 1$ in the case of positive news (i.e., ε_{t-1}< 0). Therefore, in the TGARCH (1,1) model, positive and negative shocks, have an impact, of α_1 and $\alpha_1 + \pi_1$, respectively. This in turn means that positive news has a greater effect on volatility when $\pi_i < 0$ and negative shocks predominate when $\pi_i > 0$.

ii) EGARCH (p,q)Model

To further capture this 'directional' (or asymmetric) response of volatility Nelson (1991) proposed the exponential GARCH (or EGARCH) which can be modelled as:

$$\ln h_t = \omega + [1 - \beta(L)]^{-1} [1 + \alpha(L)] \Phi(\upsilon_{t-1}) \qquad (6.10)$$

[8] The TGARCH model we have presented is also known as the GJR-GARCH (after its developers). This version of the TGARCH is distinct from the Zakoian (1994) model which models the conditional standard deviation and not the conditional variance.

where the value of $\Phi(\upsilon_t)$ is a function of both the sign and magnitude of υ_t. Because the conditional variance is exponential in parameters no sign restrictions are needed to maintain global positivity. More precisely, $\Phi(\upsilon_t) \equiv \gamma_1\upsilon_t + \gamma_2[|\upsilon_t| - E|\upsilon_t|]$ where the first term captures the sign effect and the second the magnitude effect.

iii) *APARCH (p, q) Model*

A common attribute of the models considered so far is that the conditional variance is expressed as a function of both lagged residuals and past variances. However, the imposition (or necessity) of a squared power term in the volatility equation has been disputed. Indeed, Brooks *et al* (2000) argue that by squaring the returns an artificial structure is imposed on the data and as a consequence sub-optimal modelling and forecasting performance may result in relation to other power terms. Accordingly, recognising the possibility that a squared power term may not necessarily be optimal, Ding *et al* (1993) introduced the asymmetric power ARCH (or APARCH) class of models which allows an optimal power term to be estimated from the data (instead of being pre-specified as in the GARCH and EGARCH). In addition, this model includes a leverage parameter to capture volatility asymmetry. The conditional variance, h_t, in the APARCH model is given by:

$$h_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i h_{t-i}^\delta f_i(\varepsilon_{t-i}) + \sum_{j=1}^{p} \beta_j h_{t-j}^\delta \tag{6.11}$$

where $f_i(\varepsilon_{t-i}) \equiv [|\varepsilon_{t-i}| - \Omega_i \varepsilon_{t-i}]^\delta$; in particular, Ω_i ($|\Omega_i|$<1) is the leverage parameter and δ ($\delta > 0$) represents the power term Furthermore, this model encompasses other

GARCH specifications given appropriate parameter restrictions (e.g., Ding *et al*, 1993 and Brooks *et al*, 2000).

6.4.4 Long Memory (or 'third-generation') GARCH Models

i) *FIGARCH (p, d, q) Model*

Fractionally integrated processes differ from both stationary and unit-root processes in that they are persistent but are also ultimately mean reverting, i.e., they reveal long memory behaviour (Poon, 2005). That is, the effect of the past innovations on the current conditional variance dies out at a slow-mean reverting hyperbolic rate (with the lag length).

Against this backdrop, the FIGARCH (*p*, *d*, *q*) model was developed by Baillie *et al* (1996) to capture long memory in financial market volatility. In particular, the general specification of the FIGARCH can be derived by introducing a fractional differencing parameter in the IGARCH model presented in section 6.4.2 (*ii*) such that, $\phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1-\beta(L)]\vartheta_t$, where $(1-L)^d$ denotes the fractional differencing operator; and $d \in (0,1)$; and all the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit circle. The FIGARCH nests a range of other GARCH specifications, for instance, the FIGARCH is equivalent to the GARCH and IGARCH models when the fractional differencing parameter, *d* is equal to zero and one, respectively. The FIGARCH process described above can also be written in terms of its conditional variance, as

$$h_t = \omega + [1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\}\varepsilon_t^2 \tag{6.12}$$

ii) FIEGARCH (p,d,q) Model

The long memory extension of the EGARCH is the FIEGARCH (*p*, *d*, *q*) developed by Bollerslev and Mikkelsen (1996). Under this specification, the conditional variance is modelled as:

$$\ln h_t = \omega + \phi(L)^{-1}(1-L)^{-d}[1+\xi(L)]g(\eta_{t-i}) \tag{6.13}$$

where $g(\eta_{t-i}) \equiv \kappa_1\eta_{t-i} + \kappa_2[|\eta_t| - E|\eta_t|]$ as in the EGARCH captures both the size and sign effect. The FIEGARCH therefore captures both volatility asymmetry (usually interpreted as the leverage effect) and long memory behaviour as reflected by the very slow mean-reverting hyperbolic decay of shocks to stock returns.

iii) FIAPARCH (p,d,q) Model

The long memory counterpart of the APARCH(*p*, *q*) model is the FIAPARCH (*p*, *d*, *q*) introduced by Tse (1998) is presented below:

$$h_t^{\delta} = \omega + (1-(1-\beta(L)))^{-1}\phi(L)(1-L)^{-d}(|\varepsilon_t| - \tau\varepsilon_t)^{\delta} \tag{6.14}$$

where τ_i is such that $|\tau_i| < 1$ is the leverage parameter and δ with the property that $\delta > 0$ embodies the optimal power transformation (estimated from the data). Similar to the other long memory considered the FIAPARCH captures hyperbolic decay of shocks to the conditional variance process – mirroring the highly persistent but ultimately mean-reverting nature of these shocks.

iv) Component GARCH (p,q) Model

Engle and Lee (1993) developed the CGARCH model in order to capture multiple volatility components by decomposing volatility effects into their short-run and long-run parts. In particular, this decomposition entails a separation of long-run and short-run volatility effects analogous to the Beveridge-Nelson (1981) decomposition of conditional mean ARMA models for economic time series. In particular, the CGARCH specification allows mean reversion to a time-varying long-run volatility level ℓ_t and the model can be expressed as:

$$h_t = \ell_t + \alpha\left(\varepsilon_{t-1}^2 - \ell_{t-1}\right) + \beta\left(h_{t-1} - \ell_{t-1}\right) \qquad (6.15)$$

where, $\ell_t = \omega + \rho\ell_{t-1} + \phi\left(\varepsilon_{t-1}^2 - h_{t-1}\right)$ is the long-run volatility (or permanent component in volatility). The forecast error $\left(\varepsilon_{t-1}^2 - h_{t-1}\right)$ drives the time-varying process of ℓ_t and the difference between the conditional variance and its trend $\left(h_{t-1} - \ell_{t-1}\right)$ represents the transitory component of the volatility process.[9] These short- and long-run components converge to zero and the unconditional variance with powers of $(\alpha + \beta)$ and ρ, respectively.[10]

[9] By substitution, the CGARCH model may be expressed as a GARCH (1,1) model with a time-varying intercept or as a GARCH (2,2) process such that the second-order terms of the reduced GARCH(2,2) model should be negative (e.g., Engle and Lee, 1999). For estimation we use this latter property.

[10] This process implies that the long-run volatility dominates the forecast values of the conditional variance as the forecast horizon increases.

6.5 Forecast Evaluation

In order to evaluate the forecasting performance of the various models we start by splitting our data set in half and then estimate each model for all series covering the first part of the sample and use those to forecast volatility over the sample period covered by the second half of our data. We then assess the forecast performance of each model relative to several criteria (discussed hereafter). In these exercises, we let r_t^2 (i.e., squared returns) represent our volatility proxy (or measure) and h_t^f denotes the appropriate volatility forecast.

6.5.1 Symmetric Forecast Evaluation

Two symmetric measures are used to evaluate forecast accuracy, namely, the mean absolute error (MAE) and the root mean square error (RMSE). They are defined below:

$$MAE = \frac{1}{\tau}\sum_{t=T+1}^{T+\tau}\left|h_t^f - r_t^2\right| \tag{6.17}$$

$$RMSE = \sqrt{\frac{1}{\tau}\sum_{t=T+1}^{T+\tau}\left(h_t^f - r_t^2\right)^2} \tag{6.18}$$

where τ is the number of forecast data points and r_t^2 is the proxy for volatility . Both the MAE and RMSE assume the underlying loss function to be symmetric.

Furthermore, under these evaluation criteria the model which minimises the loss function is preferred.

6.5.2 Asymmetric Forecast Evaluation

Standard error statistics assume the underlying loss function to be symmetric. Brailsford and Faff (1996) introduced the mean mixed error (MME) statistic in order to capture asymmetry in volatility realisations. For instance, from a trading and risk management perspective, it is often the case that market participants will not attach equal importance to both over- and under-predictions of stock-return volatility of similar magnitude. For instance, the value of both call and put options increases as volatility increases; hence, pricing an option involves valuing volatility.[11] More precisely, a positive relationship exists between the volatility of (underlying) stock prices and call option prices. As a consequence, an under-prediction of equity price volatility implies a downward biased estimate of the call option price. This under-estimate of the price is more likely to be unfavourable to a seller than a buyer, and vice-versus. Similarly, in Value-at-Risk (VaR) management investors have fixed VaR target which typically necessitates them to offset higher (lower) volatility with lower (higher) leverage (see Loeys and Panigirtzoglou, 2005). Accordingly, a negative contemporaneous relationship between leverage and volatility would suggest that under- and over predictions of volatility would have a significant bearing on the calculations of VaR targets.

[11] For example, in option pricing, the volatility associated with the future price of the underlying asset is the most important determinant in the pricing function (see Bollerslev *et al*, 1992). In particular, the owner of a call option benefits from price increases but has limited downside risk in the event of price decreases, since the most the investor can lose is the price of the option. On the other hand, the owner of a put option benefits from price decreases but has limited downside risk in the event of price increases. Since pricing an option involves valuing volatility, it follows that the size and frequency of under predictions or over predictions of volatility is relevant in valuing puts and calls.

Following previous research (e.g., Brailsford and Faff, 1996 and McMillan *et al*, 2000) we therefore also consider error statistics designed to account for potential asymmetry in the loss function. That is, mean mixed error statistics which penalise under predictions more considerably, i.e., MME*(U))* and the MME*(O)* which weighs over predictions more heavily, respectively:

$$MME(U) = \frac{1}{\tau}\left[\sum_{i=1}^{O}\left|h_t^f - r_t^2\right| + \sum_{i=1}^{U}\sqrt{\left|h_t^f - r_t^2\right|}\right] \tag{6.19}$$

$$MME(O) = \frac{1}{\tau}\left[\sum_{i=1}^{O}\sqrt{\left|h_t^f - r_t^2\right|} + \sum_{i=1}^{U}\left|h_t^f - r_t^2\right|\right] \tag{6.20}$$

where *O* denotes the number of over predictions and *U* the number of under predictions among the out-of-sample forecasts.

Following previous empirical work, we also present standardised values for all forecast error statistics using the forecast error statistic for the historical mean benchmark for each series. This has the benefit of allowing for greater performance comparability among the competing models (e.g., McMillan *et al*, 2000 and Yu, 2002).

6.5.3 Test of Superior Predictive Ability (SPA)

We also perform a test of SPA (Hansen, 2005) in order to further assess forecast accuracy. This method considers whether any competing model delivers greater forecast accuracy than the benchmark forecast. More precisely, this test evaluates whether any of the models k = 1,…, m, produce the smallest expected loss with respect to the benchmark.

In particular, if we let $L(Y_t, \hat{Y}_t)$ denote the loss if one had made the prediction, $\hat{Y}_t$, when the realised value turned out to be Y_t. The performance of model k relative to the benchmark model (at time t) can be expressed as:

$$X_k(t) = L(Y_t, \hat{Y}_{0t}) - L(Y_t, \hat{Y}_{kt}) \qquad k = 1, \ldots, l, \quad t = 1, \ldots, n. \tag{6.21}$$

where $X_k(t)$ denotes to the forecast accuracy of model k in relation to the benchmark model at time t. Therefore, the SPA test evaluates whether any of the models k = 1, . . .,l produce more accurate forecasts than the benchmark model. The hypothesis that the benchmark model is superior to all the alternatives can therefore be expressed as:

$$\mu_k = E[X_k(t)] \leq 0 \qquad k = 1, \ldots, l \tag{6.22}$$

Since $\mu_k > 0$ is equivalent to the case where the k*th* model outperforms the benchmark, we can test the hypothesis: $H_0 : \mu_k \leq 0$, for k = 1, . . .,l. To test this hypothesis we compute the following statistic:

$$T_n^{sm} = \max_k \frac{n^{0.5}\overline{X}_k}{\hat{\sigma}_k} \tag{6.23}$$

Where $\overline{X}_k = \frac{1}{n}\sum_{t=1}^{n} X_{k,t}$ and $\hat{\sigma}_k^2$ is a consistent estimator of $\sigma_k^2 = \mathrm{var}\left(n^{0.5}\overline{d}_k\right)$ which is estimated through a bootstrap procedure (see Hansen, 2005; Hansen and Lunde, 2005).

6.6 Out-of-sample Forecast Evaluation

6.6.1 Symmetric forecast error results

Daily Data

Table 6.1 to 6.13 presents the actual and relative forecast error statistics for each model across the four error measures for the eleven ASMs considered in this study and the two benchmark comparators sampled at the daily frequency. In particular, the first two columns of Table 6.1 to 6.13 report the forecast MAE and RMSE statistics and in parentheses are their respective standardised values (derived using the error statistic for the HM for series).[12] An examination of the forecast MAE and RMSE statistics indicates that the results (in terms of the best performing model) are very diverse; there exists wide variability in the performance of the various models being compared; and that for the most part the various model rankings are sensitive to the error statistic used to evaluate the accuracy of the forecasts.

[12] Standardisation allows the forecast errors to be more conveniently interpreted relative to a benchmark forecast.

On the basis of the forecast MAE statistic our results show that the random walk model provides the most accurate stock return volatility forecasts for the most number of ASMs. In particular, these markets are Egypt, Mauritius and Zimbabwe, where the random walk forecast model is 90 percent, 79 percent and 32 percent more accurate than the benchmark model, respectively. In addition, our results indicate that for the benchmark comparators the random walk model delivers superior forecast performance. Indeed, for the UK and US, the random walk model is 62 percent and 68 percent more accurate than the benchmark forecasts, respectively. Using the forecast MAE as a criterion we also find wide variability in the performance of the various models used to evaluate forecast performance. For instance, in Egypt the random walk model (i.e., the best performing model) has a forecast MAE of 3.10e-06 while the worst performing model (i.e., the EWMA model) has a forecast MAE of 4.25e-04. This represents a change of 99.3 percent. Furthermore, in the case of Egypt, the best five performing models in order are: RW, EGARCH, SR, APARCH and FIEGARCH and their forecast MAEs range from 3.10e-06 to 5.88e-06, which represents a percentage change of 47.3 percent. Similarly, in Mauritius, we find that the simple regression model performs worst (MAE is 5.76e-04) while random walk performs best (MAE is 4.17e-05) implying a percentage change of 92.8 percent. In Zimbabwe, this variability in model performance is also pronounced, reflecting (or perhaps in tandem with) the hyperinflationary environment in that country. For instance, the random walk (i.e., the best performing model) has a forecast MAE of 2.71e-07 while the FIAPARCH the worst performing model has a forecast MAE of 1.71e-05, implying a percentage change of 98.4 percent. In comparison our results show that for both the UK and US forecast MAE statistics are concentrated in a relatively more narrow range.

The forecast MAE statistic indicates that the GARCH model provides the most accurate results for Morocco and Tunisia. In these countries, the GARCH is 89 and 30 percent, more accurate than the benchmark model, respectively. For Morocco, our results also show that the long memory models are the worst performing. The worst model (in terms of delivering forecasting accuracy) is the FIGARCH with a forecast MAE statistic of 8.03e-04. The second, third and fourth worst forecasting models are the CGARCH (MAE is 7.93e-04), FIEGARCH (MAE is 3.81e-04) and the FIAPARCH (MAE is 3.81e-04). Indeed, taken in terms of groups, our results show that the short-memory GARCH models consistently provide the best forecast accuracy (i.e., TGARCH, EGARCH and APARCH are the second, third and fourth best models). The simple statistical models are in turn preferred over the long memory class of models in terms of providing the most accurate forecasts. This result suggests that at least in the context of Morocco long memory models do not provide accurate forecasts at the daily frequency. Indeed, this conclusion, is consistent with the expectation that long memory models should deliver more accurate forecasts over longer horizons. For Tunisia, our results are ambiguous to the extent that long memory models deliver accurate forecasts even at the daily frequency. For example, our results show that the EGARCH is the second most accurate model, providing 18 percent more accuracy than the benchmark forecast. The FIEGARCH and FIAPARCH are ranked a close third and fourth registering 17 and 16 percent more accuracy than the benchmark forecast respectively. The simple statistical models generally perform poorly in Tunisia with the exponential smoothing (ES) and the moving average (MA) models delivering the worst and second worst forecast accuracy. Meanwhile, for Kenya and Namibia our findings suggest that long memory models, in particular, the CGARCH and FIEGARCH have the best predictive value.

Indeed, these models are 78 percent and 17 percent, more accurate than the benchmark forecasts, respectively. Meanwhile, short memory models with asymmetric effects deliver the best forecast performance in Botswana and Nigeria. Specifically, the EGARCH and APARCH models register the smallest forecast MAE and are 48 percent and 41 percent more accurate than the benchmark forecasts respectively. In both Botswana and Nigeria the next best performing models are long memory models, the FIGARCH and FIAPARCH models, respectively, which are 33 percent and 18 percent more accurate than the benchmark forecasts. The worst performing models are the TGARCH and the CGARCH which are 98.1 percent and 86.4 percent less accurate than the most accurate models. In Ghana the TGARCH model provides the most accurate forecasts. This model is 75 percent more accurate than the benchmark model. The GARCH model ranks a close second and is 73 percent more accurate than benchmark forecasts. The exponential smoothing (ES) model delivers the worst forecasting performance and is 93.7 percent less accurate than the TGARCH (i.e., the best forecasting model).

When we use the forecast RMSE statistic as a criterion to evaluate forecast performance the most noticeable result we obtain relates to the consistency in terms of delivering superior forecast accuracy of a variety of models in several countries. In particular, the RMSE indicates that for Egypt, Kenya, Tunisia and Zimbabwe, the same models which were identified by the MAE as delivering the best forecast performance are also validated by the RMSE. In other words, in Egypt and Zimbabwe, the random walk (RW) model provides the most accurate forecasts (consistent with the results of the MAE test). In addition, in Kenya and Tunisia, the CGARCH and GARCH models, respectively, are again the best forecasting models.

