Global Capital Flows and Macroeconomy

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Abstract

This dissertation develops an analytical and empirical framework to understand the effect of global financial flows on a heterogeneous-agent economy. The global integration of financial markets was an important milestone that characterizes the recent history of the world economy. The financial decisions of households, firms, banks, and governments are now closely affected by various global factors as capital moves across the boundaries of nations. These economic agents are not homogeneous; thus, their responses to global financial flows are not identical. I shed light on the interaction of these heterogeneous agents to explore the linkage of global financial flows to economy-wide phenomena such as wealth inequality, corporate earnings and household balance sheet.

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To Harim, Mina and my parents

Introduction

The focus of this dissertation is developing an analytical and empirical framework to understand the effect of global financial flows on a heterogeneous-agent economy. The global integration of financial markets was an important milestone that characterizes the recent history of the world economy. The financial decisions of households, firms, banks, and governments are now closely affected by various global factors as capital moves across the boundaries of nations. These economic agents are not homogeneous; thus, their responses to global financial flows are not identical. My research sheds light on the interaction of these heterogeneous agents to explore the linkage of global financial flows to economy-wide phenomena. I use various methodologies, from model development to data inference, to analyze the macroeconomic impacts of financial globalization.

The first chapter paper develops a general equilibrium model to analyze the effect of global financial integration to rising wealth concentration among American households. I highlight the following points: 1) financial globalization raises wealth inequality in a financially-developed economy initially due to foreign capital pressing up domestic asset prices; 2) much of this increase is transitory and can be reversed as future expected returns on domestic assets fall; and 3) despite the low-interest-rate environment, newly accessed foreign capital provides incentives for affluent households to reallocate wealth toward risky assets while impoverished households increase their debt. Wealth concentration ensues only if this rebalancing effect is large enough to counteract diminished return on domestic assets.

The second chapter is a joint work with Casey Kearney. We turn our eyes to the corporate sector. US multinational companies (MNCs) play a prominent role in raising capital abroad and investing in high-yield global business opportunities. Using survey data collected by the

US Bureau of Economic Analysis on both the intensive and extensive margins of the activities of US MNCs and their foreign affiliates, we estimate the impact of MNC operations on the persistent spread between the return on assets (ROA) and the interest rate payments of firms. Our evidence indicates MNCs enjoy a 0.9% larger spread between ROA and average interest rate compared to when these same firms did not have large ownership holdings in foreign affiliates. We then introduce a model of MNC activity which can disentangle potential mechanisms to explain this spread and estimate the implied 'FDI Restrictiveness' of different regions based on observed patterns of foreign investment. While we do not test this model directly with existing data, our simulation suggests some of the variation in firm performance can be accounted for by the incomplete integration of global financial markets. Our results highlight the role of US multinationals as global arbitrageurs in addition to being global risk-takers.

The third chapter develops a general equilibrium model of financial development when households have non-homothetic preferences over risk and return. Using a continuous-time approach, I provide a quantitative framework to characterize the model's dynamics with a system of partial differential equations. The model can be used to analyze the macroeconomic effects of financial development on the wealth distribution, asset prices and household balance sheet. I then apply the methodology to quantify the effect of financial globalization on the rise in U.S. wealth inequality since 1989.

Chapter 1

Global Capital Flows and Wealth Inequality[1]

1.1 Introduction

The United States is often referred to as the "banker to the world," due to its unique role in supplying global reserve assets and funding foreign risky investment (Kindleberger 1965, Gourinchas and Rey 2007). The rapid advancement of financial globalization in recent decades has allowed U.S. economic entities to seek funding from foreign investors, created new investment opportunities, and changed the market value of domestic assets owned by American households. However, despite the U.S. being the centerpiece of global financial architecture, little research has studied the effect of capital flows on the domestic household wealth distribution.

This paper takes a first stab at the mechanism. Using new methods for modeling heterogeneous agent economies, I show that the liberalization of financial flows between the central and peripheral economies potentially accounts for 34% to 55% of the observed increase in the current top one percent wealth share in the U.S.. Yet, the model also implies that the trend in rising wealth concentration could reverse over the course of the twenty-first century.

The key factor here is a contrast between (a) the low interest rate and the inflated market

[1]I am deeply grateful to my advisors — Gita Gopinath, Pol Antràs, Kenneth Rogoff, and Jeremy Stein — for their thoughtful advice and continuous support. I would also like to thank John Campbell, Vu Chau, Thummim Cho, Xiang Ding, Emmanuel Farhi, Xavier Gabaix, Jesus Fernandez-Villaverde, Elhanan Helpman, Yosub Jung, Casey Kearney, Spencer Kwon, Andrew Lilley, Matteo Maggiori, Michael-David Mangini, Ben Moll, Giselle Montamat, Elisa Rubbo, Gea Hyun Shin, Hillary Stein, Ludwig Straub, Maria Voronina, Brian Wheaton, Paul Willen and seminar participants at Harvard University for their careful feedback on this project. I gratefully acknowledge financial support from the Samsung Scholarship Foundation. All errors are my own.

value of U.S. domestic assets due to capital inflows and (b) the expansion of new risky investment outflows such as global equity, and the foreign direct investment (FDI) of multinational firms. The future trajectory of wealth concentration depends on the relative sizes of these two forces as they dictate household investment, debt raising, and domestic asset valuation. The main contribution of this paper is to use a tractable modeling framework to elucidate the linkage of global financial flows to wealth concentration. I also offer a new angle on the rising wealth inequality in America, which is most often seen simply as a permanent trend.

This study is motivated by three major changes in the American capital market, which have drawn much attention among economists in international finance and macroeconomics:

- **Financial globalization:** Capital account liberalization has integrated global financial markets to a remarkable degree since the 1980s. The sum of foreign assets and liabilities in the U.S. scaled by GDP—a de facto measure of financial integration—surged from 48.3% in 1980 to 324% in 2017. As of 2017, foreign portfolio equity and FDI account for 40.3% of the total value of equity held by American households (see Figure 1.1a). Foreign investors own 30.0% of U.S. corporate bonds outstanding and 44.5% of U.S. treasuries (U.S. Department of the Treasury 2018).

- **Banker to the world:** The U.S. is the world's dominant supplier of global reserve assets and fixed income securities. Its cross-border asset positions, by contrast, are mainly composed of equity and FDI (see Figure 1.1b). Because of this two-way capital flow, the U.S. is often described as the "banker to the world" or even the "venture capitalist to the world." (e.g. Gourinchas and Rey 2007, Gourinchas and Rey 2010)

- **Wealth concentration & household balance sheet:** Wealth distribution and household balance sheet in the U.S. have been shifted asymmetrically across wealth groups. In terms of the distribution shape, the estimated wealth share of the top one percent of households, by wealth, rose from 24.3% to 41.8% from 1980 to 2011. (Saez and Zucman 2016) In terms of the balance sheet, those same households have substantially increased their exposure to equity, even though equity earnings yield has declined (see Figure 1.1c). The bottom 90 percent has not increased its share in equity as much, while its household debt has surged (see Figure 1.1d).

These facts naturally lead us to ask the following questions: how does financial globalization affect the return on capital owned by the wealthiest magnates, such as George Soros and Phil Knight[2], while promoting debt among the middle class? To what extent is wealth inequality driven by foreign investors pressing up U.S. domestic asset prices? Is the increased wealth concentration permanent, or rather transitory? In this paper, I develop a unified model to answer these questions quantitatively.

Three modeling ingredients deserve comment. First, I assume that households have decreasing relative risk aversion.[3] This implies that affluent households invest more heavily in risky assets than impoverished households do. More importantly — and somewhat subtly—the decreasing relative risk aversion makes affluent households readjust their balance sheet more elastically whenever there is a change in risk compensation; impoverished households are relatively stuck in safe assets or in debt.

Second, safe assets are short-term assets such as bank deposits whereas risky assets are long-term assets such as equities. An unanticipated drop in interest rates thus generates capital gains for equity holders, while deposit holders receive only contemporaneous yields.

Third, banks[4] in peripheral countries have limited abilities to issue debt, due to the inferior financial system. The supply of safe assets is thus limited in those regions. Besides, since equity financing involves a transaction cost beyond the risk premium, these banks end up facing a higher overall cost of capital. The total investment is thus restrained in peripheral economies. Essentially, the financial center country such as the U.S. is endowed with a comparative advantage in manufacturing safe assets, and with absolute advantages in both safe and risky assets. The

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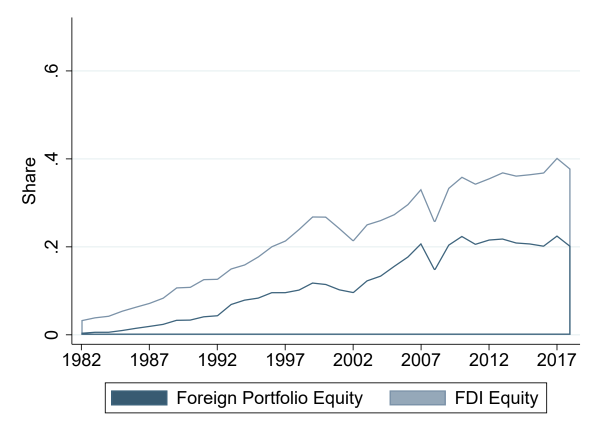
[2]George Soros is the founder of Soros Fund Management, which invests in foreign currency, equity, and fixed income markets across the globe. Phil Knight is a cofounder of Nike, Inc. As of 2017, foreign direct investment outside North America accounts for 68.8% of Nike's long-term physical assets i.e., Property, Plant and Equipment.

[3]I consider a standard HARA (hyperbolic absolute risk aversion) utility that exhibits decreasing relative risk aversion. In the paper, decreasing relative risk aversion is defined only in this narrow class of utility functions. The functional form has been used in various contexts including portfolio choice models (e.g., Litzenberger and Rubinstein 1976) and habit formation models (e.g., Campbell and Cochrane 1999). See Ogaki and Zhang (2001) and the related literature for micro-level evidence that supports decreasing relative risk aversion for portfolio choice.

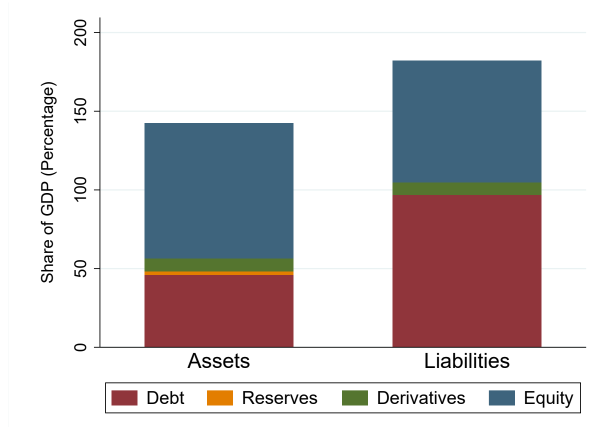
[4]Banks here can be more accurately thought of as any consolidated entity, a category encompassing private sector companies, financial intermediaries, and government.

Figure 1.1: *Financial Globalization and Household Balance Sheet*

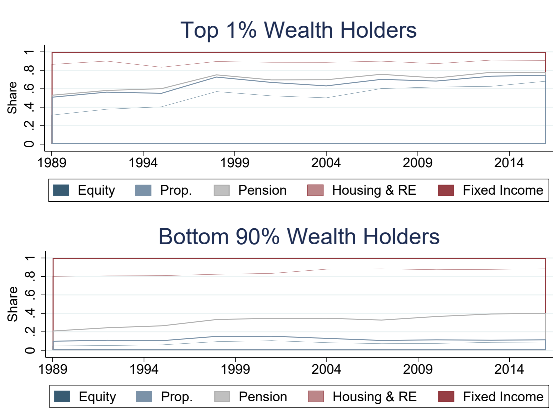
(a) *Foreign Equity as Share of Total Equity Holdings*



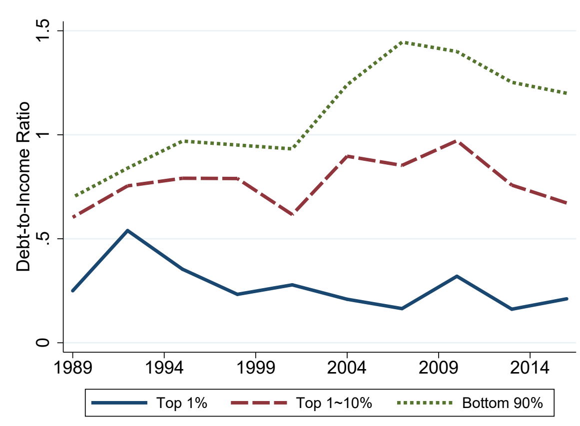
(b) *External Balance Sheet, 2017*



(c) *Asset Allocation by Wealth Group*



(d) *Debt-to-income Ratio*



Notes. Panel (a): The numerators are the estimated values of foreign portfolio equity and direct investment equity reported in IIP Table 1.2., from the Bureau of Economic Analysis. The denominator is the estimated total value of equity held by U.S. residents reported in the Fed's Financial Account (Series Code: FL153081005). Panel (b): IIP Table 1.2., Bureau of Economic Analysis. Panel (c): Survey of Consumer Finances. "Prop." represents proprietorship and partnership assets, while "RE" represents real estate. Panel (d): Survey of Consumer Finances. Mortgage and non-mortgage debts are all considered.

center country therefore exports safe assets, imports risky assets and, becomes a net debtor after crossborder investment barriers are lifted.

With this structure in place, I first show that global integration increases wealth concentration in the financial center country, initially due to foreign capital pressing up domestic asset prices (i.e., *the Revaluation Effect*). The market value of domestic equity appreciates immediately as its required rate of return has fallen. Affluent households reap more capital gains because their exposure to domestic equities is higher prior to the shock. In the model, the required expected return on domestic equity consists of the risk-free interest rate plus the domestic risk premium. After global financial markets are integrated, the overall required return for U.S. domestic equity is decreased either through a lower risk premium on the U.S. equity, or though a change in the total required return. The domestic equity price immediately reflects this change, generating capital gains for wealthy households who subsequently earn a lower expected return.

A more important point, however, is that this effect is transitory. The inflated top wealth share gradually disappears and, in the distant future, it can even be reversed (i.e., *the Decline-in-return Effect*). The basic principle of finance is the inverse relation between price and return. Whatever prompts asset-price inflation will, in turn, lower future expected returns on existing assets. The short-run effect of capital gains dissipates gradually; within a generation, households smooth their consumption in the lower-interest-rate environment. Between generations, wealth inheritance is imperfect. In the new stationary state, the top wealth share is eventually suppressed by the lower future expected return on existing domestic assets.

What might generate a persistent concentration of wealth, in the case of financial globalization, is asymmetric balance sheet readjustment amongst household groups in the new environment (i.e., *the Rebalancing Effect*). After capital gains are realized, households in the financial center country face a lower risk-free interest rate with the entry of foreign investors. At the same time, they gain access to new foreign risky assets, whether in the form of global equity or of the FDI outflow of multinational firms. Thus, the new investment portfolio, combining foreign and domestic risky assets, offers a higher risk-to-reward ratio than the pure domestic counterpart in autarky. The tilted expansion of the investment frontier offers households incentives to reallocate wealth to risky assets. Affluent households increase exposure to risky assets more elastically than impoverished households, due to decreasing risk aversion. The average return on wealth between the two household groups diverges. Wealth inequality in the financial center country if this rebalancing effect is large enough to counteract diminished returns on domestic assets.

I first illustrate these key points with a simple setup. The model is then extended to include additional features—such as household debt, entrepreneurial income, and FDI—to gain a full understanding of the distributional effects of financial globalization. These components reinforce the main idea: indebted households take on more debt as the interest rate falls. The increased household debt lowers the net worth of the bottom household group, which in turn raises the top wealth share in the financial center country. Entrepreneurial income rises as entrepreneurs face a lower cost of capital. The market-equivalent value of entrepreneurial equity is further increased by a lower required return in the financial market. Lastly, FDI outflow of domestic firms allows for more expansion of risky assets. It strengthens incentives—particularly for rich households—to reallocate wealth to risky assets. All in all, global integration provides an environment conducive to wealth concentration in the financial center country[5] although, as before, the long-run trajectory still depends on the relative magnitudes of (a) the decrease in domestic interest rates and (b) the expansion of foreign assets.

By shedding light on the architecture of global finance, this study offers a novel argument for why, amongst the developed economies, the U.S. has experienced a particularly large increase in wealth concentration. The U.S. is often referred to as the global banker, due to its exclusive role as safe asset provider in international financial markets (Gourinchas and Rey 2007, Gourinchas and Rey 2010, Eichengreen 2011). My paper suggests that this unique function of the U.S. economy played a prominent role in transforming domestic financial prices during the period of rapid global integration, thereby fostering wealth concentration amongst American households. It also provides theoretical underpinnings for cross-country studies finding that capital account liberalization was followed by increased income inequality[6] (Jaumotte *et al.* 2008, Furceri and Loungani 2015, Furceri *et al.* 2017), although the focus of this paper is more on wealth inequality

[5] Distribution effects in outside countries are less clear. Section 1.4.3 discusses cross-country implications.

[6] In particular, Jaumotte *et al.* (2008) shows that financial globalization and FDI are associated with an increase in inequality, much more so than is trade liberalization. Cross-country implications of my model are revisited in Section 1.4.3.

and on the U.S. economy. While most of the literature explores trade liberalization and wage inequality[7], I emphasize the importance of capital income and the financial side of globalization.

Methodologically, this paper contributes to the literature by proposing a tractable model for studying macro-finance implications of global financial flows. It has been widely documented that a higher foreign demand for U.S. debt securities can account for a decline in safe yields over the past decades (e.g. Caballero *et al.* 2008, Mendoza *et al.* 2009). Yet, less attention has been paid to the relationship between foreign factors and other financial variables, such as risk premium, Sharpe ratio, required expected return, corporate profits, and the portfolio frontier, which are all essential to understanding the distribution of capital income. Even studies that incorporate one or two of these elements require heavy numerical computation (Dou and Verdelhan 2015, Maggiori 2017). The model in this paper is more versatile. It allows for comparative statics with simple equilibrium solutions as well as a full quantitative analysis with numerical simulations. These modeling tools can potentially be used to understand time-varying changes in returns on various assets (Jordà *et al.* 2017), especially in the international context. This paper also relates to the literature on trade liberalization and wealth dynamics. (Chesnokova 2007, Antràs and Caballero 2010) For modeling tools, I embed global financial markets to a model of Pareto Inequality which orginates back to Champernowne (1953).[8]

The rest of the paper is organized as follows. Section 1.2 presents a stripped-down version of the model to present my core predictions. Section 1.3 develops general equilibrium foundations. Section 1.4 discusses distributional effects of security market liberalization and foreign direct investment as well as the cross-country implications. Section 1.5 concludes. Detailed proofs of propositions are referred to the appendices.

1.2 Core Model

To illustrate the mechanism, I begin by considering the simplest case of financial globalization: the set of investment opportunities available to households is fixed, and then transformed by

[7] See Helpman (2018) for the most up-to-date review on globalization and inequality.

[8] The literature on Pareto-inequality models has a long history and numerous applications. See Gabaix (2009), Jones (2015) and Benhabib and Bisin (2018) and for recent surveys of studies.

global integration of financial markets. The interest rates on these assets are exogenously given and then changed. I characterize the immediate and persistent effects of this shock on the household wealth distribution. The model is extended step by step in later sections: from market clearing conditions to labor income and household debt.

1.2.1 Setup

Households Consider a closed economy populated by a continuum of households. The measure of households is normalized to unity. Household i is endowed with initial wealth drawn from a probability density function, g_0. Time is continuous. In a closed economy, each household has access to two types of investments: a risk-free asset and a domestic risky asset. The risk-free asset yields $r^* dt$ with certainty. The domestic risky asset yields $(r^* + \sigma_1 s_1^*)dt + \sigma_1 dz_{1t}$, where $\sigma_1 s_1^*$ represents the risk premium and dz_{1t} represents the increment of a Wiener process. To conceptualize the price of risk, I decompose the risk premium into two parts: the standard deviation of returns, σ_1, and the Sharpe ratio (=an asset's risk premium divided by its standard deviation) of the domestic risky asset, s_1^*. By definition, s_1^* can be viewed as a reward for taking one unit of domestic risk. The domestic risky asset is indexed by 1. An asterisk in the superscript is used to indicate an autarky price.

The portfolio frontier spanned by the two basis assets, $\{r^*, (s_1^*, \sigma_1)\}$, plays a central role in households' saving decisions.[9] For the moment, assume that the portfolio frontier is exogenously given, so the values of r^* and s_1^* stay constant over time. We will consider a general equilibrium foundation in the next section.

Given the constant portfolio frontier, $\{r^*, (s_1^*, \sigma_1)\}$, household i maximizes lifetime utility by choosing consumption flow, c_{it}, and the share of savings in the domestic risky asset, θ_{1it}. Let δ denote the time discount rate. Household i born at time 0 seeks to maximize

$$\max_{\theta_{1it}, c_{it}} \mathbb{E}_0 \left[\int_0^\infty e^{-(\delta+m)t} \log(c_{it} - \kappa) dt \right] \tag{1.1}$$

[9]The Mutual Fund Theorem (e.g. Merton 1971) implies that $\{r^*, (s_1^*, \sigma_1)\}$ embodies 1 risk-free asset and N risky assets without loss of generality.

The budget constraint is given by

$$da_{it} = [(r^* + \sigma_1 s_1^* \theta_{1it})a_{it} - c_{it}]dt + \sigma_1 \theta_{1it} a_{it}\, dz_{1t} \tag{1.2}$$

In the core model, financial assets are the only sources of income for households. The two parameters, m and κ, in the household's problem deserve further comments.

First, m is intended to capture the death probability. As in the perpetual youth model (Yaari 1965, Blanchard 1985), a fraction m of households die and lose their wealth at every instantaneous time. These households are replaced by offsprings whose wealth endowments are re-drawn from the initial density distribution, g_0. Wealth dispersion of the newborn households, g_0, differs from wealth dispersion of the deceased households, g_t.[10] The discrepancy between the two distributions implies that wealth of parents is not fully transferred to wealth of their children. A large value of m intensifies the imperfect wealth inheritance.

Second, $\kappa > 0$ in the flow utility captures decreasing relative risk aversion.[11] This assumption implies that wealthy households not only invest in the risky asset more heavily than impoverished households, but they are also more responsive to a change in risk compensation. This property can be seen most clearly from solutions of a household's problem (Merton 1971)

$$c_{it} = \underbrace{(\delta + m)a_{it} + (r^* - \delta - m)\underline{a}}_{\text{Consumption Flow}} \quad \theta_{1it} = \underbrace{\frac{s_1^*}{\sigma_1}\left(1 - \frac{\underline{a}}{a_{it}}\right)}_{\text{Share of Risky Savings}} \tag{1.3}$$

where $\underline{a} \equiv \frac{\kappa}{r^*}$ denotes the wealth cutoff. All households choose to retain wealth above this cutoff to avoid negative consumption (i.e., $a_{it} \geq \underline{a}$). Expression (1.3) shows that the share of savings invested in the domestic risky asset increases with individual wealth, a_{it}, and falls to zero when $a_{it} = \underline{a}$. Furthermore, $\frac{\partial^2 \theta_{1it}}{\partial a_{it} \partial s_1^*} > 0$ implies that wealthy households respond to a change in s_1^* more elastically.

[10] Every k'th moment of g_0 is assumed to be finite, which implies that g_0 does not have excessively thick tails. Examples of such distributions include normal, and lognormal distributions. Also, I impose $\int_{-\infty}^{\infty} g_0(a)da \leq \int_{-\infty}^{\infty} g_t(a)da$. The strict inequality can be interpreted as a deadweight loss.

[11] The flow utility in the objective function belongs to the class of HARA (hyperbolic absolute risk aversion) utility functions. This functional form has been used in various studies such as portfolio choice models (e.g., Litzenberger and Rubinstein 1976). In my paper, decreasing relative risk aversion is defined only in this narrow class of utility functions.

There are several ways to interpret $\underline{a}$. The cutoff can be viewed as the minimum level of wealth that should be retained every period. Households whose wealth is near this threshold invest more heavily in the safe asset to meet the subsistence consumption level, κ, without any uncertainty. On the other hand, households with a sufficiently large stock of wealth are relatively free of this concern so they invest more heavily in the domestic risky asset. Note that (i) household debt and (ii) private equity are missing in the core model. I will discuss them in later sections.

Asset Price Another important assumption is that the domestic risky asset is a long-term asset, much like equity. The safe asset is assumed to be a short-term asset, much like bank deposit. To capture this aspect, let n_{it} denote the number of domestic risky asset shares invested by household i, and p_t denote the price per share. Each share is a contract that transfers $\sigma_1 dz_{1\tau}$ to its holder. The contract pledges constant future dividend xdt for all $\tau \geq t$. Given an exogenous market price $\{r^*, (s_1^*, \sigma_1)\}$, the expected return on the risky asset in the budget constraint (1.2) can be decomposed into

$$\underbrace{(r^* + \sigma_1 s_1^*)dt}_{\text{Required Expected Return}} = \underbrace{\frac{\mathbb{E}_t[dp_t]}{p_t}}_{\text{Price Change}} + \underbrace{\frac{xdt}{p_t}}_{\text{Dividend Yield}} \tag{1.4}$$

For tractability, I focus on the case where $x = r^* + \sigma_1 s_1^*$ and $p_t = 1$ for all t.[12] The price of the domestic risky asset stays constant as long as there is no unanticipated change in the economy. The savings in the domestic risky asset can then be written as $\theta_{1it} a_{it} \equiv n_{it}$. Conversely, an unanticipated change in the required expected return generates capital gains or losses for risky asset holders. Suppose, for example, the market required return drops unexpectedly to $r + \sigma_1 s_1$ at time T, with x being fixed. Then the price of the domestic risky asset should rise immediately to $\frac{r^* + \sigma_1 s_1^*}{r + \sigma_1 s_1} > 1$ to satisfy (1.4). The price remains constant thereafter. The safe asset, on the other hand, does not pledge future cash flows beyond contemporaneous yield r^*. No capital gains or losses are generated even if there is a change in r^*

[12] In other words, each share of the domestic risky asset pays variable cash flow $\sigma_1 dz_{1t}$ and fixed dividend xdt at each instantaneous time. Given this, we can rewrite the budget constraint, (1.2), as $\left[(r^* + \sigma_1 s_1^* a_{it}) a_{it} - c_{it}\right] dt + \sigma_1 \theta_{1it} a_{it} dz_{1t} \equiv [r^*(1 - \theta_{1it}) a_{it} + x\theta_{1it} a_{it} - c_{it}]\, dt + n_{it}(\sigma_1 dz_{1t} + dp_t)$.

Wealth Distribution Lastly, let me define a measure of wealth concentration in the economy. We focus on a trajectory of aggregate shocks in which $dz_{1t} = 0$ for all t. Yet, households consider ex ante volatility, $Var[\sigma_1 dz_{1t}] = \sigma_1^2 dt$, in financial markets.[13]

The c.d.f. and p.d.f. of the wealth distribution, $G_t(a)$ and $g_t(a)$, are defined over this zero trajectory, so they evolve deterministically. We can characterize the evolution of g_t by the below differential equation:[14]

$$\frac{d}{dt}g_t(a) = -\underbrace{mg_t(a)}_{\text{(i) Death}} + \underbrace{mg_0(a)}_{\text{(ii) Newborn}} - \frac{d}{da}[\underbrace{\{(r^* + \sigma_1 s_1^* \theta_1(a))a - c(a)\}}_{\text{(iii) Savings}} g_t(a)] \tag{1.5}$$

Here, $\theta_1(a)$ and $c(a)$ indicate portfolio and consumption choices of a household whose wealth level is a. The differential equation in (1.5) is often referred to as the Kolmogorov Forward Equation (KFE) in the heterogeneous-agent model literature. The intuition for the KFE is as follows: the first term on the right hand side represents the wealth distribution of households who drop out due to death. The second term is the wealth endowment distribution of new-born households. The last term represents a change in the wealth distribution driven by savings of individual households. See Appendix A.3.1 and A.3.2 for the derivation of the KFE and its convergence property.

It is worth noting that, as time passes by, g_t gradually converges to the stationary wealth distribution, g_∞. In principle, a stationary wealth distribution is defined as a wealth distribution that stays constant over the course of time, i.e. $\frac{d}{dt}g_\infty(a) = 0$, as the economy moves along the zero trajectory. Given any initial distribution, the long-term wealth distribution of the economy coincides with g_∞. So one can think of g_∞ as the distribution of wealth that will eventually arise in the distant future.

With this apparatus, I shed light on two measures of wealth inequality, Ω_t and Ω_∞, to characterize the short-run and long-run effects of financial globalization respectively.

Definition 1. *Let Ω_t denote the top one percent's wealth share in g_t. Also, let Ω_∞ denote*

[13] Similar approaches have been taken in previous studies, such as Fernández-Villaverde *et al.* (2018), Ahn *et al.* (2018) and Kaplan *et al.* (2018) to establish tractability. The notion of stationary state in my model corresponds to the stochastic steady state defined by Fernández-Villaverde *et al.* (2018).

[14] The equation has no quadratic term that typically appears in a Kolmogorov Forward Equation. This is because the model has no idiosyncratic return and the wealth distribution is only defined along the zero trajectory.

the top one percent's wealth share in the stationary state, which is approximated by the Pareto exponent[15] *of* g_∞. *In other words,*

$$\Omega_t = \frac{\int_{G_t^{-1}(0.99)}^{\infty} a g_t(a) da}{\int_{-\infty}^{\infty} a g_t(a) da} \quad \text{and} \quad \Omega_\infty \equiv 100^{\frac{1}{\xi}-1}$$

where the Pareto exponent, ξ, *is defined as a constant stemming from* $\lim_{a\to\infty} \frac{g_\infty(\tau a)}{g_\infty(a)} = \tau^{-(1+\xi)}$. *Note that the approximation is exact (i.e.,* $\lim_{t\to\infty} \Omega_t = \Omega_\infty$*) if* g_∞ *coincides with a Pareto distribution.*

These measures, Ω_t and Ω_∞, are used to investigate the short-run and long-run effects of financial globalization on wealth concentration respectively. The top 1% bracket is chosen for illustration, and can be replaced with any n% without loss of generality. Suppose now that global capital markets are integrated unexpectedly at time T. One can use $d\log\Omega_T$ to capture an immediate increase in the top one percent wealth share, and $d\log\Omega_\infty$ to capture a long-run increase.