For the remaining ASMs our results show that these model selection criteria produce differing results.

For instance in Botswana, the forecast RMSE statistic suggests that the random walk (RW) model provides the most accurate forecasts; while the EGARCH (which on the basis of the MAE delivers the best forecast performance) is found to be the worst performing model. In terms of the RMSE, the FIGARCH model provides the most accurate forecasts in Ghana. This forecast model is 30 percent more accurate than the benchmark model. Indeed, using the RMSE as a criterion Ghana is the only country were a long memory model delivers the best forecast performance at the daily frequency. For Mauritius, Nigeria and Tunisia we find that the GARCH model has the best predictive value. Indeed, the GARCH and random walk (RW) model are the best performing models in the sense that they both deliver the most accurate results for the most number of ASMs (i.e., the RW also generates the most accurate forecasts in three countries, namely, Botswana, Egypt and Zimbabwe).

In Morocco, the popular RiskMetrics (or EWMA) model delivers the best forecast performance. This model is 78 percent more accurate than the benchmark forecast. Another salient feature of the results from Morocco is the close ranking of the best performing seven models (i.e., the EWMA, CGARCH, RW, SR, APARCH, EGARCH and FIGARCH). In particular, the difference in the forecast RMSE statistic between the first and seventh position is 7.8 percent. On the other hand, the TGARCH is revealed to be the least accurate model and it is 84.2 percent less accurate than the most accurate forecasting model for Morocco (i.e., the EWMA).

Forecasting models with asymmetric features are found to be important considerations in providing accurate forecasts in Namibia and South Africa; in particular, the APARCH and EGARCH deliver the best forecast performance, respectively. These models are 39 percent and 28 percent more accurate than the benchmark forecasts, respectively. In the UK and US the forecast RMSE statistic indicates that the EGARCH and RW models generate superior forecasts. Indeed, for the US the RMSE result matches that obtained when the MAE is used as a criterion. Table 6.14 present a summary of these findings. In particular, it lists the best performing model given the assessment criteria.

Monthly Data

Table 6.15 to 6.27 presents the forecast results when the data is sampled at the monthly frequency. We are particularly concerned with the behaviour of long memory models which may potentially be useful for forecasting over long(er) horizons than the other models we have considered given their formulation (e.g., Baillie *et al*, 1996). Against this background, we find that when the forecast MAE statistic is used as a criterion long memory models provide the most accurate forecasts in five of the eleven ASMs considered. In particular, for Botswana, Kenya, and Morocco the CGARCH model provides the forecast with the smallest MAE. Indeed, in these three markets the forecasts from the CGARCH are 39 percent, 86 percent and 54 percent more accurate than the benchmark forecasts. For Botswana and Kenya, we find that fractionally integrated models as a group generally perform much better than the other class of models (i.e., both standard GARCH and simple statistical models). However, for Morocco we find that among long memory models the CGARCH is an anomaly insofar as the other variety of long memory models – the fractionally integrated models – are among the worst performing models (relative to the other class of models we have considered). For Egypt and South Africa, the FIGARCH provides the most accurate forecasts, which are 85 percent and 71 percent more accurate than the benchmark forecasts. In the case of Egypt, the difference between the FIGARCH and the next best model (the simple regression (SR)) is marginal. For example, the MAE of the FIGARCH and the SR is 0.0026 and 0.0027, respectively, while all other models perform in a more dispersed manner. For South Africa, the next best forecasting model is the FIEGARCH which is 67 percent more accurate than the benchmark forecast and the worst performing model is the moving average (MA) model, which underperforms the benchmark by 10 performs. Indeed, in South Africa, we find that in general long memory models perform best, then short memory (conditional variance) models generally rank second and simple statistical methods models generally deliver the worst forecasting performance.

For both Ghana and Zimbabwe the MAE statistic favours the random walk (RW) and this model is 36 percent and 79 percent more accurate than the benchmark model, respectively. The APARCH is the second best performing model in Ghana and is only marginally inferior to the RW model (it is 35 percent more accurate than the benchmark). Long memory models with the exception of the FIEGARCH (the third best performing model in Ghana) are among the worst performing models. In Zimbabwe, the FIAPARCH is the second best performing model and the FIGARCH is the fifth best performing model; while, the CGARCH is the second worst performing model. On the basis of the MAE forecast error statistic the APARCH, TGARCH and RiskMetrics (or EWMA) model deliver the best forecast accuracy for Namibia, Nigeria and Tunisia. For Namibia, the second best performing is the GARCH and its forecast MAE statistic is almost indistinguishable from that of the

APARCH model. Asymmetric long memory models (i.e., FIEGARCH and FIAPARCH) deliver the third and fourth best forecast accuracy. While, the FIAGARCH and CGARCH models are among the worst performing models. In Nigeria, the TGARCH is 75 percent more accurate than the benchmark model; while, the performance of all the long memory models is mediocre. In the case of Tunisia our results indicate that the RiskMetrics model considerably outperforms the competing models. Meanwhile, with the exception of the CGARCH the other long memory models are among the worst performing. For the benchmark comparators, our results indicate that models with asymmetric features deliver the best forecasts. In particular, the APARCH and FIEGARCH are the most accurate models for the UK and US, respectively. In the UK, our results show that with the exception of the FIEGARCH model (i.e., fourth best forecasting model) all other long memory models are ranked very average. In the US, the CGARCH is the second worst performing model; while, the fractionally integrated class of models are the best performing models. Indeed, the FIEGARCH, FIGARCH and FIAPARCH are the first, second and third best performing models, respectively.

Under the RMSE forecast error statistic our results indicate that this measure yields results for Botswana, Egypt, South Africa and Zimbabwe that are consistent with those obtained when the MAE is used as a criterion. In particular, for Botswana the CGARCH is also found to provide the most accurate forecasts; for both Egypt and South Africa the FIGARCH delivers the best forecast performance; while, in Zimbabwe the RW model also ranks first on the RMSE criterion. For all other countries model ranking between the forecast MAE and RMSE statistics diverge. For instance, in Ghana, the RMSE indicates that the CGARCH model provides the most accurate forecasts. Indeed, this model is 68 percent more accurate than the benchmark model. The RW model which ranked first in terms of the MAE criterion is ranked sixth when the RMSE is used. In particular, the RW is now 32 percent more accurate than the benchmark. For Kenya and Nigeria our results indicate that the GARCH model delivers superior forecasts. In Kenya, our results show that the long memory genre of models produce the poorest forecasts; while, simple statistical models generally provide the most accurate forecasts. In Nigeria, the performance of long memory models is mixed. For example, the FIGARCH and FIAPARCH are among the best forecasting models under the RMSE forecast error statistic. In contrast, the FIEGARCH and CGARCH are among the worst forecasting models. Models with asymmetric effects are found to provide the most accurate forecasts in Mauritius, Namibia and Tunisia, where, the FIAPARCH, EGARCH and APARCH models are preferred, respectively, on the basis of the forecast RMSE statistic. In the UK and US, long memory models – the FIEGARCH and FIGARCH – respectively, provide the most accurate forecasts. More precisely, the former is 59 percent and the latter is 30 percent more accurate than the benchmark, respectively. Table 6.28 presents a summary of the main results of this study. Specifically, it lists the models that provide the most accurate forecasts.

6.6.2 Asymmetric forecast error results

The common feature of both the forecast MAE and RMSE error statistic (presented in Tables 6.1 to 6.13 and 6.15 to 6.27) are that they assume that the underlying loss function is symmetric. However, market participants may not necessarily attach equal value to both over- and under-predictions of volatility of similar magnitude.[13] Therefore, to examine potential asymmetry in the loss function we evaluate the same models (previously described in Section 6.4.1) using the mean mixed error (MME) statistic. The forecast MME(*U*) and MME(*O*) statistics are presented in the last two columns of Table 6.1 to 6.13 and Tables 6.15 to 6.27.

In particular, Tables 6.1 to 6.13 report MME(*U*) and MME(*O*) statistics for all series at the daily frequency. Our results indicate that when the MME(*U*) statistic is used as a criterion simple statistical methods provide the best results for many ASMs. For instance, in Kenya, the moving average (MA) model is preferred on the MME(*U*) criterion penalising under-predictions more heavily while the FIEGARCH is ranked last. In contrast, the CGARCH model which is ranked first on the basis of both the MAE and RMSE (i.e., symmetric loss functions) is mediocre when ranked in terms of the MME(*U*). The MME(*U*) statistic in Table 6.5 indicates superiority of the RW model for Mauritius while all other models provide significantly worse forecasts. The simple regression model is preferred in both Namibia and Zimbabwe while the FIEGARCH and RW provide the worst forecasts, respectively. The RiskMetrics model provides the most accurate forecasts when the MME(*U*) is used in Nigeria while the RW is ranked last. For Botswana, Egypt, Ghana and Tunisia a variety of conditional variance models dominate forecast performance. Again, all these models are different from those selected on the basis of a symmetric loss function.

[13] For example, in the context of option pricing, an under-prediction of stock price volatility results in a downward biased estimate of the call option price. This development is more adverse on the writer of the call than the call buyer, and vice-versa.

The last column of Tables 6.1 to 6.13 presents the MME(*O*) statistic which penalises over-prediction errors more heavily than under-prediction of volatility. The most striking feature is that for most ASMs the conditional variance models provide the best forecast accuracy. Indeed, the basic GARCH model delivers the best forecast accuracy in Morocco, South Africa and Zimbabwe. The only exceptions to the dominance of conditional variance models are results from Ghana and Tunisia, where the RiskMetrics (or EWMA) and MA models deliver the most accurate forecasts, respectively. In addition, these two models are 76 percent and 90 percent more accurate than the benchmark forecasts, respectively. Once again our results show that the models that are preferred on the MME(*O*) criterion differ from those selected on all the other criterion, underscoring the sensitivity of the results to the specification of the forecast error statistic.

When monthly volatility forecasts are analysed our results offer some evidence in favour of the outperformance of long memory models. On the MME(*U*) criterion long memory models are preferred for Egypt (FIAPARCH), Ghana (FIGARCH), Namibia (CGARCH) and Zimbabwe (FIEGARCH). In the case of Egypt and Namibia, the preferred long memory models are 24 percent and 13 percent more accurate than the benchmark forecasts, respectively, and significantly outperforms all other competing models in terms of forecast accuracy. In the case of Ghana and particularly Zimbabwe, the gap in the rankings of the various models is narrower. For instance in Zimbabwe the difference between the rankings of the best three models is marginal. In particular, the FIEGARCH, exponential smoothing (ES) and EGARCH are 98 percent, 97 percent and 96 percent more accurate than the benchmark forecast, respectively. Our results also indicate a considerable performance gap in the ranking of the FIGARCH and all other competing models. For all other countries a diverse range of models deliver the most accurate forecasts. However, the GARCH model is dominant and is found to provide superior forecast accuracy on the basis of the forecast MME(*U*) statistic for Botswana, Morocco, South Africa and the UK. In Botswana, the FIAPARCH is ranked a close second (a 1.4 percent difference in their performance), while all other models are considerably outperformed. Similarly, in the UK, a marginal difference exists between the first and second (i.e., FIGARCH model) ranked models. In Morocco a 97.7 percent difference exists between the accuracy of the GARCH and the worst ranked model (the ES model). In South Africa, the forecast performance of the best five performing models, i.e., the GARCH, TGARCH, ES, RW and FIGARCH is very close. Indeed, they outperform the benchmark forecast on average by 96.2 percent.

Under the MME(*O*) forecast error statistic our results show that long memory models provide the most accurate forecasts for Kenya (CGARCH) and Nigeria (FIGARCH). In the case of Kenya the CGARCH is 75 percent more accurate than the benchmark, and the EGARCH is ranked second and outperforms the benchmark by 77 percent. Indeed, our results show that the benchmark model is the worst performing. For Nigeria, our results show that fractionally integrated genre of models – in order FIGARCH, FIAPARCH and FIEGARCH – significantly outperform all other competing models in terms of producing superior quality forecasts. For all other ASMs a diverse medley of models are preferred when overpredictions are penalised more heavily. For example, the APARCH model produces the best quality forecasts in Botswana, while the historical average (HA) model is preferred in Zimbabwe. Among the benchmark comparators the FIAPARCH provides the best forecasts for the UK while the EGARCH produces the poorest forecasts. In the US the GARCH and RW models deliver the most accurate and inaccurate forecasts, respectively, when the MME(*U*) is used as a selection criteria. Table 6.28 highlights the model which produces the best quality forecasts given the various loss functions.

6.6.3 Test of Superior Predictive Analysis (SPA)

In order to examine compare forecast performance across the various models employed in this study we conduct a test of SPA (Hansen, 2005) in order to compare the various model specifications. At the daily frequency we use the HM as our benchmark model (since it was basis of standardisation in the previous analysis using symmetric and asymmetric loss functions). Table 6.29 presents our results. Specifically, at the daily level our results suggest that the benchmark model is inferior to at least one of the alternative specifications, for most of the ASMs considered with the exception of Kenya and Morocco where our results suggest that the HM provides superior forecast accuracy. These results are mostly consistent with the results obtained from the evaluation of both symmetric and asymmetric loss which generally indicated that the HM model was mostly outperformed by the other forecasting models. Against this background, the apparent outperformance of the HM in both Kenya and Morocco is surprising. In the case of Kenya, the HM is among the worst performing models regardless of assessment criteria. In contrast, for Morocco, the HM is among the best performing models on the on the basis of the MAE criteria; however, on the basis of other evaluation criteria its performance relative to the other volatility forecasting models is mediocre.

In order to evaluate forecasting ability at the monthly frequency, we use the GARCH (1,1) since this process has been shown to be able to represent the majority of financial time series (e.g., Bera and Higgins, 1993). Our findings with respect to the performance of the GARCH model are mixed. In particular, they suggest that for most ASMs alternative models provide better forecast accuracy than the GARCH model. Nonetheless, some evidence, albeit more limited is found to support the forecast accuracy of the GARCH model in the case of Botswana, Kenya, Morocco, Nigeria and South Africa. Indeed, for Kenya and Nigeria these results are consistent with those obtained using the RMSE as an assessment criteria. These results also point to the viability of long memory models in forecasting stock return volatility over long horizons. In particular, a statistically significant difference is revealed between the performance of long memory models and the standard GARCH (1,1) model over longer horizons.

6.7 Conclusions

This research has compared and evaluated the performance of a number of volatility models, in terms of their ability to forecast in an out-of-sample setting. This endeavour has been motivated by recognition of the importance of accurate volatility forecasts in a wide range of applications including portfolio and risk management and the limited empirical evidence available to date for ASMs.

A total of fifteen volatility forecasting models are considered, comprising of six simple statistical models (HA, RW, MA, SR, ES and EWMA), five conditional variance models (GARCH, IGARCH, TGARCH and APARCH) and four long memory models (FIGARCH, FIEGARCH, FIAPARCH and CGARCH). In particular, forecast evaluations are performed at both the daily and monthly frequencies using both symmetric and asymmetric loss functions. In addition, we applied a test of superior predicative ability to further ascertain forecasting performance. Our findings are very diverse and are indeed similar to previous studies insofar as the accuracy of volatility forecasts is sensitive to the choice of evaluation criteria, loss function and the forecast horizon. In this study we have paid special attention to the quality of long memory forecasts, especially over longer horizons. In this respect, our results provide some evidence in favour of the outperformance of long memory models at the monthly frequency. In particular, this evidence is mixed, indicating that perhaps these models are more applicable when time spans of greater than one month are considered. Finally, since these results indicate that model performance is sensitive to the choice of evaluation criteria this implies that from the outset market participants must specify the context within which the forecasts will be used (e.g., the choice between a symmetric and asymmetric loss function). In addition, since some of our forecast results are very similar or constrained to a narrow range, market participants may also need to define a confidence interval around a target that would be compatible with their investment objectives. This in turn, may further guide model selection and hence forecast accuracy.

Finally, there are areas where future research might be useful. First, Granger and Poon (2003) and Poon (2005) recommend the use of realised volatility constructed from intraday high-frequency daily data as a superior proxy for volatility than squared returns which they argue are a noisy proxy for volatility. In our case, data constraints limited this option, however, future research may find the application of realised variance may produce more accurate forecasts. Second, future research may also consider exploring the relevance of stochastic volatility models in ASMs (and indeed, elsewhere) as another method to generate forecasts. Third, while we have examined a variety of time series models they only extrapolate the past and do not relate to fundamental economic developments. As such, future research may wish to further develop a fundamental framework to project future market volatility. This could shed more light on economic and structural variables that drive volatility (e.g., Loeys and Panigirtzoglou, 2005).