1.2.2 Effect of Financial Globalization

I next investigate the effect of financial globalization. Prior to global integration, all households face the portfolio frontier spanned by $\{r^*, (s_1^*, \sigma_1)\}$: there are a single risk-free asset whose return is given by r^*, and a domestic risky asset whose risk premium is given by $\sigma_1 s_1^*$. What financial globalization does is to transform the set of investment opportunities (=the portfolio frontier) available to households from the left to the right.

$$\{r^*, (s_1^*, \sigma_1)\} \quad \Rightarrow \quad \{r, (s_1, \sigma_1), (s_2, \sigma_2)\} \tag{1.6}$$

The correlation between dz_{1t} and dz_{2t} is given by $\rho \in [0, 1)$. There are two changes. First, the risk-free interest rate, and the Sharpe ratio of the domestic risky asset may take new values. Second, households gain new access to a foreign risky asset, which is indexed by 2.

The first question we ask is the following: given (1.6), how does financial globalization transform the evolution of the wealth distribution in the short run and in the long run? Essentially,

[15] The approximation method yields analytically tractable results. See Jones (2015) and Gabaix *et al.* (2016) for other studies using this conversion.

global integration leads households' budget constraints to become

$$\underbrace{da_{it}}_{\text{Change in Wealth}} = \left(\left(r + \begin{bmatrix} \theta_{1it} & \theta_{2it} \end{bmatrix} \begin{bmatrix} \sigma_1 s_1 \\ \sigma_2 s_2 \end{bmatrix} \right) a_{it} - c_{it} \right) dt$$
$$+ a_{it}\theta_{1it}\sigma_1 dz_{1t} + a_{it}\theta_{2it}\sigma_2 dz_{2t}$$

from time T onward. In this setup, I make two assumptions to avoid pathological situations. First, I only consider cases where the optimal choices of θ_{1it} and θ_{2it} are non-negative. Second, the wealth cutoff, $\underline{a}$, is assumed constant. This prevents households at the bottom from suddenly falling into negative consumption[16] (i.e. negative infinite utility) in the case of an unanticipated drop in r. To analyze the effect on wealth concentration, I examine two examples: the integration of symmetric countries, and asymmetric countries.

[Case 1] US and EM are symmetric Consider an integration of two identical countries, US and EM. As in Obstfeld (1994), the risky assets in US and EM have imperfect correlation, so the only driver for global financial flows in this case is a diversification benefit. While capital flow between the central and peripheral economies is an important theme of international finance, the integration of identical countries clarifies the key channels needed to generate a persistent increase in wealth inequality. Below, I show that financial globalization between symmetric countries only generates a short-run increase in wealth concentration.

Assume that, after global markets are integrated, the market interest rates in US are changed from $\{r^*, (s_1^*, \sigma_1)\}$ to $\{r, (s_1, \sigma_1), (s_2, \sigma_2)\}$ such that:

$$(a)\ r = r^* \qquad (b)\ s_1 = s_2 \leq \left(\frac{1+\rho}{2}\right) s_1^* \qquad (c)\ \rho < 1$$

In other words, the risk-free rate remains unchanged. The Sharpe ratio of the domestic risky asset is lowered and is identical to the Sharpe ratio of the foreign risky asset. The decrease in the Sharpe ratio is reinforced by a lower correlation between the US and EM risky assets.

[16] The constant wealth cutoff is equivalent to assuming that the subsistence level of consumption is variable and proportional to $\underline{a}$ (i.e., $\kappa = r\underline{a}$ after the shock, and $r^*\underline{a}$ before the shock). As in habit formation models (e.g. Campbell and Cochrane 1999), I treat κ as a variable rather than a constant parameter. Alternatively, one can avoid the negative infinite utility issue by assuming that the wealth distribution $g_t(\cdot)$ prior to T has no mass below $\underline{a} = \frac{\kappa}{r}$.

As will be shown in later sections, the decrease in the domestic Sharpe ratio (or equivalently, the risk premium), stems from the fact that the world economy as a whole becomes safer. Due to the diversification effect, the domestic risk premium no longer has to be high to clear the global financial markets. We will later confirm these results with a general equilibrium setup (See Corollary 2).

Given these changes in the financial markets, how does the wealth distribution evolves within a country over different time horizons? Let Ω_T^* denote the top one percent wealth share within a closed economy in the stationary state. One can verify:

Proposition 1. *The global integration of symmetric economies increases wealth inequality within a country immediately (i.e.,* $\Omega_T > \Omega_T^*$*) but decreases wealth inequality in the long run (i.e.,* $\Omega_\infty < \Omega_T^*$*).*

Proof. See Appendix A.1.1 □

The core message of Proposition 1 can be summarized with three points. First, financial globalization raises wealth inequality in the short term through domestic asset price inflation (i.e., *the Revaluation Effect*). Recall that the discount rate for the domestic risky asset consists of the risk-free rate plus the risk premium. An unexpected integration of global capital markets lowers the market required return, $(r + \sigma_1 s_1)$, due to a decrease in the domestic risk premium. The pledged future cash flow, $(r^* + \sigma_1 s_1^*)dt$, remains unchanged by contrast. The price of the domestic risky asset, p_T, rises immediately in response to this change. Since affluent households have a larger exposure to the domestic risky asset prior to financial globalization, the top one percent wealth share in the new open economy, Ω_T, is increased by the revaluation gains.

Second, this increase is only transitory and — in the new stationary state — the concentration of wealth is eventually suppressed by the lower expected return on the domestic assets. (i.e., *the Decline-in-return Effect*). The basic tenet of finance is the inverse relation between price and return. The low discount factor that offers affluent households capital gains, in turn, lowers the future expected return on the domestic risky asset. The effect of capital gains wears off gradually through consumption smoothing within a generation. Wealth inheritance is imperfect between generations. Eventually, the top one percent wealth share of the stationary wealth distribution is decreased by the lower future expected return on household wealth.

At the end, what might generate a persistent increase in wealth concentration is the asymmetric portfolio reallocation between different household groups (i.e. *the Rebalancing Effect*). Affluent households increase their investment share in risky assets more elastically, while impoverished households are relatively stuck in the safe asset. The open-economy wealth distribution eventually converges to

$$g(a) = \mathcal{C}(a)(a - \underline{a})^{-1-\frac{m}{r+2s_1^2/(1+\rho)-\delta-m}} \tag{1.7}$$

along the zero trajectory where $\mathcal{C}(a)$ is a function of a. This expression shows that the stationary wealth distribution approximately follows a Pareto distribution.[17] One may also notice that the Pareto exponent is a function of ρ. That is, a low correlation between the US and EM assets provides a stronger incentive for wealthy households to increase their investment shares in risky assets more than poor households. The heterogene portfolio rebalancing exerts an upward force to support a long-term increase in wealth inequality.

However, in the case of the integration of symmetric countries, the rebalancing effect is not strong enough to counteract the diminished return on the domestic assets. Recall that $s_1 \leq \left(\frac{1+\rho}{2}\right) s_1^*$. Due to this general equilibrium force I will revisit later, the Pareto exponent in an open economy is lower than the Pareto exponent in a closed economy.[18] At the end, the top one percent wealth share in the new stationary state is lower than the initial state, so wealth inequality follows an inverse U-shape transitional dynamics. The key take-away from this exercise is that the model needs an extra-driver to induce a persistent increase in wealth inequality.

[Case 2] US and EM are asymmetric A core idea of this paper is that the integration of the central and peripheral economies, US and EM, leads to a biased technological change in financial markets. Below, I show that this asymmetry provides an additional force to generate a long-term increase in US wealth inequality. Now assume for the moment that, after financial

[17] The existence of all k'th moments of g_0 (i.e. g_0 has a thin tail) ensures that $\mathcal{C}$ does not distort the exponent as a goes to infinity. Examples of such thin tail distributions include log-normal distribution. See Appendix A.1.1.

[18] More specifically, the result stems from $\frac{r+2s_1^2/(1+\rho)-\delta-m}{m} \leq \frac{r+(1+\rho)s_1^{*2}/2-\delta-m}{m} < \frac{r+s_1^{*2}-\delta-m}{m}$

globalization, the market interest rates in US are changed as follows

$$(a)\ r < r^*, \quad (b)\ s_1 = s_1^* < s_2, \quad (c)\ \rho < 1$$

First, households begin to face a lower risk-free interest rate, r. Second, unlike the symmetric case, the new Sharpe ratio of the domestic risky asset, s_1, does not necessarily have to be smaller than the old ratio, s_1^*. For illustration, assume that the Sharpe ratio of the domestic risky asset is unchanged. The expected return on the domestic risky asset, $r + \sigma_1 s_1$ is thus lowered. Lastly, $s_2 > s_1$ holds, which implies that the newly-added foreign asset provides a higher reward-to-risk ratio than the domestic risky asset.

In later sections, I will formally state under what conditions financial globalization results in these changes and how this relates to the asymmetry between US and EM. Before that, let me turn to the effect on wealth inequality. Proposition 2 below characterizes the short-term and long-term changes in US wealth inequality after global financial integration.

Proposition 2. *The short-term and long-term effects of financial globalization on the top one percent wealth share can be summarized as follows:*[19]

$$d \log \Omega_T = -\underbrace{\phi_1 d \log(r + \sigma_1 s_1)}_{\text{(i) Revaluation Effect}} \tag{1.8}$$

$$d \log \Omega_\infty = \underbrace{\phi_2 d \log(r + \sigma_1 s_1)}_{\text{(ii) Decline-in-return Effect}} + \underbrace{\phi_3 d \log s_2 - \phi_4 d \log \rho}_{\text{(iii) Rebalancing Effect}} \tag{1.9}$$

where ϕ_1, ϕ_2, ϕ_3, and ϕ_4 are all positive coefficients.

Proof. See Appendix A.1.1 □

Corollary 1. *Financial globalization widens wealth inequality immediately by generating capital gains. Wealth inequality is increased permanently only if s_2 is sufficiently large.*

Proposition 2 again confirms that financial globalization raises US wealth inequality in the short term through the domestic asset price inflation. The effect is greater when the required expected return drops more acutely. As in the symmetric case, term (i) and (ii) have opposite

[19]Note here that, for illustration, I take differentiation with respect to $(r + \sigma_1 s_1)$ instead of r and s_1 separately. The latter would yield different coefficients for r and s_1.

signs in Proposition 2, which implies that the long-term wealth concentration is repressed by the decline in return on domestic assets. Thus, without any countervailing forces, wealth inequality would first increase, and then revert back to a lower level, even lower than the initial stationary state.

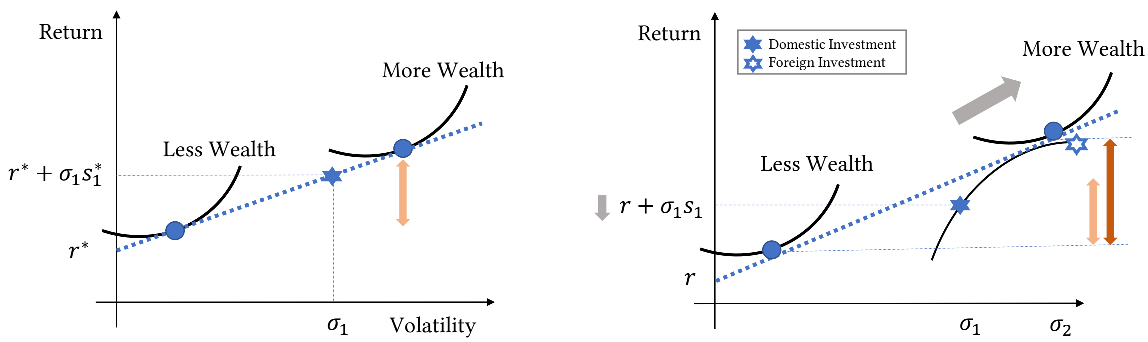
What Proposition 2 highlights is that the integration of asymmetric countries, US and EM, provides a stronger incentives countervailing force to increase the wealth inequality in the long term. Figure 1.2 illustrates the mechanism with a diagram from the static Capital Asset Pricing Model (Sharpe 1964). The key factor here is the slope of the capital allocation line — the dotted line connecting the risk-free to risky assets in the diagram. Households choose their portfolio such that their indifference curves between risk and expected return are tangent to the capital allocation line. In a closed economy, wealthy households choose a riskier portfolio due to the decreasing relative risk aversion assumption. In an open economy, the expected return on the domestic risky asset, $r + \sigma_1 s_1$, falls. Yet, an expansion of the portfolio frontier (indicated by the curve connecting stars) increases the slope of the capital allocation line. This provides incentives for households to reallocate their wealth more towards the risky domestic and foreign assets. Affluent households respond to this change more elastically than impoverished households due to decreasing relative risk aversion. Thus, capital return inequality between the US households is widened further by financial globalization, especially when the central country has comparative advantage in safe assets relative to peripheral economies.

This rebalancing effect exerts an upward force to support Ω_∞ in the long run. The expression in (1.9) suggests that the rebalancing effect is larger when a newly-accessed foreign risky asset has a higher reward-to-risk ratio, s_2, and a lower correlation with domestic assets, ρ. Financial globalization leads to persistent wealth concentration only if these forces are strong enough to counteract diminished returns on the domestic assets. Corollary 1 formalizes this intuition.

Two simplifying assumptions deserve comments. First, the core model does not feature household debt. All households choose to retain positive net worth throughout their lifespans. In section 3.4, I extend the model such that some households have negative net worth. Numerical simulations show that indebted households begin to take on more debts as r drops upon financial globalization. Net worth of households in the bottom decile falls, which raises the top one

Figure 1.2: *Capital Allocation Line*

(a) *Closed Economy* **(b)** *Open Economy*



percent's wealth share more than the illustrative model suggests. The asymmetry between US and EM strengthens this channel. Second, every household has unrestricted access to risky investment. In practice, entrepreneurial households might hold a large share of stock in certain companies and some of these holdings are private equity. In the next section, I do extend the model to allow for private equity. The extension only strengthens the main results; financial globalization increases entrepreneurial income and inflates its market-equivalent valuation.

1.2.3 Back-of-the-Envelope Calculation

One can use this core setup— before turning to a full-blown quantitative model — to roughly gauge the magnitude of the effect of financial globalization. The aim of this back-of-the-envelope calculation is to quantitatively disentangle the basic forces that transform the distribution of wealth. As the first step, let me invoke the formula for the stationary wealth distribution

$$g_\infty(a) = \mathcal{C}(a)(a - \underline{a})^{-1-\frac{m}{\tilde{r}+\tilde{s}^2-\delta-m}}$$

In a closed economy, the interest rates are given by $\tilde{r} = r^*$ and $\tilde{s} = s_1^*$. In an open economy, $\tilde{r} = r$ and $\tilde{s} = \sqrt{[\sigma_1 s_1, \sigma_2 s_2]\Sigma^{-1}[\sigma_1 s_1, \sigma_2 s_2]'}$. The latter indicates the Sharpe ratio of the open economy portfolio consisting of the domestic and foreign risky assets. As explained earlier, one can approximate the top one percent's wealth share with the Pareto exponent of the stationary wealth distribution, i.e., $\Omega_\infty = 100^{-1-\frac{m}{\tilde{r}+\tilde{s}^2-\delta-m}}$

The next step is to calculate a change in wealth concentration by calibrating the parameters

Table 1.1: *Back-of-the-envelope Calculation*

	Scenarios		
Common Parameters	(1)	(2)	(3)
Real risk-free interest (Autarky)	0.027	.	.
Sharpe ratio (Autarky)	0.29	.	.
m	0.036	.	.
δ	0.05	.	.
Portfolio weight in equity by wealth groups	0.5, 0.35, 0.15	.	.
Shock from financial globalization			
Real risk-free interest (Open)	0.01	0.01	0.005
Sharpe ratio (Open)	0.321	0.314	0.321
Results: Top 1% wealth share			
Autarky (= Data, 1989)	27.4%	27.4%	27.4%
Open (after capital gains)	+1.9%p	+3.1%p	+8.6%p
Open (stationary state)	+7.9%p	-7.6%p	-8.9%p
Data, 2016	+8.7%p	+8.7%p	+8.7%p

Notes: This table displays back-of-the-envelope calculations for changes in wealth concentration under different scenarios. A period mark indicates that the value is identical to the left column. In column (1), the values, 0.5, 0.35, and 0.19, represent portfolio weights in equity by the top 1%, the top 1-9% and the bottom 90% household groups. Data here refers to the top wealth shares estimated by the Survey of Consumer Finances.

of the core model. I pick 1989 as the benchmark year for the pre-globalization period.[20] I set $r^* = 0.027$, $s_1^* = 0.029$, and $\delta = 0.05$. r^* is calibrated from the 1-year treasury yield after inflation around 1989. s_1^* stems from the estimated Sharpe ratio for the domestic portfolio, which I will revisit in Section 3.4.1. δ is taken from the standard value for the discount rate. The remaining parameter in the Pareto exponent is m, which represents the imperfection of wealth inheritance between generations. I leave m as a free parameter; the value of m is chosen to fit the top one percent's wealth share in year 1989 from the Survey of Consumer Finances (SCF). Finally, I calibrate the average portfolio weights for the top 1 percent, the 1-9 percent and the bottom 90 percent households from the SCF.

The core part of this exercise is to change the values of $\tilde{r}$ and $\tilde{s}$ to measure the effect of a

[20] The year 1989 is chosen as the benchmark year due to data availability, such as Survey of Consumer Finances, which will be later used for a full-blown quantitative analysis. I use the same year for the back-of-the-envelope calculation to maintain consistency.

financial globalization shock on the top one percent's wealth share. Table 1.1 presents three scenarios. In scenario (1), the risk-free interest falls to 0.01 and the Sharpe ratio of the market portfolio rises 0.321. The top 1% wealth share immediately increases by 1.9%p due to capital gains. The share continues to increase because, in this scenario, the portfolio rebalancing effect outweighs the decline in return on the domestic assets. The model-implied increase, 7.9%p, is comparable in size to the actual increase in the data. By contrast, in scenario (2), the Sharpe ratio does not increase as much,[21] so the model exhibits an inverse-U shape transitional dynamics. Along this transitional dynamics, the top one percent wealth share first rises due to the revaluation effect, but eventually reverts back. The new stationary state ends up having a lower wealth inequality than the initial state. A similar pattern arises in scenario (3), where the risk-free interest falls zero. One can also see that the current trend in rising wealth concentration can reverse in the future if the expansion of foreign investment no longer increases the Sharpe ratio as much (i.e., scenario (1) $\Rightarrow$ scenario (3)), or the expected return on domestic assets falls too sharply (i.e., scenario (1) $\Rightarrow$ scenario (2)).

1.3 Closed Economy

Next, I turn to the market clearing conditions. The portfolio frontier is now determined jointly by (a) saving decisions of households and (b) funding decisions of banks. The core element here is that a country's banking system constitutes a source of comparative advantage in global financial markets. To model this feature, I specify the supply side of assets (in the banking sector) and the market clearing conditions. I first consider a closed-economy setup and then move on to an open economy. The aim of this analysis is to disentangle the basic forces that determine the market interest rates. In later sections, I shall explain how this whole structure can be embedded into quantitative analysis.

[21] One may notice that the back-of-the-envelope calculation is highly sensitive to a change in the Sharpe ratio. The sensitivity is alleviated if the utility function is replaced by one with a higher risk aversion than log utility. For example, one may assume $u(c_{it}) = \frac{(c_{it}-\kappa)^{1-\gamma}}{1-\gamma}$ and $\gamma = 2$. In this case, the Sharpe ratio should rise to 0.454 to generate the same magnitude as in Scenario (1).

1.3.1 Model Extension

Households The household side remains largely unchanged. Let $\{r_t^*, (s_{1t}^*, \sigma_1)\}$ denote an equilibrium portfolio frontier in autarky at period t. Households take these price processes as given. Their saving decisions constitute the demand side of financial assets. Define $\mathbb{S}_t \equiv A_t \equiv \int a_{it} di$ and $\mathbb{S}_{1t} \equiv \int \theta_{1it} a_{it} di$. Here, $\mathbb{S}_t$ represents the total savings invested by households, while $\mathbb{S}_{1t}$ represents the savings in the domestic risky asset. Using $\theta_{1it} = \frac{s_{1t}^*}{\sigma_1}(1 - \frac{\underline{a}}{a_{it}})$,[22] we can rewrite the closed economy saving curves as

$$\mathbb{S}_t = A_t, \qquad \mathbb{S}_{1t} = \frac{s_1^*}{\sigma_1}(A_t - \kappa) \tag{1.10}$$

Aggregate savings the in domestic risky asset increases with the Sharpe ratio. These two savings curves, along with the investment curves that will be defined momentarily, are used to pin down the equilibrium portfolio frontier in financial markets.

Banks Financial assets are manufactured by the representative bank — a consolidated entity encompassing private companies, financial intermediaries and the government. (Appendix A.1.8 provides an alternative microfoundation based on a simple endowment economy.) Every period, the bank generates

$$d\pi_t = \Phi(K_t)dt + \bar{\sigma} K_t dz_{1t}$$

by investing K_t units of capital within the boundary of a country. Production involves raw output volatility $\bar{\sigma}$ in proportion to the investment level. The production function, $\Phi(K_t)$, exhibits diminishing marginal returns.[23]

Funding decisions of the bank constitute the supply side of assets. The bank creates assets by converting its future cash flow into risk-free and risky tranches. The bank is a price taker and there is no adjustment cost in changing K_t. Thus, they simply maximize contemporaneous

[22] As in the core model, I assume that $\underline{a}$ is given as constant to avoid the negative infinite utility issue associated with an unanticipated structural change. (See footnote 16) Also, it will later turn out that (r_t, s_{1t}) is a function of the aggregate state variable A_t in equilibrium, which itself follows a stochastic process. Unlike Merton (1971), this feature adds extra complexity to the Hamiltonian-Jacobi-Bellman equation. See Appendix A.2 for more details.

[23] That is, $\Phi' > 0$, $\Phi'' < 0$, $\lim_{K\to 0} \Phi'(K) = -\infty$ and $\lim_{K\to\infty} \Phi'(K) = 0$. In this sense, my model is a variant of Cox *et al.* (1985), which assumes constant return to capital (i.e. $\Phi(K_t) = \alpha K_t$ and $dK_t = \alpha K_t + \bar{\sigma} K_t dz$)

profit[24]

$$\underbrace{V_t^* dt}_{\text{Private Equity Income}} \equiv \max_{K_t, D_t, E_t} \left\{ d\pi_t - \underbrace{r_t^* D_t dt}_{\text{Debt Income}} - \underbrace{(r_t^* + \sigma_1 s_{1t}^* + \tau) E_t dt}_{\text{Public Equity Income}} - \underbrace{\sigma_1 E_t dz_{1t}}_{\text{Public Equity Volatility}} \right\} \tag{1.11}$$

subject to the constraints

$$K_t \equiv D_t + E_t, \quad \sigma_1 = \bar{\sigma} K_t / E_t, \quad D_t \leq \lambda K_t \tag{1.12}$$

where D_t is the value of debt, E_t is the value of equity and σ_1 is the standard deviation of returns per unit of equity outstanding.

Let me first elaborate on the objective function. $V_t^* dt$ represents the excess profit that stems from the gap between the average physical rate of returns and the average cost of capital. I simply assume that $V_t^* dt$ is not distributed to households as there is a separate entrepreneur of the bank who monopolizes technology. The entrepreneur is in effect a hand-to-mouth agent who consumes $V_t^* dt$ immediately. The bank relies on outside capital. Debt holders receive the risk-free rate. Equity holders are compensated with the risk premium in proportion to the risk per unit of equity. Equity financing involves a deadweight transaction cost, τ, besides the risk premium, so the bank has incentives to rely on debt as a cheaper means of capital raising.

Turning to the balance sheet conditions, the first constraint in (3.3) implies that the value of assets should be equal to the value of debt and equity outstanding. The next constraint implies that the issuance of debt-like securities scales up the risk per unit of equity. Essentially, risky asset holders bear all the risk. Lastly, the maximum leverage is limited up to λK_t. λ indicates a country's capacity to create safe assets by tranching. Later on, I will assume that peripheral countries have a lower value of λ than the financial center country, which implies that the ability to manufacture safe assets is not identical across economies. Besides, because of τ, firms in peripheral countries end up facing a higher overall cost of capital. The total investment is thus restrained in peripheral economies.

We can derive the aggregate supply curves of the domestic assets from the funding decisions

[24] Appendix A.3.3 studies the associated decision makings in a discrete time framework over the interval $[t, t+h]$, and the model here corresponds to the limit case when h converges to 0.

of the bank: $\mathbb{I}_t = K_t$ and $\mathbb{I}_{1t} = E_t$, with $\sigma_1 = \bar{\sigma} K_t / E_t$. Solving the bank's optimization problem, it is easy to show[25]

$$\mathbb{I}_t = \Phi'^{-1}(r_t^* + \bar{\sigma} s_{1t}^* + \tau - \tau\lambda), \qquad \mathbb{I}_{1t} = (1-\lambda)\mathbb{I}_t \tag{1.13}$$

The volatility per share is $\sigma_1 = \frac{\bar{\sigma}}{1-\lambda}$. A quick inspection shows that both of these curves are downward sloping in terms of the market funding cost, $r_t + \bar{\sigma} s_{1t}$, and the Sharpe ratio of the country's risky asset, s_{1t}^*. Figure 1.3 displays these two downward-sloping investment curves in tandem with the saving curves.

1.3.2 Market Clearing Conditions

A closed economy equilibrium is defined as a path of price vectors that clear local financial markets. So each country has its own interest rates before the integration of global financial markets. Formally speaking, I make the following definition.

Definition 2. *A closed economy equilibrium is a stochastic process, $\{r_t^*, (s_{1t}^*, \sigma_1)\}_{t\geq 0}$, which clears local financial markets: $\mathbb{S}_t = \mathbb{I}_t$ and $\mathbb{S}_{1t} = \mathbb{I}_{1t}$ for all t.*

Given this setup, we can solve for the equilibrium values of r^* and s_1^* by using the two market clearing conditions. Merging (1.10) and (1.13), we can write the solutions as

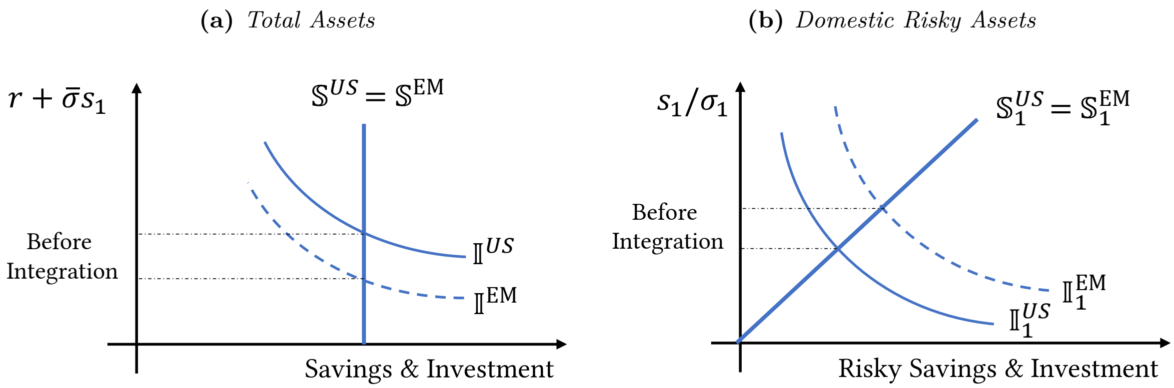
$$s_1^*(A) = \bar{\sigma} A / (A - \underline{a}) \tag{1.14}$$

$$r^*(A) = \Phi'(A) - \bar{\sigma}^2 A / (A - \underline{a}) - \tau + \tau\lambda \tag{1.15}$$

when the aggregate wealth stock is given by $A_t = A$. Notice that the market clearing interest rates can be expressed as a function of the total wealth stock. Essentially, A_t acts as the state variable of the economy. I henceforth use the notation $r_t^* \equiv r^*(A_t)$ and $s_{1t}^* \equiv s_1^*(A_t)$. After all, A_t evolves according to a stochastic process that will be discussed shortly, and so do the interest rates. The (long-term) domestic risky asset is modeled as a contingent security that pledges

[25] The total investment, K_t, is simply pinned down by the first order condition, $\Phi'(K_t) = r + \bar{\sigma} s_1 + \tau(1-\lambda)$. The issuance of equity is pinned down by the binding constraint, $E_t = (1-\lambda)K_t$, as equity financing involves extra costs besides the risk premium.

Figure 1.3: *Closed Economy Equilibrium*



future dividend $x_t \equiv r^*(A_t) + s_1^*(A_t)\sigma_1$ for all $\tau \geq t$ as in Section 1.2. The dividend stream now depends on the realization of the state variable.

1.3.3 Comparative Statics

Consider two countries: US and EM. In this two-economy world, US represents the financial center country, while EM represents the rest of the world. The key difference[26] between the central and peripheral economies is their ability to create safe assets in the banking sector. I assume $\lambda^{US} > \lambda^{EM}$, so US has comparative advantage in manufacturing safe assets. Along with the imperfect correlation between dz_{1t} and dz_{2t} (= diversification benefit from the foreign risky asset), the different size of λ^{US} is the minimal building block to provide microfoundation for the biased change in the portfolio frontier in Section 1.2. Additionally, one may assume that the output volatility is lower in the financial center country (i.e. $\bar{\sigma}^{US} < \bar{\sigma}^{EM}$), which helps improve a quantitative fit to the data in later sections.

Before turning to dynamics, we can use the solutions in (1.14) and (1.15) to conduct comparative static analysis. By log-differentiating the two, it is straightforward to prove the following statement:

Proposition 3. *Suppose that US and EM are identical in size (i.e., $A_{1t} = A_{2t} = A$). Then, in*

[26]Later on, I will also talk about the case where two identical countries are integrated. The only driver for global financial flows in this case is diversification benefit. See Remark 2 in Section 1.4.

autarky, EM has a lower risk-free rate and a higher Sharpe ratio than US. That is,

$$(a)\ d\log r^* = \underbrace{\phi_5 d\log\lambda}_{\textit{Financial Friction}} - \underbrace{\phi_6 d\log\bar{\sigma}}_{\textit{Output Volatility}}$$

$$(b)\ d\log s_1^* = \underbrace{\phi_7 d\log\bar{\sigma}}_{\textit{Output Volatility}}$$

where ϕ_5, ϕ_6 and ϕ_7 are positive coefficients. Furthermore, EM has a lower required return on risky assets, and a lower excess profit in autarky:

$$d\log(r^* + \bar{\sigma}s_1^*) = \underbrace{\phi_8 d\log\lambda}_{\textit{Financial Friction}} \qquad d\log(V^*) = \underbrace{\phi_9 d\log\lambda}_{\textit{Financial Friction}}$$

where ϕ_8 and ϕ_9 are again positive coefficients.