DAILY FORECAST RESULTS

In all tables presented below '*' indicates the preferred forecasting model

Table 6.1: Botswana – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	3.08e-05 (1.00)	4.12e-04 (1.00)	0.0013 (1.00)	0.0035 (1.00)
RW	2.31e-04 (7.50)	1.71e-04 (0.41)*	0.0018 (1.13)	0.0027 (0.77)
MA	5.94e-05 (1.93)	3.26e-04 (0.79)	0.0229 (17.8)	0.0033 (0.93)
SR	1.35e-04 (4.38)	1.68e-04 (0.41)	0.0238 (18.4)	0.0019 (0.54)
ES	8.64e-05 (2.81)	4.21e-04 (1.02)	0.0036 (2.80)	0.0009 (0.24)
EWMA	5.98e-04 (19.4)	3.83e-04 (0.93)	0.0028 (2.15)	0.0221 (6.24)
GARCH	3.15e-05 (1.02)	8.81e-04 (2.14)	0.0003 (0.21)*	0.0062 (1.75)
IGARCH	1.14e-04 (3.70)	2.05e-04 (4.98)	0.0038 (2.92)	0.0097 (2.77)
EGARCH	1.65e-05 (0.54)*	1.71e-04 (0.41)	0.0255 (19.8)	0.0099 (2.79)
TGARCH	8.62e-04 (28.0)	5.37e-04 (1.30)	0.0053 (4.11)	0.0067 (1.89)
APARCH	2.94e-05 (0.95)	1.82e-04 (0.44)	0.0010 (0.79)	0.0008 (0.24)
FIGARCH	2.08e-05 (0.67)	1.84e-04 (0.45)	0.0033 (2.57)	0.0010 (0.28)
FIEGARCH	1.79e-05 (0.58)	1.81e-04 (0.44)	0.0589 (45.7)	0.0065 (1.84)
FIAPARCH	4.79e-05 (1.56)	1.74e-04 (0.42)	0.0076 (5.91)	0.0005 (0.13)
CGARCH	2.65e-05 (0.86)	4.66e-04 (1.13)	0.0009 (0.67)	0.0003 (0.09)*

Table 6.2: Egypt – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	3.00e-05 (1.00)	8.59e-04 (1.00)	0.0017 (1.00)	0.0019 (1.00)
RW	3.10e-06 (0.10)*	3.41e-04 (0.40)*	0.0022 (1.25)	0.0026 (1.33)
MA	3.90e-04 (13.0)	9.55e-04 (1.11)	0.0006 (0.36)	0.0056 (2.92)
SR	5.88e-06 (0.20)	5.53e-04 (0.64)	0.0026 (1.50)	0.0036 (1.87)
ES	6.75e-06 (0.23)	9.16e-04 (1.07)	0.0012 (0.71)	0.0002 (0.08)
EWMA	4.25e-04 (14.2)	9.62e-04 (1.12)	0.0050 (2.88)	0.0032 (1.66)
GARCH	1.81e-05 (0.60)	4.65e-04 (0.54)	0.0003 (0.19)*	0.0051 (2.66)
IGARCH	4.77e-04 (15.9)	8.71e-04 (1.01)	0.0072 (4.24)	0.0056 (2.95)
EGARCH	3.70e-06 (0.12)	4.74e-04 (0.55)	0.0022 (1.28)	0.0004 (0.21)
TGARCH	5.63e-05 (1.88)	5.14e-04 (0.60)	0.0065 (3.75)	0.0001 (0.06)
APARCH	5.88e-06 (0.20)	5.87e-04 (0.68)	0.0078 (4.50)	0.0012 (0.63)
FIGARCH	8.70e-05 (2.90)	4.24e-04 (0.49)	0.0041 (2.39)	0.0001 (0.06)*
FIEGARCH	7.87e-06 (0.26)	6.74e-04 (0.78)	0.0015 (0.85)	0.0058 (3.02)
FIAPARCH	8.60e-05 (2.87)	8.61e-04 (1.00)	0.0003 (0.20)	0.0016 (0.85)
CGARCH	4.45e-05 (1.48)	8.18e-04 (0.95)	0.0007 (0.42)	0.0005 (0.25)

Table 6.3: Ghana – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	8.80e-04 (1.00)	2.48e-03 (1.00)	0.0057 (1.00)	0.0043 (1.00)
RW	4.28e-04 (0.49)	3.80e-03 (1.53)	0.0006 (0.11)	0.0021 (0.48)
MA	1.18e-04 (0.13)	6.77e-03 (2.73)	0.0012 (0.21)	0.0014 (0.32)
SR	4.42e-04 (0.50)	4.50e-03 (1.82)	0.0008 (0.15)	0.0015 (0.35)
ES	3.46e-03 (3.93)	2.12e-03 (0.86)	0.0026 (0.46)	0.0034 (0.78)
EWMA	5.01e-04 (0.57)	9.62e-03 (3.89)	0.0019 (0.33)	0.0010 (0.24)*
GARCH	2.34e-04 (0.27)	4.32e-03 (1.75)	0.0006 (0.11)	0.0058 (1.34)
IGARCH	8.26e-04 (0.94)	3.15e-03 (1.27)	0.0042 (0.74)	0.0053 (1.23)
EGARCH	3.39e-04 (0.39)	8.17e-03 (3.30)	0.0038 (0.67)	0.0079 (1.82)
TGARCH	2.18e-04 (0.25)*	3.74e-03 (1.51)	0.0055 (0.97)	0.0036 (0.84)
APARCH	5.20e-03 (0.59)	2.38e-03 (0.96)	0.0002 (0.03)*	0.0018 (0.42)
FIGARCH	2.82e-04 (0.32)	1.74e-03 (0.70)*	0.0096 (1.68)	0.0026 (0.61)
FIEGARCH	8.69e-04 (0.99)	3.02e-03 (1.22)	0.0009 (0.16)	0.0013 (0.31)
FIAPARCH	5.03e-04 (0.57)	2.46e-03 (0.99)	0.0075 (1.32)	0.0062 (1.44)
CGARCH	4.36e-04 (0.49)	2.47e-03 (1.00)	0.0005 (0.08)	0.0009 (0.20)

Table 6.4: Kenya – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	1.85e-03 (1.00)	9.22e-03 (1.00)	0.0566 (1.00)	0.0087 (1.00)
RW	5.19e-03 (2.81)	7.85e-03 (0.85)	0.0075 (0.13)	0.0075 (0.86)
MA	3.77e-04 (0.20)	8.20e-03 (0.89)	0.0018 (0.03)*	0.0055 (0.64)
SR	2.67e-03 (1.44)	2.68e-02 (2.91)	0.0024 (0.04)	0.0476 (5.50)
ES	7.19e-03 (3.89)	8.10e-03 (0.88)	0.0700 (1.24)	0.0065 (0.75)
EWMA	2.89e-04 (0.16)	8.31e-03 (0.90)	0.0810 (1.43)	0.0134 (1.54)
GARCH	5.43e-03 (2.94)	7.93e-03 (0.86)	0.0033 (0.06)	0.0055 (0.64)
IGARCH	2.98e-03 (1.61)	5.03e-03 (0.55)	0.0512 (0.90)	0.0068 (0.78)
EGARCH	1.26e-03 (0.68)	8.26e-03 (0.90)	0.0664 (1.17)	0.0020 (0.23)
TGARCH	5.62e-04 (0.30)	9.74e-03 (1.06)	0.0032 (0.06)	0.0630 (7.27)
APARCH	4.13e-04 (0.22)	8.08e-03 (0.88)	0.0607 (1.07)	0.0036 (0.41)
FIGARCH	8.42e-03 (4.55)	1.18e-02 (1.27)	0.0073 (0.13)	0.0021 (0.25)
FIEGARCH	9.52e-03 (5.15)	7.93e-03 (0.86)	0.0866 (1.53)	0.0026 (0.30)
FIAPARCH	7.03e-04 (0.38)	2.14e-02 (2.32)	0.0067 (0.12)	0.0018 (0.20)*
CGARCH	3.89e-04 (0.21)*	7.04e-03 (0.76)*	0.0322 (0.57)	0.0047 (0.55)

Table 6.5: Mauritius – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	1.97e-04 (1.00)	7.85e-05 (1.00)	0.0013 (1.00)	0.0154 (1.00)
RW	4.17e-05 (0.21)*	8.97e-04 (11.4)	0.0005 (0.42)*	0.0144 (0.93)
MA	4.84e-05 (0.25)	8.98e-05 (1.14)	0.0224 (17.7)	0.0037 (0.24)
SR	5.76e-04 (2.93)	6.41e-04 (8.16)	0.0019 (1.50)	0.0048 (0.31)
ES	4.39e-04 (2.23)	4.35e-04 (5.54)	0.0941 (74.4)	0.0117 (0.76)
EWMA	2.22e-04 (1.13)	1.06e-04 (1.35)	0.0328 (26.0)	0.0022 (0.14)
GARCH	1.06e-04 (0.54)	2.59e-05 (0.33)*	0.0653 (51.7)	0.0020 (0.13)
IGARCH	4.36e-04 (2.21)	3.17e-04 (4.04)	0.0659 (50.7)	0.0085 (0.55)
EGARCH	5.31e-05 (0.27)	3.91e-04 (4.98)	0.0084 (6.65)	0.0016 (0.10)*
TGARCH	6.24e-05 (0.32)	5.08e-05 (0.65)	0.0206 (16.3)	0.0081 (0.53)
APARCH	1.36e-04 (0.69)	6.65e-04 (8.47)	0.0125 (9.87)	0.0208 (1.35)
FIGARCH	5.09e-04 (2.59)	1.15e-04 (1.46)	0.0059 (4.64)	0.0068 (0.44)
FIEGARCH	8.34e-04 (4.24)	7.83e-04 (9.97)	0.0341 (27.0)	0.0117 (0.76)
FIAPARCH	8.91e-05 (0.45)	5.02e-05 (0.64)	0.0017 (1.31)	0.0089 (0.58)
CGARCH	3.78e-04 (1.92)	4.48e-04 (5.70)	0.0759 (60.0)	0.0137 (0.89)

Table 6.6: Morocco – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	1.02e-04 (1.00)	4.08e-03 (1.00)	0.0020 (1.00)	0.0045 (1.00)
RW	2.01e-04 (1.96)	9.33e-04 (0.23)	0.0013 (0.62)	0.0029 (0.65)
MA	2.83e-04 (2.77)	3.36e-03 (0.82)	0.0020 (0.98)	0.0018 (0.14)
SR	2.53e-04 (2.47)	9.46e-04 (0.23)	0.0012 (0.57)	0.0035 (0.79)
ES	2.32e-04 (2.27)	1.50e-03 (0.37)	0.0009 (0.44)*	0.0044 (0.98)
EWMA	2.76e-04 (2.70)	9.06e-04 (0.22)*	0.0027 (1.33)	0.0066 (1.46)
GARCH	1.13e-05 (0.11)*	1.72e-03 (0.42)	0.0023 (1.13)	0.0005 (0.11)*
IGARCH	1.78e-05 (0.17)	2.93e-03 (0.72)	0.0126 (6.30)	0.0094 (2.09)
EGARCH	4.41e-05 (0.43)	9.80e-04 (0.24)	0.0017 (0.81)	0.0099 (2.20)
TGARCH	1.29e-05 (0.13)	5.74e-03 (1.41)	0.0037 (1.81)	0.0022 (0.48)
APARCH	5.51e-05 (0.54)	9.77e-04 (0.24)	0.0029 (1.43)	0.0011 (0.24)
FIGARCH	8.03e-04 (7.85)	9.83e-04 (0.24)	0.0040 (1.96)	0.0039 (0.86)
FIEGARCH	3.81e-04 (3.73)	4.25e-03 (1.04)	0.0059 (2.89)	0.0069 (1.54)
FIAPARCH	2.72e-04 (2.67)	4.14e-03 (1.01)	0.0075 (3.68)	0.0070 (1.56)
CGARCH	7.93e-04 (7.76)	9.25e-04 (0.23)	0.0193 (9.46)	0.0106 (2.35)

Table 6.7: Namibia – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	3.74e-06 (1.00)	2.63e-04 (1.00)	0.0011 (1.00)	0.0080 (1.00)
RW	3.38e-06 (0.90)	1.69e-04 (0.64)	0.0081 (7.46)	0.0086 (1.08)
MA	5.40e-06 (1.44)	5.58e-04 (2.12)	0.0002 (0.21)	0.0037 (0.46)
SR	7.08e-06 (1.89)	3.05e-04 (1.16)	8.6e-06 (0.008)*	0.0032 (0.40)
ES	8.67e-05 (23.2)	1.79e-04 (0.68)	0.0054 (4.98)	0.0550 (6.89)
EWMA	1.54e-05 (4.11)	4.58e-05 (0.17)	0.0012 (1.08)	0.0057 (0.70)
GARCH	3.04e-05 (8.13)	3.12e-04 (1.18)	0.0024 (2.20)	0.0066 (0.83)
IGARCH	5.06e-05 (13.5)	2.17e-04 (0.83)	0.0007 (0.64)	0.0050 (0.63)
EGARCH	7.44e-05 (19.9)	3.78e-04 (1.44)	2.9e-05 (0.03)	0.0040 (0.50)
TGARCH	4.38e-06 (1.17)	5.35e-04 (2.03)	0.0046 (4.25)	0.0045 (0.54)
APARCH	4.24e-06 (1.13)	1.62e-04 (0.61)*	0.0015 (1.41)	0.0003 (0.03)*
FIGARCH	6.22e-05 (16.6)	2.13e-04 (0.81)	0.0030 (2.71)	0.0012 (0.16)
FIEGARCH	3.11e-06 (0.83)*	2.99e-04 (1.14)	0.0098 (9.02)	0.0090 (1.12)
FIAPARCH	8.89e-06 (2.38)	3.37e-04 (1.28)	0.0011 (0.98)	0.0004 (0.05)
CGARCH	8.77e-06 (2.34)	2.28e-04 (0.86)	0.0034 (3.13)	0.0022 (0.28)

Table 6.8: Nigeria – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	1.86e-04 (1.00)	5.88e-04 (1.00)	0.0661 (1.00)	0.0521 (1.00)
RW	8.10e-04 (4.35)	4.43e-04 (0.75)	0.0792 (1.20)	0.0341 (0.65)
MA	2.30e-04 (1.24)	9.52e-04 (1.62)	0.0379 (0.57)	0.0983 (1.89)
SR	3.22e-04 (1.73)	8.19e-04 (1.39)	0.0604 (0.91)	0.0674 (1.29)
ES	3.40e-04 (1.83)	7.31e-04 (1.24)	0.0166 (0.25)	0.0177 (0.34)
EWMA	3.53e-04 (1.90)	4.86e-04 (0.83)	0.0110 (0.17)*	0.0159 (0.30)
GARCH	3.42e-04 (1.84)	4.17e-04 (0.71)*	0.0146 (0.22)	0.0962 (1.85)
IGARCH	7.44e-04 (4.00)	6.27e-04 (1.07)	0.0048 (0.07)	0.0427 (0.82)
EGARCH	5.41e-04 (2.91)	6.18e-04 (1.05)	0.0206 (0.31)	0.0559 (1.07)
TGARCH	3.80e-04 (2.04)	5.71e-04 (0.97)	0.0152 (0.23)	0.0307 (0.59)
APARCH	1.10e-04 (0.59)*	7.25e-04 (1.23)	0.0581 (0.88)	0.0510 (0.98)
FIGARCH	3.00e-04 (1.61)	5.19e-04 (0.88)	0.0622 (0.94)	0.0190 (0.36)
FIEGARCH	4.62e-04 (2.48)	6.23e-04 (1.06)	0.0239 (0.36)	0.0131 (0.25)*
FIAPARCH	1.53e-04 (0.82)	5.17e-04 (0.88)	0.0149 (0.23)	0.0132 (0.25)
CGARCH	8.06e-04 (4.33)	7.97e-04 (1.36)	0.0320 (0.48)	0.0246 (0.47)

Table 6.9: South Africa – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(U)	MME(O)
HA	7.44e-04 (1.00)	1.93e-03 (1.00)	0.0414 (1.00)	0.0492 (1.00)
RW	2.30e-04 (0.31)	2.80e-03 (1.45)	0.0114 (0.28)	0.0969 (1.97)
MA	3.97e-04 (0.53)	2.50e-03 (1.29)	0.0357 (0.86)	0.0254 (0.52)
SR	6.01e-04 (0.81)	3.11e-03 (1.61)	0.0423 (1.02)	0.0569 (1.16)
ES	2.30e-04 (0.31)*	1.63e-03 (0.85)	0.0983 (2.37)	0.0097 (0.20)
EWMA	6.42e-04 (0.86)	1.46e-03 (0.75)	0.0697 (1.68)	0.0670 (1.36)
GARCH	7.61e-04 (1.02)	2.05e-03 (1.06)	0.0721 (1.74)	0.0013 (0.03)*
IGARCH	7.85e-04 (1.06)	2.23e-03 (1.16)	0.0493 (1.19)	0.0022 (0.04)
EGARCH	2.31e-04 (0.31)	1.39e-03 (0.72)*	0.0897 (2.17)	0.0458 (0.93)
TGARCH	1.39e-03 (1.87)	7.31e-03 (3.78)	0.0207 (0.50)*	0.0320 (0.65)
APARCH	1.01e-03 (1.36)	5.90e-03 (3.05)	0.0371 (0.90)	0.0502 (1.02)
FIGARCH	6.45e-04 (0.87)	1.62e-03 (0.84)	0.0592 (1.43)	0.0215 (0.44)
FIEGARCH	7.03e-04 (0.94)	1.84e-03 (0.95)	0.0418 (1.01)	0.0295 (0.60)
FIAPARCH	7.11e-04 (0.96)	4.08e-03 (2.11)	0.0531 (1.28)	0.0243 (0.49)
CGARCH	3.46e-04 (0.46)	3.79e-03 (1.96)	0.0255 (0.62)	0.0477 (0.97)

Table 6.10: Tunisia – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(U)	MME(O)
HA	3.42e-03 (1.00)	1.77e-02 (1.00)	0.1060 (1.00)	0.0185 (1.00)
RW	4.27e-03 (1.25)	2.52e-02 (1.43)	0.1284 (1.21)	0.0149 (0.80)
MA	8.48e-03 (2.48)	8.17e-03 (0.46)	0.2028 (1.91)	0.0019 (0.10)*
SR	8.36e-03 (2.44)	8.86e-02 (5.00)	0.3151 (2.97)	0.0041 (0.22)
ES	8.84e-03 (2.58)	1.76e-02 (1.00)	1.3815 (13.0)	0.0136 (0.74)
EWMA	6.60e-03 (1.93)	1.07e-02 (0.60)	1.0413 (9.83)	0.0064 (0.35)
GARCH	2.39e-03 (0.70)*	6.21e-03 (0.35)*	0.1547 (1.46)	0.0044 (0.24)
IGARCH	5.52e-03 (1.61)	2.08e-02 (1.18)	1.0158 (9.58)	0.0036 (0.19)
EGARCH	2.80e-03 (0.82)	8.84e-03 (0.50)	0.1261 (1.19)	0.0128 (0.69)
TGARCH	7.61e-03 (2.22)	6.20e-02 (3.50)	0.7157 (6.75)	0.0044 (0.24)
APARCH	4.14e-03 (1.21)	6.69e-03 (0.38)	0.5139 (4.85)	0.0033 (0.18)
FIGARCH	5.28e-03 (1.54)	6.32e-03 (0.36)	0.0517 (0.49)	0.0135 (0.73)
FIEGARCH	9.69e-03 (2.83)	8.37e-03 (0.47)	0.0559 (0.53)	0.0037 (0.20)
FIAPARCH	2.86e-03 (0.84)	7.52e-03 (0.42)	0.1124 (1.06)	0.0078 (0.42)
CGARCH	6.62e-03 (1.94)	7.87e-03 (0.44)	0.0337 (0.32)*	0.0081 (0.44)