The core message of Figure 1.3 and the associated proposition is that peripheral economies tend to have an equilibrium in which the risk-free interest rate is low, the expected required return on the risky asset is low, and the Sharpe ratio of the domestic risky asset is high. The pledgeability of future cash flows plays a central role. The production sector's limited ability to promise a fixed return dictates the use of costly fund raising. The supply of safe contractual claims is limited, but risky contractual claims are relatively more abundant due to $\bar{\sigma}$. To clear the market, a higher compensation should be offered to those who hold risky assets. The expected return on the domestic risky asset, $r^* + \bar{\sigma}s_1^*$, and the excess profit, V^* are also lower when λ is small.

Remark 1: Dynamics The earlier results compare two economies with identical sizes of wealth stocks. As time passes by, the wealth stock of each economy grows and the stationary state level of wealth is affected by various parameters including λ and $\bar{\sigma}$. In the stationary state, as it turns out, the wealth stock is lower in EM than in US. In Appendix A.1.2, I show that most of the results in Proposition 3 remain intact even when we compare stationary state interest rates of the two economies, provided that λ is sufficiently small. The appendix also presents the equilibrium law of motion for wealth stock.

1.4 Financial Globalization

The next step of the analysis is to explore changes in the US portfolio frontier when the two economies, US and EM, become integrated. In this section, I decompose financial globalization into two stages: (i) security market liberalization and (ii) FDI liberalization.

1.4.1 Security Market Liberalization

Security market liberalization allows US households to invest in assets issued by the foreign bank. The interest rates of the existing domestic assets are changed. The foreign risky asset, characterized by (s_{2t}, σ_2), is also added to the portfolio frontier, so US households are given more investment opportunities. To specify the market clearing conditions, define

$$\mathcal{S}_{1t}^k \equiv \int \theta_{1it}^k a_{it}^k di, \qquad \mathcal{S}_{2t}^k \equiv \int \theta_{2it}^k a_{it}^k di \qquad \mathcal{S}_t^k \equiv \int a_{it}^k di$$

for each origin country $k \in \{US, EM\}$. In the above expressions, $\mathcal{S}_{1t}^k$ represents country k's savings in the US risky asset, while $\mathcal{S}_{2t}^k$ represents country k's savings in the EM risky asset. Finally, $\mathcal{S}_t^k$ is country k's savings in all types of assets.

I define an open economy equilibrium as a path of the interest rates that clears the entire global financial markets. After security market liberalization, the two economies are coordinated by a common set of interest rates. The market clearing conditions pin down the equilibrium values of r, s_1 and s_2.[27] When the wealth stocks of the two countries are identical in size, one can verify that security market liberalization transforms the portfolio frontier of the US economy as described by Proposition 4, thereby altering the country's wealth distribution.

Definition 3. *An open economy equilibrium is a stochastic process,* $\{r_t, (s_{1t}, \sigma_1), (s_{2t}, \sigma_2)\}_{t\geq 0}$, *which clears the global financial markets:* $\sum_{k\in\{EM,US\}}(\mathbb{S}_t^k - \mathbb{I}_t^k) = 0$, $\sum_{k\in\{EM,US\}} \mathbb{S}_{1t}^k = \mathbb{I}_{1t}^{US}$ *and* $\sum_{k\in\{EM,US\}} \mathbb{S}_{2t}^k = \mathbb{I}_{2t}^{EM}$.

Proposition 4. *(i) After security market liberalization, US becomes the exporter of safe asset*

[27] In an open economy, $A_{1t} + A_{2t}$ acts as the state variable of the economy where $A_{1t} \equiv \int a_{it}^{US} di$ and $A_{2t} \equiv \int a_{it}^{EM} di$. Thus, one can write $r_t \equiv r(A_{1t} + A_{2t})$, $s_{1t} \equiv s_1(A_{1t} + A_{2t})$, $s_{2t} \equiv s_2(A_{1t} + A_{2t})$ and $V_t \equiv V(A_{1t} + A_{2t})$.

and the net importer of global risky assets. US households face[28]

$$(a)\ r < r^* \qquad (b)\ r + \sigma_1 s_1 < r^* + \sigma s_1^* \qquad (c)\ V > V^* \qquad (d)\ s_{mix} \geq s_1$$

where s_{mix} is the Sharpe ratio of the optimal portfolio combining the foreign and domestic risky assets. (ii) The Shape ratio of the domestic risky asset rises (i.e. $s_1 > s_1^*$) if and only if $\bar{\sigma}^{US} < \rho\bar{\sigma}^{EM}$

Proof. Appendix A.1.4 □

Corollary 2. *The integration of symmetric countries lead to $r = r^*$, $s_1 = \left(\frac{1+\rho}{2}\right) s_1^*$, and $V > V^*$.*

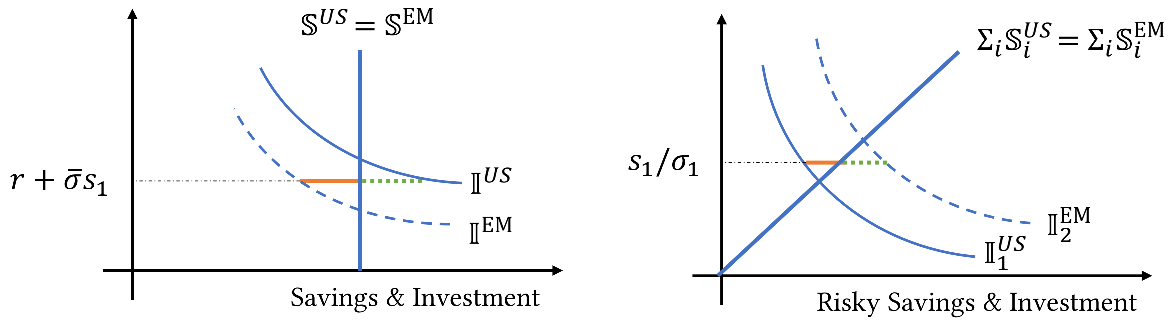
Proposition 4(i) states that security market liberalization offers new risky investment opportunities for US households, while simultaneously decreasing the required expected returns on the US domestic assets. Figure 1.4 illustrates the basic intuition by examining a special case $\rho = 1$ (i.e., the foreign and domestic risky assets are perfect substitutes.) The supply of safe assets is limited in the EM. Besides, since equity financing involves a deadweight transaction cost beside the risk premium, the EM bank faces a higher overall cost of capital. The total investment is thus restrained. After security market liberalization, US should sell its assets to EM and become a net debtor to clear the global financial markets. The excess demand from EM —as indicated by the solid horizontal line in Figure 1.4a— exerts a downward force on $r + \sigma_1 s_1$ and r in the US economy. It is now easy to show the excess profit increases (i.e., $V > V^*$) as the US bank faces a lower average cost of capital. $s_{mix} \geq s_1$ is also straightforward.

How does financial globalization change the risk premium (or the Sharpe ratio) of the US domestic risky asset? While the core model assumed $s_1^* = s_1$, Proposition 4(ii) gives a sharper prediction. The change in s_1 depends on the risk profile of the outside world. If EM is significantly riskier, $\bar{\sigma}^{US} < \rho\bar{\sigma}^{EM}$, US households should bear more risk after security market liberalization, so the domestic Sharpe ratio rises to clear the market. Figure 1.4b illustrates this point when $\rho = 1$. If this is not the case (e.g. $\bar{\sigma}^{US} = \bar{\sigma}^{EM}$ and $\rho < 1$), the domestic Sharpe ratio falls after global integration. This is because diversification from the foreign risky asset helps create a safer portfolio, so a lower value of s_1 is enough to induce US households to clear the markets.

[28]In the statement, $r < r^*$ is a simplified expression for $r(A_{1t} + A_{2t}) < r^*(A_{1t})$ where $A_{1t} = A_{2t} = A$ (i.e., US and EM are identical in size.) The same notation is applied to the other variables.

Figure 1.4: *Open Economy Equilibrium*

(a) *Total Assets* **(b)** *Risky Assets (if $\rho = 1$ and $\bar{\sigma}^{US} < \bar{\sigma}^{EM}$)*



These forces can raise wealth concentration in the US economy through domestic asset price inflation in the short run, and through asymmetric portfolio rebalancing in the long run. The core model in Section 1.2 confirmed theses channels when the supply of assets is perfectly elastic. This section provides microfoundation for the change. To quantitatively measure the effect on wealth inequality with this general equilibrium setup, one needs to turn to numerical simulations.

1.4.2 FDI Liberalization

In this subsection, I briefly show that FDI provides an additional driver for wealth concentration in US as it further expands risky investment opportunities available in global economy.[29] To incorporate foreign direct investment, I embed a simplified model of Holmstrom and Tirole (1997) and Antràs *et al.* (2009) into my framework. A poor contracting environment in the EM gives rise to the need for US multinational firms as they serve as *de facto* financial intermediary in global capital markets.

The extended model has three more ingredients. (Appendix A.1.6 provides a more detailed explanation about the associated optimal contract problem.) First, the entrepreneur in the EM can misbehave in pursuit of private benefits. The misbehavior lowers the expected earning from $\Phi(K_{2t})dt$ to $\pi_L \Phi(K_{2t})dt$, where $\pi_L \in [0, 1)$. The misbehavior gives private benefit to the EM entrepreneur. Second, the US bank can choose to to invest K_{2t}^{FDI} to create a joint venture with

[29]FDI outflow of US multinational firms currently accounts for 43 percent of foreign equity holdings by American households in terms of estimated market value. (Bureau of Economic Analysis, 2017)

the EM entrepreneur. As a compensation, the US bank receives a designated share of profit. The US bank monitors the EM entrepreneur to make sure that the full profit is reached. Finally, the remaining portion of the investment, $K_{2t}^{Local} \equiv K_{2t} - K_{2t}^{FDI}$, is funded through the local bank in the EM. The banks in the two economies raise funds from investors as before, which constitute the supply side of assets.

Given this extension, in Appendix A.1.6, I compare the three stages of financial globalization: (i) autarky (ii) security market liberalization and (iii) FDI and security market liberalization. For each stage of financial globalization, the equilibrium interest rates are denoted with the superscript (i), (ii), and (iii) respectively. We can then confirm that foreign direct investment provides an additional expansion of the foreign risky asset, thereby exerting an upward force on wealth concentration in US.

Proposition 5. *The liberalization of security markets and FDI transforms the US portfolio frontier such that*

$$(a)\ \ s_1^{(iii)} > s_1^{(ii)} > s_1^{(i)} \qquad (b)\ \ (V + V^{FDI})^{(iii)} > V^{(iii)} > V^{(i)}$$

if $\bar{\sigma}^{US} < \rho\bar{\sigma}^{EM}$ and π_L is sufficiently small. Furthermore, US becomes the exporter of safe assets, and the net importer of the foreign portfolio and direct investment assets.

Proof. See Appendix A.1.6. □

The intuition for Proposition 5 is as follows. Even after the global security markets are liberalized, the EM entrepreneur still has limited ability to raise funds due to their low pledgeability (i.e., $\pi_L < 1$). Investment in the EM is more limited than investment in US. What FDI does is to let the US bank become the parent company and monitors the EM subsidiary. By doing so, the joint venture opens up the full potential of investment in the EM. The investment in the EM is essentially riskier than the investment in US. Thus, a higher value of s_1 is required when both FDI and security markets are liberalized, to induce US households to bear more risks and clear the markets. The US bank gains an additional excess profit, in the form of $V^{FDI}dt$.

1.4.3 Cross-country implications

One implication of the model is that financial globalization increases wealth concentration most prominently within a financially-developed economy (at least in the short run) by changing the country's equilibrium interest rates. Two channels have been discussed: domestic asset price inflation in the short run and portfolio reallocation in the long run. By shedding light on the global finance architecture, the model offers a novel argument for why the U.S. has experienced a particularly large increase in wealth concentration among developed economies. The US economy is often quoted as the world's banker due to its special role in international financial markets. The model suggests that the country's special function would have played a central role in transforming domestic interest rates, thereby increasing wealth inequality among American households.

By contrast, in peripheral economies, the effect depends on specific circumstances. First, suppose that FDI is shut off and the foreign risky asset is a perfect substitute for the domestic risky asset. In this case, whatever happens in US, the opposite will happen in the EM. Second, if the foreign and domestic risky assets provide diversification benefits for each other, US and EM will both experience the heterogeneous portfolio rebalancing between the rich and the poor. Finally, wealth inequality can be increased in the both regions if FDI plays the most significant role in the process of financial globalization. This is because the liberalization of FDI can expand risky investment opportunities not only for advanced economies but also for emerging markets.

1.5 Conclusion

Financial globalization was an important milestone for the U.S. capital market. The U.S. economy has experienced a dramatic increase in capital flows over the past decades, yet the expansion of external balance sheet was asymmetric, owing to the country's special role in the global financial system. On the one hand, American investors gained new access to high-yield risky assets in the form of global equity, and FDI of U.S. multinational firms. On the other hand, an increasingly large proportion of U.S. debt securities is being held by foreign investors seeking safe returns.

This paper developed a model to analyze the effect of financial globalization on rising wealth concentration in the U.S. I showed that capital account liberalization around the globe can change the market value of net worth for American households and reshape the way their wealth is subsequently reinvested. Quantitatively, about one-third to one-half of the increase in the top one percent's wealth share could potentially be accounted for by global financial flows between the financial center and peripheral economies. At the same time, the model indicates that a future trajectory of wealth concentration is dependent on the relative magnitudes of the drop in domestic interest rates and the expansion of new risky assets. Declining yields of global and U.S. domestic assets in recent periods suggest that a reversal of the trend in rising wealth inequality is not impossible in the future.

Studying the international dimension of capital market would help us understand the evolution of the wealth distribution and to design distribution policies. This paper takes one step, but the following areas deserve further investigation: First, there is still a computational difficulty in modeling asset prices with heterogeneous agents at a large scale. The literature has room for improvement. Second, I simplify the linkage between housing markets and global financial flows. Third, no active tax policy is explored in this paper. I leave these topics for future research.

Chapter 2

Multinationals as Global Financiers

2.1 Introduction

The U.S. economy is often quoted as the "global venture capitalist" due to its exclusive role in international financial markets (Gourinchas and Rey 2007). US multinational companies (MNCs), in particular, play a prominent role in raising capital abroad and investing in high-yield business opportunities across the globe. Yet, the focus of recent studies on MNCs has largely prioritized their outsize role in international trade. Relatively little attention has been paid to study the effect of global investment activity on financial performance of an individual firm.

This paper explores the role of MNC activity on the wider return spread between American firms with foreign operations. Our evidence indicates US multinational firms enjoy a 0.9% larger spread between their return on asset (i.e., profits divided by book value of assets) and average interest rate compared to when these same firms were not engaged in substantial overseas investment. This spread suggests that US multinationals on average generate higher profits relative to their invested capital and pay lower interest rates on their liabilities. Motivated by this evidence, we develop a quantifiable model to further decompose several possible causal channels of this gap, such as incomplete financial markets and risk premia on global investment. We examine this model with simulated data, and our simulation suggests some of the variation in this spread can be accounted for by the first channel; cross-country investment barriers allow MNCs to borrow at a lower interest rate and earn a higher return, relative to domestic-oriented companies.

Our analysis highlights the role of US multinationals as a global arbitrageur that exploits return differentials across countries. The main contribution of this paper is to provide a unified framework to quantitatively decompose various channels that account for the superior performance of US multinational firms. We shed light on the international finance dimension of multinational firms as a crucial driver for the recent changes in the US corporate sector.

This paper begins by reporting that there is a widening spread between the weighted averages of firms' physical rate of return and interest rate amongst non-financial firms in the United States[1]. The first measure is a widely used metric for evaluating capital's profitability. The second measure accounts for the cost of debt capital. They both reflect foreign operations, if any, as well as domestic earnings of US headquartered firms. The standard neoclassical theory suggests that the spread between the average return on capital[2] and interest rate should be constant.

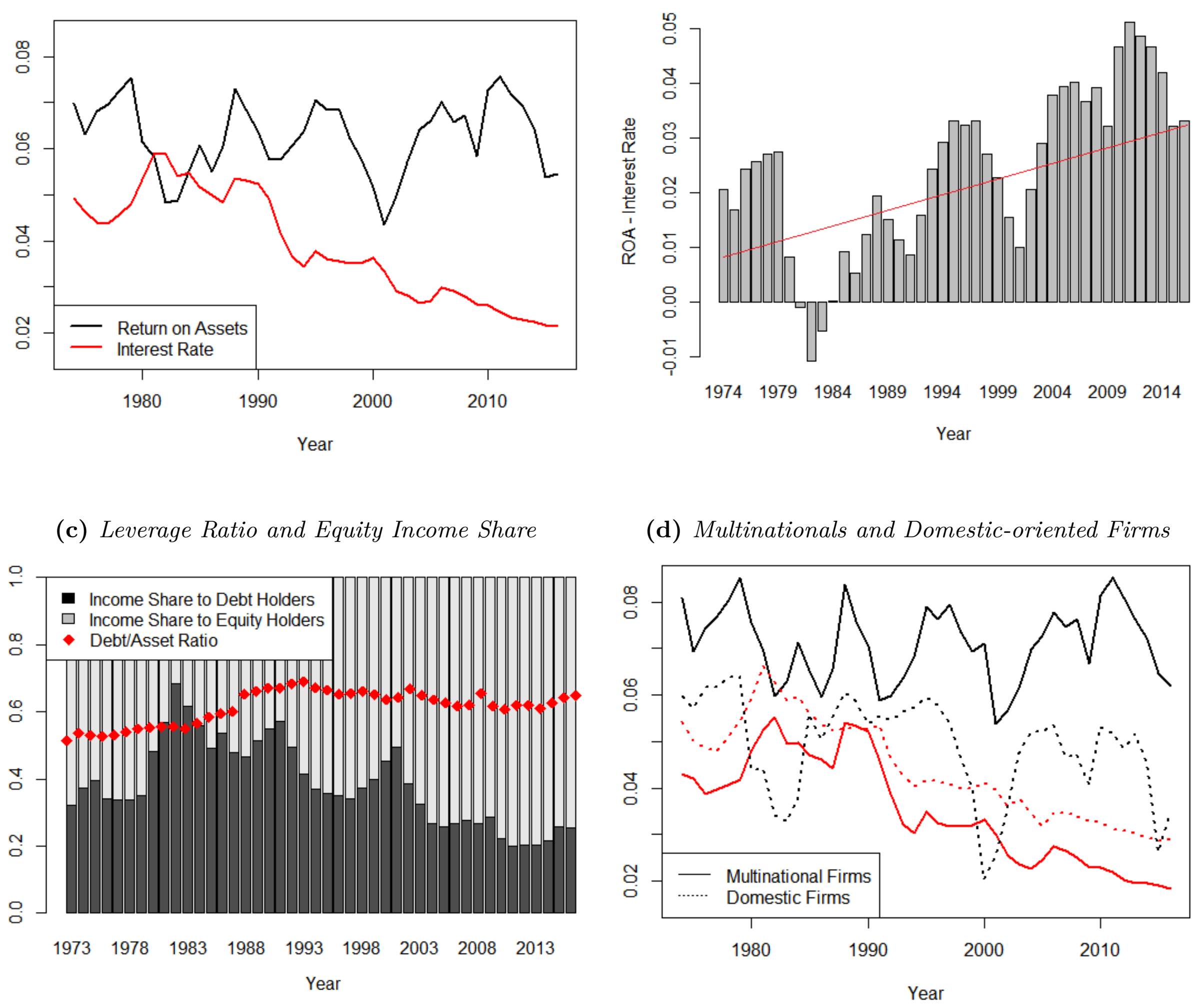
What is striking about these measures is that over the last 40 years Return on Assets (ROA) has fluctuated around a relatively stable constant value, while the interest rate actually paid by these firms has steadily declined since 1980s. This divergence has resulted in a growing spread between firm ROAs and average cost of capital, which appears in Panel (a) of Figure 2.1. The data exhibits a fluctuating, but still steadily increasing difference between ROA and interest rate, with the largest differential emerging in years immediately preceding at the onset of the global financial crisis (with interest rates briefly exceeding ROA in the years around the Volcker Shock). Two natural questions emerging from this figure are the following: what accounts for this increased return gap, and where are the excess profits going?

Turning to where the excess profits are going, we see that over the same period income shares are increasingly being paid out to equity holders. (See Figure 2.1 plot c). This increase in the income share to equity holders does not appear to be related to any systematic changes in firms' aggregate capital structure, as the debt-to-asset ratio has remained approximately constant over

[1]Source: Compustat data. We restrict our sample to the post-1972 period to cover all publicly traded firms.

[2]Under constant return-to-scale production function, the average return on capital is equal to the marginal return on capital.

Figure 2.1: *Motivating Evidence*



(a) *Return on Asset and Interest Rate*

(b) *Spread between Return on Asset and Interest Rate*

(c) *Leverage Ratio and Equity Income Share*

(d) *Multinationals and Domestic-oriented Firms*

Notes. Panel (a): The black line displays a time series of the average return on asset among publicly-traded non-financial US firms, while the red line displays a time series of the average interest rate paid by these firms. Both measures are weighted by firm sizes. Panel (b): Bar plot displays the spread between the average return on asset and interest rate. Panel (c): Red dots indicate the average debt-to-asset ratio. Black bars are defined as the total interest expenses divided by the total earnings. Gray bars are one minus the values from the black bars. Panel (d): The solid lines (black and red) indicate the average return on asset and interest rate paid among multinational firms. The dotted lines indicate the average return on asset and interest rate paid by domestic-oriented firms. A firm is coded as being an MNC if it has a non-zero value for foreign income tax paid. Source: Compustat data.

this time period.

As for the source of this increased return gap, the differential appears more pronounced amongst US Multinational Companies (see Figure 2.1 plot d)[3]. The dotted lines display the average ROA and interest rate of US firms that pay no foreign income taxes, which we use as a proxy for activities abroad. Later sections in this paper confirm this result holds for recent years with more accurate measures of MNC activity at both the extensive and intensive margins obtained from confidential microdata collected by the US Bureau of Economic Analysis (BEA). Solid lines indicate the average ROA (in black) and interest rate (in red) of US multinational firms, at the fully consolidated level. This same analysis for non-MNCs is presented with dashed lines. The gap between ROA and interest rate has been increasing among multinationals over the past decades. Domestic firms, on the other hand, have shown relatively parallel trends between the average ROA and interest rate. MNCs appear to have been able to generate persistently higher ROAs than their domestic counterparts while also benefiting from a larger reduction in borrowing costs.

Motivated by these patterns, we develop theory and empirics to understand the effect of MNC activity on the spread between the ROA and interest rate of a firm. The aim of this analysis is twofold: first, identify the key channels of this spread such as risk premia and market imperfections. Second, quantify the relative magnitudes of these different channels. Our analysis proceeds in two steps.

First, we explore parent firm-level evidence to determine if multinationals do in fact earn a higher accounting return on capital and pay a lower interest rate. Our baseline analysis employs fixed-effect regressions. Our evidence indicates MNCs enjoy a 0.9% larger spread between ROA and average interest rate compared to when these same firms operated only domestically. Multinationals engaged in primarily vertical FDI exhibit a 1.2% larger spread, while horizontally oriented MNCs enjoy a slightly lower spread of 0.8%.

Second, we develop a model to explore the channels that account for the widened spread between ROA and interest rate of multinational firms. We describe two types of MNC premia in the model. The first is a simple risk-premium story: if MNCs engage in riskier investments, they

[3] Source: Compustat data. A firm is coded as being an MNC if it is has a non-zero value for foreign income tax.

demand higher returns to compensate for these risks. MNCs face several new risks in the form of potential supply chain disruptions, adverse currency movements, and in some sectors outright expropriation. The second channel we consider is the role of incomplete financial markets: if MNCs have greater access to foreign investment opportunities (either through direct market access or differential credit constraints), these companies can use this access to pursue different ideal portfolio and leverage compositions which allow these firms to generate greater returns. We build a multi-sector model that incorporates sector-specific FDI potentials and fixed costs across different regions of the world to predict observed patterns of foreign investment (although to-date we only test this model with simulated data).

The structure of the model enables separable identification of the FDI potential and fixed cost of entering markets. Our three-step estimation method follows from Antràs *et al.* (2017). Unlike these authors, whose focus is primarily on input sourcing decisions, we shed light on the investment and funding decisions of a multinational firm. Our simulation results show that a large portion of the MNC premia could be potentially attributed to the financial market incompleteness. For this paper, we do not use any BEA microdata to directly test this model model. Instead we use pseudo-data to assess the validity of our estimation strategy and investigate qualitative patterns of the model. Our simulated model suggests that multinational firms on average can take a lower risk due to the global diversification effect. Despite the lower volatility, their intrinsic advantage in market access generates a higher profitability and lower funding cost in the partially segmented global financial market.

Our results highlight the role of US multinationals as global arbitrageurs in addition to being global risk-takers. Previous studies have documented return differentials of foreign assets and liabilities, including foreign direct investment, at the macro-economy level (e.g. Caballero *et al.* 2008, Gourinchas and Rey 2007, Gourinchas and Rey 2010). Yet, little attention has been paid to micro-level sources of the return differentials. Our paper sheds light on new channels to account for these differentials. On the international trade side, conventional trade models have largely focused on the advantages of high productivity firms with respect to exporting products (Melitz 2003), importing inputs (Antràs *et al.* 2017) or both concurrently (Bernard *et al.* 2018). In this paper, we extend this conceptual framework to the context of global investment and funding. Namely, firms are economic entities that import foreign assets and export domestic liability in global capital markets. The key contribution of this paper is to develop a unified framework to understand the role of MNCs in global financial markets, which has been studied largely independently in the literature on international trade and international finance.

The rest of our paper is organized as follows. Section 2.2 describes the data and presents motivating evidence. Section 2.3 introduces the baseline model. Section 2.4 extends this model and provides an estimation strategy. Section 2.5 displays quantitative results with simulated data. Section 2.6 concludes. Detailed proofs and computation algorithms not appearing in the main text are included in the appendix.

Literature Review Through this study, we aim to contribute to three strands of the academic literature. First, as previously mentioned, we shed light on firm-level analysis of global capital flows. Unlike previous studies focusing on aggregate statistics (e.g. Caballero *et al.* 2008, Gourinchas and Rey 2007, Gourinchas and Rey 2010, Curcuru *et al.* 2013), we bring to the fore the importance of firm-level analysis in studying global capital movements and their returns. We provide a novel mechanism to account for the return differentials, which stems from the incomplete integration of global capital markets. Ours is distinct from previous channels such as intangible assets (McGrattan and Prescott 2010) and risk premium (Fillat and Garetto 2015). Our methodology allows us to decompose the quantitative magnitudes of these channels.

We also contribute to the literature on the financing activities of MNCs. Most of this literature focuses on the sources of financing for their foreign affiliates (Manova *et al.* 2015), financial frictions faced by them (Bilir *et al.* 2019) and the role of these firms as *de facto* financial intermediaries (Antràs *et al.* 2009). Our paper also relates to the literature on intra-firm credit spillovers and borrowings within a multinational firm (e.g. Desai *et al.* 2004, Manova *et al.* 2015). We shed light on a mechanism that MNCs' abilities to tap into local foreign credit markets promote their financial performance and excess profits.

Third, there is a growing interest in the rise of "superstar" firms and its implications on income share and corporate inequality (e.g. Karabarbounis and Neiman 2013, Autor *et al.* 2017). Most of these studies focus on domestic market shares, domestic markups and wages. In contrast, we augment this literature by investigating the international asset and liability sides of U.S. multinational firms. We hope to gain an understanding of how the global expansion of U.S. firms has affected the sub-components of capital income such as interest rates, risk premium and excess profits. The macro-level trend in the U.S. domestic market has been documented recently (Caballero *et al.* 2017, Farhi and Gourio 2018) but there is a lack of understanding on theoretical channels, micro-level evidence and connections to globalization. Our model provides a quantitative framework highlighting the role of financial globalization in the trends.

2.2 Motivating Evidence

A key limitation of the data presented in Figure 1 is the coarse definition of MNC status obtained from Compustat. To address this issue, our analysis merges the key ROA and interest rate measures available from Compustat with more accurate measures of MNC activity collected by the US Bureau of Economic Analysis (BEA) in its annual and benchmark survey data on U.S. Direct Investment Abroad.

2.2.1 Data Description

We merge firm-level financial data from Compustat with confidential micro-data collected by the United States Bureau of Economic Analysis on the extensive and intensive margins of activities of foreign affiliates of US MNCs.

Compustat provides financial accounting data for US firms on a fully consolidated basis. We use this data to calculate our firm level measures of return on assets and interest rate. This data also includes operating sector indicators (4-digit) and a wide array of other accounting measures, including Total Assets which we use to create a measure of a firm's relative size in its sector. Our primary outcome variables are calculated using exclusively this data as: (i) roa = (Earnings Before Interest and Taxes (EBIT) - Tax Expense)/Total Assets, (ii) interest rate = Interest Expense / Total Liabilities and (iii) spread = roa - interest rate. These heuristic measures are intended to capture return on investment and cost of debt capital at the firm level.

While Compustat provides the accounting information used to calculate roa and interest rate at the fully consolidated level, we use the annual (Form BE-11) and benchmark (Form BE-10) BEA survey data on U.S. Direct Investment Abroad to identify U.S. firms with foreign affiliates and to identify those MNEs that are primarily engaged in horizontal and vertical FDI. This data is derived from information collected in surveys of U.S. multinational enterprises and surveys of U.S. affiliates of foreign multinational enterprises that are conducted by BEA, and the data includes annual financial and operating data of the U.S. reporter and its foreign affiliates. U.S. parents must report on the operations of both majority and minority owned foreign affiliates that are sufficiently large.[4] Additionally, every five years the BEA conducts a benchmark survey (Form BE-10, the most recent benchmark survey with available data was in 2014). These benchmark surveys provide greater coverage as a response is required from entities subject to the reporting requirements of the BE-10, whether or not they are contacted by BEA. Our reported results are not materially changed if we restrict the sample to only these years when the more complete benchmark survey was conducted.

With this microdata, we obtain a variety of measures for the extensive and intensive margins of foreign activities at the reporting parent level, including:

- Total number of foreign affiliates
- Sales of foreign affiliates broken down by industry classification
- Sales of foreign affiliates by destination of sale (host country, US, or rest of the world)
- Sales of foreign affiliates by whether the purchaser is an affiliated party

The regression results reported in the next section use the BEA data collected above to create three variables of MNC activity at the firm-level[5]

- *FDI* - A binary indicator of whether firm i reported at least one foreign affiliate in the

[4]For the 2015 BE-11, the most recent filing year used in this analysis, a U.S. parent has to report information on a foreign affiliate if the foreign affiliate has a value of more than \$60 million for any of the following: total assets; sales or gross operating revenues, excluding sales tax; or net income after provision for foreign income taxes.

[5]These binary indicators are based on data for affiliates that are sufficiently large to report sales by affiliation. We do not include foreign affiliates which only completed a form BE-11D (BE-10D for benchmark years) filing. For the 2015 BE-11, form D was filed for a foreign affiliate established or acquired during a fiscal year that the affiliate has total assets, sales or gross operating revenues, or net income of more than \$25 million (positive or negative) but for which none of these exceed \$60 million (positive or negative) at the end of the affiliate's fiscal year.

annual (or benchmark) BEA survey data on U.S. Direct Investment Abroad collected in year t

- *VerticalFDI* - A binary indicator of whether firm i reported at least one foreign affiliate in the annual (or benchmark) BEA survey data on U.S. Direct Investment Abroad collected in year t *and* the percent of total foreign affiliate sales to related parties (summing across all foreign affiliates) exceeded 25%
- *HorizontalFDI* - A binary indicator of whether firm i reported at least one foreign affiliate in the annual (or benchmark) BEA survey data on U.S. Direct Investment Abroad collected in year t *and* the percent of total foreign affiliate sales to related parties (summing across all foreign affiliates) did not exceed 25%

While we recognize this is a coarse measure of vertically and horizontally oriented FDI, our measure does capture the potential for firms to use sales with related parties to achieve internal cost-savings or benefit from transfer pricing decisions. This measure could however assign a firm as being engaged in primarily vertically oriented FDI if it has a large volume of sales to related parties, but little product transformation (such as sales to affiliated local dealers who resell a product with no additional transformation). Given the model in this paper does not rely on a distinction between vertical and horizontal FDI, and the fact that our regression results in the next section do not suggest there is evidence of the effect of MNC status differing significantly across this classification, we use this as prima facie evidence to justify looking at aggregate FDI values in our structural model. While we do not analyze this distinction further in this paper, subsequent work could use more traditional measures of FDI type, such as International Surveys Industry (ISI) product transformations between affiliates or direct shipments to the parent alone to measure vertically-oriented FDI.

The outcome variables (roa, interest rate and spread) are all calculated as outlined above using only Compustat data. Finally, it is well known that firm-size is an important predictor of profitability. To control for this, we use Compustat data exclusively to calculate each firm's asset quartile by year. This calculation is done by first assigning firms to one of five sectors (manufacturing, retail, wholesale, services or other) and calculating asset quartiles at the sector-year level.

2.2.2 Fixed-effects Regression

The aim of this paper is to explore a relation between the expansion of multinational operations and a higher spread between return on asset and interest rate. To this end, our initial regression framework implements the following fixed effect specification for firm i at time t

$$y_{it} = \alpha_i + \alpha_t + \beta FDI_{it} + \gamma X_{it} + \varepsilon_{it}$$

where $FDI_{i,t}$ is the binary measure of whether a given parent firm i completed a BEA filing for at least one foreign affiliate in year t. X_{it} is a matrix of time-varying company controls. α_i is the firm-level fixed effect, α_t is time fixed effect. $FDI_{i,t}$ can be (a) binary or (b) the number of countries that firm i has entered. Here, we focus on the binary indicator; the sign and magnitude of our results are robust to using these alternative measures of foreign activity. Our sample period uses data from 1998 to 2015. While data is available in earlier years, earlier surveys do not all collect the same intensive margin measures we initially considered.

Table 2.1: *Simple Regression - Full Sample*

	Dependent variable: ROA Interest Spread		
	(1)	(2)	(3)
FDI	0.013** (0.002)	0.196** (0.005)	0.050** (0.003)
1st Asset Quartile			-0.495** (0.011)
2nd Asset Quartile			-0.058** (0.005)
3rd Asset Quartile			0.018** (0.005)
4th Asset Quartile			0.032** (0.005)
Fixed Effects	None	Year	Year
Observations	68,949	68,949	68,949

Notes: This table presents results from the simple linear regression. Column (1) indicates the values when only the binary indicator is used. Column (2) adds the year fixed effect. Column (3) controls for size quartiles. Size quartiles are calculated on a yearly basis with firms grouped into five broad sectors (manufacturing, retail, wholesale, services and other). The standard errors are clustered at the firm-level. *p<0.05; **p<0.01;

2.2.3 Results

Table 2.1 displays the results from a simple linear regression, where we add year fixed effects and controls for company size. This model confirms the graphical intuition presented in the motivating evidence; firms engaged in FDI exhibit greater spreads between their ROA and interest rate. In column (1), we run a simple linear regression. FDI does generate a positive gap in this simplest setup. In column (2) and (3), we add year-fixed effects and time-varying sizes of firms. The last column suggests that the spread between the roa and interest rate is a higher among multinational firms by 5.0 percentage points. Note that the coefficient estimate for FDI_{it} in the simple regression can be interpreted as the average gap between multinational firms and domestic firms at year t. This specification does not allow us to rule out unobservable differences between multinational and domestic firms, such as productivity and industry-level characteristics, could act as confounding factors. To address this concern, we next employ a fixed effects regression framework.

Table 2.2 displays results from the full fixed effect specification. The coefficient on FDI

Table 2.2: *Fixed Effect Regression - Full Sample*

	Dependent variable: ROA Interest Spread	
	(1)	(2)
FDI	0.009* (0.004)	
Vertical FDI		0.012** (0.004)
Horizontal FDI		0.008** (0.004)
1st Asset Quartile	-0.256** (0.013)	-0.256** (0.013)
2nd Asset Quartile	-0.053** (0.006)	-0.053** (0.006)
3rd Asset Quartile	-0.005 (0.003)	-0.005 (0.003)
4th Asset Quartile	-0.065** (0.005)	-0.065** (0.005)
Fixed Effects	Firm and Year	Firm and Year
Observations	68,949	68,949

Notes: This table presents results from the fixed-effects regression. Column (1) indicates the values when only the binary indicator is used. Column (2) decomposes this explanatory variable into two: horizontal FDI and vertical FDI. Size quartiles are calculated on a yearly basis with firms grouped into five broad sectors (manufacturing, retail, wholesale, services and other). The standard errors are clustered at the firm-level. *p<0.05; **p<0.01;

indicates an increase in the spread when we compare the same firms prior to and after initiating foreign direct investment, thereby controlling for static firm-level characteristics. The model in column (1) estimates that after a company begins engaging in FDI, these firms on average generate a 0.9% larger spread between their ROA and interest rate compared to these same firms when they did not report any foreign ownership. Column (2) classifies a parent's foreign affiliates as being primarily engaged in vertical or horizontal FDI based on the percentage of total affiliate sales which are to related parties (both domestic and international related parties). If the percent of affiliate sales to related parties is less than 25%, a parent firm is coded as being engaged in primarily horizontal FDI. There does not appear to be evidence based on this measurement that the effect of MNC status on the estimated ROA interest spread is systemically different for vertical or horizontal MNCs.

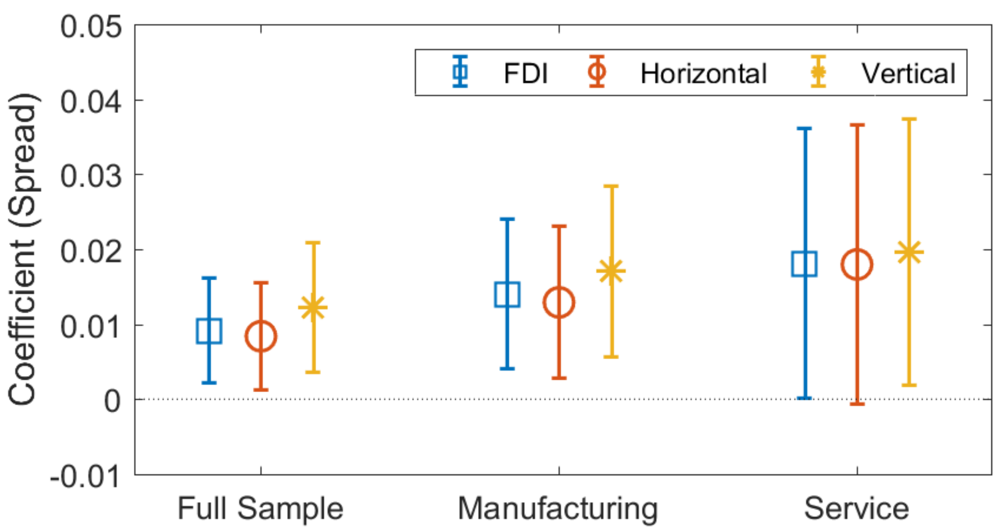
Next, we replicate this same fixed effect analysis across different sectors of the data and for outcome variables focusing on only the ROA generated by or only the interest rate paid by these firms. The following three plots, along with Table 2.3 display these results across three data samples (the full sample, manufacturing firms only, and services firms only) and for our three outcome variables of interest. Across industry sectors, MNCs generate both higher ROAs and pay lower interest costs. Interest rate channels appear smaller in magnitude yet statistically more robust across different specifications. These findings motivate further examining how MNCs are different on both their operating and financing channels.

Table 2.3: *Summary of Regression Coefficients*

Sample	Type	Dependent Variable (Spread)	(ROA)	(Interest Rate)
Full	FDI	0.0091* (0.0035)	0.0071* (0.0034)	-0.0020** (0.0005)
	Horizontal	0.0084* (0.0037)	0.0064 (0.0035)	-0.0020** (0.0005)
	Vertical	0.0122** (0.0043)	0.0100* (0.0041)	-0.0022**(0.0006)
Manufacturing	FDI	0.0140** (0.0051)	0.0117* (0.0049)	-0.0023** (0.0008)
	Horizontal	0.0129* (0.0052)	0.0108* (0.0050)	-0.0021** (0.0008)
	Vertical	0.0170** (0.0058)	0.0144* (0.0055)	-0.0026** (0.0009)
Service	FDI	0.0181* (0.0091)	0.0155 (0.0090)	-0.0026** (0.0011)
	Horizontal	0.0180 (0.0094)	0.0151 (0.0093)	-0.0028** (0.0011)
	Vertical	0.0196* (0.0090)	0.0190* (0.0085)	-0.0006 (0.0019)

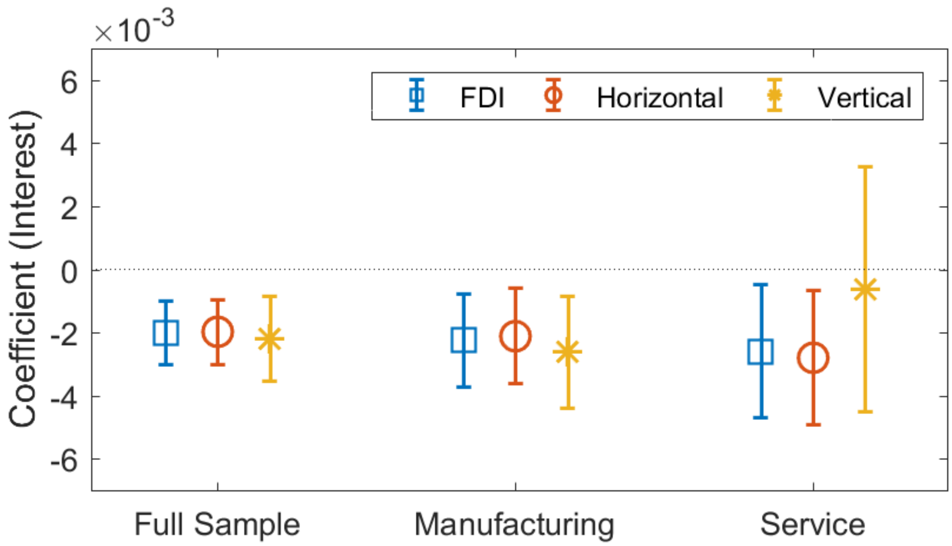
Notes: This table presents results from the fixed-effects regression along three dimensions: sector subsamples, main explanatory variables (types of foreign direct investment) and dependent variables, controlling for time-varying sizes of firms. *p<0.05; **p<0.01.

Figure 2.2: *Spread - FDI Regression Coefficient*



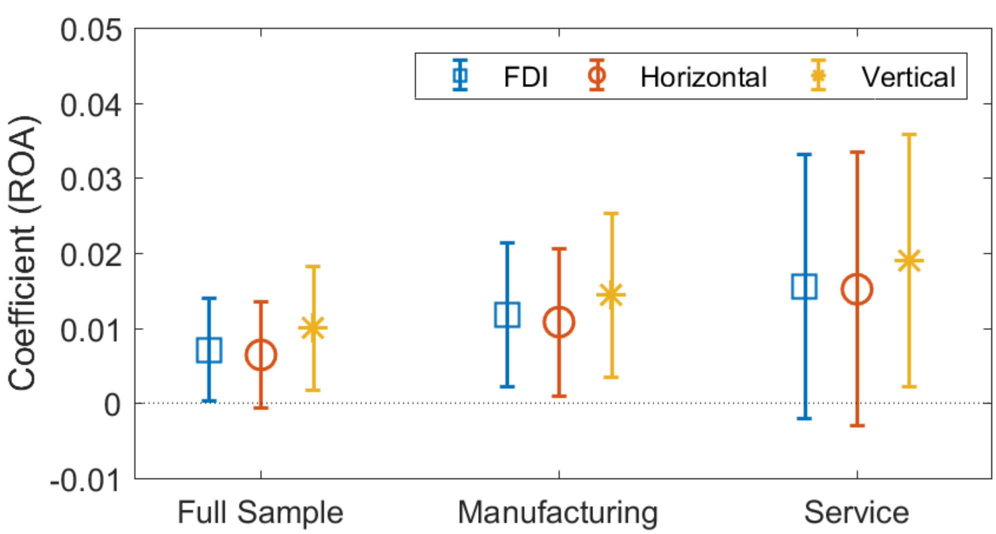
Notes: This figures displays 95 percent confidence intervals for regression coefficients when the dependent variable is given by the spread between return on asset and interest rate. Intervals on the manufacturing (service) column indicate confidence intervals when only manufacturing (service) firms are considered in a subsample. The blue bar indicates general foreign direct investment, while red and yellow bars indicate horizontal and vertical FDI defined before.

Figure 2.3: *Interest Rate - FDI Regression Coefficient*



Notes: This figures displays 95 percent confidence intervals for regression coefficients when the dependent variable is given by the average interest rate. Intervals on the manufacturing (service) column indicate confidence intervals when only manufacturing (service) firms are considered in a subsample. The blue bar indicates general foreign direct investment, while red and yellow bars indicate horizontal and vertical FDI defined before.

Figure 2.4: *ROA - FDI Regression Coefficient*



Notes: This figures displays 95 percent confidence intervals for regression coefficients when the dependent variable is given by the average return on asset. Intervals on the manufacturing (service) column indicate confidence intervals when only manufacturing (service) firms are considered in a subsample. The blue bar indicates general foreign direct investment, while red and yellow bars indicate horizontal and vertical FDI defined before.

2.3 Model

In this section, we describe two types of MNC premia. The first hypothesis is a risk exposure story, in which foreign investment involves intrinsically higher risk than domestic investment. The second hypothesis is an incomplete integration, under which multinational firms serve as global arbitrageurs through investment; multinationals raise funds at a lower interest rate country and, at the same time, invest in countries with a higher marginal product of capital. Finally, we assess a possibility that some of the spread between return on asset and cost of debt capital is driven by measurement errors of intangible assets.

2.3.1 Setup

The global economy consists of N countries. Each country is denoted by $n = 1, ..., N$ and has two types of entities: households and firms. Households invest in risk-less domestic deposits at period $t = 0$ and receive interest at period $t = 1$. Firms take deposits from households, invest in physical capital and make profits. Capital is the only factor of production. Capital markets are isolated prior to globalization so each country faces its own interest rate and risk premium. Upon financial globalization, firms can pay a fixed cost to tap into foreign capital markets. Below, we elaborate on details of households, firms and our equilibrium concept.

Households The representative household in country $n \in \{1, 2, ..., N\}$ is endowed with W_n units of consumption goods in period 0. The price of a consumption good in country 0 acts as the numeraire. All values are measured in terms of these goods. For expository purposes, we often call country 1 as the US and their currency as the dollar. Besides consumption goods, households are endowed with K_n units of capital goods. The price of a capital good in period 0 is given by q_n. Households sell these capital goods to firms as they have no production technology. We model the household's problem in country n as maximizing a two-period utility

$$u(C_{0n}) + \delta u(C_{1n})$$

subject to

$$C_{0n} = W_n + q_n K_n - D_n$$

$$\tilde{x}_n C_{1n} = \tilde{x}_n (1 + r_n) D_n$$

Here, $\tilde{x}_i$ represents the period-1 exchange rate against the dollar, which is a stochastic variable. Assume $\mathbb{E}_0[\tilde{x}_n] = 1$. The period-0 exchange rate is given by 1. Essentially, households can save only in risk-less deposits D_n, denominated in their home currency. δ is the time discount factor.

Firms There are I firms in the global economy. Each firm is managed by a single entrepreneur denoted by $i = 1, ..., I$. I use index i to indicate a firm and its entrepreneur interchangeably. Firm i has non-movable headquarters at country $n_i \in \{0, 1, 2, ..., n\}$, which we call its nationality. The entrepreneur who manages firm i is endowed with e_i units of goods for investment in period 0. Their role is to raise outside capital and invest by purchasing capital goods.

Unlike households, firms have production technology. Let $\alpha_i \equiv (\alpha_{i,0}, ..., \alpha_{i,N})'$ denote a vector of capital goods invested by firm i across different countries. Technology is assumed linear. That is, firm i generates

$$f(\alpha_i) \equiv \sum_{n=1}^{N} (\pi_i + \pi_n + \sigma_n \tilde{z}_n)\, \alpha_{k,i}$$

units of consumption goods in period 1 where π_n is a country-specific component of the return, π_i an idiosyncratic return of firm i and $\tilde{z}_n$ represents a random component. The random variable is drawn from the standard normal distribution with σ_n being the standard deviation. Since the price of a capital good is q_n, the expected return on capital is $\frac{\pi_i + \pi_n}{q_n}$ when firm i invests in country n. The expected return on bearing one unit of risk is $s_{i,n} \equiv \frac{\pi_i + \pi_n}{q_n \sigma_n}$.

With this technology in place, the decisions makings of firms are two-fold. First, firm i chooses a set of countries, $X_i \subseteq \{1, 2, ..., N\}$, in which to operate their business. The firm has to pay a fixed cost $\tau_i f_n$ to enter foreign country n. Only after paying this fixed cost, the firm is able to buy physical assets or issue debt securities in country n outside their home country. No cost is incurred for home country investment. Second, the firm determines a business portfolio weight $\alpha_i \equiv [\alpha_{i,1}, ..., \alpha_{i,N}]'$ and debt weight $\beta_i \equiv [\beta_{i,1}, ..., \beta_{i,N}]'$ over different countries. Naturally, the investment country set can be expressed as $X_i \equiv \{n : \alpha_{i,n} \neq 0 \text{ or } \beta_{i,n} \neq 0\}$. All firms are price takers.

An entrepreneur who owns a firm has exponential utility over period-1 consumption and the risk aversion parameter is given by γ. Thus, the objective function of entrepreneurs can be simply transformed to $\mathbb{E}[C^e_{i0}] - \frac{\gamma}{2}\mathbb{V}[C^e_{i1}]$. In the model, firms face two types of risks: production risks and exchange rate risks. Production risks stem from $\sum_n \alpha_{i,n}\sigma_n\tilde{z}_n$, while exchange rate risks stem from $\sum_n \beta_{i,n}(1+r_n)\tilde{x}_n$. Let Ω_0 denote the variance-covariance matrix stemming from a vector of random variables $[\sigma_1\tilde{z}_1, ..., \sigma_N\tilde{z}_N, (1+r_1), (1+r_2)\tilde{x}_2, ..., (1+r_N)\tilde{x}_N]'$ in country 0. Putting all these ingredients together, we can state an entrepreneur's problem in country 0 as

$$\max_{\{\alpha_i,\beta_i\}} \left\{ \underbrace{\left[\sum_{n\in X_i} \alpha_{i,n}\,(\pi_i + \pi_n) - \sum_{n\in X_i} \beta_{i,n}(1+r_n) \right]}_{\text{(i) Expected Return on Levered Capital}} - \underbrace{\frac{\gamma}{2}[\alpha_i', \beta_i']\Omega_0 \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix}}_{\text{(ii) Volatility of Returns}} \right\} - \underbrace{\sum_{n\in X_i\setminus\{0\}} \tau_i f_n}_{\text{(iii) Fixed Costs}}$$

subject to

$$\sum_{n\in X_i} q_i\alpha_{i,n} = e_i + \sum_{n\in X_i} \beta_{i,n} \tag{2.1}$$

$$\alpha_{i,n} \geq 0, \beta_{i,n} \geq 0 \text{ for all } n \tag{2.2}$$

The second constraint rules out short-selling of productive capital and net positive savings in deposits. The former is a natural assumption. The latter limits the ability of firms as arbitrageurs in global debt markets.

Market Clearing Conditions We define a market equilibrium as a price vector such that the supply and demand for all assets are equalized. The price of debt securities is clear: we can regard the interest rate, r_n, as the price that coordinates savings of households and debt issuance of firms. As for productive capital, we use q_n as the main variable that determines compensation for taking one unit of risk in country n.

Definition 4. *Equilibrium is defined as a price vector* $\{q_n, r_n\}_{n=1}^N$ *that clears financial markets in all countries: (i)* $\sum_{i=1}^I \beta_{i,n} = D_n$ *and (ii)* $\sum_{i=1}^I \alpha_{i,n} = K_n$ *for all* $n = 0, ..., N$.

Condition (i) means that the total debt issuance in country i is equal to the demand for deposits in country i. Condition (ii) shows that the total market value of productive capital should be equal to the total investment of firms in each country.

2.3.2 Discussion on the Model

Three features of the model are worth noting. First, our model can be extended to incorporate intangible assets, which may act as a confounding factor in our analysis. Firms with intangible assets can be thought of as having a higher π_i. It provides excess profits for firms relative to others with the same investment strategy.

Second, the objective function of firm i can also be interpreted as capturing technological complementary/substitutability of multinational investment. A typical way to interpret the CARA utility is that the agent likes a higher expected payoff but hates its volatility. Instead, in our context, one can view the whole objective function as profits so term (ii) in the objective function of a firm represents an additional incentive for a firm to invest in a country-specific asset which increases payoffs of assets in other countries. A good real-world analogy is a firm acquiring a warehouse in Brazil to serve marketing units in other Latin American countries. Later, we will try to back out this technological complentarity in the empirical section.

Third, our framework dispenses with pricing decision (e.g. mark-ups) in the goods market but it still capture competitive forces between firms operating in the same country. As more firms enter country n by paying a fixed cost, the price of productive capital, q_n, rises in equilibrium so that incumbent firms in country n begin to yield a lower expected rate of return on their investment. We view that the expected return on productive capital is a sufficient statistics summarizing various profitability factors in country n, including mark-ups and technological efficiency.

2.3.3 Main Results

The questions we are going to be most interested in are the followings: (i) what are the sources of MNC premia in the spread between RoA and CoC and (i) how it varies with sector-level and firm-level characteristics? To answer these questions in a more stark way, we begin with a stylized model in which cost structure, $\{\tau_i, f_n\}_{\forall i,n}$, is simplified. We next extend the model to match with data and do quantitative analysis.

Suppose that, in country 1, there are only two firms, a multinational and a domestic-oriented company. Their indexes are denoted by m and d. Assume for the moment that there is no idiosyncratic return differentials i.e. $\pi_m = \pi_d = 0$. The only difference between these two types of firms is a technological barrier to foreign investment i.e. $\tau_d > \tau_m$. This feature reflects the fact that, as we will see in data analysis, firms in certain industries face higher barriers for entering foreign markets. To illustrate the main point, we first consider three extreme cases

[Case 1.] Financial markets are disintegrated. Both firm m and d face a infinitely large entry cost for foreign investment.

[Case 2.] Financial markets are fully integrated i.e. $\tau_m = \tau_d = 0$. But firm m and d have different investment profiles as their risk aversions differ i.e $\gamma_m < \gamma_d$.

[Case 3.] Financial markets are incompletely integrated. Multinational firms have $\tau_m = 0$ while domestic oriented firms have an infinitely large τ_d.

In Case 1, no firm is able to initiate foreign operation so all have the same rate of returns. In Case 2, multinational firms may a higher rate of returns as they bear a higher risk than domestic oriented firms. What about Case 3? In the model, we can compute return on assets and the average interest rate as

$$Roa_i = \frac{\sum_{n \in X_i} \pi_n \alpha_{i,n}}{\sum_{n \in X_i} q_n \alpha_{i,n}}, \quad \text{and} \quad Int_i = \frac{\sum_{n \in X_i} r_n \beta_{i,n}}{\sum_{n \in X_i} \beta_{i,n}}$$

Let $\mathcal{S}_i = Roa_i - Int_i$ denote the spread between the two. The following proposition presents a basic decomposition of MNC premia in the case where global capital markets are incompletely integrated.

Proposition 6. *In country* 1*, MNC premia in the spread between return on assets and interest rate on debt can be decomposed into three parts*

$$\mathbb{E}_0[\mathcal{S}_m] - \mathbb{E}_0[\mathcal{S}_d] = \underbrace{\Big\{ \sum_{n \in X_m} s_n \sigma_n \omega_{\alpha,n} - s_1 \bar{\sigma}_m \Big\}}_{\textit{(i-a) Incomplete Integration}} - \underbrace{\Big\{ \sum_{n \in X_m} r_n \omega_{\beta,i} - r_1 \Big\}}_{\textit{(i-b) Incomplete Integration}} + \underbrace{\{ s_1 \bar{\sigma}_m - s_1 \sigma_1 \}}_{\textit{(ii) Difference in Risk}}$$

where $\omega_{\alpha,n} = \frac{q_n \alpha_{m,n}}{\sum_{i \in X_m} q_n \alpha_{m,n}}$ $\omega_{\beta,n} = \frac{\beta_{m,n}}{\sum_{n \in X_m} \beta_{m,n}}$, *and* $\bar{\sigma}_m \equiv \sqrt{[\alpha'_m, \beta'_m]\Omega_0[\alpha'_m, \beta'_m]'}$ *is the average volatility faced by multinational firm* m.

The core message of Proposition 1 is that an increase in MNC premia that we saw in the previous section can stem from two broad factors. One is simply that multinational firms bear more risks so they are compensated by a higher expected return on investment. This is consistent with the view proposed by Fillat and Garetto (2015) and many others that multinational investment is riskier. The second channel, which is novel in the literature, presents the view that multinational firms are global financiers. Because entry is restricted, some countries provide a higher rate of return on capital relative risks than other countries. Bonds markets are also incompletely integrated due to exchange rates so r_n is different across countries. What multinational firms do is to take arbitrage between countries that provide a higher risk compensation and countries that have a lower interest rate.

Remark 1: Intangible Asset Another factor one may consider is firms' abilities to generate excess profits from their investment, namely intangible assets. High productivity firms self-select into foreign markets, so the higher return on asset among multinational firms could be attributed to intangible assets rather than risk premium or return differentials. Our model can accommodate this feature as profits of firms have different idiosyncratic components i.e. π_i. The formula below handles this case.

Proposition 7. *Suppose that firm* m *has intangible assets providing* $\pi_m > 0$. *MNC premia can now be decomposed into*

$$\begin{aligned}\mathbb{E}_0[\mathcal{S}_m] - \mathbb{E}_0[\mathcal{S}_d] = &\underbrace{\sum_{n \in X_m} \pi_m \omega_{\alpha,n}/q_m}_{\text{(0) Intangible Assets}} + \underbrace{\Big\{\sum_{n \in X_m} s_n \sigma_n \omega_{\alpha,n} - s_1 \bar{\sigma}_m\Big\}}_{\text{(i-a) Incomplete Integration}} - \underbrace{\Big\{\sum_{n \in X_m} r_n \omega_{\beta,n} - r_1\Big\}}_{\text{(i-b) Incomplete Integration}} \\ &+ \underbrace{\{s_1 \bar{\sigma}_m - s_1 \sigma_1\}}_{\text{(ii) Difference in Risk}}\end{aligned}$$

Term (0) is added to the previous decomposition.

This formula shows that intangible assets provide an additional margin for the MNC premia due to the market expansion. Firms with a higher idiosyncratic return are more likely to initiate multinational operation after global capital markets are integrated. Essentially, foreign expansion allows firms to replicate their high-yield business in different markets. This effect is captured by the first term, $\pi_m \sum_{i \in X_m} \alpha_{m,i}$. Capital market integration allows firm m to increase investment share in capital goods in different markets. The rest of the terms, incomplete integration and differences in risk-takings, remain unchanged.

2.4 Quantitative Analysis

In this section, we develop a quantitative model to measure the contribution of the three channels, which we identified in the previous section, to the rising premium of US multinational firms. In this draft, we use pseudo-data artificially generated by a simulation which was blind to any microdata collected by the BEA. The focus of this simulation exercise is to study identification issues, conduct sensitivity checks and investigate quantitative patterns of the model.

Given the simulated data, the estimation proceeds in three steps. First, we extend the model to fit to the data. We consider a multi-sector model that incorporates sector-specific FDI potentials and fixed costs across different regions of the world. Second, with this extension in place, we estimate the sector-specific FDI potentials with the data. Finally, we run a method of simulated moments to compute estimates for fixed costs and derive standard errors. Our three-step estimation method follows from Antràs *et al.* (2017). Unlike these authors, whose focus is primarily on input sourcing decisions, we shed light on the investment and funding decisions of a multinational firm.

2.4.1 Data Generating Process

We maintain the two-period model structure. An implicit assumption behind this framework is that corporate decisions only take into account current state variables, which characterize all the past and present information, and future expectation about business environment. An application of our methodology begins by picking a baseline year for quantitative analysis. A natural choice could be a benchmark year in which the BEA conducts its benchmark survey. These surveys contain more detailed information on affiliate-level variables than regular annual surveys.

Geographically, we consider six regions: US, European Union, China, Mexico, Canada and the rest of the world. US acts as the home country and is indexed by 1. The other regions are indexed from 2 through 6 respectively. The variance-covariance matrix Ω is defined among these regions. In the simulation exercise, we assume that Ω is know to researchers a priori. In practice, one can use covariances between the US gdp growth with exports growth rates to the other regions as a proxy for investment risks. Similarly, one can use region-level exchange rates to calibrate currency risks.

In the simulation, we investigate two non-financial sectors: manufacturing and service. As will be shown later, our estimation strategy can easily accommodate an arbitrary number of sectors with minimal computational burdens. In actual data, for example, one may turn on 3-digit non-financial SIC industries. Whichever layer we use, we assume that the equity size of each firm is randomly drawn from a lognormal distribution. The mean and standard deviation of the natural logarithm of sizes are given by $(\mu_k, \sigma_k^2)_{k=1}^K$ where k indexes industry and K is the number of sectors. Each sector is characterized by a pair of these parameters.