Table 6.11: Zimbabwe – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	3.96e-07 (1.00)	3.35e-06 (1.00)	4.97e-07 (1.00)	3.78e-08 (1.00)
RW	2.71e-07 (0.68)*	2.21e-06 (0.66)*	8.26e-06 (16.6)	3.99e-07 (10.6)
MA	6.22e-06 (15.7)	4.88e-05 (14.6)	2.58e-06 (5.19)	3.68e-07 (9.74)
SR	6.67e-06 (16.8)	2.34e-05 (7.00)	2.27e-08 (0.05)*	6.97e-07 (18.4)
ES	5.18e-06 (13.1)	4.65e-06 (1.39)	2.23e-06 (4.49)	3.83e-07 (10.1)
EWMA	9.32e-06 (23.5)	1.60e-05 (4.78)	1.80e-05 (36.2)	4.31e-08 (1.14)
GARCH	1.38e-05 (34.9)	3.34e-06 (1.00)	2.98e-06 (6.00)	3.77e-08 (1.00)*
IGARCH	3.09e-06 (7.80)	1.88e-05 (5.61)	3.17e-05 (63.8)	4.94e-07 (13.1)
EGARCH	8.08e-06 (20.4)	2.53e-05 (7.55)	5.32e-06 (10.7)	4.24e-08 (1.12)
TGARCH	2.13e-06 (5.38)	3.71e-06 (1.11)	4.05e-06 (8.15)	5.33e-07 (14.1)
APARCH	7.62e-06 (19.2)	2.73e-06 (0.81)	5.43e-07 (1.09)	6.08e-07 (16.1)
FIGARCH	4.32e-06 (10.9)	1.93e-05 (5.76)	8.83e-08 (0.18)	1.88e-07 (4.97)
FIEGARCH	1.29e-05 (32.6)	2.28e-05 (6.81)	3.43e-06 (6.90)	2.90e-07 (7.67)
FIAPARCH	1.71e-05 (43.2)	3.39e-06 (1.01)	2.92e-07 (0.59)	2.78e-07 (7.35)
CGARCH	1.05e-05 (26.4)	3.14e-06 (0.94)	3.36e-06 (6.76)	1.59e-07 (4.20)

Table 6.12: UK – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	4.94e-04 (1.00)	3.72e-04 (1.00)	0.0052 (1.00)	0.0331 (1.00)
RW	1.87e-04 (0.38)*	4.72e-05 (0.13)	0.0157 (3.04)	0.0224 (0.68)
MA	6.84e-04 (1.38)	3.70e-04 (0.99)	0.0307 (5.95)	0.0083 (0.25)
SR	2.24e-03 (4.53)	3.65e-04 (0.98)	0.0193 (3.74)	0.0110 (0.33)
ES	2.05e-03 (4.14)	4.22e-04 (1.13)	0.0354 (6.85)	0.0087 (0.26)
EWMA	1.50e-03 (3.04)	7.32e-04 (1.97)	0.0070 (1.35)	0.0253 (0.76)
GARCH	3.95e-04 (0.80)	5.33e-05 (0.14)	0.0405 (7.83)	0.0049 (0.15)
IGARCH	7.11e-04 (1.44)	3.37e-04 (0.91)	0.0086 (1.65)	0.0139 (0.42)
EGARCH	6.62e-04 (1.34)	3.15e-05 (0.08)*	0.0072 (1.40)	0.0039 (0.12)*
TGARCH	8.26e-04 (1.67)	6.71e-05 (0.18)	0.0047 (0.91)*	0.0055 (0.17)
APARCH	2.02e-04 (0.41)	7.64e-05 (0.21)	0.0357 (6.91)	0.0188 (0.57)
FIGARCH	4.63e-04 (0.94)	5.08e-05 (0.14)	0.0105 (2.03)	0.0107 (0.32)
FIEGARCH	1.25e-04 (0.25)	5.51e-05 (0.15)	0.0096 (1.86)	0.0090 (0.27)
FIAPARCH	9.17e-04 (1.86)	3.20e-05 (0.08)	0.0321 (6.22)	0.0065 (0.20)
CGARCH	4.64e-04 (0.94)	7.33e-05 (0.20)	0.0161 (3.11)	0.0201 (0.61)

Table 6.13: US – Daily Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	1.55e-04 (1.00)	2.16e-04 (1.00)	0.0039 (1.00)	0.0022 (1.00)
RW	5.01e-05 (0.32)*	6.20e-05 (0.29)*	0.0052 (1.35)	0.0006 (0.29)
MA	3.72e-04 (2.39)	6.88e-04 (3.19)	0.0179 (4.62)	0.0019 (0.83)
SR	1.64e-04 (1.05)	8.11e-04 (3.75)	0.0074 (1.91)	0.0067 (3.01)
ES	2.59e-04 (1.67)	9.03e-04 (4.18)	0.0062 (1.60)	0.0832 (37.2)
EWMA	6.21e-05 (0.40)	8.89e-05 (0.41)	0.0073 (1.89)	0.0054 (2.39)
GARCH	7.01e-05 (0.45)	7.42e-05 (0.34)	0.0037 (0.96)*	0.0037 (1.67)
IGARCH	6.18e-05 (0.40)	4.06e-05 (0.19)	0.0043 (1.10)	0.0011 (0.50)
EGARCH	2.34e-04 (1.51)	3.87e-04 (1.79)	0.0262 (6.76)	0.0030 (1.33)
TGARCH	2.51e-05 (0.16)	8.95e-05 (0.41)	0.0530 (13.7)	0.0010 (0.46)
APARCH	1.95e-05 (0.13)	6.48e-05 (0.30)	0.0278 (7.18)	0.0003 (0.12)*
FIGARCH	5.71e-05 (0.37)	6.65e-05 (0.31)	0.0057 (1.47)	0.0226 (10.1)
FIEGARCH	3.67e-04 (2.37)	4.02e-04 (1.86)	0.0094 (2.43)	0.0167 (7.47)
FIAPARCH	6.66e-05 (0.43)	8.04e-05 (0.37)	0.0065 (1.68)	0.0132 (5.92)
CGARCH	8.60e-04 (5.55)	9.98e-04 (4.62)	0.0100 (2.57)	0.0016 (0.70)

Table 6.14: Summary of Best Performing Models at Daily Frequency				
	Model		Model	
	MAE	RMSE	MME(*U*)	MME(*O*)
Botswana	EGARCH	RW	APARCH	CGARCH
Egypt	RW	RW	GARCH	FIGARCH
Ghana	TGARCH	FIGARCH	APARCH	EWMA
Kenya	CGARCH	CGARCH	MA	FIAPARCH
Mauritius	RW	GARCH	RW	EGARCH
Morocco	GARCH	EWMA	ES	GARCH
Namibia	FIEGARCH	APARCH	SR	APARCH
Nigeria	APARCH	GARCH	EWMA	FIEGARCH
South Africa	ES	EGARCH	TGARCH	GARCH
Tunisia	GARCH	GARCH	CGARCH	MA
Zimbabwe	RW	RW	SR	GARCH
UK	RW	EGARCH	TGARCH	EGARCH
US	RW	RW	GARCH	APARCH

MONTHLY FORECAST RESULTS

In all tables presented below '*' indicates the preferred forecasting model

Table 6.15: Botswana – Monthly Frequency

	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	2.71e-03 (1.00)	6.00e-02 (1.00)	0.0785 (1.00)	0.0527 (1.00)
RW	3.92e-03 (1.45)	5.04e-03 (0.08)	0.0355 (0.45)	0.0632 (1.19)
MA	3.43e-03 (1.26)	7.00e-03 (0.12)	0.0876 (1.12)	0.0658 (1.24)
SR	5.02e-03 (1.85)	5.25e-02 (0.87)	0.0653 (0.83)	0.0540 (1.02)
ES	3.18e-03 (1.17)	5.67e-03 (0.09)	0.0581 (0.74)	0.0421 (0.79)
EWMA	2.43e-03 (0.89)	8.82e-03 (0.14)	0.0637 (0.81)	0.0469 (0.88)
GARCH	3.72e-03 (1.37)	5.02e-02 (0.83)	0.0072 (0.09)*	0.0565 (1.07)
IGARCH	2.53e-03 (0.93)	4.01e-02 (0.67)	0.0395 (0.50)	0.0544 (1.03)
EGARCH	3.13e-03 (1.15)	8.86e-03 (0.14)	0.0721(0.92)	0.0687 (1.30)
TGARCH	3.75e-03 (1.38)	4.88e-02 (0.81)	0.0714 (0.91)	0.0485 (0.92)
APARCH	4.89e-03 (1.80)	8.52e-03 (0.14)	0.0730 (0.92)	0.0419 (0.80)*
FIGARCH	3.32e-03 (1.22)	5.86e-03 (0.09)	0.0754 (0.96)	0.0456 (0.86)
FIEGARCH	1.89e-03 (0.69)	4.41e-03 (0.07)	0.0689 (0.87)	0.0736 (1.39)
FIAPARCH	2.84e-03 (1.04)	9.75e-04 (0.02)	0.0073 (0.09)	0.0479 (0.90)
CGARCH	1.66e-03 (0.61)*	8.69e-04 (0.01)*	0.0719 (0.91)	0.0529 (1.00)

Table 6.16: Egypt – Monthly Frequency

	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	0.0179 (1.00)	0.0553 (1.00)	0.0016 (1.00)	0.0150 (1.00)
RW	0.0151 (0.84)	0.0366 (0.66)	0.0017 (1.06)	0.0022 (0.14)
MA	0.0492 (2.74)	0.0305 (0.55)	0.0180 (11.2)	0.0019 (0.13)
SR	0.0027 (0.15)	0.0600 (1.08)	0.0131 (8.12)	0.0043 (0.29)
ES	0.0065 (0.36)	0.0665 (1.20)	0.0039 (2.43)	0.0019 (0.12)
EWMA	0.0084 (0.47)	0.0810 (1.46)	0.0775 (48.2)	0.0074 (0.49)
GARCH	0.0054 (0.30)	0.0111 (0.20)	0.0238 (14.8)	0.0121 (0.80)
IGARCH	0.0043 (0.24)	0.0091 (0.16)	0.0077 (4.81)	0.0056 (0.37)
EGARCH	0.0036 (0.20)	0.0092 (0.16)	0.0124 (7.77)	0.0018 (0.12)*
TGARCH	0.0153 (0.85)	0.0049 (0.08)	0.0189 (11.7)	0.0102 (0.67)
APARCH	0.0150 (0.85)	0.0060 (0.11)	0.0140 (8.73)	0.0138 (0.92)
FIGARCH	0.0026 (0.15)*	0.0033 (0.06)*	0.0418 (25.9)	0.0119 (0.79)
FIEGARCH	0.0147 (0.82)	0.0194 (0.35)	0.0358 (22.3)	0.0123 (0.82)
FIAPARCH	0.0149 (0.83)	0.0184 (0.33)	0.0012 (0.76)*	0.0030 (0.19)
CGARCH	0.0151 (0.84)	0.0405 (0.73)	0.0277 (17.2)	0.0034 (0.22)

Table 6.17: Ghana – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	5.19e-04 (1.00)	1.75e-02 (1.00)	0.5050 (1.00)	0.3067 (1.00)
RW	3.34e-03 (0.64)*	1.20e-02 (0.68)	0.5209 (1.03)	0.0210 (0.07)*
MA	8.99e-04 (1.73)	7.73e-02 (4.41)	0.8689 (1.72)	0.0470 (0.15)
SR	1.19e-03 (2.29)	1.73e-02 (0.98)	0.2911 (0.58)	0.3239 (1.05)
ES	3.26e-03 (6.28)	8.67e-02 (4.95)	0.7912 (1.57)	0.0422 (0.14)
EWMA	7.81e-04 (1.50)	7.17e-03 (0.41)	0.6027 (1.19)	0.0507(0.17)
GARCH	6.07e-04 (1.16)	7.72e-03 (0.44)	0.3083 (0.61)	0.0776 (0.25)
IGARCH	3.47e-04 (0.67)	6.19e-03 (0.35)	0.1702 (0.34)	0.0327 (0.11)
EGARCH	5.40e-04 (1.04)	1.92e-02 (1.09)	0.5611 (1.11)	0.0885 (0.29)
TGARCH	1.78e-03 (3.43)	5.39e-02 (3.08)	0.1743 (0.35)	0.3193 (1.04)
APARCH	3.42e-04 (0.65)	2.03e-01 (11.5)	0.8453 (1.67)	0.2570 (0.84)
FIGARCH	2.50e-03 (4.81)	8.99e-03 (0.51)	0.0976 (0.19)*	0.0971 (0.32)
FIEGARCH	4.85e-04 (0.93)	2.26e-02 (1.29)	0.9654 (1.91)	0.0886 (0.29)
FIAPARCH	1.67e-03 (3.21)	1.19e-02 (0.68)	0.4631 (0.92)	0.0512 (0.16)
CGARCH	1.65e-03 (3.17)	5.59e-03 (0.32)*	0.7465 (1.48)	0.0536 (0.17)

Table 6.18: Kenya – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	7.13e-03 (1.00)	3.42e-02 (1.00)	0.0091(1.00)*	0.0073 (1.00)
RW	4.39e-03 (0.62)	9.08e-03 (0.26)	0.0400 (4.38)	0.0043 (0.58)
MA	2.34e-03 (0.33)	3.40e-02 (0.99)	0.0426 (4.67)	0.0063 (0.85)
SR	5.30e-03 (0.74)	3.29e-02 (0.96)	0.0468 (5.13)	0.0046 (0.62)
ES	6.40e-03 (0.89)	3.23e-02 (0.94)	0.0274 (3.00)	0.0037 (0.50)
EWMA	2.05e-02 (2.87)	2.23e-02 (0.56)	0.0992 (10.9)	0.0021 (0.29)
GARCH	4.41e-03 (0.62)	7.12e-03 (0.21)*	0.0243 (2.66)	0.0029 (0.39)
IGARCH	1.34e-03 (0.19)	8.15e-03 (0.24)	0.0501 (5.51)	0.0078 (1.07)
EGARCH	1.71e-03 (0.24)	4.60e-02 (1.34)	0.0164 (1.79)	0.0020 (0.27)
TGARCH	4.10e-03 (0.57)	3.15e-02 (0.92)	0.0670 (7.34)	0.0054 (0.74)
APARCH	1.74e-02 (2.44)	3.15e-02 (0.92)	0.0132 (1.45)	0.0043 (0.59)
FIGARCH	1.61e-03 (0.23)	4.69e-02 (1.37)	0.0378 (4.14)	0.0043 (0.59)
FIEGARCH	2.93e-03 (0.41)	8.82e-02 (2.58)	0.0498 (5.45)	0.0053 (0.72)
FIAPARCH	1.52e-03 (0.21)	1.28e-02 (3.74)	0.0183 (2.00)	0.0028 (0.38)
CGARCH	1.01e-03 (0.14)*	3.37e-02 (0.99)	0.0296 (3.23)	0.0019 (0.25)*

Table 6.19: Mauritius – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	2.06e-03 (1.00)	3.06e-03 (1.00)	0.0603 (1.00)	0.0438 (1.00)
RW	8.97e-03 (4.36)	2.19e-03 (0.72)	0.0192 (0.32)	0.0465 (1.06)
MA	5.97e-03 (2.90)	3.10e-02 (10.1)	0.0132 (0.22)	0.0407 (0.93)
SR	1.87e-03 (0.91)	2.72e-03 (0.89)	0.0090 (0.15)	0.0143 (0.33)
ES	6.70e-03 (3.26)	2.96e-03 (0.97)	0.0428 (0.71)	0.0878 (2.01)
EWMA	2.67e-03 (1.30)	3.15e-02 (10.3)	0.0224 (0.37)	0.0111 (0.25)
GARCH	1.08e-03 (0.53)*	2.89e-03 (0.95)	0.0063 (0.10)	0.0104 (0.24)*
IGARCH	1.11e-03 (0.54)	2.37e-03 (0.77)	0.0094 (0.16)	0.0353 (0.81)
EGARCH	3.08e-03 (1.50)	7.93e-03 (2.60)	0.0054 (0.09)*	0.0120 (0.27)
TGARCH	6.84e-03 (3.32)	3.40e-02 (11.1)	0.0116 (0.19)	0.0511 (1.17)
APARCH	4.41e-03 (2.14)	3.01e-03 (0.99)	0.0645 (1.07)	0.0172 (0.39)
FIGARCH	1.38e-03 (0.67)	2.88e-03 (0.94)	0.0021 (0.36)	0.0239 (0.55)
FIEGARCH	2.63e-03 (1.28)	3.66e-03 (1.20)	0.0143 (0.24)	0.0357 (0.82)
FIAPARCH	1.13e-03 (0.55)	2.01e-03 (0.66)*	0.0115 (0.19)	0.0200 (0.46)
CGARCH	1.34e-03 (0.65)	2.97e-03 (0.97)	0.0337 (0.56)	0.0238 (0.54)

Table 6.20: Morocco – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	6.50e-03 (1.00)	8.75e-02 (1.00)	0.0090 (1.00)	0.0887 (1.00)
RW	3.42e-03 (0.53)	7.59e-02 (0.87)	0.0021 (0.23)	0.0828 (0.93)
MA	8.46e-03 (1.30)	9.12e-02 (1.04)	0.0063 (0.70)	0.0029 (0.03)*
SR	3.77e-03 (0.58)	8.23e-02 (0.94)	0.0084 (0.94)	0.0077 (0.09)
ES	3.00e-02 (4.61)	9.88e-03 (0.11)	0.0436 (4.84)	0.1717 (1.94)
EWMA	2.37e-02 (3.65)	3.80e-02 (0.43)	0.0081 (0.88)	0.0618 (0.70)
GARCH	3.00e-02 (4.61)	7.99e-02 (9.14)	0.0010 (0.11)*	0.0655 (0.74)
IGARCH	3.40e-03 (0.52)	2.78e-02 (0.32)	0.0013 (0.14)	0.0829 (0.93)
EGARCH	3.09e-02 (4.75)	8.63e-02 (0.99)	0.0398 (4.42)	0.0087 (0.10)
TGARCH	3.44e-02 (5.30)	9.77e-03 (0.11)*	0.0047 (0.52)	0.1039 (1.17)
APARCH	3.48e-02 (5.35)	5.53e-03 (6.32)	0.0029 (0.33)	0.1023 (1.15)
FIGARCH	3.70e-02 (5.69)	4.90e-04 (0.56)	0.0360 (4.00)	0.0031 (0.03)
FIEGARCH	1.40e-02 (2.15)	8.31e-03 (9.50)	0.0919 (10.2)	0.0924 (1.04)
FIAPARCH	4.03e-02 (6.21)	5.54e-04 (0.63)	0.0089 (0.99)	0.0067 (0.08)
CGARCH	3.01e-03 (0.46)*	8.05e-03 (9.21)	0.0073 (0.81)	0.0041 (0.05)