Our data generating process is the followings: each firm draws their sector, equity size and idiosyncratic preferences, $\varepsilon_{i,n}$, over geographic locations. To model the idiosyncratic preferences, note that the first-order condition of the firms' maximization problem in Section 2.3.1 are given by

$$\begin{bmatrix} \alpha_{i,1} q_1 \\ \dots \\ \beta_i \end{bmatrix} = \frac{\Omega^{-1}}{\gamma} \begin{bmatrix} s_{n,1}\sigma_1 - r_1 \\ \dots \\ r_1 - r_N \end{bmatrix}$$

We assume that a firm's actual investment and funding choices are given by $\{\alpha_{i,1} q_1 e^{\varepsilon_{i,1}}, \dots, \beta_i e^{\varepsilon_{i,2N}}\}$ where $\varepsilon_{i,n}$ follows a normal distribution with mean zero and variance σ_p^2. Parameter σ_p determines the variability of idiosyncratic investment and funding choices.

Finally, we extend the baseline model to improve the fit. We make two additional assumptions. First, we assume that the risk aversion of a firm is proportional to the equity size of the firm. In the baseline model, firms are mean-variance maximizers, and have constant absolute risk aversion.

Table 2.4: *List of Simulation Parameters*

Parameter	Description	True Value
γ	Risk aversion	3
f_e	Fixed operation cost	0.06
σ_p	Variability of idiosyncratic preferences	0.2
Ω	Variance-Covariance Matrix	See *Notes*
$(f_n)_{n=2}^{6}$	Foreign market entry costs	$(5,7,6,6,6)$
$(\mu_k, \sigma_k^2)_{k=1}^{2}$	Log-normal distribution of equity sizes	(2.2, 1): Manu. (2.2, 1): Serv.
$(s_n^k)_{\forall n,k}$	FDI potentials	(1.1, 1.4, 1.5, 2.1, 1.4, 1.3): Manu. (1.4, 1.4, 1.6, 1.8, 2.1, 1.3): Serv.
$(r_n)_{n=1}^{6}$	Interest rates	(0.03, 0.15, 0.02, 0.02, 0.02, 0.02)

Notes: This table presents the list of parameters in our structural model. True values represent parameter values used for our simulation exercise. Manu. indicates manufacturing sector, while Serv. indicates service sector. In the simulation exercise, we draw a randomly-generated symmetric and positive semi-definite matrix for Ω.

This assumption implies that these firms, no matter how large they are, conduct the same amount of risky investment in absolute terms. Second, we assume that firms pay an additional fixed cost to maintain their business. The additional cost is given by $f_e \sum_{n \in X_i} q_i \alpha_{i,n}$, which increases proportionally with the size of assets, This additional fixed cost helps match the model-implied average ROA of firms to the actual average of ROA in the data. We will revisit these two points, the risk aversion and fixed cost, in more detail later.

Table 2.4 provides a summary of parameters that are used for our simulation exercise. The aim of this quantitative extension is to quantify the channels we identified in Proposition 1 in light of the estimates from the simulated data.

2.4.2 Step 1: External Calibration

The first step of quantitative analysis is to calibrate primitive parameters. Among others, the parameters below are calibrated externally from the data. In the simulation exercise, we assume that these parameters are known to researchers a priori.

$$\{r_1, s_1, \Omega;\ (\mu_k, \sigma_K^2)_{k=1}^{K}\}$$

r_1 is the real risk-free interest rates in the U.S., s_1 is the Sharpe ratio of real investment in the U.S. corporate sector, Ω is the variance-covariance matrix and (μ_k, σ_k^2) are the parameters that determine the distribution of equity size in sector k. When using actual data, one can externally calibrate $(\mu_k, \sigma_K^2)_{k=1}^K$ from Compustat dataset. One may also calibrate r_1 from the average yield of U.S. corporate bond index, and s_1 from the mean divided by the standard deviation of profits in the U.S. corporate sector. The remaining model parameters are internally estimated as we explain below.

2.4.3 Step 2: Estimation of FDI Potentials

The next step is to estimate FDI potentials, $(s_n^k)_{\forall n,k}$, and average interest rates, $(r_n)_{n=1}^6$ across regions. Our estimation framework allows to estimates these parameters separately from other parameters, thereby involving less computation. Consider a firm, indexed by i in sector k, which engages in foreign direct investment across all regions. Note that the first-order conditions of the firm's maximization problem can be rearranged as

$$\underbrace{\log(\alpha_{i,n}) - \log(\alpha_{i,0})}_{\text{log (Asset in country i / Asset in the US)}} = \log\left[\Omega^{-1}\pi^k\right]_n - \log\left[\Omega^{-1}\pi^k\right]_0 + \varepsilon_{i,n}$$

Table 2.5: *Estimates for FDI Potentials and Interest Rates*

	Region				
Sector	EU ($\hat{s}_2$)	China ($\hat{s}_3$)	Mexico ($\hat{s}_4$)	Canada ($\hat{s}_5$)	ROW ($\hat{s}_6$)
Manufacturing	1.2113	1.5296	1.6017	2.0873	1.5712
	(0.0234)	(0.0195)	(0.0272)	(0.0205)	(0.0181)
Service	1.2113	1.1577	1.3474	1.4566	1.6682
	(0.0136)	(0.0157)	(0.0174)	(0.0248)	(0.0135)
Sector	EU ($\hat{r}_2$)	China ($\hat{r}_3$)	Mexico ($\hat{r}_4$)	Canada ($\hat{r}_5$)	ROW ($\hat{r}_6$)
Manu. & Service	0.0170	0.0170	0.0205	0.0211	0.0215
	(0.0008)	(0.0004)	(0.0005)	(0.0005)	(0.0005)

Notes: This table presents estimates for FDI potentials and interest rates from non-linear least-squared estimation. Standard errors are based on 100 bootstrap samples drawn with replacement.

where $\boldsymbol{\pi} \equiv [s_1^k\sigma_1 - r_1, ..., s_N^k\sigma_N - r_1, r_1 - r_2, ..., r_1 - r_N]$ represents the risk premia, $[\boldsymbol{x}]_i$ represents i's element of a vector $\boldsymbol{x}$ and $\varepsilon_{i,n}$ is an error term that arises due to idiosyncratic preferences we defined earlier. If a firm enters only a subset of countries, one can extract the corresponding columns and rows from Ω and rewrite the above empirical specification.

Using these first order conditions, we employ non-linear least squares to estimate $\hat{\boldsymbol{s}}^{\boldsymbol{k}}$ for each sector k, and $\{r_n\}_{n=1}^N$. The property that these estimates are not dependent on the values of other parameters reduces computational burden of the quantitative analysis. Standard errors of these estimates are jointly estimated when we run a simulated method of moments in the next subsection. The estimated values are reported in Table 2.5.

2.4.4 Step 3: Estimation of Fixed Costs

The final step of the quantitative analysis is to estimate the remaining parameters, denoted by $\hat{\boldsymbol{\eta}} \equiv \{\hat{f}_e, (\hat{f}_n)_{n=1}^N, \hat{\gamma}, \hat{\sigma}_p\}$. We run the simulated method of moments to estimate $\hat{\boldsymbol{\eta}}$ to match quantitative patterns in the data. We use two sets of empirical moments to estimate the data. First, we utilizes the share of firms that enter each region n (i.e. $\frac{1}{I}\sum_{i=1}^I \mathbb{I}_{i,n}$) where $\mathbb{I}_{i,n}$ is an indicator variable that equals one if firm i engages in investment in country n. These moments help identify the size of fixed market entry cost in each region. Second, we use the average return on assets, the average equity/asset ratio and the standard deviation of . The third moment is used to find f_x. The final moment helps identify the common risk aversion shifter, γ. The simulated

Table 2.6: *Estimates for the Fixed Costs, Risk Aversion and Variability*

	Parameters			
	$\hat{\gamma}$	$\hat{f}_e$	$\hat{\sigma}_p$	EU ($\hat{f}_2$)
Estimates	2.8723 (0.1142)	0.0594 (0.0052)	0.2332 (0.0149)	5.6536 (0.3541)
	China ($\hat{f}_3$)	Mexico ($\hat{f}_4$)	Canada ($\hat{f}_5$)	ROW ($\hat{f}_6$)
Estimates	6.3520 (0.3502)	5.0829 (0.3087)	5.1976 (0.3052)	5.9054 (0.3986)

Notes: This table presents estimates for the fixed costs, risk aversion and variability from the simulated method of moments. Standard errors based on 100 bootstrap samples drawn with replacement.

moments under $\hat{\eta}$ are denoted by $\hat{m}(\hat{\eta})$. Essentially, we select the model parameters that minimize $\hat{\eta} = \text{argmin}_{\eta}\ [m - \hat{m}(\eta)]'W[m - \hat{m}(\eta)]$ where W is an identity (weighting) matrix. Standard errors based on 100 bootstrap samples drawn with replacement. The results are reported in Table 2.6. These estimates allow us to conduct counter-factual analysis in the following section.

2.5 Simulation Results

2.5.1 Fit of the Model

This subsection evaluates the general fit of our model to the simulated data. Figure 2.5 suggest that our estimation procedure overall provides a good fit to the simulated data. In Figure 2.5, the dark color bars represent model-implied values, while the light color bars represent values generated by the simulated data. More specifically, Panel (a) displays the shares of firms that engage in foreign direct investment hosted by each country, which we often call as extensive margins. Panel (b) displays the share of investment aggregated across different regions, or namely intensive margins. The barplot in Panel (c) groups firms by the number of countries they have invested in. Panel (d) provides a comparison between the model and the simulated data in terms of their average returns on assets and interest rates The length of each grey interval represents the spread between the average return on asset and interest rate.

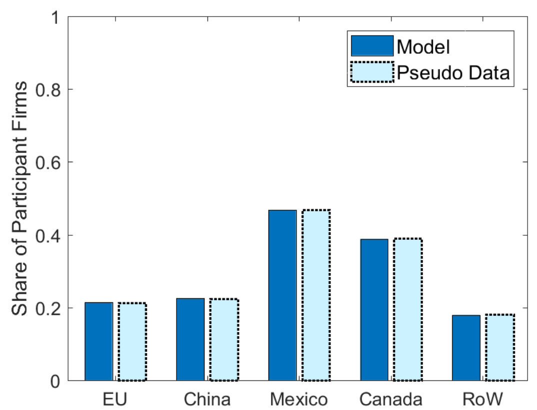
Table 2.7: *Fit of the Model*

	Moments			
	(EU)	(China)	(Mexico)	(Canada)
Model	0.2135	0.2255	0.4680	0.3885
Simulated Data	0.2130	0.2240	0.4675	0.3890
	(ROW)	(Leverage)	(ROA)	(Variability)
Model	0.1795	0.5636	0.0615	0.3251
Simulated Data	0.1810	0.5631	0.0619	0.3264

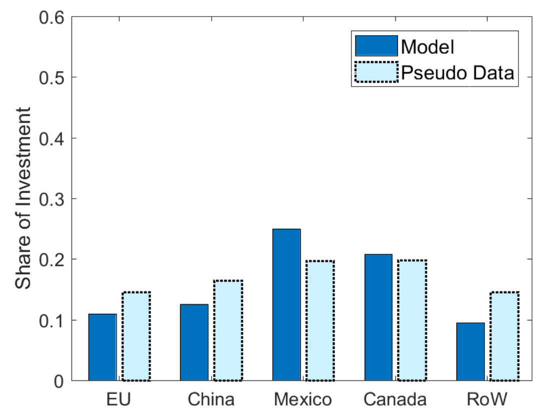
Notes: This table presents the fit of the model to the simulated data. The first five columns represent the share of firms that enter each geographic region. The sixth column represents the average equity/asset ratio. The seventh column represents the average ROA and the last column represents the variance of investment shares in the US.

Figure 2.5: *Fit of the Model*

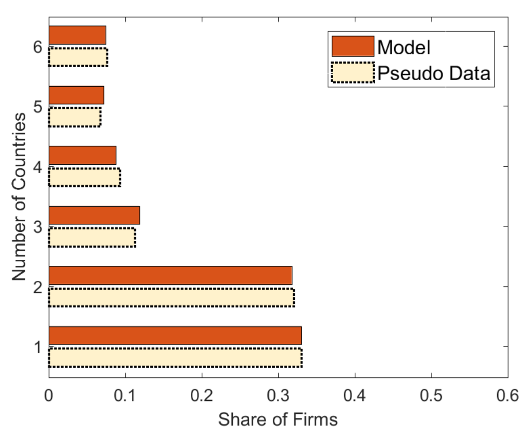
(a) *Extensive Margins*



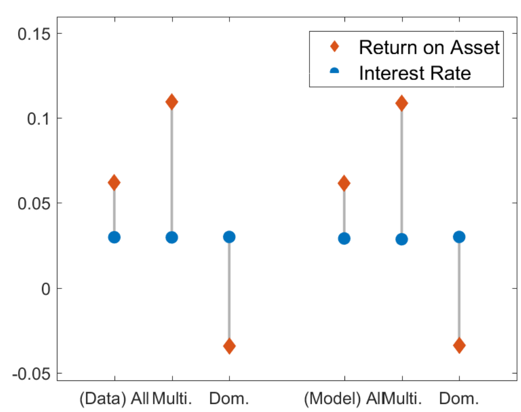
(b) *Intensive Margins*



(c) *Number of Invested Countries*



(d) *Multinationals and Domestic-oriented Firms*



Notes. In the figures, the dark color bars represent model-implied values, while the light color bars represent values in the simulated data. Panel (a) displays the share of firms that engage in foreign direct investment hosted by each country. Panel (b) displays the share of investment aggregated across different regions. Panel (c) groups firms by the number of countries they have invested in. Panel (d) compares the average return on assets and interest rates between the model and simulated data.

Table 2.7 displays the moments associated with the fit of the model. The first five columns represent the share of firms that enter each geographic region. Each of these columns corresponds to the values in Panel (a) of Figure 2.5. The sixth column represents the average equity/asset ratio, which we use to estimate the general risk aversion of shareholders in the U.S. corporate sector. The seventh column represents the average ROA and the last column represents the variance of investment shares in the US. The overarching message of this subsection is that our identified parameters generate a good fit to the simulated data.

2.5.2 Counterfactual Analysis: Disintegration of Global Capital Markets

Given these estimates, we can conduct counterfactual simulations to quantify the contribution of various channels to the gap between multinational and domestic firms. Recall that we derived the formula for decomposing the MNC premia. In the baseline setup, we can write the formula as

$$\mathbb{E}_0[\mathcal{S}_m] - \mathbb{E}_0[\mathcal{S}_d] = \underbrace{\Big\{ \sum_{n \in X_m} s_n \sigma_n \omega_{\alpha,n} - s_1 \bar{\sigma}_m \Big\}}_{\text{(i-a) Incomplete Integration}} - \underbrace{\Big\{ \sum_{n \in X_i} r_n \omega_{\beta,n} - r_1 \Big\}}_{\text{(i-b) Incomplete Integration}} + \underbrace{\{ s_1 \bar{\sigma}_m - s_1 \sigma_1 \}}_{\text{(ii) Difference in Risk}}$$

where $\omega_{\alpha,n} = \frac{q_n \alpha_{m,n}}{\sum_{i \in X_m} q_n \alpha_{m,n}}$ $\omega_{\beta,n} = \frac{\beta_{m,n}}{\sum_{n \in X_m} \beta_{m,n}}$, and $\bar{\sigma}_m \equiv \sqrt{[\alpha'_m, \beta'_m]\Omega[\alpha'_m, \beta'_m]'}$ is the average volatility of multinational investment. In the general setup, the formula should be modified to incorporate fixed market entry costs. These costs are subtracted from (i-a) as they reduce the numerator (=earnings) in ROA.

We compute each term through counterfactual simulations. Let m denote the index of a multinational firm. Index d in the above formula corresponds to the same firm when it faces infinitely large fixed costs for entering foreign markets. One implicit assumption behind this exercise is that the general equilibrium effect on $\{s_n, r_n\}_{n=1}^N$, stemming from a change in the global supply and demand upon financial integration, is negligibly small. Essentially, our analysis here only captures partial equilibrium effects.

Table 2.8 presents the estimated values of these terms. In our simulation, (i-a) and (ii-b) account for 14.5%p and 0.03%p respectively, indicating that multinational firms can indeed benefit from its special position as global arbitrageurs. Quantitatively, the sum of these two terms can account for most of the gap between the pre- and post-globalization spreads among firms. In the simulation, the return-on-asset component on average plays an outsize role while its variability is much higher than that of the interest rate component. These patterns are consistent with our observations in Section 2.2 qualitatively, although the quantitative magnitudes are off the range as we use artificially-generated data in this draft. It is also worth noting that (ii) could have a negative value, which is the case in our simulation exercise. Multinational firms on average can bear a lower volatility due to geographic diversification.

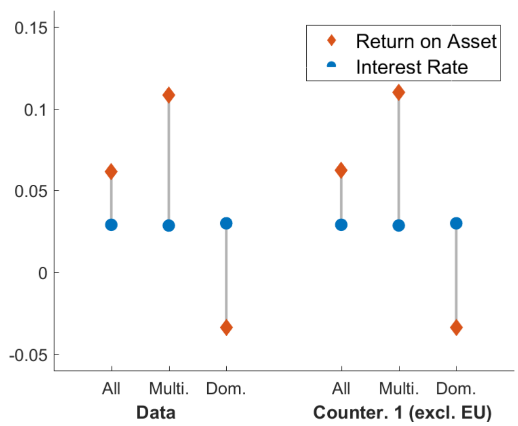
Table 2.8: *Decomposition*

Term	(i-a)	(i-b)	(ii)
Values	14.5%p	0.03%p	-0.2%p

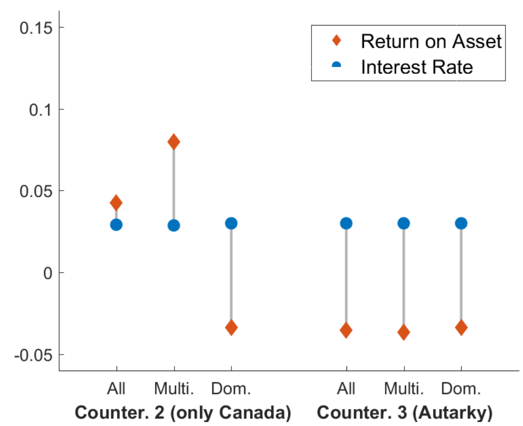
Figure 2.6 displays the effect of access to global capital on the MNC premia. We present three counterfactural simulations in this analysis, each of which represents a gradual process of global capital market integration. In the first scenario, we only exclude one region, the EU, from the global markets. Panel (a) shows that it has a relatively mild effect on the average size of the MNC premia. This is because, in our simulation, the EU is set to have a lower FDI potential than the other regions. In the second scenario, only Canada is left in the global capital markets. Panel (b) shows that this has a much greater impact as it drives out regions with a higher marginal product capital. In the final scenario in which the US is a financial autarky, all firms have a similar range of financial performances regardless of their sizes or their multinational status in the full integration case. Panel (c) and (d) confirm these effects with histograms. These results indicate that global capital market integration can account for the rising MNC premia in the spread between the return on asset and the interest rate.

Figure 2.6: *Counterfactual Analysis*

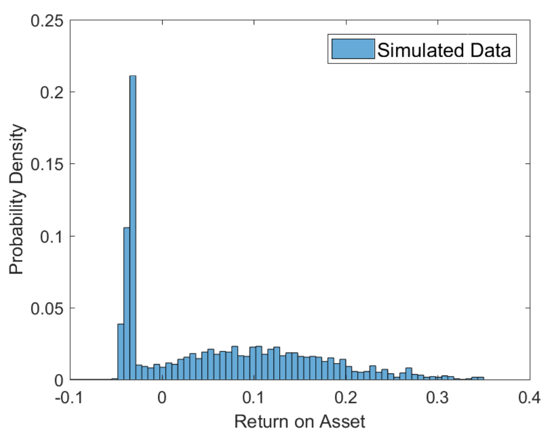
(a) *Extensive Margins*



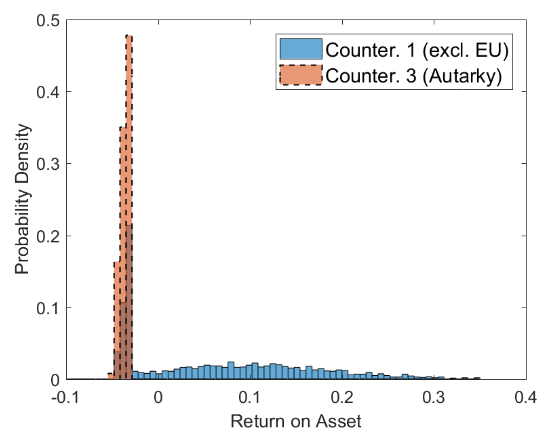
(b) *Intensive Margins*



(c) *Open Economy*



(d) *Counter-factual Cases*



Notes. This figure displays the effect of access to global capital on the MNC premia. We consider three counterfactural simulations which represents a gradual process of global capital market integration. Panel (a) shows that excluding one region has a relatively mild effect on the average size of the MNC premia. Panel (b) shows that driving out regions with a higher marginal product capital has a much greater impact. Panel (c) and (d) confirm these effects with histograms.

2.5.3 Further Applications

Our results and data provide several avenues for future research. First, the results from the strucutral estimation can be used as both an outcome variable and an independent in future work. As a LHS variable, we can relate our parameter estimates derived from observed patterns of MNC investment to a host of country and sector level characteristics to better understand what makes regions most attractive for foreign investment.

Second, we have provided evidence that MNCs pay lower interest rates. One source of risk mitigation for MNCS may be their ability to use their global relationships and supply chain networks to mitigate the negative impact of trade and credit shocks. In times when credit is unavailable, MNCs with robust affiliate relations may be better equipped to maintain their cash balances by increasing the amount of trade receivables outstanding with their foreign affiliates. We have begun an analysis of how these trade balances, collected on a quarterly basis, respond to exogenous credit shocks to test the degree to which MNCs rely on this type of internal financing.

2.6 Conclusion

US multinational firms have acted as a de facto financial intermediary in the world economy. On the one hand, multinational firms provide a vehicle for foreign investment and access to foreign returns that would otherwise be unavailable to individuals. On the other hand, they provide access to foreign debt issuance by tapping into local financial markets. The rapid advancement of financial globalization over the past decades has allowed an increasing number of US firms to take these advantages while, due to the sectoral-specific barriers, other firms still remain in domestic markets.

Using confidential data collected by the US Bureau of Economic Analysis, this paper documents a wider spread between the average return on assets and interest rates in the US corporate sector for multinational enterprises. These firms on average pay a lower interest rate and earn a higher return on investment. Motivated by these patterns, we develop a quantifiable model to assess the extent to which this gap is driven by global arbitrage opportunities in real investment, rather than risk exposure. Our simulation results show that a sizable portion could be due to the incomplete integration channel, suggesting that US multinational firms can be characterized as global arbitrageurs in addition to being risk takers.

Financial globalization is an important milestone in the history of the U.S. corporate sector. It is our hope that this paper promotes a better understanding of the recent trends surrounding US multinational firms, their size distribution and capital income shares. This deeper understanding would help government authorities to better evaluate the decision making processes of these large firms and armed with this knowledge design corresponding policies.

Chapter 3

A Heterogeneous-agent Model of Financial Development

3.1 Introduction

Technological advances in financial intermediation reduced cost of funding investment around the globe, and transformed our economic landscape. At the macroeconomy level, the size of private capital market relative to a country's gross domestic product has grown substantially over the past decades. At the individual level, consumers have gained access to increasingly diverse forms of personal finance and investment products. The wealth of nations in the contemporary world is different, along many dimensions, from that of previous generations

What can economists say about financial development and macroeconomy? The implications of financial development have long been the subject of research; most of the attention has been paid to its effects on aggregate variables such as growth, volatility and income inequality of an economy. While there is debate on the extent to which financial systems matter, many studies suggest that advances in financial instruments tend to spur economic growth, lessen macroeconomic volatility, and improve income inequality.[1] In particular, a number of empirical studies such as Beck *et al.* (2007), Beck *et al.* (2010), and Levine *et al.* (2014) have documented

[1]See Aghion *et al.* (2018) for a survey of the literature on financial development, including the relation to economic growth, volatility and income inequality. See Aghion *et al.* (2004), Aghion *et al.* (2009), Aghion *et al.* (2010) for more discussions on volatility.

that credit expansion increases earnings of low-income individuals, and thus lowers inequality.

This paper sheds light on a different angle: how does financial development transforms the distribution of wealth within a country? Wealth is stock, while income is the flow adding to the stock. In principle, a household's wealth is the market value of all financial assets less liabilities owned by an individual family. Financial instruments are a vehicle for wealth accumulation. Arguably, advances in financial technologies would have impacts on the distribution of wealth through various channels such as asset revaluation, portfolio diversification and debt rebalancing. The disparity in household wealth is economically important because it is closely related to macro-finance stability and the long-term welfare differences across households. Yet, our understanding of the linkage between financial development and the distribution of wealth is fairly limited despite the importance.

In this context, I take a first step in developing a quantitative model to analyze the effects of financial development on the evolution of household wealth distribution. The key focus of this paper is a methodology per se; the model aims to provide a general setup that is applicable to studying various technological changes in financial systems. The modeling framework can be used to analyze the macroeconomic effects of financial development on the wealth distribution, asset prices and household balance sheet. After introducing the framework, I turn to its application: quantitative analysis on financial globalization. The analysis allows us to discuss the extent to which opening up global capital flows, as a part of the broader financial development, promotes the rise in U.S. wealth inequality over the past three.

The paper begins by presenting a general setup. I consider a continuous-time economy in which households earn labor and capital incomes. One of the key ingredients is that households have non-homethetic preferences over risk and return. This property induces wealthier households to invest disproportionately more in risky assets than impoverished households. The setup departs from the constant relative risk-aversion utility function, which generates an identical portfolio weight across different wealth holders. The model can thus generate rich dynamics, such as capital return inequality and asymmetric rebalancing of assets across households, which are absent in the standard models.

Next, I provide a solution method for numerically solving the model. The solution method presented in this paper is a continuous-time analogue of Krusell and Smith (1998). Two features make the computation tractible. First, I assume that households only take into account a finite number of moments of the wealth distribution when they form future expectations of the economy. This assumption makes it feasible to numerically guess and verify the law of motion for the state variables as it reduces the dimensions of the state space to a finite number. Second, the continuous-time modeling helps reduce computational burdens. In the continuous-time setup, the households' saving decisions and the evoluation of wealth distribution are characterized by a system of partial differential equations. There exists a computationally efficient algorithm to solve for the system of equations numerically (Ahn *et al.* 2018, Fernández-Villaverde *et al.* 2018 and Kaplan *et al.* 2018). The algorithm boosts the speed for finding the law of motion for the state variables.

Finally, I present an application of the method. I turn to financial globalization as a part of the broader financial development. Extending the model in Chapter 1, I address the following questions: quantitatively, to what extent does financial globalization explain the recent change in U.S. wealth distribution? How much of that change is permanent? I estimate the model's key parameters from various sources of data—ranging from the Fed's Survey of Consumer Finances to the BEA's National Economic Accounts—to assess the quantitative magnitude of the effect of financial globalization. The national accounts data are used to construct a time series of average realized returns on various asset classes, including cross-border assets and liabilities of the U.S. economy. Each asset class is linked with its fundamentals—such as dividends, earnings, and rents at the macro-economy level—so as to estimate their risk premia, Sharpe ratios (i.e., risk premium divided by its standard deviation), and capital gains. With these estimates, I apply the solution method developed in this paper.

The simulation shows that the effect on the wealth distribution is quantitatively sizable; in the calibrated model, a global integration shock alone accounts for 34% to 55% of the observed increase in the top one percent's wealth share in the United States since 1989.[2] At least over the past three decades, the portfolio rebalancing effect, which generates a permanent increase

[2] I choose 1989 as the benchmark year for the pre-globalization economy due to data availability, such as Survey of Consumer Finances. See Section 3.4 for more details.

in wealth inequality, appears to outweigh diminished returns on domestic assets in light of the estimates in the data. Yet, the model leaves room for a reversal. The recent decline of yields on U.S. domestic assets suggests that a reversal of the trend in rising wealth inequality is not impossible in the upcoming future. Later on, I decompose the contributions of several factors to rising wealth concentration. For the top one percent's wealth share, widened wage inequality has a smaller impact than global financial integration. This is because, in the calibrated model, a major source of income for the wealthy is capital, not labor.

This paper contributes to two strands of literature. Methodologically, this study adds a new angle to the literature on heterogeneous-agent macroeconomics. I treat capital income more carefully than existing studies on wealth inequality. I also elaborate on the underlying mechanism behind a structural change in capital income. Since Piketty (2014) was published, capital income has gained much attention as a crucial driver for rising wealth inequality in advanced economies. Many studies concur (e.g. Hungerford 2011, Bach *et al.* 2018, Fagereng *et al.* 2018). Yet, the standard models in the literature simply abstract capital income into the rental rate r of physical capital (Aiyagari 1994), the risk-free interest rate (Bewley 1983, Huggett 1993), or the profits of non-public firms (Quadrini 2000, Cagetti and De Nardi 2006) with little modeling of financing instruments such as portfolio diversification. More recent studies emphasize return heterogeneity (Hubmer *et al.* 2018, Kacperczyk *et al.* 2018[3]). Recently, Gomez (2019) studies interactions between asset price and wealth inequality. Unlike these studies, I shed light on structural determinants of financial variables — including risk premium, Sharpe ratio, capital gains, and portfolio frontier—and how changes in these variables transform wealth distribution over different time horizons. This framework can be applied to structural changes in financial markets, such as financial globalization, financial innovation and capital tax reform.

This paper also relates to the large literature on the interplay between financial development and income inequality. On the theory front, the literature suggests that capital market imperfections inhibit economic opportunities of low wealth households, thereby exacerbating income inequality (Banerjee and Newman 1993, Galor and Zeira 1993, Aghion and Bolton 1997,

[3] Kacperczyk *et al.* (2018) agree on the importance of endogenous portfolio choices in driving wealth inequality in a closed economy, although their study focuses on the advancement of information acquisition technology and does not consider the revaluation gain channel.

Piketty 1997). Greenwood and Jovanovic (1990) argues that the distributional effect of financial development may differ across stages; in the early stage of capital market development, only the wealthy can access credits so income inequality is rather increased. In the later stage, income inequality is lessened as all agents gain equal access to capital markets. Unlike these studies this paper focuses on wealth inequality, which is more closely related to the financial health of individual households.