Table 6.21: Namibia – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	3.17e-02 (1.00)	2.51e-03 (1.00)	0.0026 (1.00)	0.0082 (1.00)
RW	2.93e-02 (0.93)	4.57e-02 (18.2)	0.0188 (7.19)	0.0070 (0.86)
MA	4.99e-03 (0.16)	9.87e-02 (39.3)	0.0567 (21.6)	0.0055 (0.67)
SR	2.40e-02 (0.76)	2.44e-02 (9.74)	0.2191 (83.6)	0.0048 (0.59)
ES	6.29e-03 (0.20)	8.75e-02 (34.9)	0.0818 (31.2)	0.0017 (0.21)
EWMA	5.23e-03 (0.17)	2.72e-02 (10.8)	0.0079 (3.01)	0.0022 (0.27)
GARCH	1.45e-03 (0.05)	3.93e-02 (15.7)	0.0715 (27.3)	0.0018 (0.22)
IGARCH	1.58e-03 (0.05)	4.20e-03 (1.67)	0.0091 (3.50)	0.0238 (2.90)
EGARCH	2.74e-03 (0.06)	2.48e-03 (0.99)*	0.0884 (33.7)	0.0007 (0.09)*
TGARCH	6.52e-03 (0.21)	7.80e-03 (3.11)	0.0051 (1.93)	0.0067 (0.82)
APARCH	1.45e-03 (0.05)*	2.94e-02 (11.7)	0.0397 (15.2)	0.0060 (0.73)
FIGARCH	7.23e-03 (0.23)	4.76e-02 (19.0)	0.0472 (18.0)	0.0044 (0.53)
FIEGARCH	1.72e-03 (0.05)	3.92e-03 (1.56)	0.0256 (9.77)	0.0029 (0.36)
FIAPARCH	1.89e-03 (0.06)	5.24e-02 (20.9)	0.0799 (30.5)	0.0016 (0.19)
CGARCH	8.20e-03 (0.26)	4.95e-02 (19.7)	0.0023 (0.87)*	0.0023 (0.28)

Table 6.22: Nigeria – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	2.34e-04 (1.00)	1.76e-03 (1.00)	0.0094 (1.00)	0.0009 (1.00)
RW	1.41e-04 (0.60)	8.03e-03 (4.56)	0.0745 (7.90)	0.0083 (9.23)
MA	6.07e-04 (2.60)	4.11e-03 (2.33)	0.0728 (7.72)	0.0684 (76.1)
SR	1.33e-03 (5.69)	5.28e-02 (30.0)	0.0918 (9.74)	0.0139 (15.4)
ES	2.88e-03 (12.3)	1.70e-02 (9.66)	0.0056 (0.59)	0.0255 (28.3)
EWMA	2.58e-04 (1.10)	1.19e-02 (6.76)	0.0022 (0.23)*	0.0548 (60.9)
GARCH	5.22e-03 (22.3)	7.26e-04 (0.41)*	0.0064 (0.68)	0.0075 (8.29)
IGARCH	1.71e-04 (0.73)	8.13e-04 (0.46)	0.0218 (2.32)	0.0028 (3.11)
EGARCH	8.40e-04 (3.60)	2.54e-02 (14.4)	0.0095 (1.00)	0.0062 (6.92)
TGARCH	5.91e-05 (0.25)*	1.71e-03 (0.97)	0.0580 (6.16)	0.0010 (1.08)
APARCH	6.39e-03 (27.3)	1.66e-03 (0.94)	0.0469 (4.98)	0.0375 (41.7)
FIGARCH	6.70e-04 (2.87)	1.69e-03 (0.96)	0.0679 (7.21)	0.0005 (0.56)*
FIEGARCH	3.01e-04 (1.29)	3.97e-02 (22.6)	0.0053 (0.56)	0.0008 (0.83)
FIAPARCH	8.03e-04 (3.44)	1.69e-03 (0.96)	0.0153 (1.63)	0.0007 (0.79)
CGARCH	6.75e-04 (2.89)	1.70e-02 (9.67)	0.0099 (1.05)	0.0053 (5.89)

Table 6.23: South Africa – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	8.98e-03 (1.00)	2.21e-03 (1.00)	0.0461 (1.00)	0.0019 (1.00)
RW	7.40e-03 (0.82)	6.77e-03 (3.06)	0.0017 (0.04)	0.0014 (0.73)*
MA	9.85e-03 (1.10)	5.15e-03 (2.33)	0.0059 (0.13)	0.0040 (2.16)
SR	6.21e-03 (0.69)	4.28e-03 (1.94)	0.0810 (1.76)	0.0028 (1.52)
ES	7.79e-03 (0.87)	2.35e-03 (1.06)	0.0016 (0.04)	0.0018 (1.00)
EWMA	5.11e-03 (0.57)	9.95e-03 (4.50)	0.0489 (1.06)	0.0026 (1.41)
GARCH	3.70e-03 (0.41)	8.98e-03 (4.06)	0.0014 (0.03)*	0.0019 (1.02)
IGARCH	4.06e-03 (0.45)	6.87e-03 (3.11)	0.0033 (0.07)	0.0015 (0.79)
EGARCH	4.83e-03 (0.54)	3.75e-03 (1.70)	0.0745 (1.62)	0.0047 (2.52)
TGARCH	4.69e-03 (0.52)	1.53e-03 (0.69)	0.0016 (0.03)	0.0015 (0.82)
APARCH	4.31e-03 (0.50)	9.80e-03 (4.43)	0.0122 (0.26)	0.0025 (1.35)
FIGARCH	2.58e-03 (0.29)*	1.43e-03 (0.65)*	0.0022 (0.05)	0.0041 (2.19)
FIEGARCH	2.98e-03 (0.33)	3.98e-03 (1.80)	0.0411 (0.89)	0.0030 (1.64)
FIAPARCH	8.45e-03 (0.94)	9.88e-03 (4.47)	0.0192 (0.42)	0.0021 (1.13)
CGARCH	4.36e-03 (0.49)	4.31e-03 (1.96)	0.0556 (1.21)	0.0123 (6.62)

Table 6.24: Tunisia – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	5.03e-03 (1.00)	3.01e-03 (1.00)	0.0182 (1.00)	0.0046 (1.00)
RW	2.28e-03 (0.45)	6.98e-03 (2.32)	0.0212 (1.17)	0.0013 (0.28)
MA	8.67e-03 (1.72)	9.19e-03 (3.05)	0.0102 (0.56)	0.0026 (0.56)
SR	2.52e-03 (0.50)	9.55e-03 (3.17)	0.0922 (5.07)	0.0029 (0.63)
ES	8.31e-03 (1.65)	5.51e-03 (1.83)	0.0052 (0.29)	0.0022 (0.49)
EWMA	7.52e-04 (0.15)*	2.78e-03 (0.92)	0.0134 (0.74)	0.0013 (0.28)*
GARCH	2.13e-03 (0.42)	2.58e-03 (0.86)	0.0033 (0.18)	0.0013 (0.29)
IGARCH	4.10e-03 (0.82)	8.68e-03 (2.88)	0.0060 (0.33)	0.0075 (1.63)
EGARCH	4.96e-03 (0.99)	3.43e-03 (1.14)	0.0845 (4.65)	0.0060 (1.30)
TGARCH	8.98e-03 (1.79)	2.65e-03 (0.88)	0.0011 (0.06)*	0.0018 (0.39)
APARCH	6.46e-03 (1.28)	2.01e-03 (0.67)*	0.0107 (0.59)	0.0092 (2.00)
FIGARCH	7.40e-03 (1.47)	3.10e-03 (1.03)	0.0053 (0.29)	0.0023 (0.50)
FIEGARCH	8.47e-03 (1.68)	2.36e-03 (0.78)	0.0026 (0.14)	0.0062 (1.34)
FIAPARCH	7.86e-03 (1.56)	2.28e-03 (0.75)	0.0172 (0.95)	0.0038 (0.83)
CGARCH	4.51e-03 (0.90)	4.20e-03 (1.40)	0.0811 (4.46)	0.0067 (1.46)

Table 6.25: Zimbabwe – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	2.65e-03 (1.00)	3.62e-04 (1.00)	7.53e-05 (1.00)	7.19e-07 (1.00)*
RW	5.63e-04 (0.21)*	2.32e-05 (0.06)*	1.51e-04 (2.00)	2.20e-06 (3.06)
MA	4.02e-03 (1.52)	2.45e-04 (0.68)	1.10e-05 (0.15)	1.28e-06 (1.79)
SR	9.07e-03 (3.42)	2.66e-05 (0.07)	1.94e-05 (0.26)	8.55e-07 (1.19)
ES	1.26e-03 (0.48)	8.20e-05 (0.23)	2.38e-06 (0.03)	3.13e-06 (4.36)
EWMA	7.46e-03 (2.81)	9.89e-04 (2.73)	2.37e-05 (0.31)	8.29e-07 (1.15)
GARCH	3.08e-03 (1.17)	3.27e-04 (0.90)	1.79e-05 (0.24)	1.19e-06 (1.65)
IGARCH	6.22e-04 (0.23)	9.64e-04 (2.66)	1.16e-05 (0.15)	2.81e-06 (3.91)
EGARCH	5.99e-03 (2.26)	4.09e-04 (1.13)	2.70e-06 (0.04)	8.24e-07 (1.14)
TGARCH	1.46e-03 (0.55)	4.37e-04 (1.21)	9.59e-05 (1.27)	1.44e-06 (2.01)
APARCH	6.32e-03 (2.39)	2.92e-04 (0.81)	3.82e-04 (5.07)	8.25e-07 (1.15)
FIGARCH	2.52e-03 (0.95)	6.82e-04 (1.89)	7.05e-05 (0.94)	9.13e-07 (1.27)
FIEGARCH	4.24e-03 (1.60)	2.37e-04 (0.65)	1.68e-06 (0.02)*	2.01e-06 (2.80)
FIAPARCH	7.42e-04 (0.28)	5.90e-04 (1.63)	1.92e-05 (0.25)	4.79e-07 (6.67)
CGARCH	9.04e-03 (3.42)	9.75e-05 (0.27)	2.33e-05 (0.31)	2.05e-06 (2.84)

Table 6.26: UK – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	7.73e-03 (1.00)	8.49e-03 (1.00)	9.16e-03 (1.00)	3.02e-03 (1.00)
RW	8.41e-03 (1.09)	9.61e-03 (1.13)	7.01e-03 (0.77)	4.52e-03 (1.50)
MA	7.65e-03 (0.99)	7.61e-03 (0.90)	2.10e-03 (0.23)	1.53e-03 (0.51)
SR	9.84e-03 (1.27)	9.87e-03 (1.16)	7.27e-03 (0.79)	1.97e-03 (0.65)
ES	7.26e-03 (0.94)	8.59e-03 (1.01)	4.86e-03 (0.05)	2.24e-03 (0.74)
EWMA	6.97e-03 (0.90)	4.73e-03 (0.56)	7.52e-04 (0.08)	1.28e-03 (0.42)
GARCH	5.47e-03 (0.71)	6.45e-03 (0.76)	2.35e-04 (0.03)*	1.93e-03 (0.64)
IGARCH	5.33e-03 (0.69)	7.10e-03 (0.84)	2.97e-04 (0.03)	3.05e-03 (1.01)
EGARCH	6.91e-03 (0.89)	7.42e-03 (0.87)	1.85e-03 (0.20)	8.81e-03 (2.91)
TGARCH	7.76e-03 (1.00)	4.28e-03 (0.50)	5.38e-03 (0.59)	4.46e-03 (1.48)
APARCH	4.85e-03 (0.63)*	3.82e-03 (0.45)	5.32e-03 (0.58)	3.74e-04 (0.12)
FIGARCH	7.33e-03 (0.95)	8.43e-03 (0.99)	2.62e-04 (0.03)	4.92e-03 (1.63)
FIEGARCH	6.25e-03 (0.81)	3.47e-03 (0.41)*	9.83e-03 (1.07)	2.63e-03 (0.87)
FIAPARCH	7.13e-03 (0.92)	7.35e-03 (0.87)	7.62e-03 (0.83)	2.10e-04 (0.07)*
CGARCH	7.25e-03 (0.94)	3.71e-03 (0.44)	6.19e-04 (0.07)	3.85e-03 (1.27)

Table 6:27: US – Monthly Frequency				
	Forecast Error Statistic		Mean Mixed Forecast Error Statistic	
Model	MAE	RMSE	MME(*U*)	MME(*O*)
HA	4.27e-03 (1.00)	5.89e-03 (1.00)	3.41e-02 (1.00)	1.21e-03 (1.00)
RW	6.70e-04 (1.56)	6.98e-03 (1.19)	2.09e-03 (0.06)*	8.23e-03 (6.80)
MA	3.13e-03 (0.73)	9.53e-03 (1.62)	5.20e-03 (0.15)	1.69e-03 (1.40)
SR	1.01e-03 (0.24)	9.92e-03 (1.68)	9.88e-02 (2.90)	7.12e-04 (0.59)
ES	1.43e-03 (0.33)	5.18e-03 (0.88)	2.23e-03 (0.07)	8.66e-04 (0.72)
EWMA	1.58e-03 (0.37)	5.05e-03 (0.86)	5.51e-03 (0.16)	1.95e-04 (0.16)
GARCH	6.67e-04 (0.16)	4.82e-03 (0.82)	8.81e-03 (0.26)	1.18e-04 (0.10)*
IGARCH	3.91e-03 (0.92)	8.70e-03 (1.48)	7.33e-03 (0.21)	5.52e-03 (4.56)
EGARCH	6.60e-04 (0.15)	4.88e-03 (0.83)	4.10e-02 (1.20)	2.43e-04 (0.20)
TGARCH	3.81e-03 (0.89)	4.96e-03 (0.84)	7.70e-03 (0.23)	1.20e-03 (0.99)
APARCH	9.17e-04 (0.21)	4.63e-03 (0.79)	4.74e-02 (1.39)	1.70e-03 (1.41)
FIGARCH	6.21e-04 (0.15)	4.15e-03 (0.70)*	6.41e-03 (0.19)	1.79e-03 (1.48)
FIEGARCH	5.16e-04 (0.12)*	5.02e-03 (0.85)	2.39e-03 (0.07)	8.34e-04 (0.69)
FIAPARCH	6.30e-04 (0.15)	4.46e-03 (0.76)	1.44e-02 (0.42)	2.22e-03 (1.84)
CGARCH	6.63e-04 (1.55)	5.63e-03 (0.96)	7.23e-03 (0.21)	1.54e-03 (1.28)

Table 6.28: Summary of Best Performing Models at Monthly Frequency				
	Model		Model	
	MAE	RMSE	MME(*U*)	MME(*O*)
Botswana	CGARCH	CGARCH	GARCH	APARCH
Egypt	FIAPARCH	FIGARCH	FIAPARCH	EGARCH
Ghana	RW	CGARCH	FIGARCH	RW
Kenya	CGARCH	GARCH	HA	CGARCH
Mauritius	GARCH	FIAPARCH	EGARCH	GARCH
Morocco	CGARCH	TGARCH	GARCH	MA
Namibia	APARCH	EGARCH	CGARCH	EGARCH
Nigeria	TGARCH	GARCH	EWMA	FIGARCH
South Africa	FIGARCH	FIGARCH	GARCH	RW
Tunisia	EWMA	APARCH	TGARCH	EWMA
Zimbabwe	RW	RW	FIEGARCH	HA
UK	APARCH	FIEGARCH	GARCH	FIAPARCH
US	FIEGARCH	FIGARCH	RW	GARCH

Table 6.29: Results of the Tests of Superior Predictive Ability		
	Daily	Monthly
Country	SPA *p*-values	SPA *p*-values
Botswana	0.0297 (0.00, 0.05)	0.1183 (0.05, 0.14)
Egypt	0.0329 (0.01, 0.09)	0.0559 (0.02, 0.98)
Ghana	0.0105 (0.00, 0.05)	0.0326 (0.01, 0.07)
Kenya	0.2064 (0.01 0.72)	0.2730 (0.15, 0.38)
Mauritius	0.0088 (0.00, 0.10)	0.0117 (0.00, 0.05)
Morocco	0.6185 (0.28, 0.87)	0.1054 (0.07, 0.19)
Namibia	0.0366 (0.01, 0.08)	0.0091 (0.00, 0.18)
Nigeria	0.0348 (0.01, 0.08)	0.6385 (0.33, 0.90)
South Africa	0.0174 (0.00, 0.06)	0.1293 (0.07, 0.32)
Tunisia	0.0227 (0.00, 0.06)	0.0267 (0.01, 0.10)
Zimbabwe	0.0402 (0.02, 0.11)	0.0082 (0.00, 0.05)
UK	0.0386 (0.01, 0.08)	0.0439 (0.01, 0.10)
US	0.0429 (0.02, 0.10)	0.0525 (0.03, 0.12)

Notes: Entries are *p*-values for the Hansen (2005) test of superior predictive ability. A significant *p*-value indicates that the benchmark model can be outperformed by a competing model on the basis of a standard loss function, the mean absolute deviation (MAD). The numbers in parenthesis are the lower and upper bounds for the *p*-values.