On the empirical front, the literature investigates the relation between financial reforms and income inequality. Beck *et al.* (2010) uses the timing of bank deregulation to estimate the effect on incomes of low earning households. Their results suggest that the deregulation of branching restrictions improves income inequality primarily by booting the demand for low-skilled labor. Levine *et al.* (2014) revisits this issue in the context of racial inequality. On the other hand, cross-country studies show mixed results. Earlier studies, such as Clarke *et al.* (2006) and Beck *et al.* (2007), suggest that financial development improves various measures of income inequality at the country level. Dollar and Kraay (2002) find that changes in national institutions, including capital market reforms, explains income growth of the poor only through their effect on aggregate growth. Other studies, such as Haan and Sturm (2017) and Furceri and Loungani (2015), present a negative correlation between financial liberalization and income inequality.

The remaining parts of the paper are organized as follows. Section 3.2 presents a general setup of the model and discusses different forms of capital market development. Section 3.3 elaborates on a solution method. Section 3.4 applies this methodology in the context of financial globalization. Section 3.5 discusses the results. Section 3.6 concludes. Details on the computational algorithm are referred to the appendix.

3.2 General Setup

3.2.1 Households

Consider an economy populated by measure one of households. The lifetime utility of households born at time τ is given by

$$\mathbb{E}_\tau \left[\int_\tau^\infty e^{-(\delta+m)t} u(c_{it}) dt \right] \tag{3.1}$$

We can write household i's budget constraint as

$$da_{it} = [(r_t^* + \theta_{1it}\sigma s_t^*)a_{it} + \underbrace{w_t^* l_{it}}_{\text{(i) Labor Income}} - c_{it} + \underbrace{r_t^h h_{it}}_{\text{(ii) Housing Return}})dt + \sigma_1 \theta_{1it} a_{it}\, dz_{1t}$$

where w_t^* denotes wage, l_{it} labor productivity of household i, h_{it} the value of housing endowment, and r_t^h return on housing assets. Individual wealth is defined as the sum of net financial assets and non-financial assets i.e. $a_{it} + h_{it}$. The maximum amount of household debt is limited by $a_{it} \geq \underline{a}$ with $\underline{a}$ being a negative constant. Households die at the rate of m, as in the perpetual youth model (Yaari 1965, Blanchard 1985), lose all their wealth and are replaced by newborn households whose endowments and labor productivities are randomly drawn from a distribution I will specify below.

I assume that labor productivity consists of two elements: $l_{it} \equiv \ell_i + \epsilon_{it}$. Here, ℓ_i represents permanent skill, which is drawn from a lognormal distribution when a household is born. The temporary shock ϵ_{it} follows an AR(1) process $d\epsilon_{it} = -\beta_\epsilon \epsilon_{it} + q_{it} dJ_{it}$. The process drifts towards zero at rate β_ϵ and jumps arrive at a Poisson arrival rate ζ_ϵ. When a jump occurs q_{it} is drawn from $N(\mu_\epsilon, \sigma_\epsilon^2)$. The p.d.f. of the normal distribution is denoted by $\phi(\epsilon)$. This setup — long-run and short-run components — is in line with previous works such as Kaplan *et al.* (2018) and fits well with the actual earning processes in the data.

Furthermore, I simplify the housing problem by assuming that $h_{it} \equiv f^h(a_{it})$. Here, f^h will be calibrated to fit the data non-parametrically. Essentially, housing plays a passive role in the model — r_t^h is exogenously taken from the data and remains unaffected by financial development. Housing is incorporated only to calibrate the size of the main effect. Finally, I use the standard functional forms:

$$\begin{aligned}
\text{Utility}: \quad & u(c_{it}) = \frac{(c_{it} - \kappa)^{1-\gamma} - 1}{1-\gamma} \\
\text{Technology}: \quad & \Phi(K_t) = Z\, K_t^\alpha L_t^{1-\alpha} \\
\text{Endowment:} \quad & \begin{bmatrix} \log(a_{i0} + \bar{a})) \\ \log(\ell_i + \bar{\ell}) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \bar{}_a \\ \bar{}_\ell \end{bmatrix}, \begin{bmatrix} \Sigma_{aa} & \Sigma_{a\ell} \\ \Sigma_{\ell a} & \Sigma_{\ell\ell} \end{bmatrix} \right)
\end{aligned}$$

where $L_t \equiv \int_{[0,1]} l_{it} di$ is the total labor that is shared by a measure 1 of firms. Let $g_t(a, \ell, \varepsilon)$

denote a probability density function of households whose asset level is given by a, skill by ℓ and temporary earning shock by ε. $g_0(a_{0i}, \ell, \epsilon)$ represents a probability distribution of newborn households. The last line implies that $\int g_0(a_{0i}, \ell, \epsilon) d\epsilon$ is a variant of the bivariate lognormal distribution. $\bar{a}$ and $\bar{\ell}$ act as shifting parameters. Negative skill levels are truncated by zero. Negative net worth is considered as debt.

Note here that $\kappa > 0$ in the flow utility function captures decreasing relative risk aversion; when the expected consumption is close to κ, a household becomes extremely risk averse. The functional form belongs to the class of HARA (hyperbolic absolute risk aversion) utility functions and has been used in various contexts including portfolio choice models (e.g., Litzenberger and Rubinstein 1976). This assumption induces wealthy households to invest in the risky asset more heavily than poor households and, as noted in Chapter 1, makes the rich more responsive to a change in risk compensation.

Under this setup, I follow methodologies developed in heterogeneous-agent macroeconomics. Transitional dynamics of the economy is characterized by a system of two differential equations: the first component is often called the Hamiltonian-Jacobian-Bellman equation, which governs households' saving decisions. Households' saving decisions depend upon g_t as well as individual state variables $(a_{it}, \ell_i, \varepsilon_{it})$. Following Ahn *et al.* (2018), I use $J_t(a, \ell, \varepsilon) \equiv J(a, \ell, \varepsilon, g_t)$ to denote the value function associated with the household's problem. The second component is the Kolmogorov Forward Equation which governs the evolution of the wealth distribution. See Ahn *et al.* (2018) for an introductory guide to this approach. See Kaplan *et al.* (2018) and Fernández-Villaverde *et al.* (2018) for its applications and variants.

We focus on the trajectory in which $dz_{1t} \equiv 0$ in autarky. Yet, households still take into account ex ante volatility, $Var[\sigma_1 dz_{1t}]dt$, in the economy. The c.d.f. and p.d.f. of the wealth distribution, $G_t(a)$ and $g_t(a)$, are defined over this zero trajectory, so they evolve deterministically. This approach is similar to the method proposed by Fernández-Villaverde *et al.* (2018); the stationary state in this model is conceptually analogous to the stochastic steady state defined in the paper. Given this setup, the evolution of wealth distribution can be characterized by the following system of partial differential equations:

Proposition 8. *The wealth distribution evolves according to the following differential equations*

$$(\delta+m)J_t = \max_{c,\theta_1}\Big\{u(c)+\frac{\partial J_t}{\partial a}v_t(a,\ell,\epsilon)+\frac{1}{2}\frac{\partial^2 J_t}{\partial a^2}(\sigma_1\theta_1 a)^2+\frac{\partial J_t}{\partial \ell}(-\beta\ell^s_{it})$$
$$+\zeta\int_{-\infty}^{\infty}(J_t(a,\ell,x)-J_t(a,\ell,\epsilon))\phi(x)dx+\frac{1}{dt}\mathbb{E}_t[dJ_t]\Big\} \tag{1.HJB}$$
$$\frac{d}{dt}g_t(a,\ell,\epsilon) = -mg_t(a,\ell,\epsilon)+mg_0(a,\ell,\varepsilon)-\frac{d}{da}[v_t(a,\ell,\epsilon)g_t(a,\ell,\epsilon)]$$
$$-\zeta g_t(a,\ell,\epsilon)+\zeta\phi(\epsilon)\int_{-\infty}^{\infty}g_t(a,\ell,x)dxd\ell \tag{2.Kolmogorov}$$

along with the market clearing conditions. Here, $v_t(a,\ell,\varepsilon)\equiv(r^*_t+\theta_{1it}\sigma_1 s^*_{1t})a_{it}+w^*_{1t}l_{it}-c_{it}+r^h_t h_{it}$ *represents the saving function, and* $\frac{1}{dt}\mathbb{E}_t[dJ_t]$ *is short-hand notation for* $\lim_{s\searrow 0}\mathbb{E}_t[J_{t+s}-J_s]/s$*.*

3.2.2 Financial Development

The supply side of assets is similar to that of Chapter 1. The representative bank manufactures financial assets. Every period, the bank generates

$$d\pi_t = \Phi(K_t)dt+\bar{\sigma}K_t dz_{1t}$$

by investing K_t units of capital. Production involves raw output volatility $\bar{\sigma}$ in proportion to the investment level. The production function, $\Phi(K_t)$, exhibits diminishing marginal returns: $\Phi'>0$, $\Phi''<0$, $\lim_{K\to 0}\Phi'(K)=-\infty$ and $\lim_{K\to\infty}\Phi'(K)=0$. Funding decisions of the bank constitute the supply side of assets: risk-free debt and risky equity. The bank simply maximizes contemporaneous profit

$$\underbrace{V^*_t dt}_{\text{Private Equity Income}} \equiv \max_{K_t,D_t,E_t}\Big\{d\pi_t-\underbrace{r^*_t D_t dt}_{\text{Debt Income}}-\underbrace{(r^*_t+\sigma_1 s^*_{1t}+\tau)E_t dt}_{\text{Public Equity Income}}-\underbrace{\sigma_1 E_t dz_{1t}}_{\text{Public Equity Volatility}}\Big\} \tag{3.2}$$

Their constraints are given by

$$K_t\equiv D_t+E_t,\qquad \sigma_1=\bar{\sigma}K_t/E_t,\qquad D_t\leq\lambda K_t \tag{3.3}$$

where D_t is the value of debt, E_t is the value of equity and σ_1 is the standard deviation of returns per unit of equity outstanding. Equity financing involves additional transaction costs τ. The first constraint implies that, in the balance sheet, the total value of assets should equal to the total value of shareholder's equity and debt. The second constraint implies that debt scales up the risk per unit of equity. The third constraint implies that the maximum amount of safe assets issued by the bank is limited by λ. An equilibrium of the economy is defined as a stochastic process, $\{r_t^*, (s_{1t}^*, \sigma_1)\}_{t\geq 0}$, which clears local financial markets: $\mathbb{S}_t = \mathbb{I}_t$ and $\mathbb{S}_{1t} = \mathbb{I}_{1t}$ for all t.

Example 1: Reduction in Transaction Costs Advances in financial systems reduce transaction costs involved for funding investments. The model accommodate this feature by lowering τ from period T onward. Essentially, a reduction in transaction costs narrows the wedge between borrowing costs and investment returns. A new trajectory of interest rates, $\{r_t, (s_{1t}, \sigma_1)\}_{t\geq T}$, captures general equilibrium effects, which then influences the evolution of wealth distribution across households.

Example 2: Access to New Investments Financial development may offer new investment opportunities for households. Consider a new bank, indexed by 2. The investment offered by this bank does not exist prior to time T, possibly due to incomplete contract problems. Suppose that the incompleteness is resolved at time T. The portfolio frontier becomes $\{r_t, (s_{1t}, \sigma_1), (s_{2t}, \sigma_2)\}_{t\geq T}$, which then influences the wealth distribution. .

Example 3: Financial Globalization One can extend this framework to model the integration of global financial markets. Consider two economies: US and EM. The two economies may have different parameter values for τ, which represents capital market imperfections, and $\bar{\sigma}$, which represents the overall volatility. An open economy equilibrium is a stochastic process, $\{r_t, (s_{1t}, \sigma_1), (s_{2t}, \sigma_2)\}_{t\geq 0}$, which clears the global financial markets: $\sum_{k\in\{EM,US\}}(\mathbb{S}_t^k - \mathbb{I}_t^k) = 0$, $\sum_{k\in\{EM,US\}} \mathbb{S}_{1t}^k = \mathbb{I}_{1t}^{US}$ and $\sum_{k\in\{EM,US\}} \mathbb{S}_{2t}^k = \mathbb{I}_{2t}^{EM}$.

3.3 Solution Method

I use the differential equations in Proposition 8 to derive transitional dynamics of the wealth distribution. The core challenge in numerically solving these equations is that households' saving decision depends on a realization of the cross-sectional distribution $g_t(a, \ell, \varepsilon)$, which is an infinitely dimensional object. One should reduce the dimensionality of the state variable space to solve the model. A bounded rationality assumption is one of the solutions to this problem (Krusell and Smith 1998) and has been used widely in the literature. In a similar light, I impose a restriction on households' decision makings regarding portfolio choices.

Assumption 1. *The functional form of households' portfolio choice, θ_{1it}, is given by*

$$\theta_{1it} = \frac{s_{1t}}{\chi_1 \sigma_1}\left(1 - \frac{\chi_2}{a_{it}} - \frac{\chi_3}{a_{it}\ell_i}\right) \tag{3.4}$$

where χ_1, χ_2, and χ_3 are constants calibrated from data. In open economy, $\frac{s_1}{\sigma_1}$ is replaced with $\Sigma^{-1}[\sigma_1 s_1; \sigma_2 s_2]$. Those whose wealth levels are below $\chi_2 + \frac{\chi_3}{\ell_i}$ have $\theta_{it} = 0$.

It is worth noting that the above functional form is an approximation to the actual endogenous portfolio choice by households. In the baseline Meron's model, this approximation was exact: χ_1 equals the risk-aversion parameter, χ_2 equals $\underline{a}$ and χ_3 equals 0. The portfolio weight in the full quantitative model deviates from this closed-form solution as it embodies labor income. Instead of solving it numerically, I simplify the interaction between labor income and portfolio choice as described by (3.4) for tractability. Indeed, most studies in the literature on heterogeneous-agent macroeconomics (e.g. Hubmer *et al.* 2018) assume exogenous portfolio heterogeneity.

There are two advantages of Assumption 1. First, the dimension of the state variables that determine the current equilibrium prices is reduced dramatically; all equilibrium prices, r_t, s_{1t}, s_{2t} and w_t, are now expressed as a function of a finite number of state variables thanks to its nice aggregation property. One way to see the aggregation property is to look at the total demand for risky assets. For example, in a closed economy where all households have $a_{it} \geq \chi_2 + \frac{\chi_3}{\ell_i}$ (the baseline model was one of these cases), it is easy to show

$$\int_i a_{it}\theta_{1it} di = \frac{s_{1t}}{\chi_1 \sigma_1}\left(A_{1t} - \chi_2 - \chi_3 \int_i \frac{1}{\ell_i} di\right)$$

where A_{1t} denotes $\int_i a_{it} di$. Essentially, the mean of the wealth distribution, instead of the entire distribution, acts as a sufficient statistics for the demand for the risky asset in autarky. Other market clearing conditions are also simplified. (See Appendix C.1) Second, the fit of $\theta_{1it} + \theta_{2it}$ to the data is good. Assumption 1 has properties consistent with the fact that (i) wealthier households invest more heavily in equity; (ii) the marginal increase in investment share in equity is diminishing in wealth; and (iii) the investment share in equity converges to an upper bound as the wealth level rises. I confirm these in the later section.

In light of this property, I run a continuous-time analogue of Krusell and Smith (1998). I begin with a guess for the law of motion for the state variable,[4] which is now presumed to be A_{1t} in autarky. Through simulations, I verify that the proposed motion is indeed consistent with the model's predictions. The algorithm proceeds in four steps.

(Step 1) Guess the law of motion for the state variable. In actual practice, $d \log A_{1t} = (\psi_1 - 1) \log A_{1t} + \psi_2$ works well in autarky. Begin by guessing ψ_1 and ψ_2

(Step 2) Under this law of motion, numerically solve the HJB equation and compute individual saving decisions.

(Step 3) Using these saving decisions, compute the evolution of the wealth distribution.

(Step 4) Verify that the proposed law of motion is indeed consistent with Step 3. Return to Step 1 if not consistent.

More detailed explanations about the algorithm, especially when there is household debt, are referred to Appendix C.1.[5]

3.4 Application: Financial Globalization

In this section, I turn to an application of the method. I assess whether global financial integration, as a broader part of financial development, between the central and peripheral economies is likely to be an important factor behind the observed increase in US wealth inequality. I also examine the extent to which the current increase in US wealth concentration can persist in the future. The quantitative analysis proceeds in three steps. First, I extend the baseline model by adding new ingredients, such as labor income and housing wealth. Second, I present the target moments of the model and estimates for the key variables. Lastly, I present results.

[4] As in Krusell and Smith (1998), one implicit assumption is that only a finite moments of the wealth distribution matter in the law of motion for the state variables, and thus for the future equilibrium prices.

[5] For alternative methods, see Ahn *et al.* (2018) or Fernández-Villaverde *et al.* (2018). Compared with their works, the numerical approach here provides more transparent interpretations about the law of motion for the state variable (rather than leaving it as a black box). But the range of applications is limited to wealth inequality problems as this framework cannot incorporate TFP shocks.

We consider a shock in which US makes a transition from financial autarky to open economy from period T onwards. The aim is to quantify the effect on the wealth distribution. Consider two economies, US and EM. As in the baseline model, the only differences between the two economies are their banking technology, $\lambda^{US} > \lambda^{EM}$, and output volatility, $\bar{\sigma}^{US} < \bar{\sigma}^{EM}$.

3.4.1 Calibration Strategy

Notable target moments Given this setup, I calibrate the model to fit data. I use year 1989 as the benchmark year for financial autarky due to data availability.[6] Among other parameters, the ones below deserve further comments.

$$\left\{\lambda^{US}, \bar{\sigma}^{US}, \lambda^{EM}, \bar{\sigma}^{EM}, \rho; \mu_a, \Sigma_{aa}, \Sigma_{al}\right\}$$

First, I choose $\{\lambda^{US}, \bar{\sigma}^{US}, \lambda^{EM}, \bar{\sigma}^{EM}, \rho\}$ to adjust the equilibrium portfolio frontier in autarky and in open economy, $\{r_t^*, s_{1t}^*, r_t, s_{1t}, s_{2t}\}$, to fall within a reasonable range according to the historical patterns I will explain below. Next, I pick the values of μ_a, Σ_{aa}, and Σ_{al} to match the model-implied stationary wealth distribution with the actual wealth distribution in 1989 reported in the Survey of Consumer Finances. The target moments are the top 1%, the top 5% and the bottom 90% wealth shares.

One consideration is to generate a modest size of risk premium. It has been well known that the standard neoclassical model performs poorly in matching the risk premium in the data. This paper is no exception. My model has three features that help reconcile this issue partially. First, many households do not own risky assets. Households whose wealth and labor productivity fall short of a threshold in (3.4) take on debt. By reducing the demand for risky assets and increasing the supply of safe assets, the model-implied risk premium takes on a higher value. The mechanism is similar to Mankiw and Zeldes (1991). Second, χ_1 in (3.4) can be set differently from the elasticity of intertemporal substitution implied by utility function. My model has one more degree of freedom to inhibit investment on the risky assets. Finally, I leave $\bar{\sigma}^{US}$ and $\bar{\sigma}^{EM}$ as free parameters to match the Sharpe ratios.

[6] Since 1989, the Federal Reserve has provided Survey of Consumer Finances that are consistent across years. (The earlier surveys are less comparable.) The data moments, such as top wealth and wage shares, are taken from these datasets. Alternatively, one can look at tax-based estimates on top wealth and wage shares, which allows researchers to trace back to older data series.

Estimation of the interest rates To discipline the model, we investigate a change in the risk-free interest rate, capital gains, and the Sharpe ratio of US domestic and foreign assets. I perform the estimation in two steps. First, I use national accounts data to construct a time series of average realized returns on various asset classes in the U.S. economy. Following Saez and Zucman (2016),[7] I compute a macroeconomic yield of each asset class by dividing the flow payment reported in Gross National Income by the market value reported in the Fed's Financial Accounts. In the case of equities, for instance, the average dividend yield in 2005 is defined as the ratio between the total dividend paid to households during 2005 and the total value of equity holdings at the end of 2004. I then compute capital gains as an increase in the market value that exceeds the net issuance of equities during the year. Finally, the average return on equities is measured as the average dividend yield plus the average rate of capital gain. A similar methodology can be applied to other asset classes such as fixed income assets, housing and non-corporate business, and also to sub-asset classes such as foreign equity.[8]

Panel (a) of Figure 3.1 displays the average realized returns on equities, fixed income and 3-month treasury bills in the United States, smoothed over twenty years to eliminate cyclical variations. Each line represents the geometric average of real returns over a twenty-year horizon centered on the x coordinate. What this graph shows is that the average real returns on safe assets have been declining over the past decades, which many studies (e.g. Caballero *et al.* 2008, Mendoza *et al.* 2009) associated with the foreign demand for US safe assets. On the other hand, the realized return on equities began to diverge substantially from other safe assets since 1980s as we smooth out the cyclical variation. Fixed income assets exhibit a similar pattern. We should investigate these patterns with care as the realized returns on long-term assets are jointly affected a change in the discount rate and expected cash flows.

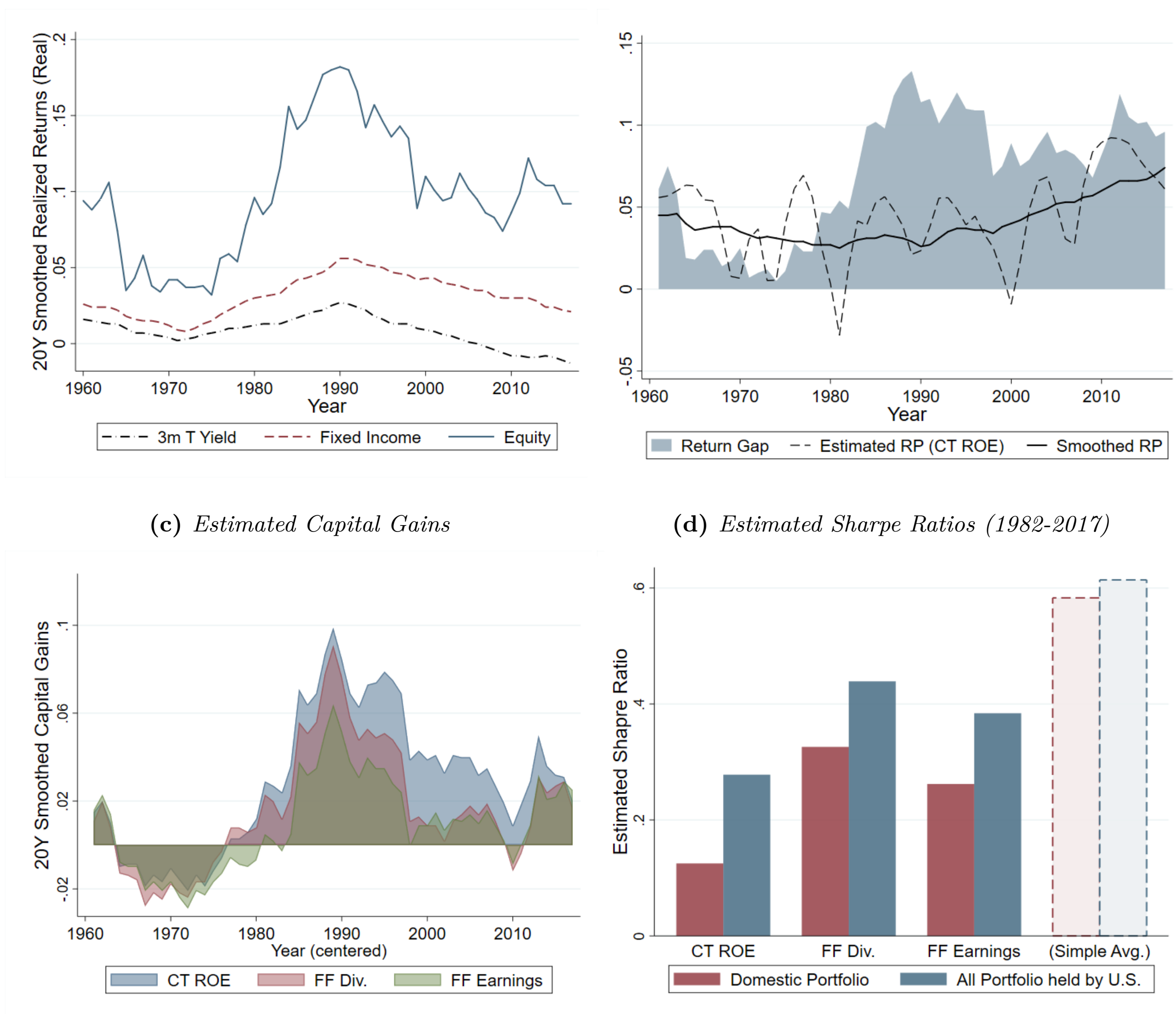
The next step of the analysis is to estimate capital gains, expected returns and Sharpe ratio so that we can examine how financial globalization has increased the relative reward for holding risky assets. To this end, I employ a simple estimation method proposed by Fama and

[7] This method has been used in many contexts including Mian *et al.* (2013), Saez and Zucman (2016) and Piketty *et al.* (2018) to compute asset returns that are consistent with macroeconomic statistics.

[8] The Fed provides the estimated value of closely held stock by matching it with the market value of publicly traded firms with similar characteristics.

Figure 3.1: *Interest Rates and Returns*

(a) *20-year Smoothed Realized Returns* **(b)** *Estimated Risk Premium*



(c) *Estimated Capital Gains* **(d)** *Estimated Sharpe Ratios (1982-2017)*

Notes: All returns and gains are measured in real terms. Smoothed returns stand for the geometric average of returns over twenty year period. The right-hand side of the time horizon is shorter in the last ten data points. Due to the data constraint, the Sharpe ratio is only calculated during 1982-2017. Simple average stands for the mean return over the standard deviation of returns, with no consideration of capital gains. CT ROE indicates the estimated risk premium based on accouting return on equity (Campbell 2008 and Campbell and Thompson 2008). FF Div and FF Earnings are based on dividend growth rates and earning growth rates respectively. (Fama and French 2002)

French (2002), Campbell (2008) and Campbell and Thompson (2008). The central idea is that fundamentals such as dividends, earnings and profitability can be used for estimating ex ante expected stock returns. The simplest form of these, for example, is dividend yield plus expected dividend growth rate. See Appendix C.1.3 for more details about these estimation methods

Table 3.1: *Related Statistics for Equity Premium and Shape Ratio*

	r_t^f	Mixed Portfolio R_t	D_{t+1}/P_t	E_{t+1}/P_t	GD_t	GE_t	RoE_t	D_t/E_t
Means of Annual Values								
1951 - 2017	0.85	9.64	4.92	10.60	3.63	4.00	7.97	48.74
1982 - 2017	0.93	13.11	5.30	9.92	4.44	4.23	7.87	56.46
Standard Deviation								
1951 - 2017	2.04	16.93	8.35	3.98	8.35	11.50	1.26	9.73
1982 - 2017	2.24	17.01	9.98	3.94	9.88	10.94	1.15	11.37
		Domestic Portfolio						
		R_t	D_{t+1}/P_t	E_{t+1}/P_t	GD_t	GE_t	RoE_t	D_t/E_t
Means of Annual Values								
1951- 2017					3.54	3.01	7.30	51.03
1982 - 2017		13.43	3.90	9.85	4.62	3.44	6.71	62.87
Standard Deviation								
1951 - 2017					15.18	14.98	1.48	17.59
1982 - 2017		19.73	1.54	5.45	19.92	15.32	1.30	15.87

Notes: Some of the series are not available during 1951-1981 due to the data constraint. r_t^f is the real return on 3 month treasury bills rolled over at each quarter, GD_t is the dividend growth rate, GE_t is the earning growth rate, and RoE_t is accounting return on equity for year t. All variables are measured in real terms and expressed as percents.

Panel (b) and (c) of Figure 3.1 suggest that a substantial portion of realized returns on equities can stem from cumulative capital gains over the past decades. In panel (b), the shaded area displays the gap between realized returns on equities and 3-month treasury bills smoothed over twenty years. Some portion of it could be accounted for by the equity premium. The 20-year smoothed equity premium is indicated by the solid line. The upper part of the blue area above the lines therefore indicates the average rate of capital gains.

In Panel (c), I present the 20-year moving averages of capital gains implied by the three different estimations. Regardless of which method I choose, the estimated capital gains account for a significant part of realized returns. This is what the baseline model predicts: as the required expected return on assets has fallen and excess profits have increased, possibly due to globalization, the overall price of equities rises as a consequence of revaluation gains. This is also consistent with the view of Fama and French (2002), who claim that a significant part of the post-war realized returns on the stock index appear to have come from a large capital gain.

Furthermore, panel (d) of Figure 3.1 provides evidence that the expansion of foreign investment opportunities helped increase the Sharpe ratio of U.S. households' portfolio. I compare two equity portfolio, mixed and domestic, indicated by the blue and red bars in Panel (d). The mixed portfolio represents the actual portfolio owned by U.S. households. The underlying dividends and earnings originate from foreign entities as well as domestic firms, which are reported in *Gross National Income.* On the other hand, the domestic portfolio is based solely on profits generated by domestic investment and therefore reported in *Gross Domestic Product.* I use estimated market values, dividends, earnings and other fundamentals reported in Fed's Financial Account.

As in Fama and French (2002), I measure the Sharpe ratio of each aggregate portfolio as the estimated risk premium over the sample standard deviation of the realized returns. The blue and red bars present the estimated Sharpe ratios based on the three approaches. The sample period in this exercise is restricted to 1982-2017 due to the data constraint. Panel (d) shows that, indeed, the U.S. households enjoy a higher risk-return trade-off than the one generated in the U.S. domestic sectors. This benefit stems from a higher return on global investment and diversification effect. Table 3.1 provides related statistics.

3.5 Quantitative Analysis

3.5.1 Model Fit

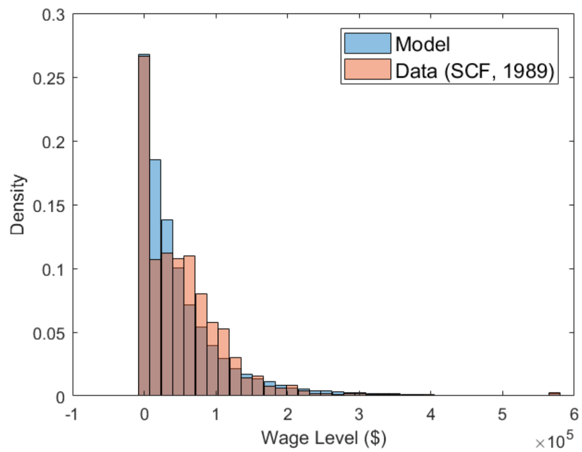
Table 3.2 presents the calibrated parameters and their target moments. Figure 3.2 displays the fit of the model. First, the model is able to generate a Pareto tail in the wealth distribution and the top wealth shares are generally in line with the estimated top wealth share in 1989. Second, the model can match wage inequality and take into account its effect on the top wealth share. Later on, I compare the contribution of financial globalization with that of rising wage inequality. Finally, we can check portfolio choices of different wealth holders as shown in Panel (e) and (f). Affluent households invest more heavily on equity whereas the middle class invest more heavily on safe assets and housing.

Table 3.2: *Calibrated Parameters*

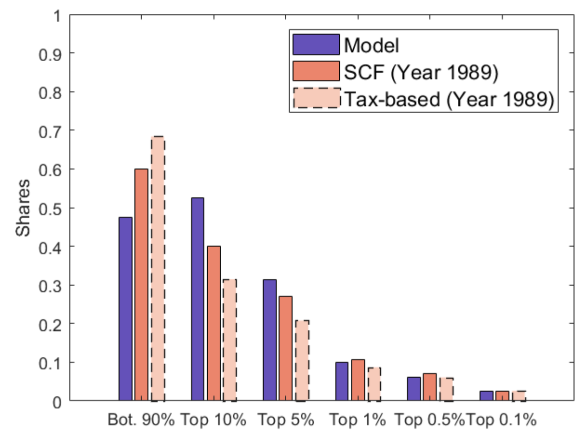
Param.	Description	US	ROW	Source / Target
Preferences				
δ	Discount Rate	0.05	.	
γ	EIS	2	.	
χ_1, χ_2, χ_3	Portfolio Choice	2, 1, 1.5	.	Survey of Consumer Finances
Production				
α	Capital Share	0.3	.	NIPA (2014)
Z	Aggregate Productivity	1	.	US Autarky wage normalized to 1
$\bar{\sigma}$	Aggregate Volatility	0.125	0.15	Internally Calibrated
ρ	Global Correlation	0.7	.	Internally Calibrated
Financial Frictions				
λ	Pledgeability	0.5	0.43	Internally Calibrated
τ	Additional Equity Cost	0.034	.	
$\underline{a}$	Maximum Allowable Debt	-1.9	.	
Idiosyncratic Shocks				
m	Death Rate	0.02	.	Average Adult Life Span: 50 years
$\mu_\ell, \Sigma_{\ell\ell}$	Labor Productivity	0.3, 0.85	.	Internally Calibrated
$\lambda_\epsilon, \beta_\epsilon, \sigma_\epsilon$	Time-varying Productivity	0.03, 0, 0	.	
μ_a, Σ_{aa}	Inherited Wealth	0.55, 0.7	.	Internally Calibrated
$\Sigma_{a\ell}$	Covariance between a_0 & ℓ	0.03	.	Internally Calibrated
$\bar{a}, \bar{\ell}$	Shifting Paramters	2, 0.8	.	Internally Calibrated

Figure 3.2: *Model Fit*

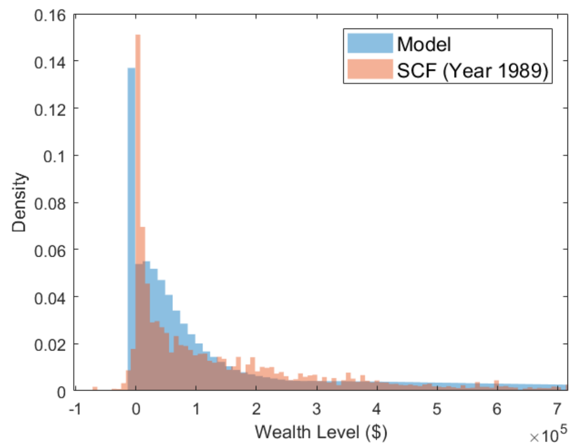
(a) *Wage Distribution*



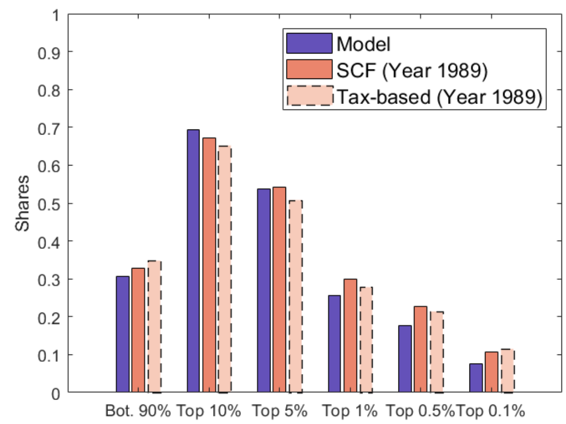
(b) *Top Wage Share*



(c) *Wealth Distribution*

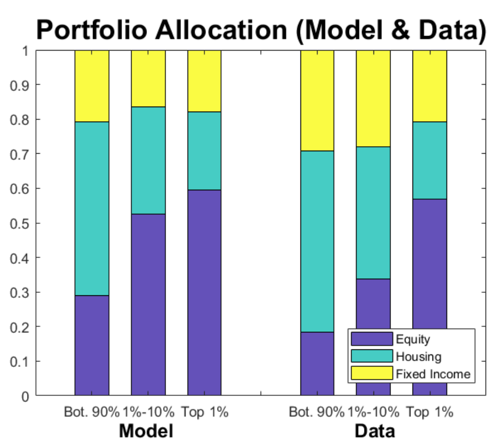


(d) *Top Wealth Share*



(e) *Portfolio Allocation (Model)*

(f) *Portfolio Allocation (Model & Data)*



3.5.2 Result 1: Transitional Dynamics

Panel (a) and (b) in Figure 3.3 plot transitional dynamics of the US wealth distribution after global capital flows transform interest rates and portfolio frontier in the economy. These figures confirm the basic logic developed in Chapter 1. After global capital flows are liberalized, low discount factors lead to capital gains of long-term assets in the financial center country. As can be seen from the figure, indebted households increase their debt level while the upper tail of the wealth distribution becomes thicker. The Pareto exponent of the distribution has increased. Panel (c) and Panel (d) illustrate portfolio shifting behaviors. These diagrams show that, even in a numerical simulation, capital return inequality is widened thanks to a higher slope of the capital allocation line. This portfolio rebalancing effect helps counteract the decline in return on the domestic assets. A higher Sharpe ratio provides incentives for rich households to increase investment share in the risky assets.

Table 3.3 presents quantitative magnitude of these effects. Over the 30 years, the model generates a 4.8%p increase in the top one percent wealth share. This accounts for $55\% = \frac{4.8\%p}{8.7\%p}$ of the observed increase in the top one percent wealth share in the Survey of Consumer Finances. If one uses estimates from Saez and Zucman (2016), the calibrated model accounts for $34\% = \frac{4.8\%p}{14.1\%p}$ of the increase in the wealth share. The transitional dynamics can be divided into two steps.

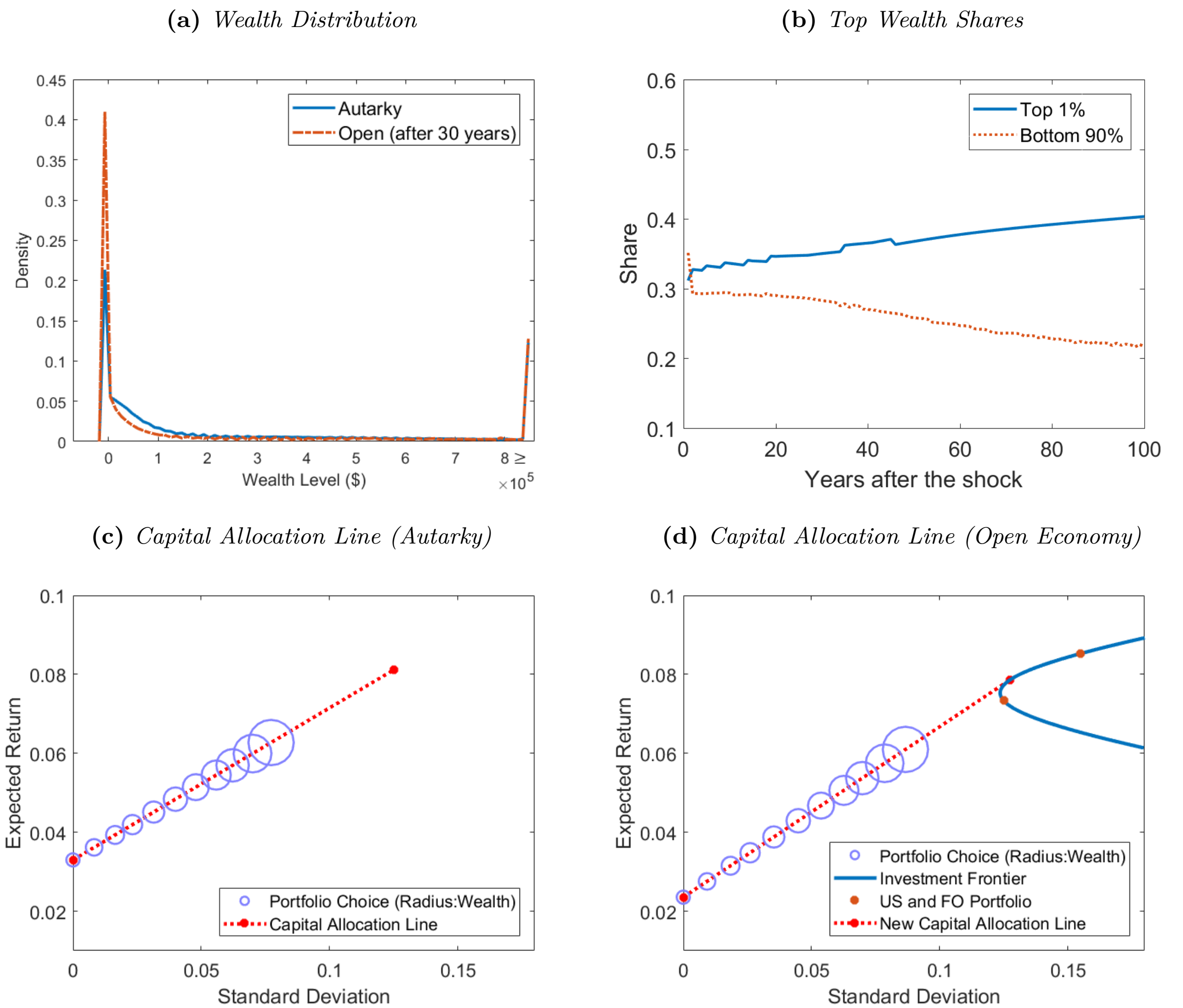
Table 3.3: *Transitional Dynamics*

Wealth Shares	Bottom 90%	Top 10%	Top 5%	Top 1%
Autarky	0.351	0.649	0.537	0.312
Data Estimates (1989)	0.329	0.671	0.542	0.299
Open (after 0 year)	-5.8%p	+5.8%p	+5.2%p	+1.5%p
Open (after 30 years)	-7.0%p	+7.0%p	+7.7%p	+4.8%p
Data Estimates (2016)	-10.0%p	+10.0%p	10.9%p	+8.7%p
Open (New Stationary State)	-13.4%p	+13.4%p	+13.7%p	+8.8%p

Equilibrium Prices	r	s_1	$r + \sigma_1 s_1$	s_2
Autarky	0.033	0.191	0.081	
Open (stationary)	0.023	0.399	0.073	0.43

Notes: This table displays transitional dynamics of the wealth distribution and equilibrium prices. Data estimates are computed from the Survey of Consumer Finances.

Figure 3.3: *Transitional Dynamics*



Notes: Panel (a) plots the model-implied wealth distribution prior to the shock and 30 years after the shock. Panel (b) displays transitional dynamics of the wealth distribution. Panel (c) presents capital allocation line in autarky. Panel (d) shows the balance sheet reallocation induced by financial globalization.

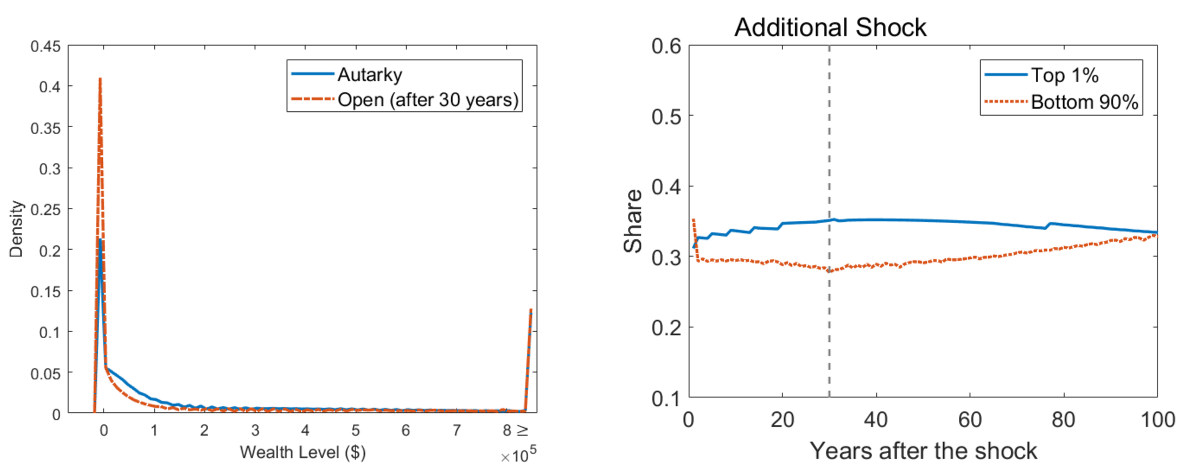
Immediately after the globalization shock, the top one percent's wealth share is increased from 0.312 to 0.327 due to capital gains on the existing domestic risky asset. The asymmetric portfolio rebalancing arises, thereby generating a gradual increase in wealth concentration. In the new stationary state, the top wealth share is increased as large as to 0.405.

3.5.3 Result 2: Reversal of the Trend

A core message of this study is that the long-run trajectory of wealth concentration on the relative magnitudes of (a) the decrease in domestic interest rates and (b) the asymmetric balance sheet reallocation across different wealth holders. The previous analysis suggests that, at least over the past three decades, the effect from (b) appears to outweigh the effect from (a). As in Section 1.2, the model still leaves room for a possible reversal in the future if financial globalization no longer provides enough diversification benefit to counteract diminished return on domestic assets. Figure 3.4 presents one such case. In this exercise, Financial globalization is modeled as two-stage shocks. The first wave of financial globalization is the same as before. In the second wave, the Sharpe ratio of the optimal portfolio is reduced due to a decrease in $\bar{\sigma}^{EM}$. These shocks, as Panel (b) shows, result in an inverse-U shape transitional dynamics of the top one percent wealth share.

Figure 3.4: *Reversal of the Trend*

(a) *Wealth Distribution* **(b)** *Top Wealth Shares*



3.5.4 Result 3: Factor Decomposition

Next, we explore different factors that could potentially contribute to rising wealth concentration in the U.S. over the past three decades. Several factors have been proposed to account for the recent change in the wealth distribution in the U.S., including widened wage inequality and tax

Table 3.4: *Factor Decomposition*

Wealth Shares	Bottom 90%	Top 10%	Top 5%	Top 1%
Autarky	35.1%	64.9%	53.7%	31.2%
Effect 1: Wage Inequality Only	-1.3%p	+1.3%p	+1.8%p	+1.0%p
Effect 2: Global Capital Flows Only	-6.8%p	+6.8%p	+8.2%p	+4.8%p
(1 and 2 combined)	(-8.2%p)	(+8.2%p)	(+9.1%p)	(+4.5%p)
Effect 3: Residuals	-4.0%p	+4.0%p	+2.3%p	+3.0%p
Data Estimates (2016)	-12.2%p	+12.2%p	+11.4%p	+7.4%p

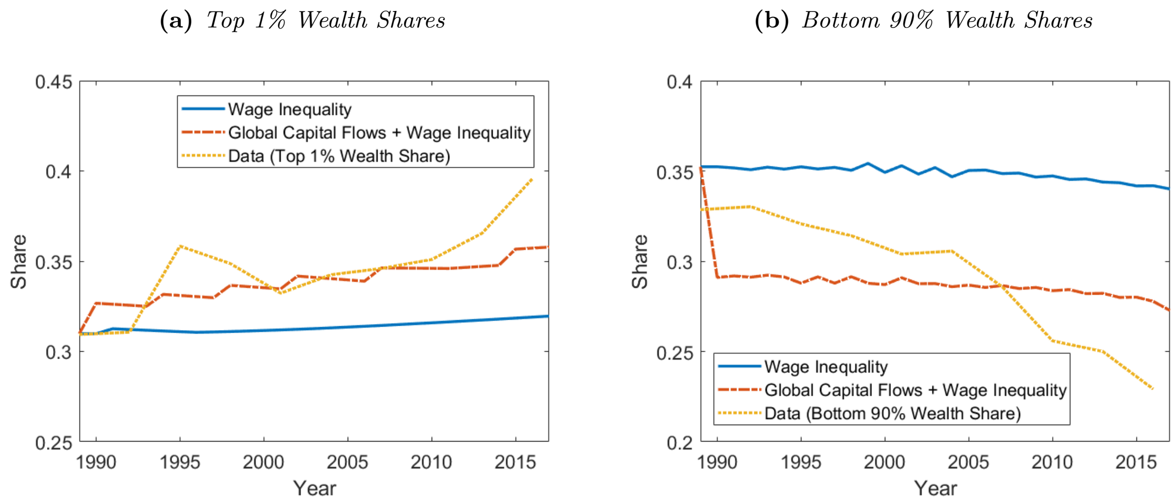
Notes: This table displays changes in the wealth distribution over 30 years under different scenarios. Effect 1 represents a case where the economy experiences changes in wage inequality while other parameters being constant. Effect 2 presents the corresponding estimates in the previous analysis. Numbers in the parenthesis indicate the joint effect. Effect 3 is defined as the gap between Data Estimates (2016) and (1 and 2 combined).

reforms. In this subsection, we compare the magnitudes of these effects and assess the extent to which global capital flows matter to changes in the U.S. wealth distribution from 1989 to 2016.

The exercise proceeds as follows. In the first scenario, I consider a situation where the economy experiences an increase in wage inequality, with every other parameter held constant. The widened wage inequality is simply captured by an change in the value of Σ_{ll}. I adjust μ_l to make sure that the total labor supply remains unchanged. These parameters are calibrated such that — prior to the shock —- the implied top wage shares are consistent with the observed shares in 1989. The post-shock wage distribution is matched to the 2016 data. In the second scenario, I assume that global capital flows are the only structural change over this period with all else staying the same. This scenario is identical to the one presented in Section 3.5.2. I also analyze the combined effect between the two factors. Finally, I compute residuals — the portion that is not accounted for by these two factors. I interpret the size of these residuals as the extent to which other factors such as changing taxes contributed to rising wealth concentration.

Table 3.4 displays the results. The first conclusion is that a change in wage inequality has a relatively small impact on the top wealth holders. This finding is consistent with what Hubmer *et al.* (2018) reported. A major source of income of the top wealth holders is capital, not labor. The widened wage inequality has limited effects on how these wealth holders reinvest their capital.

Figure 3.5: *Transitional Dynamics in Different Scenarios*



Notes: Panel (a) displays transitional dynamics of the top 1% wealth shares under different scenarios. Panel (b) plots the corresponding graphs for the bottom 90% wealth shares. Data estimates are computed from the Survey of Consumer Finances.

By contrast, global financial flows make a more immediate and sizable impact on the wealth distribution. Global capital flows alone can explain a 4.8%p increase in the top 1% wealth share out of 7.4%p and a -6.8%p decrease in the bottom 90% wealth share out of -12.2%p. Figure 3.5 plots the related transitional dynamics. The quantitative analysis here suggests that global capital flows play an outsized role in reshaping the wealth distribution of the U.S. — the central country in global financial architecture — possibly more so than other factors.

3.6 Conclusion

This paper presents a general framework to analyze its distributional effects when households have nonhomothetic preferences over risk and return. Unlike previous studies focusing on flow incomes, I shed light on the determinants of financial wealth, such as risk premium and interest rate. I emphasize the role of finance in shaping the distribution of wealth across different households in the economy.

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Appendix A

Appendix to Chapter 1

A.1 Detailed Proofs

A.1.1 Proof of Proposition 1 and 2

In this subsection, I provide a proof for Proposition 2. Proposition 1 follows immediately by plugging $s_1 = s_2 = \frac{1+\rho}{2} s_1^*$ into (A.3) below.

Result 1. $d \log \Omega_T$

It follows from the differential equation (1.4) that the price of domestic equities rises to $p_T = \frac{r^* + \sigma_1 s_1^*}{r + \sigma_1 s_1} > 1$. Using this, we can write Ω_T as

$$\Omega_T = \frac{\int_{a=G^{*-1}(0.99)}^{\infty} \left\{ a(1-\theta_1(a)) + \left(\frac{r^* + \sigma_1 s_1^*}{r + \sigma_1 s_1} \right) a\theta_1(a) \right\} g^*(a)da}{\int_{a=\underline{a}}^{\infty} \left\{ a(1-\theta_1(a)) + \left(\frac{r^* + \sigma_1 s_1^*}{r + \sigma_1 s_1} \right) a\theta_1(a) \right\} g^*(a)da}$$

where $G^*(\cdot)$ and $g^*(\cdot)$ are c.d.f. and p.d.f. of the wealth distribution prior to the shock. The numerator is the average net worth of the top 1 percent households; the cutoff is given by $G^{*-1}(0.99)$. The denominator is the average net worth of the entire population. Recall that all households retain their wealth above $\underline{a}$, so no mass exists below this threshold. Multiplying

$r + \sigma_1 s_1$ and taking log, one can obtain

$$\log \Omega_T = \log\left(\int_{a=G^{*-1}(0.99)}^{\infty} \{a(1-\theta_1(a))(r+\sigma_1 s_1) + a\theta_1(a)\,(r^* + \sigma_1 s_1^*)\}\, g^*(a)da\right) - \log\left(\int_{a=\underline{a}}^{\infty} \{a(1-\theta_1(a))(r+\sigma_1 s_1) + a\theta_1(a)\,(r^* + \sigma_1 s_1^*)\}\, g^*(a)da\right)$$

Differentiating the both sides with respect to $x \equiv r + \sigma_1 s_1$, we have

$$d\log \Omega_T = -\phi_1 \frac{d(r+\sigma_1 s_1)}{r+\sigma_1 s_1} = -\phi_1 d\log(r+\sigma_1 s_1)$$

where

$$\phi_1 = -\frac{(r+\sigma_1 s_1)\int_{a=G^{*-1}(0.99)}^{\infty} \{a(1-\theta_1(a))\}\, g^*(a)da}{\int_{a=G^{*-1}(0.99)}^{\infty} \left\{a(1-\theta_1(a))(r+\sigma_1 s_1) + a\theta_1(a)\left(r^* + \sigma_1 s_1^*\right)\right\} g^*(a)da} + \frac{(r+\sigma_1 s_1)\int_{a=\underline{a}}^{\infty} \{a(1-\theta_1(a))\}\, g^*(a)da}{\int_{a=\underline{a}}^{\infty} \left\{a(1-\theta_1(a))(r+\sigma_1 s_1) + a\theta_1(a)\left(r^* + \sigma_1 s_1^*\right)\right\} g^*(a)da}$$

Dividing each term by its numerator, we can re-write ϕ_1 as

$$\phi_1 = -\frac{1}{1 + \frac{r^*+\sigma_1 s_1^*}{r+\sigma_1 s_1} \frac{\int_{a=G^{*-1}(0.99)}^{\infty} a\theta_1(a) g^*(a)da}{\int_{a=G^{*-1}(0.99)}^{\infty} a(1-\theta_1(a)) g^*(a)da}} + \frac{1}{1 + \frac{r^*+\sigma_1 s_1^*}{r+\sigma_1 s_1} \frac{\int_{a=\underline{a}}^{\infty} a\theta_1(a) g^*(a)da}{\int_{a=\underline{a}}^{\infty} a(1-\theta_1(a)) g^*(a)da}} > 0$$

The positive sign of ϕ_1 follows from Lemma 1 below, which suggests that

$$\frac{\int_{a=G^{*-1}(0.99)}^{\infty} a\theta_1(a) g^*(a)da}{\int_{a=G^{*-1}(0.99)}^{\infty} a(1-\theta_1(a)) g^*(a)da} > \frac{\int_{a=\underline{a}}^{\infty} a\theta_1(a) g^*(a)da}{\int_{a=\underline{a}}^{\infty} a(1-\theta_1(a)) g^*(a)da}$$

This proves the first part of the proposition. Intuitively speaking, the top 1 percent households, on average, invest more heavily in risky assets prior to the shock so they get more revaluation gains than the average population.

Lemma 1. $\frac{d}{dx} \frac{\int_{x}^{\infty} a\theta_1(a) g^*(a)da}{\int_{x}^{\infty} a(1-\theta_1(a)) g^*(a)da} > 0$ *for all* $x \geq \underline{a}$.

Proof. It is easy to show

$$\frac{\int_{a=x}^{\infty} a\theta_1(a) g^*(a)da}{\int_{x}^{\infty} a(1-\theta_1(a)) g^*(a)da} = \frac{\int_{x}^{\infty} a\theta_1(a) g^*(a)da / \int_{x}^{\infty} ag^*(a)}{1 - \int_{x}^{\infty} a\theta_1(a) g^*(a)da / \int_{a=x}^{\infty} ag^*(a)}$$

So one can see that the sign of $\frac{d}{dx} \frac{\int_{a=x}^{\infty} a\theta_1(a) g^*(a)da}{\int_{a=x}^{\infty} a(1-\theta_1(a)) g^*(a)da}$ is identical to the sign of $\frac{d}{dx} \frac{\int_{a=x}^{\infty} a\theta_1(a) g^*(a)da}{\int_{a=x}^{\infty} ag^*(a)}$.

Taking a derivative with respect to x, we obtain

$$\begin{aligned}
&\frac{1}{\left(\int_x^\infty ag^*(a)\right)^2}\left[-x\theta_1(x)g^*(x)\int_x^a ag^*(a)da + xg^*(x)\int_x^a a\theta_1(a)g^*(a)da\right] \\
=&\frac{xg^*(x)\theta_1(x)}{\left(\int_x^\infty ag^*(a)\right)^2}\left[-\int_x^a ag^*(a)da + \int_x^a a\frac{\theta_1(a)}{\theta_1(x)}g^*(a)da\right] \\
>&0
\end{aligned}$$

where the last inequality results from $\frac{\theta_1(a)}{\theta_1(x)} > 1$ for all $a \geq x$. □

Result 2. $d\log\Omega_\infty$

Next, let me turn to Ω_∞. Let $g(a)$ denote the stationary distribution in closed economy. Substituting $c(a)$ and $\theta_1(a)$ from (1.3), the Kolmogorov Forward equation in (1.5) can be expressed as

$$0 = -mg_0(a) + mg(a) + \frac{d}{da}\left[(r^* + (s_1^*)^2 - \delta - m)(a - \underline{a})g(a)\right]$$

Arranging the terms, one can restate the differential equation as

$$\frac{dg(a)}{da} + \frac{r^* + (s_1^*)^2 - \delta}{r^* + (s_1^*)^2 - \delta - m}\frac{g(a)}{a - \underline{a}} = \frac{mg_0(a)}{(r^* + (s_1^*)^2 - \delta - m)(a - \underline{a})}$$

Multiplying the both sides by $(a - \underline{a})^{\frac{r^*+(s_1^*)^2-\delta}{r^*+(s_1^*)^2-\delta-m}}$, we have

$$\frac{d}{da}\left[g(a)(a - \underline{a})^{\frac{r^*+(s_1^*)^2-\delta}{r^*+(s_1^*)^2-\delta-m}}\right] = (a - \underline{a})^{\frac{r^*+(s_1^*)^2-\delta}{r^*+(s_1^*)^2-\delta-m}-1}\frac{mg_0(a)}{(r^* + (s_1^*)^2 - \delta - m)}$$

Taking an integral in terms of a, we can obtain

$$g(a) = \left[\int_{\underline{a}}^a (x - \underline{a})^{\frac{r^*+(s_1^*)^2-\delta}{r^*+(s_1^*)^2-\delta-m}}\frac{mg_0(x)}{r^* + (s_1^*)^2 - \delta - m}dx + \mathcal{C}\right](a - \underline{a})^{-\frac{r^*+(s_1^*)^2-\delta}{r^*+(s_1^*)^2-\delta-m}}$$

where $\mathcal{C}$ is the constant of integration. $\mathcal{C}$ is pinned down by the condition $\int_{\underline{a}}^\infty g(a)da = 1$. Recall that every k'th moment of g_0 is finite by assumption, so the limit of the first term in the bracket is finite. That is,

$$\lim_{a\to\infty}\int_{\underline{a}}^a (x - \underline{a})^{\frac{r^*+(s_1^*)^2-\delta}{r^*+(s_1^*)^2-\delta-m}}\frac{mg_0(x)}{r^* + (s_1^*)^2 - \delta - m}dx = \mathcal{C}_1 < \infty$$

for some constant $\mathcal{C}_2$. Thus, it is easy to show

$$\lim_{a\to\infty} \frac{g(\tau a)}{g(a)} = \tau^{-\left(1+\frac{m}{r^*+(s^*)^2-\delta-m}\right)}$$

The Pareto exponent in closed economy is given by $\frac{r^*+(s_1^*)^2-\delta-m}{m}$.

Next, we turn to the stationary wealth distribution and its Pareto exponent in open economy. To establish an analogous result, we use the following lemma.

Lemma 2. *In open economy, household i's portfolio choice functions, $\theta_{it} = [\theta_{1it}, \theta_{2it}]'$, are characterized by*

$$\begin{bmatrix} \theta_{1it} \\ \theta_{2it} \end{bmatrix} = \Sigma^{-1} \begin{bmatrix} \sigma_1 s_1 \\ \sigma_2 s_2 \end{bmatrix} \left(1 - \frac{\underline{a}}{a_{it}}\right) \tag{A.1}$$

where $\Sigma \equiv [\sigma_1^2, \rho\sigma_1\sigma_2; \rho\sigma_1\sigma_2, \sigma_2^2]$ is the variance-covariance matrix.