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7 Value-at-Risk Estimation in African Stock Markets: Comparative Evidence from Symmetric, Asymmetric and Long Memory GARCH Models

7.1 Introduction

African stock markets (ASMs) have experienced rapid and substantial growth as investors take advantage of the opportunity to diversify their portfolios internationally in search of the highest potential returns to their investments. These developments have motivated empirical analysis into various aspects of stock return behaviour in these markets. An important and topical area of research concerns the calculation of value-at-risk (VaR) in financial markets. This methodology is widely used by financial institutions and regulatory agencies to measure, monitor and manage market risk (Jorion, 2007) and is indeed the central tenet of the market risk amendment to the Basle Accord (BIS, 1996). Furthermore, VaR estimation underlies a range of risk controls including margin requirements and position limits (IMF, 2007). In addition, economic risk capital (ERC) models are based on VaR principles, but incorporate a wider set of risks (e.g., credit, liquidity and operational risks) assessed at higher confidence intervals.[1]

Against this background, recent empirical research focuses on the comparison of the performance of alternative volatility forecasting methods under VaR modelling given the precepts of the Basle Committee adequacy criteria. While most of the research in this area has concentrated on the world's major stock markets and some of the more

[1] ERC models measure the amount of capital required to absorb losses from the occurrence of rare events over long time horizons. For instance, typical ERC models use confidence intervals of up to 99.97 percent (compared to 95 to 99 percent for VaR models) and horizons of up to one year (relative to 10 days for standard VaR models).

recent studies cover markets in Asia (e.g., McMillan and Speight, 2007; So and Yu, 2006). These issues have not been examined so far for ASMs, therefore this paper attempts to fill the gap, and in so doing provide new insights into these markets and complement existing studies on other emerging markets, by addressing the following questions.

First, this paper extends the research by considering stock index return data for eleven ASMs for which there appear to be no or limited evidence with respect to the evaluation of a spectrum of volatility forecasting models in the computation of VaR estimates such that we are able to evaluate and then select the best model in terms of minimising the number of exceptions (i.e., days when the VaR is insufficient to cover actual portfolio losses). In addition, we employ data from the US and UK for comparative purposes. Second, we compare the performance of the various GARCH-based models in providing accurate VaR measures. Among others these include symmetric and asymmetric GARCH models, their long memory extensions and models incorporating multiple volatility components. The base model for our analysis is the RiskMetrics model as it has become a widely accepted quantitative measure of market risk within many financial institutions. Third, we perform diagnostic tests in order to verify the adequacy of the VaR estimates. In particular, we implement the Kupiec LM test (1995) and the dynamic quartile (DQ) test of Engle and Manganelli (2004) which are tests of unconditional and conditional VaR accuracy, respectively. The former examines whether there is an excessive number of exceptions and the latter tests for autocorrelation in the sequence of exceptions.

While, a number of recent studies compare and contrast the performance of a variety of volatility forecasting models in the computation of VaR estimates in a variety of contexts including Alexander and Leigh (1997), Jackson *et al* (1998) Brooks and Persand (2000a, 2000b) Brooks *et al* (2000), Longin (2000), Vlaar (2000), Lopez and Walter (2001), Berkowitz (2001), Brooks and Persand (2003), So and Yu (2006) and McMillan and Speight (2007). However, empirical analysis of VaR estimation in emerging markets appear to be limited especially in the context of Basle Committee rules; indeed, for ASMs the extant literature points to a dearth of research.

Nonetheless empirical analysis of other emerging equity markets is instructive. For instance, Brooks and Persand (2003) examine the impact of asymmetric effects in equity data on the evaluation and accuracy of VaR estimates on the stock markets of five Southeast Asian economies and the S&P 500 index for comparative purposes. The models they consider include the RiskMetrics, semi-variance, GARCH, TGARCH, and EGARCH models and the multivariate extensions of the GARCH-type models considered. Their results suggest that the incorporation of an asymmetric effect in the modelling framework generates improved volatility forecasts which in turn produce improved VaR estimates. In particular, the semi-variance model, which allows for asymmetry, delivers the most stable and reliable method for calculating VaR. So and Yu (2006) also examine the performance of a variety of GARCH models including two long memory (or fractionally integrated) GARCH models in the context of VaR estimation. They examine both long and short investment positions on nine stock market indices of Asian economies and two US equity indices (NASDAQ and S&P 500) and UK (FTSE 100) in order to assess the accuracy of each model in estimating VaR at various confidence levels. Their results show that both stationary and fractionally integrated GARCH models are superior to the RiskMetrics model in calculating VaR at the 99 percent probability level. Furthermore, the authors show the existence of asymmetric behaviour in the equity data and that *t*-error models provide more accurate VaR estimates than normal-error models for long positions, but not for short positions. In a similar study, McMillan and Speight (2007) examine eight emerging stock markets in Asia, in addition to US and UK benchmark comparators in order to evaluate the performance of nine volatility forecasting methods under VaR modelling in the context of the Basle Committee regulatory rules. The authors broaden the class of GARCH processes to include asymmetric and long memory features. Their results indicate that models which include both asymmetric and long memory characteristics provide improved VaR estimates on both an in-sample and out-of-sample basis.

The analysis adopted in this study is similar in spirit to Brooks and Persand (2003), So and Yu (2006) and McMillan and Speight (2007). The innovation contained in this study relates to the application of a broader array of GARCH models than hitherto the case; the application of diagnostic checks in order to assess the adequacy of the estimated models and examine how well the fitted model accords with the observed data; and we focus on ASMs where these issues appear not to have been addressed.

To summarise our results from the outset, we find that both symmetric GARCH and RiskMetrics models are generally outperformed by the alternative models. Beyond this, our conclusions are diverse and for a few markets sensitive to the choice of the stipulated probability level. However, for nine of the ASMs considered, we find that the following models provide robust VaR measure in terms of minimising the number of exceptions across all probability levels we analysed: FIEGARCH (Botswana and Namibia), EGARCH (Ghana), IGARCH (Egypt), APARCH (Morocco) and the CGARCH (Kenya, Nigeria, Tunisia and Zimbabwe). In general, we find that allowing for multiple volatility components (i.e., separation of long run and short volatility effects), asymmetric and long memory behaviour are important for delivering accurate volatility forecasts and hence more precise VaR estimates. Finally, these results may be of interest to regulators and investors with respect to delivering more effective risk assessments, and may potentially serve as an integral part of an early warning system in a risk management framework. In particular, these findings may enhance supervisory forebearance and allow risk managers to institute appropriate response mechanisms to avoid (or deal with) the consequences of excessive exposure (to market risk) and the punitive measures they may entail such as prohibition from using an internal risk model and/or increases in their capital requirements.

7.2 Calculating and Evaluating Measures of VaR

VaR is a widely used measure to capture the exposure of a portfolio to market risk. In addition, VaR provides a mechanism for investors to value their market exposure in terms of risk, thereby providing them with a basis to allocate risk more efficiently (Engle and Manganelli, 2004). More formally, it is defined as the maximum potential loss for a given portfolio within a prescribed holding period at a specified probability (Jorion, 2007). In other words, VaR at a given confidence level, α, is defined as the maximum loss expected to occur with probability, $p = (1-\alpha)$.[2] For instance, a one-day estimated VaR of USD10 million, at a confidence level of 95 percent implies that the maximum market loss a firm or portfolio could expect on 19 out of 20 days is USD10 million.[3] The conventional holding period adopted in VaR estimation ranges from one to 10 days (depending on how long it takes to close or hedge a position). Indeed, the Basle regulatory framework stipulates that for the purpose of determining regulatory market risk capital, a 99 percent (one tailed) confidence interval be used over a 10-day horizon. The most important assumption underlying VaR estimation relates to the distribution of stock returns. As such, it is important to recognise that VaR is appropriately utilised to quantify portfolio risks under typical (or normal) market conditions (IMF, 2007).[4] Indeed, for this reason, the preponderance of previous academic literature and professional practice make use of the assumption of asymptotic normality to characterise the distribution of expected price changes by appealling to the central limit theorem. In addition, this parametric approach to stock index return VaR estimates has oftentimes been shown to be more preferable even in environments where the assumption of normality does not appear to be tenable (e.g., Jorion 1995a, b).

[2] Related to the concept of VaR is the notion of the minimum capital risk requirement which is defined as the minimum value of capital needed to offset the bulk of expected future losses; with the exception of a specified and small proportion of anticipated losses.

[3] This may also be interpreted to mean that a one-day estimated VaR of USD10 million, at a confidence level of 95 percent indicates that a loss of at least USD10 million is expected on 5 trading days out of 100.

[4] While VaR is a widely used measure of market risk within most financial institutions its disadvantages are well known. First, the VaR methodology may not be appropriate for asset-return characterised by 'fat' and 'superfat' tails For example, these attributes are manifested by portfolios having stocks that are mostly non-actively traded but occasionally jump in price (e.g., Danielsson *et al*, 2006). As a result, recent academic enquiry has focused on improving VaR estimation through the use of different distributional assumptions and extreme value theory (e.g., Neftci, 2000 and Brooks and Persand, 2002). Second, it is widely recognised that standard VaR estimation does not adequately quantify market risks under atypical market conditions. In order to counteract these deficiencies, VaR measures are typically used in conjunction with a variety of stress tests (which encompass scenario or sensitivity analysis or both) (BIS, 2005; IMF, 2007).

The VaR paradigm was first used by major financial institutions in the late 1980s. This risk management system was made widely available by the introduction of JPMorgan's RiskMetrics VaR methodology in 1994. Indeed, VaR is now widely used as an internal risk management tool by financial institutions and as a regulatory measure of risk exposure (ECB, 2007). Furthermore, Krause (2003) points out that the VaR methodology has evolved from a mechanism to measure risk to an important ingredient in active risk management. In addition, the VaR method is the cornerstone of the 1996 market risk amendment to the Basle Accord (1996). In particular, this framework stipulates that VaR estimates from a financial institution form the basis for a financial institutions market risk regulatory capital requirement.

The Basle Accord prescribes the VaR method in order that financial institutions can meet the capital requirements to cover the market risk they incur in the process of their daily business operations. Under this framework, operational evaluation takes the form of backtesting volatility forecasts and exception reporting. In particular, the BIS guidelines stipulate that VaR be computed as the higher of the preceding daily calculation or VaR, or the average estimated daily VaR over 60 business days subject to a scaling factor of between 3 and 4, with the exact numeral depending on the supervisory authorities appraisal of the quality and accuracy of the financial institution's approach to VaR estimation. Under this approach, the validity of a financial institutions internally modelled VaR is evaluated by performing a number of validation exercises (referred to as backtesting) are performed in order to check that the number of loss exceptions to the estimated VaR is consistent with the model's intended construction. For example, at a confidence level of 95 percent, losses exceeding the one-day VaR should occur on around five days out of 100.

Furthermore, the Basle Accord stipulates that for the purpose of calculating regulatory market risk capital it is required that VaR estimates be calculated at the 99 percent probability level using daily data over a minimum sample period of at least one business year (equivalent to 250 trading days) and that these estimates be updated at least every quarter (i.e., 60 trading days).

A standard market practice is to evaluate VaR through factor models, such as RiskMetrics (1996), which entails the multiplication of forecast volatility by the value of the portfolio and (given the assumption of conditional normality) by the appropriate standard normal deviate. While there are various approaches to calculate VaR it is usual to split them into two broad categories, namely: full and local valuation methods. Their applicability depends on a number of conditions including the existence of nonlinear payoffs, dynamic trading and long holding periods (e.g., Jorion, 2007). While, the former is potentially the most accurate since it requires less stringent assumptions, it is typically data intensive, difficult validate through backtesting, and hard to communicate to investors and supervisory authorities (ECB, 2007; IMF, 2007). Accordingly, recent research has therefore focused on improving VaR through the use of alternative distributional assumptions and extreme value theory (e.g., Bervas, 2006 and ECB, 2007).[5] On the other hand, Bams *et al* (2005) compare the performance of these studies against more standard techniques; however, their results suggests that the use of greater complexity in tail modelling produces less accurate VaR estimates or more uncertainty in VaR estimation given the paucity of underlying tail observations for precise estimates.

[5] The ECB (2007) provide a detailed review of alternatives to VaR and Bervas (2006) presents an overview of recent work on VaR measures that incorporate liquidity risk (“L-VaRs”).

Against this background, local valuation methods may be preferred. In particular, the archetype is represented by the delta-normal (or variance-covariance) model in which returns are jointly normally distributed. The normality assumption makes the VaR computation convenient since only the mean and variance-covariance matrix of returns are necessary in order to calculate the maximal loss at a given level of statistical confidence. Accordingly, the delta-normal VaR is given by

$$VaR = N_{\alpha} h^{f} 3V \tag{7.1}$$

where h^{f} is the forecast of volatility, V is the value of the portfolio, the scaling factor of 3 is the minimum regulatory Basle multiplicative factor and N_{α} is the appropriate standard normal deviate. While, financial institutions have been advised by the Basle Committee to apply a 99 percent confidence level to determine the minimum regulatory capital needed to protect against market risk; however, in most risk management settings and empirical studies the statistical confidence is usually set at between 95 and 99 percent – the higher the level, the more cautious the measure. Therefore, in order that our results are consistent with the requirements of the Basle Accord and in line with previous empirical analysis we also focus on the 99 percent, 97.5percent and 95 percent probability levels (which means that N_{α} = 2.326, 1.96 and 1.645, respectively).

7.3 Volatility Modelling, Forecasting and Diagnostic Testing

The accuracy of the generated out-of-sample forecasts is key for producing accurate VaR estimates. As such we compare the performance of a variety of models in order to ascertain which volatility forecasting models produce the most accurate VaR estimates. In particular, we examine which volatility forecasting technique delivers the minimum number of exceptions and hence more precise VaR estimates. In this endeavour, operational evaluation takes the form of backtesting volatility following BIS guidelines. In our estimation strategy we follow previous studies by using the five most recent years of data (where data permit) to calculate volatility of the series, which is then used as the forecast volatility over the next evaluation sample period, which the BIS prescribe to 60 days. The sample period is then rolled forward by another 60 observations and the volatility measure again updated, and, this process is repeated until the entire sample is considered. The base model for our analysis is the RiskMetrics model introduced by JPMorgan (1994) since it is a widely used criterion in many risk management systems.

The volatility modelling and forecasting methodology used in this chapter is described in section 6.4 (Volatility Modelling and Forecasting) in the previous chapter. In particular, the ten models used in this analysis are presented in the following equations in chapter 6: RiskMetrics model (equation 6.6) in subsection 6.4.1; the GARCH and IGARCH models are presented by equation 6.7 and 6.8, respectively in subsection 6.4.2. The asymmetric models represented by the TGARCH (6.9), EGARCH (6.10) and APARCH (6.11) are outlined in subsection

6.4.3. The long memory models utilised are the: FIGARCH (6.12); FIEGARCH (6.13) and FIAPARCH (6.14) and the CGARCH (6.15) presented in subsection 6.4.4.

Diagnostics

In econometric analysis, diagnostic checks are required to assess the adequacy of the model and to examine how well the fitted model accords with the observed data. If the model is misspecified, then it can yield false inference; for this reason model checking is crucial to statistical analysis. In order to analyse the performance of the various models in providing VaR estimates we have examined the number of exceptions (i.e., the frequency in which the actual loss exceeds the estimated VaR) generated by each volatility forecasting model. In particular, we have evaluated forecasting performance in terms of which model produces the minimum number of exceptions as a criteria to determine which model delivers the most accurate VaR measure. In addition, to further ascertain the accuracy of our results we perform the Kupiec (1995) test which determines the accuracy of VaR estimates by examining the equality of the empirical failure rate to the specified statistical level. In other words, if the VaR model is correctly specified then the number of exceptions must occur at the specified rate. Under the null hypothesis, $H_0 : p = \hat{p}$ where p is the probability of failure on any of the independent trials and $\hat{p}$ is the probability of failure under the null hypothesis. The likelihood ratio test of the null hypothesis is given by

$$-2\log\left[(1-\hat{p})^{n-x}(\hat{p})^{x}\right]+2\log\left[\left(1-\frac{x}{n}\right)^{n-x}\left(\frac{x}{n}\right)^{x}\right] \tag{7.2}$$

where n is the sample size and x is the number of failures in the sample. This test has a chi-square distribution with one degree of freedom.

To further verify the accuracy of our VaR estimates we also implement the Dynamic Quartile test proposed by Engle and Manganelli (2004). This test assesses whether the exceptions are independent and identically distributed. This property is verified by means of evaluating the performance of the function

$$\mathrm{Hit}_k = \mathrm{I}\,(\mathrm{r}_k < -\mathrm{VaR}_k) - \alpha \qquad (7.3)$$

where Hit_k assumes the value (1-α) everytime returns, r_k , are less than the VaR quantile and -α otherwise and $E(Hit_k) = 0$, i.e., the expected value of the function is zero. In particular, Hit_k should be uncorrelated with its own lagged values and with VaR_k and must have an expected value of zero. If these assumptions on the behaviour of the hit sequence Hit_k hold then this means that; first, the hits are uncorrelated (i.e., no autocorrelation in the VaR exceptions); second, no measurement error; and third, the model will capture the specified proportion of exceptions.[6] To conduct this test the hit sequence Hit_k is regressed on its lagged values and the current value of VaR. In particular, the DQ test statistic is calculated as $DQ = \hat{\beta}' X' X \hat{\beta} / \alpha(1-\alpha)$ where X is a vector of independent (or explanatory) variables and $\hat{\beta}$ the OLS estimates. The DQ test is χ^2 distributed with the degrees of freedom corresponding to the number of parameters.

[6] This third criteria is the same property evaluated by the Kupiec test.

7.4. Data and VaR Evaluation Results

As previously described in Chapter 3 (Table 3.1 in particular) the sample sizes vary considerably. For the markets with relatively longer time series (i.e., Egypt, Kenya, Nigeria, South Africa, Tunisia, Zimbabwe and benchmark comparators) we use the first 5 years of data for initial model parameter estimation and the remaining sample observations are used for the construction and testing of VaR measures under the volatility models described in chapter 6 (equation 6.6 to 6.15). Model parameters are calculated using maximum likelihood estimation methods and the estimation package is G@RCH 5.2 Ox developed by Laurent and Peters (2005). For all other countries (since the samples are smaller) we use the first 2 year period to derive initial model parameters and the remaining for construction and evaluation of VaR measures. [7]

In order to estimate VaR measures we use the rolling window (or updating) procedure enunciated in the Basle framework and the volatility forecasting models indicated in section 7.3 (and described in chapter 6). In particular, initial volatility forecasts and VaR measures are constructed over a 60 trading day interval; then, the initial estimation sample is updated every 60 observations before the next set of volatility forecasts are produced. This method produces a number of sub-samples of 60 days over which VaR performance is evaluated. This assessment is performed through appraisal of the in-sample VaR failure rates associated with VaR measures constructed using the fitted value of the volatility measure from the estimated models, and the out-of-sample VaR failure rates associated with VaR measures constructed

[7] While a variety of sample period lengths for parameter estimation have been recommended in the literature (e.g., Kupiec, 1995 and Berkowitz, 2001) we note that in this study we employ data in excess of the minimum requirements under the Basle Accord stipulation of at least one year.

using the forecast values of the relevant volatility measures. Our focus on the out-of-sample performance stems from risk management processes because the risk manager typically uses parameters obtained from an already observed sample in order to evaluate the risks associated with current and future random movements in risk factors (Neftci, 2000; McMillan and Speight, 2007). In particular, comparing the performance of the competing volatility forecasting models outside the sample used to estimate the underlying parameters in order to analyse which method delivers most accurate VaR estimates. This study therefore concentrates on stock return forecastability (i.e., out-of-sample estimation) as a criterion of model selection rather than on stock return predictability (i.e., in-sample estimation).

Tables 7.1 to 7.3 reports the in-sample VaR test results for each of the ten volatility forecasting models for each of the thirteen markets (i.e., eleven ASMs and two benchmark comparators) in terms of the percentage number of days for which there was an exceedance of the VaR estimate in the backtest over the respective sub-samples, in the sense that the calculated VaR would have been insufficient to cover trading losses. These tests are conducted at three probability levels. In particular, we examine the 99 percent probability level stipulated by the Basle Accord, and the lower probability levels of 97.5 percent and 95 percent that have been evaluated in previous research. Our findings are very diverse and highlight that in many of the stock markets considered the forecasting model that minimises the percentage number of daily VaR exceedances is sensitive to the specification of the probability level. When the Basle Committee rules are applied (i.e., the 99 percent probability level) our results indicate that the widely used RiskMetrics method provides the exceedance-minimising method for only South Africa. The TGARCH is preferred in both

Botswana and Mauritius (although in the case of Mauritius the IGARCH performs as well as the TGARCH at the 99 percent level). The IGARCH model is preferred in both Egypt and Kenya (although the CGARCH and FIAPARCH perform equally well in Kenya). In the case of Ghana and Namibia the FIEGARCH delivers the exceedance-minimising forecasts. For Morocco, Nigeria, Tunisia and the UK our results suggest that the APARCH model provides the exceedance-minimising method in these stock markets. In addition, this result also indicates that the APARCH is the most successful model in terms of providing the exceedance-minimising method in our sample. In the case of Nigeria, the CGARCH performs just as well as the APARCH at the 99 percent probability level. The CGARCH model is preferred in Zimbabwe (and as already mentioned is equally preferred in Kenya and Nigeria where it performs just as well as a variety of other models). Morocco and Zimbabwe are the only economies where a single model, the APARCH and CGARCH, respectively, are preferred across the three probability levels we have analysed. Long memory models are preferred in the US (i.e., the FIGARCH) and as already mentioned long memory models are preferred in Ghana (FIEGARCH), Kenya (FIAPARCH and CGARCH along with the IGARCH), and Namibia (FIEGARCH). At the 97.5 and 95 percent probability levels our results are equally mixed. The standard RiskMetrics is preferred in Egypt at both the 97.5 and 95 percent probability levels. Beyond this, the in-sample VaR failure results, do not lend much support to the viability of the RiskMetrics model and our results show that this method is outperformed by a range of GARCH formulations.

Table 7.1 VaR Failure Rates – in-Sample

	Botswana			Egypt		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0285	0.0327	0.0425	0.0116	0.0207*	0.0328*
GARCH	0.0226	0.0331	0.0463	0.0093	0.0172	0.0289
CGARCH	0.1283	0.1301	0.1386	0.0098	0.0233	0.0384
TGARCH	0.0023*	0.0023*	0.0127	0.0177	0.0265	0.0412
EGARCH	0.0169	0.0307	0.0423	0.0109	0.0191	0.0355
APARCH	0.0193	0.0230	0.0367	0.0139	0.0217	0.0337
IGARCH	0.0272	0.0293	0.0389	0.0114*	0.0229	0.0337
FIGARCH	0.0308	0.0366	0.0392	0.0149	0.0247	0.0344
FIEGARCH	0.0028	0.0069	0.0122	0.0122	0.0230	0.0357
FIAPARCH	0.0023*	0.0070	0.0105*	0.0117	0.0227	0.0383
	Ghana			Kenya		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0088	0.0153	0.0179	0.0115	0.0248	0.0391
GARCH	0.0056	0.0072	0.0107	0.0113	0.0244	0.0353
CGARCH	0.0811	0.0894	0.0923	0.0088*	0.0175	0.0282*
TGARCH	0.0851	0.0902	0.0964	0.0115	0.0289	0.0448
EGARCH	0.0018	0.0025	0.0032*	0.0107	0.0294	0.0423
APARCH	0.0025	0.0043	0.0051	0.0102	0.0268	0.0327
IGARCH	0.0043	0.0055	0.0085	0.0088*	0.0167*	0.0291
FIGARCH	0.0065	0.0108	0.0123	0.0097	0.0167*	0.0291
FIEGARCH	0.0017*	0.0023*	0.0042	0.0107	0.0228	0.0383
FIAPARCH	0.0092	0.0116	0.0153	0.0088*	0.0245	0.0365
	Mauritius			Morocco		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0164	0.0256	0.0308	0.0222	0.0349	0.0409
GARCH	0.0277	0.0307	0.0415	0.0222	0.0286	0.0413
CGARCH	0.0140	0.0204	0.0254	0.0156	0.0247	0.0373
TGARCH	0.0133*	0.0312	0.0419	0.0230	0.0286	0.0390
EGARCH	0.0144	0.0246	0.0339	0.0230	0.0289	0.0410
APARCH	0.0146	0.0213	0.0317	0.0012*	0.0025*	0.0066*
IGARCH	0.0133*	0.0202	0.0259	0.0157	0.0244	0.0357
FIGARCH	0.0169	0.0202	0.0251*	0.0169	0.0253	0.0329
FIEGARCH	0.0173	0.0214	0.0306	0.0195	0.0268	0.0390
FIAPARCH	0.0148	0.0177*	0.0260	0.0158	0.0274	0.0321

Note: Asterisks indicate preferred models

Table 7.2 VaR Failure Rates – in-Sample

	Namibia			Nigeria		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0158	0.0224	0.0381	0.0096	0.0117	0.0216
GARCH	0.0039	0.0070*	0.0103*	0.0102	0.0122	0.0314
CGARCH	0.0052	0.0078	0.0099	0.0072*	0.0129	0.0209
TGARCH	0.0136	0.0227	0.0422	0.0115	0.0155	0.0306
EGARCH	0.0067	0.0089	0.0106	0.0119	0.0171	0.0232
APARCH	0.0043	0.0133	0.0278	0.0072*	0.0114	0.0255
IGARCH	0.0068	0.0116	0.0208	0.0103	0.0178	0.0299
FIGARCH	0.0141	0.0335	0.0444*	0.0088	0.0172	0.0227
FIEGARCH	0.0020*	0.0078	0.0103	0.0102	0.0183	0.0206*
FIAPARCH	0.0104	0.0169	0.0224	0.0131	0.0191	0.0258
	South Africa			Tunisia		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0073*	0.0157	0.0298	0.0161	0.0294	0.0486
GARCH	0.0088	0.0265	0.0282	0.0082	0.0251	0.0332
CGARCH	0.0127	0.0208	0.0342	0.0088	0.0203	0.0328
TGARCH	0.0253	0.0315	0.0406	0.0124	0.0231	0.0453
EGARCH	0.0193	0.0242	0.0227	0.0123	0.0231	0.0320
APARCH	0.0077	0.0143*	0.0217	0.0079*	0.0186	0.0312
IGARCH	0.0077	0.0175	0.0217	0.0124	0.0188	0.0404
FIGARCH	0.0135	0.0164	0.0293	0.0101	0.0222	0.0395
FIEGARCH	0.0105	0.0145	0.0187*	0.0162	0.0213	0.0287
FIAPARCH	0.0133	0.0205	0.0254	0.0088	0.0145*	0.0271*

	Zimbabwe		
Model	99%	97.5%	95%
EWMA/RM	0.0153	0.0232	0.0351
GARCH	0.0089	0.0136	0.0273
CGARCH	0.0074*	0.0122*	0.0190*
TGARCH	0.0135	0.0211	0.0343
EGARCH	0.0156	0.0210	0.0255
APARCH	0.0104	0.0164	0.0216
IGARCH	0.0122	0.0176	0.0265
FIGARCH	0.0092	0.0208	0.0212
FIEGARCH	0.0104	0.0173	0.0283
FIAPARCH	0.0092	0.0177	0.0216

Note: Asterisks indicate preferred models

Table 7.3 VaR Failure Rates – in-Sample

	US			UK		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0195	0.0327	0.0544	0.0188	0.0359	0.0603
GARCH	0.0097	0.0286	0.0461	0.0106	0.0227	0.0386
CGARCH	0.0188	0.0378	0.0508	0.0202	0.0268	0.0440
TGARCH	0.0160	0.0252	0.0447	0.0205	0.0318	0.0369
EGARCH	0.0094	0.0212*	0.0386	0.0096	0.0193	0.0372
APARCH	0.0107	0.0220	0.0395	0.0095*	0.0178	0.0266
IGARCH	0.0138	0.0286	0.0445	0.0107	0.0251	0.0313
FIGARCH	0.0091*	0.0205	0.0274*	0.0107	0.0255	0.0307
FIEGARCH	0.0107	0.0217	0.0274*	0.0135	0.0203	0.0266
FIAPARCH	0.0112	0.0212*	0.0332	0.0102	0.0191*	0.0272*

Note: Asterisks indicate preferred models

Table 7.4 to 7.6 presents the out-of-sample VaR failure rates for each of the ten volatility forecasting models for each of the ASMs and benchmark comparators. As discussed in Section 7.2, these tests are performed at the 99% probability level as stipulated under the Basle rules. Also, in line with previous research we also consider performance at the 97.5% and 95% probability levels, respectively. First, our results show that the RiskMetrics and standard GARCH models are generally outperformed by the alternative models, implying that forecasts from these models are outperformed by forecasts from more elaborate models. Second, our results show that for nine ASMs a single model provides the most accurate VaR measures in terms of minimising the number of exceptions across the three probability levels that we have considered. In particular, we find that the component-GARCH (CGARCH) specification produces the most accurate VaR measures for Kenya, Nigeria, Tunisia and Zimbabwe at all three probability levels of 95%, 97.5% and 99%. For Tunisia, the APARCH performs just as well as the CGARCH in terms of delivering accurate VaR performance. For Botswana and Namibia we find that the fractionally integrated EGARCH (FIEGARCH) model delivers the most accurate VaR across all three confidence levels. The Integrated GARCH (IGARCH), exponential GARCH (EGARCH) and the Asymmetric Power ARCH

(APARCH) consistently provide the most precise VaR estimates for Egypt, Ghana and Morocco, across all three confidence levels we have examined. In contrast, for South Africa and Mauritius we find that (volatility forecasting) model performance is sensitive to the specified probability level. For instance, in both markets our results show that the IGARCH model provides the most accurate VaR measure at the 99% probability level. However, at the 97.5% probability level, both the EGARCH and FIAPARCH models are preferred in South Africa; while, in the case of Mauritius both the CGARCH are FIAPARCH are preferred. Meanwhile, at the 95% probability level, our results indicate that in South Africa and Mauritius, the FIEGARCH and CGARCH models, respectively, are preferred. Finally, for the benchmark comparators – the US and UK - we find that long memory models are preferred. For example, in the UK the FIAPARCH is preferred at both the 99% and 97.5% confidence levels; while in the US, the FIGARCH and FIAPARCH produce the best VaR estimates. At the 95% probability level, our results indicate that FIAPARCH and FIGARCH are specifications generate the most accurate forecasts and hence superior VaR estimates, for US and UK, respectively.

Table 7.4 VaR Failure Rates – Out-of-Sample

	Botswana			Egypt		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0277	0.0333	0.0468	0.0120	0.0230	0.0373
GARCH	0.0222	0.0301	0.0404	0.0134	0.0198	0.0373
CGARCH	0.1142	0.1174	0.1214	0.0103	0.0230	0.0380
TGARCH	0.0251	0.0366	0.0422	0.0121	0.0230	0.0366
EGARCH	0.0214	0.0325	0.0460	0.0119	0.0206	0.0373
APARCH	0.0261	0.0357	0.0484	0.0126	0.0214	0.0349
IGARCH	0.0206	0.0261	0.0404	0.0119*	0.0190*	0.0341*
FIGARCH	0.0246	0.0325	0.0380	0.0134	0.0222	0.0404
FIEGARCH	0.0031*	0.0076*	0.0111*	0.0119*	0.0238	0.0349
FIAPARCH	0.0034	0.0079	0.0111*	0.0119*	0.0222	0.0388
	Ghana			Kenya		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0095	0.0142	0.0190	0.0119	0.0261	0.0436
GARCH	0.0071	0.0087	0.0126	0.0119	0.0253	0.0365
CGARCH	0.0888	0.0904	0.0944	0.0103*	0.0182*	0.0293*
TGARCH	0.0107	0.0159	0.0183	0.0137	0.0241	0.0371
EGARCH	0.0015*	0.0023*	0.0034*	0.0150	0.0277	0.0444
APARCH	0.0031	0.0039	0.0047	0.0134	0.0253	0.0436
IGARCH	0.0047	0.0071	0.0087	0.0111	0.0206	0.0357
FIGARCH	0.0071	0.0095	0.0126	0.0134	0.0246	0.0368
FIEGARCH	0.0023	0.0023*	0.0041	0.0119	0.0261	0.0428
FIAPARCH	0.0087	0.0119	0.0166	0.0111	0.0253	0.0373
	Mauritius			Morocco		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0182	0.0253	0.0317	0.0230	0.0325	0.0428
GARCH	0.0285	0.0285	0.0412	0.0222	0.0293	0.0436
CGARCH	0.0150	0.0198*	0.0253*	0.0150	0.0261	0.0380
TGARCH	0.0169	0.0228	0.0392	0.0103	0.0228	0.0327
EGARCH	0.0158	0.0253	0.0357	0.0230	0.0293	0.0452
APARCH	0.0150	0.0206	0.0309	0.0007*	0.0023*	0.0023*
IGARCH	0.0142*	0.0206	0.0287	0.0174	0.0269	0.0396
FIGARCH	0.0174	0.0206	0.0261	0.0198	0.0285	0.0404
FIEGARCH	0.0166	0.0222	0.0309	0.0222	0.0293	0.0444
FIAPARCH	0.0158	0.0198*	0.0261	0.0206	0.0309	0.0388

Note: Asterisks indicate preferred models

Table 7.5 VaR Failure Rates – Out-of-Sample

	Namibia			Nigeria		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0230	0.0444	0.0658	0.0126	0.0158	0.0333
GARCH	0.0341	0.0341	0.0341	0.0095	0.0182	0.0309
CGARCH	0.0095	0.0095	0.0103	0.0087*	0.0150*	0.0277*
TGARCH	0.0118	0.0203	0.0249	0.0142	0.0271	0.0361
EGARCH	0.0039	0.0039	0.0055	0.0126	0.0222	0.0380
APARCH	0.0103	0.1860	0.0322	0.0111	0.0214	0.0388
IGARCH	0.0158	0.0246	0.0460	0.0119	0.0238	0.0349
FIGARCH	0.0349	0.0452	0.0571	0.0103	0.0190	0.0349
FIEGARCH	0.0023*	0.0031*	0.0031*	0.0134	0.0222	0.0333
FIAPARCH	0.0285	0.0301	0.0309	0.0119	0.0150*	0.0269
	South Africa			Tunisia		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0174	0.0325	0.0333	0.0174	0.0309	0.0523
GARCH	0.0150	0.0261	0.0309	0.0150	0.0261	0.0476
CGARCH	0.0166	0.0277	0.0277	0.0126*	0.0230*	0.0436*
TGARCH	0.0166	0.0314	0.0354	0.0143	0.0285	0.0437
EGARCH	0.0150	0.0238*	0.0380	0.0158	0.0246	0.0452
APARCH	0.0162	0.0246	0.0388	0.0150	0.0230*	0.0444
IGARCH	0.0134*	0.0269	0.0349	0.0150	0.0261	0.0476
FIGARCH	0.0163	0.0253	0.0349	0.0166	0.0277	0.0492
FIEGARCH	0.0139	0.0246	0.0269*	0.0158	0.0269	0.0468
FIAPARCH	0.0172	0.0238*	0.0317	0.0142	0.0246	0.0468

	Zimbabwe		
Model	99%	97.5%	95%
EWMA/RM	0.0166	0.0246	0.0325
GARCH	0.0134	0.0206	0.0301
CGARCH	0.0111*	0.0158*	0.0246*
TGARCH	0.0205	0.0273	0.0419
EGARCH	0.0134	0.0214	0.0325
APARCH	0.0126	0.0182	0.0309
IGARCH	0.0150	0.0222	0.0325
FIGARCH	0.0134	0.0230	0.0333
FIEGARCH	0.0150	0.0214	0.0333
FIAPARCH	0.0150	0.0222	0.0309

Note: Asterisks indicate preferred models

Table 7.6 VaR Failure Rates – Out-of-Sample

	US			UK		
Model	99%	97.5%	95%	99%	97.5%	95%
EWMA/RM	0.0222	0.0380	0.0571	0.0230	0.0380	0.0595
GARCH	0.0190	0.0325	0.0507	0.0158	0.0301	0.0507
CGARCH	0.0277	0.0412	0.0642	0.0198	0.0341	0.0571
TGARCH	0.0218	0.0326	0.0551	0.0214	0.0392	0.0574
EGARCH	0.0142	0.0269	0.0507	0.0158	0.0285	0.0507
APARCH	0.0126*	0.0277	0.0484	0.0158	0.0285	0.0507
IGARCH	0.0166	0.0317	0.0476	0.0182	0.0285	0.0507
FIGARCH	0.0126*	0.0301	0.0476	0.0142	0.0277	0.0444*
FIEGARCH	0.0174	0.0253	0.0539	0.0150	0.0285	0.0523
FIAPARCH	0.0134	0.0222*	0.0404*	0.0134*	0.0253*	0.0468

Note: Asterisks indicate preferred models

Diagnostics

In the previous section we used VaR failure rates as a criteria to rank (volatility forecasting) model performance. However, it is important to perform diagnostic checks in order to assess the adequacy of the preferred models. If the model is invalid, then it can yield false inference, which may have a bearing on the quality and hence applicability of our VaR results; for this reason model checking is crucial to our analysis. In particular, we implement the Kupiec (1995) LM test which evaluates the equality of the VaR failure rate to the specified statistical level; and, the dynamic quartile (DQ) test for autocorrelation in VaR exceptions (Engle and Manganelli, 2004). Table 7.7 below presents the results of these tests.

Table 7.7: Kupiec and Dynamic Quantile Results

Country	Model	99%	97.5%	95%
Botswana	FIEGARCH	0.0044 (1.0000)	0.0004 (1.000)	0.0000 (1.0000)
Egypt	IGARCH	0.5094 (0.0839)	0.1580 (0.6421)	0.0062 (0.3722)
Ghana	EGARCH	0.0001 (1.0000)	0.0004 (1.0000)	0.0000 (1.0000)
Kenya	CGARCH	0.9102 (0.4301)	0.1074 (0.5823)	0.0002 (0.0575)
Mauritius	IGARCH*	0.1508 (0.3909)	0.3063 (0.4191)	0.0001 (0.0041)
Morocco	APARCH	0.0019 (0.0262)	0.0041 (0.0133)	0.0037 (0.0691)
Namibia	FIEGARCH	0.0010 (1.0000)	0.0004 (1.0000)	0.0000 (1.0000)
Nigeria	CGARCH	0.6433 (0.9947)	0.0150 (0.2598)	0.0082 (0.0464)
South Africa	IGARCH*	0.0918 (0.5782)	0.9282 (0.0232)	0.8968 (0.1704)
Tunisia	CGARCH	0.3554 (0.9389)	0.6475 (0.9723)	0.2909 (0.5419)
Zimbabwe	CGARCH	0.6969 (0.3456)	0.0262 (0.1118)	0.0000 (0.0002)
UK	FIGARCH*	0.1627 (0.0957)	0.1074 (0.0350)	0.0039 (0.3805)
US	FIAPARCH*	0.2368 (0.8657)	0.9282 (0.8657)	0.6014 (0.9930)

Notes: Entries are *p*-values associated with the Kupiec (1995) LM test which tests the equality of the empirical failure rate to the specified statistical level; while, those in brackets are the *p*-values associated with the DQ test for autocorrelation in VaR exceptions. The ‘*’ in the model column indicates that the specified model is the preferred model at only the 99% probability level. For Mauritius, CGARCH preferred at 97.5% and 95% probability levels. For South Africa, 97.5% and 95% probability levels refer to FIAPARCH and FIGARCH models, respectively. For UK, 97.5% and 95% probability levels indicate FIAPARCH and FIGARCH models; while, in the US both 97.5% and 95% probability levels both refer to the FIAPARCH model.

In terms of the Kupiec test evidence of the null hypothesis due to an excessive number of exceptions in the preferred models are generally mixed across all markets and across all probability levels. In particular, we find that this test is very sensitive to the choice of percentile chosen; hence, practitioners need to be aware of this ambiguity in their evaluation of VaR measures. For Botswana, Ghana and Namibia we are unable to obtain meaningful results for the DQ test (perhaps owing to the nature of the sample; i.e., relatively short data spans). However, like the Kupiec test the results of the DQ test are ambiguous in that they are sensitive to the level of confidence interval chosen and hence the absence of autocorrelation in VaR exceptions for ASMs must be interpreted cautiously. Nonetheless, for the most part our diagnostic checks point to adequate model specification.

7.5 Conclusion

An important and topical application of volatility modelling and forecasting involves the computation of VaR measures. VaR models where developed to estimate the exposure of a portfolio to market risk. This concept derives from modern finance methodologies which were developed in order to evaluate the risks of financial failure. In particular, the VaR measure focuses on the maximal potential losses of a portfolio or trading position. Furthermore, the Basle market risk framework, endorses the use of VaR methods in relation to the application of sound risk management practices for use by supervisory authorities and financial institutions. In addition VaR forms the basis for a variety of risk controls including margin requirements and position limits which are widely used by financial institutions in their trading activities.

Against this background, this paper examines the performance of a ten volatility forecasting models as a basis to provide accurate VaR estimates in light of the key role that VaR modelling plays in measuring market risk and hence its relevance to financial institutions and their regulators. We focus on ten GARCH-based models in order to generate out-of-sample forecasts which we use to calculate and evaluate the performance of VaR at three extreme percentiles, captured by the 99%, 97.5% and 95% probability levels.

Our results show that more complex GARCH-based specifications outperform the standard GARCH model and the popular RiskMetrics approach in terms of producing precise VaR estimates. This finding emphasises the importance of considering a broader class of models for modelling and forecasting volatility models in ASMs. In particular, our empirical analysis found that the CGARCH model provides the best out-of-sample forecasts, and as a consequence, delivers the best VaR measure in terms of minimising the number of exceptions across all three probability levels for Kenya, Nigeria, and Zimbabwe. For Tunisia, the CGARCH provides the exceedance-minimising forecasts at the 99 percent probability level. This result suggests that in order to derive accurate volatility forecasts in these markets it is important to separate long-run and short-run volatility effects, which in turn may reflect the ongoing structural developments in their financial markets and economic environment. For example, Zimbabwe is characterised by ongoing macroeconomic instability (specifically a hyperinflationary environment) which may shape long-run volatility behaviour while the lack of alternative investment opportunities (due to the imposition of various exchange controls) may potential drive short-run volatility dynamics. For Botswana, Ghana and Namibia volatility forecasting models that capture an asymmetric response to volatility are found to be important. This finding may highlight the extent to which leverage is important in these markets (at least over the sample period). For Botswana and Namibia, volatility has two facets: returns have an asymmetric impact on volatility and a long memory component. The long memory attribute may reflect the persistently large number of non-actively traded shares in the market or it may be a (specious) consequence of a relatively small sample which is used to capture long memory behaviour. For Morocco, the APARCH model delivers the minimum VaR failure rates across all three probability levels. This means that in this market the standard power transformation inherent in GARCH-type models would deliver sub-optimal forecasts; in addition, leverage is an important explanatory factor in providing accurate volatility forecasts and hence accurate VaR measures. For Egypt, the IGARCH model delivers the minimum VaR failure rate. This means that volatility in this market is such that any shock to volatility is permanent and the unconditional variance is infinite. For South Africa and Mauritius the IGARCH delivers the minimum VaR failure rate at only the 99% probability level. The relevance of the IGARCH process may also reflect a variety of factors that influence the processing of new information, such as prevalent illiquidity in these markets or other structural characteristics. For our benchmark comparators our results indicate that long memory attributes consistently provide the minimum VaR failure rates. The results of the diagnostic tests are ambiguous in some cases but still lend credence to the validity of our results. In addition, our analysis highlights the importance of using the relevant probability level stipulated by regulatory framework, and of employing out-of-sample forecast evaluation methods for the accurate identification of forecasting methods and models that provide the most accurate VaR measures in terms of minimising the number of exceptions, that is, when the minimum capital requirement identified by the VaR methodology would have fallen short of actual trading losses. The analysis also supports the superiority of more complex GARCH models; in particular, models that incorporate multiple volatility components, asymmetric effects and long memory attributes were found to be important in delivering the minimum VaR failure rates. In sum, our results imply that market participants and researchers with a focus on ASMs need to have available to them a range of models in order to allow them to capture the volatility profiles of these markets, in particular, multiple volatility components, asymmetric effects, and long memory attributes.

Finally, while our analysis calculates standard VaR measures in the context of ASMs, further analysis could be usefully conducted in a number of directions. One extension would be to include a wider spectrum of sensitivity tests (i.e., VaR at different horizons, confidence intervals, and using different underlying factor data). Another possibility would be to investigate the relevance of liquidity VaRs (L-VaRs) in ASMs given the thin trading conditions in these markets. In particular, standard VaR measures may fail to adequately capture the liquidity risks associated with some funds' strategies and positions. At their most basic these models are designed to manage liquidity risk on trading positions based on the markets' underlying turnover (ratio). More sophisticated variants of L-VaRs models incorporate adjustments to the volatility and correlation of the VaR to the degree of (illiquid) trading conditions (IMF, 2007). A third extension would be to explore the viability of other volatility proxies (e.g., intraday volatility and realised volatility) which may provide more precise volatility forecasts and hence VaR estimates (e.g., Poon, 2005).

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8 Final Conclusions

8.1 Summary of Thesis

The essays contained in this thesis examine different aspects of long memory behaviour in the context of ASMs. The implications of long memory in financial markets were first investigated by Mandelbrot (1971). This research motivated the introduction of fractionally integrated models, for example, the ARFIMA (*m*, *d*, *n*) process of Granger and Joyeux (1980) and Hosking (1981) and the FIGARCH (*p*, *d*, *q*) class of models proposed by Baillie *et al* (1996) and their various extensions. In these models the fractional differencing parameter, *d*, which measures the order of integration, is not an integer value (0 or 1) but rather a fractional value. In particular, nonzero values of *d* imply dependence between distant observations, and as a result, recent research has focused on the analysis of fractional dynamics in asset return data. The examination of the long memory properties in financial time series data has concentrated on the well-developed financial markets; however, little is known about the long memory attributes of emerging securities markets, particularly, ASMs.

This research attempts to fill this gap in the empirical literature by examining the following questions. First, we investigate the extent to which ASMs are efficient in pricing securities, given that long memory provides evidence against the weak-form version of the EMH. Second, we re-examine evidence of persistence in variance and long memory in light of the existence of, and failure to account for structural breaks, which in turn may have a bearing on the accuracy of our volatility estimates. In particular, structural breaks in the equity data may lead to spurious long memory conclusions. Third, we compare and contrast the forecasting performance of a variety of volatility models over both the daily and monthly frequency, in order to primarily ascertain if long memory models deliver superior forecast accuracy over longer horizons. Fourth, we evaluate the performance a spectrum of alternative volatility forecasting methods under VaR modelling in the context of the Basle market risk framework by broadening the class of GARCH models to include a variety of asymmetric and long memory models and using out-of-sample forecasts as a criterion to select volatility forecasting models which minimise the realisation of occasions when the minimum capital requirement identified by the VaR framework would have fallen short of actual trading losses. Finally, there are a range of findings, which we draw together here to present broad conclusions regarding the stock markets under review.

Against this background, this thesis is structured as follows. Chapter 1 introduces long memory in time series data and offers a synopsis of the thesis. Chapter 2 provides descriptive information on ASMs, emphasising the structural characteristics of these markets and the recent trends they have followed. Chapter 3 reviews the data used throughout this research and presents various descriptions of it, including a range of summary statistics. Chapter 4 deals with the efficiency of ASMs. Our results indicate that ASMs mostly display a predictable component in returns which raises evidence against the weak form version of the EMH; while evidence of long memory in stock return volatility is mixed. In comparison, evidence from the benchmark comparators (i.e., UK and US) suggests short memory in returns; while, evidence of long memory in volatility is mixed. We also suggest remedial measures to improve the efficiency of ASMs. In particular, to boost liquidity and market efficiency a viable policy option would be the development of a regional capital market which may potentially help alleviate the disadvantages associated with low liquidity and in turn promote market efficiency. Indeed, as noted in chapter 4, a number of other researchers have advanced financial market integration in order to boost stock market development and recent evidence points to the beneficial impacts of cross-listings as a means to improve liquidity and market efficiency. A second area of reform involves the need to modernise trading, clearing, settlement and custody procedures which could be improved by automation and thereby ensure more rapid transmission of information. More generally, this area of reform relates to the implementation of measures designed to improve the overall quality of information dissemination on the performance of listed companies. In addition, it is recommended that the relevant authorities need to ensure that the regulatory and supervisory framework governing ASMs, and the procedures relating to the conduct of investment business (e.g., transparency requirements), evolve in line with international best practice. In total, these improvements may help promote the efficiency of ASMs. Furthermore our results show that the behaviour of equity market returns and volatility are dissimilar across markets and this may have implications for portfolio diversification and risk management strategies. Chapter 5, re-examines evidence of volatility persistence and long memory taking account of structural breaks in the equity data. Specifically, we focus on potential time-variation in the unconditional mean of the volatility series. The results suggest that persistence and long memory in volatility are overestimated when regime shifts are not accounted for. In particular, application of breakpoint tests and a moving average procedure suggest that unconditional volatility display substantial time-variation. Finally a modification of the standard GARCH model to allow for time-variation in the unconditional variance generates improved volatility forecasting performance for some ASMs. In total, we find that allowing for structural breaks in the volatility process may improve the accuracy of volatility estimates (in particular, volatility persistence and long memory measures) and indeed forecast performance (although not in all cases) and should therefore be considered useful to both market participants (in the context of risk measurement, derivative pricing and formulation of trading strategies) and other researchers. Chapter 6 examines volatility forecasting in ASMs using a variety of models, at both daily and monthly frequency under both symmetric and asymmetric loss functions. Our results indicate that the various model rankings are sensitive to the specification of the error statistic used to assess the accuracy of the forecasts. This result implies that the choice of error measure needs to be consistent with an underlying loss function which in turn depends on the end use of the forecasts. For example, a call option buyer and seller would prefer the MME(*O*) and MME(*U*) statistics, respectively. We also analyse the forecast superiority of long memory models (especially over longer horizons) and obtain mixed results. In particular, we find that long memory models do not consistently outperform all the other alternative models, although as a class of models they perform better than both short memory GARCH models and the simple statistical methods used. The test of superior predictive ability also offers mixed evidence in favour of long memory models. In total, these results have potential importance in terms of the appropriate modelling and forecasting procedures for volatility and may be used to improve both portfolio management strategies and policy making. Chapter 7 examines the performance of a range of volatility forecasting models in Value-at-Risk (VaR) estimation within the context of the Basle market risk framework. These models are then assessed to examine the accuracy of VaR estimates at various confidence levels. This research demonstrates the importance of using out-of-sample forecasts, for the identification of volatility models which minimise the incidence of when the minimum capital requirement identified by the VaR methodology would have been insufficient to cover the actual trading losses. With respect to the range of forecasting models evaluated under these conditions our findings are diverse; however, in the main, the results show that models which capture long memory dynamics (especially multiple volatility components) and asymmetric effects are important considerations in delivering improved VaR estimates. Furthermore, we find that all the models considered generally outperform the RiskMetrics and standard GARCH method in estimating the VaR at the three extreme percentiles we consider. We also perform some diagnostic checks to assess the adequacy of our selected models. The model verification exercise generally indicates that our chosen models are well specified. In total, this analysis may inform operational risk measurement and as a consequence mitigate the probability of financial distress on the part of financial institutions or specific equity portfolios. Furthermore, this study emphasises the importance of using fully out-of-sample forecasting methods for the identification of statistical and econometric models that minimise the occurrence of VaR exceptions given stringent probability guidelines. These results are therefore potentially useful to market participants who have exposure (or investments) in ASMs and may also be instructive to both regulatory institutions and other researchers.

8.2 Future work

This research has covered a wide range of issues relating to the long memory properties of return volatility in ASMs; in addition, the results of this study may potentially be used to inform portfolio and policy analysis. Some caveats to our results exist, however. First, the increased availability of intraday data may potentially result in the construction of improved daily volatility forecasts. More specifically, volatility forecasts based on models that use intraday data adjusted for its various components (e.g., intraday periodicity) and from models that aggregate intraday data to form daily volatility and a host of daily realised volatility measures can be compared in order to ascertain which method which produces the most accurate volatility forecasts. This could be an area for future research which may potentially provide more accurate volatility forecasts and in turn lead to improved forecasts in a range of settings including portfolio and risk management and derivative pricing in the context of ASMs. Another possibility for future research relates to the use of a variety of multivariate GARCH processes that parameterise the covariance between multiple time-series. In particular, the multivariate approach allows for the construction of a weighted portfolio based upon investor holdings of African equities, for example, and its associated volatility interactions. This approach allows estimation of time-varying volatility correlations and covariances between various stocks, which may better describe the overall portfolio variance of investment funds dedicated to ASMs. A third extension involves the impact of volatility in one market on another. In particular, the increasing interdependence between some ASMs (e.g., South Africa and its neighbouring countries) suggests the possibility of the transmission of financial stress from one equity market to another. As a result, the examination of the contemporaneous relationship between stock return volatilities and associated correlations among returns in ASMs may provide market participants and policy makers with a better understanding of volatility spillovers in ASMs.

In sum, the findings of this research are premised on univariate methods and use of daily squared returns to proxy daily volatility. Future research may therefore expand upon the results contained in this studying by employing a variety of alternative volatility proxies (from intraday data) in order to provide potentially more accurate volatility forecasts and employ multivariate GARCH models to assess volatility covariances and correlations of different stocks and across ASMs. These innovations would further enhance our understanding of volatility in equity markets and provide useful information to market participants, policymakers and other researchers.

Appendix A

Publications and Conference Papers

A.1. Publications

A.1.1. Publications (Refereed)

McMillan, D. G. and P. Thupayagale, 2008, "Efficiency of the South African Equity Market," *Applied Financial Economics Letters*, Vol. 4, No. 5, pp.327 - 330.

McMillan, D. G. and P. Thupayagale, 2009, "The Efficiency of African Stock Markets," *Studies in Economics and Finance*, Vol. 26, No. 4, pp. 275-292.

McMillan, D. G. and P. Thupayagale, 2009, "Measuring Volatility Persistence and Long Memory in the Presence of Structural Breaks," *Journal of Managerial Finance* (forthcoming).

A.1.2. Publications (not refereed)

McMillan, D. G. and P. Thupayagale, 2008, "Forecasting Volatility Using Long Memory Models: Evidence from African Stock Markets" School of Management Working Paper (University of St Andrews)

McMillan, D. G. and P. Thupayagale, 2009, "Value-at-Risk Estimation in African Stock Markets: Comparative Evidence from Symmetric, Asymmetric and Long Memory GARCH Models" School of Management Working Paper (University of St Andrews)

A.2. Conference Papers (not refereed)

Thupayagale, P., 2007, "Long Memory in the Returns and Volatility of an Emerging Market: The Case of South Africa," Scottish Doctoral Management Conference, St Andrews, June 2007.

Thupayagale, P., 2008, "Volatility Persistence, Long Memory and Time-Varying Unconditional Mean: Evidence from the Johannesburg Securities Exchange," Scottish Doctoral Management Conference, St Andrews, June 2008.

Thupayagale, P., 2009, "Evaluating Stock Index Return Value-at-Risk Estimates in South Africa: Comparative Evidence for Symmetric, Asymmetric and Long Memory GARCH Models," Scottish Doctoral Management Conference, St Andrews, June 2009.