Proof. See Appendix A.2.1. □

Lemma 2 implies that the relative portfolio weight between the two risky assets are given by

$$\Sigma^{-1} \begin{bmatrix} \sigma_1 s_1 \\ \sigma_2 s_2 \end{bmatrix} = \frac{1}{1-\rho^2} \begin{bmatrix} \frac{s_1}{\sigma_1} - \frac{\rho s_2}{\sigma_1} \\ \frac{s_2}{\sigma_2} - \frac{\rho s_1}{\sigma_2} \end{bmatrix}$$

Again, we only consider cases where the optimal choices of θ_{1it} and θ_{2it} are non-negative. The formula above shows that a sufficient and necessary condition for this is

$$\rho < \min\left\{\frac{s_1}{s_2}, \frac{s_2}{s_1}\right\} \tag{A.2}$$

We take this condition as given. Substituting θ_{it} from (A.1) into the Kolmogorov Forward equation, we can obtain

$$mg(a) + \frac{d}{da}\left[(r + R'\Sigma^{-1}R - \delta - m)(a - \underline{a})g(a)\right] = mg_0(a)$$

where $R = [\sigma_1 s_1; \sigma_2 s_2]$. We can then solve for the stationary wealth distribution by following the same procedure as in closed economy. The Pareto exponent in open economy is given by

$$\begin{aligned}\frac{1}{\xi} =& \frac{r + R'\Sigma^{-1}R - \delta - m}{m} \\ =& \frac{r + \frac{1}{1-\rho^2}(s_1^2 - 2\rho s_1 s_2 + s_2^2) - \delta - m}{m}\end{aligned} \tag{A.3}$$

Totally differentiating $\Omega_\infty = 100^{\frac{1}{\xi}-1}$, we have

$$d\log\Omega_\infty = \phi_2 d\log(r + \sigma_1 s_1) + \phi_3 d\log s_2 - \phi_4 d\log\rho$$

where

$$\begin{aligned}\phi_2 &= \frac{(r+\sigma_1 s_1)\log 100}{m} > 0 \\ \phi_3 &= \frac{2s_2\log 100}{m(1-\rho^2)}(s_2 - \rho s_1) > 0 \\ \phi_4 &= \frac{2\rho\log 100}{m(1-\rho^2)^2}\left(\rho^2 s_1 s_2 - \rho(s_1^2 + s_2^2) + s_1 s_2\right) > 0\end{aligned}$$

The second inequality results from (A.2). The last inequality follows from

$$-\rho(s_1^2 + s_2^2) + s_1 s_2 = s_1 s_2 \rho\left(-\frac{s_1}{s_2} - \frac{s_2}{s_1} + \frac{1}{\rho}\right) > 0$$

in view of the condition in (A.2).

A.1.2 More Details on Dynamics

As time passes by, wealth stock grows and the stationary state level of wealth is affected by various technological parameters. Let me first define the stationary state of this economy. By plugging $\theta_{1it} = \frac{s_{1t}^*}{\sigma_1}\left(1 - \frac{\underline{a}}{a_{it}}\right)$ and $c_{it} = (\delta + m)a_{it} + (r_t^* - \delta - m)\underline{a}$ into the households' budget constraint, integrating them with i and incorporating the overlapping generation structure, we can derive the evolution of wealth stock A_t as follows

$$dA_t = \left[(r^*(A_t) + s_1^*(A_t)^2 - \delta - m)(A_t - \underline{a}) + m(A_t - A_0)\right]dt + s_1^*(A_t)A_t dz \tag{A.4}$$

where $r(\cdot)$ and $s(\cdot)$ are the solutions given by (1.14) and (1.15), and A_0 is the mean wealth of new-born households. Let A^s denote the stock of wealth such that $\mathbb{E}_t[dA_t] = 0$ and $A^s > \underline{a}$. One can interpret A^s as the stationary state level of wealth stock in that the expected growth rate is zero. The next proposition states that this feature does little to alter Proposition 3 when $\underline{a}$ is small.

Corollary 3. *In the stationary state, a developing country exhibits a smaller wealth stock A^s, a lower risk-free rate $r^*(A^s)$, and a higher Sharpe ratio $s_1^*(A^s)$, a lower cost of capital $r^*(A^s) + \bar{\sigma} s_1^*(A^s)$ and a lower excess profit $V^*(A^s)$ when $\underline{a} = 0$*

A.1.3 Proof of Collorary 3

As we discussed in the draft, A_t evolves according to a stochastic process

$$dA_t = \left[(r(A_t) + s_1(A_t)^2 - \delta - m)(A_t - \underline{a}) + m(A_t - A_0) \right] dt + s_1(A_t) A_t dz$$

where

$$s_1(A) = \bar{\sigma} A / (A - \underline{a}) \tag{A.5}$$

$$r(A) = \Phi'(A) - \bar{\sigma}^2 A / (A - \underline{a}) - \tau + \tau\lambda \tag{A.6}$$

The stationary wealth stock, A^s, is pinned down by

$$r(A^s) + s(A^s)^2 = \delta + m + \frac{m(A_t - A_0)}{A_t - \underline{a}}$$

Substituting $r(A)$ and $s_1(A)$ from (A.5) and (A.6), one can obtain

$$\Phi'(A^s) + \bar{\sigma}^2 \left(\left(\frac{A^s}{A^s - \underline{a}} \right)^2 - \frac{A^s}{A^s - \underline{a}} \right) - \tau + \tau\lambda = \delta + m + \frac{m(A^s - A_0)}{A^s - \underline{a}}$$

Consider the case where $\underline{a} = 0$. One can then write the above expression as

$$\Phi'(A^s) - \tau + \tau\lambda = 2m + \delta - \frac{mA_0}{A^s}$$

The left-hand side is decreasing in A^s, while the right-hand side is increasing in A^s. So A^s is uniquely pinned down by this condition. Invoking the Implicit Function Theorem, we have

$$\left(\Phi''(A^s) - \frac{mA_0}{(A^s)^2} \right) \frac{\partial A^s}{\partial \lambda} + \tau = 0$$

which leads to $\frac{\partial A^s}{\partial \lambda} > 0$ due to diminishing marginal return. Therefore, EM has a lower A^s in autarky. For other variables, it is easy to show that $s_1(A) = \bar{\sigma}$ is increasing in $\bar{\sigma}$ and that

$$r(A^s) = -\bar{\sigma}^2 - \tau + \tau\lambda$$

is decreasing in $\bar{\sigma}$ and increasing in λ. So EM has a higher s_1 and a lower r. Finally, the cost of capital. $r(A^s) + \bar{\sigma}^{US} s(A^s) + \tau - \tau\lambda = \Phi'(A^s)$, is higher in a developing country as well. This leads to lower excess profits, V_{1t}.

A.1.4 Proof of Proposition 4

Case 1. $\rho \in (0, 1)$

Suppose that $\rho < 1$. In Appendix A.2.2, I show that the portfolio choice functions can be written as

$$\begin{bmatrix} \theta_{1it} \\ \theta_{2it} \end{bmatrix} = \Sigma^{-1} \begin{bmatrix} \sigma_1 s_{1t} \\ \sigma_2 s_{2t} \end{bmatrix} \left(1 - \frac{\underline{a}}{a_{it}}\right)$$

and the market clearing conditions are

$$r_t = \Phi'(K_{1t}) - \bar{\sigma}^{US} s_{1t} - \tau(1 - \lambda^{US}) \tag{A.7}$$

$$r_t = \Phi'(K_{2t}) - \bar{\sigma}^{EM} s_{2t} - \tau(1 - \lambda^{EM}) \tag{A.8}$$

$$K_{1t} = \frac{(s_{1t} - \rho s_{2t})}{\bar{\sigma}^{US}(1 - \rho^2)} (2A - 2\underline{a}) \tag{A.9}$$

$$K_{2t} = \frac{(s_{2t} - \rho s_{1t})}{\bar{\sigma}^{EM}(1 - \rho^2)} (2A - 2\underline{a}) \tag{A.10}$$

$$K_{1t} + K_{2t} = 2A \tag{A.11}$$

conditional on $A_{1t} = A_{2t} = A$. Using the system of equations, (A.9) and (A.10), we can solve for s_{1t} as follows

$$s_{1t} \equiv \frac{\bar{\sigma}^{US} K_{1t} + \rho \bar{\sigma}^{EM} K_{2t}}{2A - 2\underline{a}}$$

Substituting K_{1t} from (A.11), one can write the above formula as

$$s_{1t} = \bar{\sigma}^{US} \left(\frac{2A - (1 - \rho\bar{\sigma}^{EM}/\bar{\sigma}^{US})K_{2t}}{2A - 2\underline{a}} \right) \tag{A.12}$$

Since $\rho\bar{\sigma}^{EM} > \bar{\sigma}^{US}$, we have

$$\begin{aligned} s_{1t} &= \bar{\sigma}^{US} \left(\frac{2A - (1 - \rho\bar{\sigma}^{EM}/\bar{\sigma}^{US})K_{2t}}{2A - 2\underline{a}} \right) \\ &> \bar{\sigma}^{US} \left(\frac{2A}{2A - 2\underline{a}} \right) \\ &= s_{1t}^* \end{aligned}$$

Next, I compare between r_t and r_t^* conditional on the realization of $A_{1t} = A_{2t} = A$. To show that the risk-free interest in financial center country rises after global integration, first note that

$$\Phi'^{-1}(r_t^* + \bar{\sigma}^{US} s_{1t}^* + \tau(1 - \lambda^{US})) = A \tag{A.13}$$

Suppose now, to get a contradiction, $r_t \geq r_t^*$. I already showed $s_{1t} > s_{1t}^*$, so one can obtain

$$r_1 + \bar{\sigma}^{US} s_{1t} + \tau(1 - \lambda^{US}) > r_1^* + \bar{\sigma}^{US} s_{1t}^* + \tau(1 - \lambda^{US})$$

Using the first order condition (A.13) and $\Phi''(\cdot) < 0$, it is now straightforward to see that $K_{1t} < A$. This leads to $K_{2t} > A$ due to (A.11). We can then obtain

$$\begin{aligned} &\Phi'(K_{1t}) > \Phi'(K_{2t}) \\ \Leftrightarrow\ & r_t + \bar{\sigma}^{US} s_{1t} + \tau(1 - \lambda^{US}) > r_t + \bar{\sigma}^{EM} s_{2t} + \tau(1 - \lambda^{EM}) \\ \Leftrightarrow\ & \bar{\sigma}^{EM} s_{2t} < \bar{\sigma}^{US} s_{1t} - \tau(\lambda^{US} - \lambda^{EM}) \end{aligned}$$

in view of (A.7) and (A.8). Subtracting $\rho\bar{\sigma}^{EM} s_{1t}$ from the both sides of the last inequality, we have

$$\begin{aligned} \bar{\sigma}^{EM}(s_{2t} - \rho s_{1t}) &< \bar{\sigma}^{US} s_{1t} - \rho\bar{\sigma}^{EM} s_{1t} - \tau(\lambda^{US} - \lambda^{EM}) \\ &< 0 \end{aligned}$$

The last inequality follows from $\rho\bar{\sigma}^{EM} > \bar{\sigma}^{US}$ and $\lambda^{US} > \lambda^{EM}$. This inequality is a contradiction to $K_{2t} > A$ because it implies

$$K_{2t} = \frac{(s_{2t} - \rho s_{1t})}{\bar{\sigma}^{EM}(1-\rho^2)}(2A - 2\underline{a})$$
$$< 0 \tag{A.14}$$

in equilibrium. Thus, $r_t < r_t^*$ must hold when $A_{1t} = A_{2t} = A$ is given.

Finally, I turn to proving $r_t + \bar{\sigma}^{US} s_{1t} < r_t^* + \bar{\sigma}^{US} s_{1t}^*$ and $V_{1t} > V_{1t}^*$. The first inequality is straightforward to prove because

$$\begin{aligned} r_t + \bar{\sigma}^{US} s_{1t} &= \Phi'(K_{1t}) - \tau(1-\lambda^{US}) \\ &< \Phi'(A) - \tau(1-\lambda^{US}) \\ &= r_t^* + \bar{\sigma}^{US} s_{1t}^* \end{aligned}$$

The line equality follow from (A.7). The second line results from $K_{1t} > A$; recall from (A.14) that assuming $K_{1t} < A$ leads to a contradiction as it implies $K_{1t} < A < K_{2t}$. The final line is due to (A.13). Since the cost of capital becomes lower after financial globalization, excess profits to entrepreneurs rise i.e. $V_{1t} > V_{1t}^*$. This inequality can be analytically proved by applying the Envelope theorem to the firm's profit maximization problem. More specifically, we can write its objective function as

$$\begin{aligned} V_{1t} \equiv \max_{K_{1t}, D_{1t}, E_{1t}} \Big\{ &\Phi(K_{1t}) - r_t D_{1t} - (r_t^* + \sigma_1 s_{1t}^* + \tau) E_{1t} \\ &+ \zeta_1(K_{1t} - D_{1t} - E_{1t}) + \zeta_2(\lambda K_{1t} - D_{1t}) \Big\} \end{aligned}$$

where ζ_1 and ζ_2 are the Lagrangian multipliers. Also, recall $\sigma_1 = \frac{\bar{\sigma}^{US}}{1-\lambda}$. So we can restate the problem as

$$V_{1t} = \max_{K_{1t}} \Big\{ \Phi(K_{1t}) - (r_t + \bar{\sigma}^{US} s_{1t}) K_{1t} \Big\}$$

Given that the firm is a price taker, let $x \equiv r_t + \bar{\sigma} s_{1t}$. Invoking the Envelope Theorem, we have $\frac{dV_{1t}}{dx} < 0$ for all x. Thus, we can see that $V_{1t} > V_{1t}^*$.

Case 2. æ = 1

Now let me turn to the limit case where $\rho = 1$. While the risks per unit of equity are different (i.e. $\sigma_1 = \frac{\bar{\sigma}^{US}}{1-\lambda^{US}}$ and $\sigma_2 = \frac{\bar{\sigma}^{EM}}{1-\lambda^{EM}}$), domestic and foreign risky assets provide identical stochastic returns up to normalization. Essentially, foreign risky assets act as perfect substitutes to domestic risky assets. A single price clears the market for risky assets. I shall use $s_t \equiv s_{1t} = s_{2t}$ to denote the common Sharpe ratio. Conditional on the realization of $A_{1t} = A_{2t} = A$, one can write the market clearing conditions as

$$r_t = \Phi'(K_{1t}) - \bar{\sigma}^{US} s_t - \tau(1 - \lambda^{US}) \tag{A.15}$$

$$r_t = \Phi'(K_{2t}) - \bar{\sigma}^{EM} s_t - \tau(1 - \lambda^{EM}) \tag{A.16}$$

$$(2A - \underline{a}_1 - \underline{a}_2) s_t = \bar{\sigma}^{US} K_{1t} + \bar{\sigma}^{EM} K_{2t} \tag{A.17}$$

$$2A = K_{1t} + K_{2t} \tag{A.18}$$

Note here that (A.9) and (A.10) are now replaced by (A.17), which consists of the supply and demand functions for risky assets. The left-hand side of (A.17) represents the total aggregate amount of risks borne by households, while the right-hand represents the total aggregate amount of risks generated by domestic and foreign firms. In fact, we can rewrite condition (A.17) as

$$s_t = \frac{\omega_1 \bar{\sigma}^{US} + \omega_2 \bar{\sigma}^{EM}}{1 - (\underline{a}_1 + \underline{a}_2)/(2A)}$$

where $\omega_1 \equiv \frac{K_{1t}}{2A}$ and $\omega_2 \equiv \frac{K_{2t}}{2A}$. The sum of ω_1 and ω_2 equals 1 due to (A.18). It is then straightforward to see that

$$\begin{aligned} s_t &= \frac{\omega_1 \bar{\sigma}^{US} + \omega_2 \bar{\sigma}^{EM}}{1 - (\underline{a}_1 + \underline{a}_2)/(2A)} \\ &> \frac{\bar{\sigma}^{US}}{1 - \underline{a}_1 / A_{1t}} \\ &= s_t^* \end{aligned}$$

since $\bar{\sigma}^{EM} > \bar{\sigma}^{US}$ and $\underline{a}_1 < \underline{a}_2$. This proves $s_t > s_t^*$. Moving on, it follows from (A.15) and (A.16) that $K_{1t} > K_{2t}$. This implies that $K_{1t} > A$ so one obtains

$$\begin{aligned}
&\Phi(K_{1t}) > \Phi(A) \\
\Leftrightarrow\ & r_t + \bar{\sigma}^{US} s_t + \tau(1 - \lambda^{US}) < r_t^* + \bar{\sigma}^{US} s_t^* + \tau(1 - \lambda^{US}) \\
\Leftrightarrow\ & r_t + \bar{\sigma}^{US} s_t < r_t^* + \bar{\sigma}^{US} s_t^*
\end{aligned}$$

Combined with $s_{1t} > s_{1t}^*$, this leads to $r_t < r_t^*$. Also, invoking the Envelope Theorem as before, one can obtain $V_{1t}^* < V_{1t}$ when $A_{1t} = A_{2t} = A$ is given.

A.1.5 Proof of Corollary 2

It follows from (A.12) that, when two countries are symmetric, the Sharpe ratio in an open economy becomes

$$s_{1t} = \bar{\sigma}^{US} \left(\frac{(1+\rho)A}{2A - 2\underline{a}} \right)$$

The risk-free rate remains unchanged. The excess profit, V_{1t}, is increased as the bank now faces a lower cost of capital.

A.1.6 More Details on Foreign Direct Investment

Putting all the ingredients in Section 1.4.2 together, we can state the US firm's problem as maximizing the profit $V_{1t} + V_{1t}^{FDI}$ generated in the two countries. V_{1t} is domestic excess profit defined in (3.2), while V_{1t}^{FDI} stems from the optimal contract problem:

$$V_{1t}^{FDI} dt \equiv \max_{\substack{\{\phi^{Local}, \phi^{FDI}, c, \\ K_{2t}^{Local}, K_{2t}^{FDI}\}}} \left[\phi^{FDI} \Phi(K_{2t}) - R_1 K_{2t}^{FDI} - c\Phi(K_{2t}) \right] dt \tag{A.19}$$

subject to

$$K_{2t} = K_{2t}^{Local} + K_{2t}^{FDI} \tag{A.20}$$

$$(1-\pi_L)(1-\phi^{FDI}-\phi^{Local})\Phi(K_{2t}) \geq B(\eta)\Phi(K_{2t}) \tag{A.21}$$

$$(1-\pi_L)\phi^{FDI}\Phi(K_{2t}) \geq \eta\Phi(K_{2t}) \tag{A.22}$$

$$(1-\phi^{Local}-\phi^{FDI})\Phi(K_{2t}) \geq 0 \tag{A.23}$$

$$\phi^{Local}\Phi(K_{2t}) \geq R_2 K_{2t}^{Local} \tag{A.24}$$

where $R_1 \equiv r_t + \bar{\sigma}^{EM}s_{2t} + \tau^{FDI}(1-\lambda^{US})$ and $R_2 \equiv r_t + \bar{\sigma}^{EM}s_{2t} + \tau(1-\lambda^{EM})$ are funding costs per unit of capital in US and EM respectively. Under this contract, the EM entrepreneur, US firm and local financial intermediary receive $(1-\phi^{FDI}-\phi^{Local})$, ϕ^{FDI} and ϕ^{Local} shares of profits at each instantaneous time respectively.

Turning to constraints associated with the contract, the first line represents the balance sheet of the EM firm. It has two funding sources: the U.S. parent firm, K_{2t}^{FDI}, and local financial intermediary, K_{2t}^{Local}. The second condition is the incentive-compatible constraint for the EM entrepreneur. π_L is the profit loss from the misbehavior of the EM entrepreneur. Private benefits to the entrepreneur is assumed to be $B(\eta)\Phi(K_{2t})$ where η is the monitoring level by the U.S. firm and $B'(\eta) < 0$.[1] From the EM entrepreneur's viewpoint, increased payoffs from the good behavior should be greater than the private benefit. The third condition represents incentive-compatible constraint to induce the US firm to pay a monitoring cost $\eta\Phi(K_{2t})$. It implies that the benefit of monitoring should outweigh the cost. The fourth condition corresponds to the participation constraint of the EM entrepreneur. The last condition is associated with the funding cost in the EM ; the share of profits allocated to local financial intermediary should be greater than or equal to the equilibrium funding cost. Local financial intermediary breaks even.

Finally, recall that the funding cost for K_{2t}^{FDI} and K_{2t}^{Local} are given by $R_1 dt$ and $R_2 dt$ respectively

[1] To guarantee an interior solution, I assume $B''(\eta) > 0$, $\lim_{\eta\to 0} B'(\eta) = 0$ and $\lim_{\eta\to\infty} B(\eta) = \infty$.

and we can express them as

$$R_1 \equiv r_t + \bar{\sigma}^{EM} s_{2t} + \tau^{FDI}(1 - \lambda^{US})$$
$$R_2 \equiv r_t + \bar{\sigma}^{EM} s_{2t} + \tau(1 - \lambda^{EM})$$

One assumption I make here is that $\tau^{FDI} > \tau$ so as to make $R_1 > R_2$ arise. This condition would make local funding preferable when there were no monitoring benefit from FDI. In equilibrium, the firm chooses a mix of K_{2t}^{local} and K_{2t}^{FDI} to equalize the marginal benefit of FDI with its opportunity cost. As it turns out later, this structure leads to the coexistence of foreign portfolio equity and FDI in global capital flows. With this apparatus in place, open economy equilibrium is now defined as follows:

Definition 5. *An open economy equilibrium is a stochastic process,* $\{r_t, (s_{1t}, \sigma_1), (s_{2t}, \sigma_2)\}_{t\geq 0}$, *which clears global financial markets:* $\sum_{k\in\{EM,US\}}(\mathbb{S}_t^k - \mathbb{I}_t^k) = 0$, $\sum_{k\in\{EM,US\}} \mathbb{S}_{1t}^k = \mathbb{I}_{1t}^{US}$ *and* $\sum_{k\in\{EM,US\}} \mathbb{S}_{2t}^k = \mathbb{I}_{2t}^{EM}$ *where* $\mathbb{I}_{2t}^{EM} = K_{2t}^{Local} + K_{2t}^{FDI}$. *The rest is same as in Section 1.4.1.*

A.1.7 Proof of Proposition 5

I take a similar approach to Antràs *et al.* (2009). Note here that the participation constraint of the EM entrepreneur, (A.23), never binds in optimal contract due to some informational rent; if it were to bind, the left-hand side of (A.21) would become negative so the EM entrepreneur would always shirk. We can then write the Lagrangian associated with the U.S. entrepreneur's problem as

$$\begin{aligned}\mathcal{L} =& \phi^{FDI}\Phi(K_{2t}) - R_1 K_{2t}^{FDI} - \eta\Phi(K_{2t}) + \mu_1[K_{2t}^{Local} + K_{2t}^{FDI} - K_{2t}] \\ &+ \mu_2\left[(1-\pi_L)(1-\phi^{FDI} - \phi^{Local}) - B(\eta)\right] \\ &+ \mu_3[\phi^{FDI} - \eta/(1-\pi_L)] + \mu_5\left[\phi^{Local}\Phi(K_{2t}) - R_2 K_{2t}^{Local}\right]\end{aligned}$$

where μ_k represents the Lagrangian multiplier for the k'th constraint. The first order conditions yield

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_{2t}} &= (\phi^{FDI} - \eta)\Phi'(K_{2t}) - \mu_1 + \mu_5 \phi^{Local}\Phi'(K_{2t}) = 0 && \text{(A.25)} \\
\frac{\partial \mathcal{L}}{\partial K_{2t}^{FDI}} &= -R_1 + \mu_1 = 0 \\
\frac{\partial \mathcal{L}}{\partial K_{2t}^{Local}} &= \mu_1 - R_2\mu_5 = 0 \\
\frac{\partial \mathcal{L}}{\partial \phi^{FDI}} &= \Phi(K_{2t}) - \mu_2(1-\pi_L) + \mu_3 = 0 \\
\frac{\partial \mathcal{L}}{\partial \phi^{Local}} &= -\mu_2(1-\pi_L) + \mu_5\Phi(K_{2t}) = 0 \\
\frac{\partial \mathcal{L}}{\partial \eta} &= -\Phi(K_{2t}) - \mu_2 B'(\eta) - \frac{\mu_3}{1-\pi_L} = 0 && \text{(A.26)}
\end{aligned}
$$

Merging these conditions, it is easy to show

$$
\mu_1 = R_1 > 0, \quad \mu_5 = \frac{R_1}{R_2} > 0, \quad \mu_2 = \frac{R_1}{R_2}\frac{\Phi(K_{2t})}{1-\pi_L} > 0, \quad \mu_3 = \Phi(K_{2t})\left(\frac{R_1}{R_2} - 1\right) > 0 \qquad \text{(A.27)}
$$

where the last inequality results from the assumption I made earlier: $R_1 \equiv r_t + \bar{\sigma}^{EM}s_{2t} + \tau^{FDI}(1 - \lambda^{US}) > r_t + \bar{\sigma}^{EM}s_{2t} + \tau(1-\lambda^{US}) \equiv R_2$. These results imply that constraint (A.20), (A.21), (A.22), and (A.24) must be binding.

Next, substituting μ_2 and μ_3 from (A.27), one can convert condition (A.26) into

$$
B'(\eta) = -\frac{R_2}{R_1}\left(\frac{R_1}{R_2} - \pi_L\right) < 0
$$

which pins down the optimal value of monitoring, η, when the market funding costs are given. Also, the optimal level of investment is determined by condition (A.25). Substituting μ_1 and μ_5 from (A.27), we obtain

$$
\begin{aligned}
\Phi'(K_{2t}) &= \frac{R_1}{\phi^{FDI} - \eta - \frac{R_1}{R_2}\phi^{Local}} \\
&= \frac{R_1}{\frac{\eta\pi_L}{1-\pi_L} + \frac{R_1}{R_2}\left(1 - \frac{B(\eta)+\eta}{1-\pi_L}\right)} && \text{(A.28)}
\end{aligned}
$$

Let K_{2t}^* and η^* denote the level of investment and monitoring determined by these conditions.

The rest of control variables are pinned down by the constraints as follows

$$\phi^{FDI*} = \frac{\eta^*}{1-\pi_L} \tag{A.29}$$

$$\phi^{Local*} = 1 - \frac{B(\eta^*) + \eta^*}{1-\pi_L} \tag{A.30}$$

$$K_{2t}^{Local*} = \frac{\phi^{Local*}\Phi(K_{2t}^*)}{R_2} \tag{A.31}$$

$$K_{2t}^{FDI*} = K_{2t}^* - K_{2t}^{Local*} \tag{A.32}$$

where the first line follows from (A.23), the second line from (A.21), the third line from (A.24) and the last line from (A.20). The total payoff obtained by the US entrepreneur is

$$\begin{aligned} V_{1t}^{FDI} \equiv & \phi^{FDI*}\Phi(K_{2t}^*) - R_1 K_{2t}^{FDI*} - \eta^*\Phi(K_{2t}^*) \\ = & (\phi^{FDI*} - \eta^*)\Phi(K_{2t}^*) - R_1(K_{2t}^* - K_{2t}^{Local*}) \\ = & (\phi^{FDI*} - \eta^*)\Phi(K_{2t}^*) - R_1 K_{2t}^* + \frac{R_1}{R_2}\phi^{Local*}\Phi(K_{2t}^*) \\ = & R_1\left(\frac{\Phi(K_{2t}^*)}{\Phi'(K_{2t}^*)} - K_{2t}^*\right) \end{aligned} \tag{A.33}$$

Next, consider the opposite case: the U.S. firm does not conduct FDI whether by their own choice or by investment barriers. (e.g. security market liberalization) Obviously, the US entrepreneur has no incentive to pay monitoring costs any more. It is also clear that the EM entrepreneur receive no share of profits. The optimal contract problem in this case can be simply written as

$$\max_{\phi^{FDI}, K_{2t}, K_{2t}^{FDI}, K_{2t}^{Lo}} \quad \phi^{FDI}\pi_L\Phi(K_{2t}) - R_1 K_{2t}^{FDI}$$

subject to the constraints

$$\begin{aligned} K_{2t} &= K_{2t}^{FDI} + K_{2t}^{Local} \\ \pi_L(1-\phi^{FDI})\Phi(K_{2t}) &\geq R_2 K_{2t}^{Local} \\ \phi^{FDI} &\leq 0 \end{aligned}$$

So we can easily see that $\phi^{Local} = 1$ and $\phi^{FDI} = 0$ must hold. The optimal level of investment is determined by the first order condition of the EM entrepreneur:

$$\Phi'(K_{2t}^{**}) = \Phi'(K_{2t}^{Local**}) = \frac{R_2}{\pi_L}$$

Note that as $\pi_L \to 0$, K_{2t}^{**} converges to zero, while K_{2t}^{*} converges to a positive value because (A.28) becomes $\Phi'(K_{2t}^{*}) = \frac{R_1}{(1-B(\eta^*)-\eta^*)}$, provided that $B(\eta^*) + \eta^* < 1$. I assume that this is taken as given due to the functional form of B.

The final step of the proof is to compare equilibrium prices under the full integration, in which K_{2t}^{*} arises, and under security market liberalization, in which K_{2t}^{**} arises. Returning back to the market clearing conditions in Proposition 4, the supply function in (A.8) is now replaced by

$$r_t = \pi_L \Phi'(K_{2t}) - \bar{\sigma}^{EM} s_{2t} - \tau(1 - \lambda^{EM}) \tag{A.34}$$

$$\text{Or,} \quad r_t = (1 - B(\eta^*) - \eta^*)\Phi'(K_{2t}) - \bar{\sigma}^{EM} s_{2t} - \tau(1 - \lambda^{EM}) \tag{A.35}$$

The first line shows up in the case of security market liberalization, while the second line shows up in the case of full integration. In any case, one can derive

$$s_{1t} = \bar{\sigma}^{US} \left(\frac{2A - (1 - \rho\bar{\sigma}^{EM}/\bar{\sigma}^{US})K_{2t}}{2A - \underline{a}_1 - \underline{a}_2} \right)$$

when $\rho < 1$ as in (A.12). Since $K_{2t}^{*} > K_{2t}^{**} > 0$ when π_L is sufficiently small, we have $s_{1t}^{(iii)} > s_{1t}^{(ii)} > s_{1t}^{(i)}$. This proves the first part of the proposition. Lastly, it is easy to extend the proof of Proposition 4 to show $V_{1t}^{(iii)} > V_{1t}^{(i)}$. Also, $V_{1t}^{FDI} > 0$ follows from (A.33), (A.28) and the Inada conditions associated with $\Phi(\cdot)$.

A.1.8 Alternative Microfoundation: Safe Assets

The household side remains unchanged. On the supply side, the economy is endowed with a net quantity Γ of safe assets. Risky assets, on the other hand, are endogenously created in the real side of the economy. All countries have an identical technology. By investing K_t units of capital, the representative firm earns $\Phi(K_t)dt + \bar{\sigma}K_t dz_{1t}$ with diminishing marginal return. Given a market price vector $\{r_t^*, s_{1t}^*\}$, the profit maximization problem of the firm is

$$V_{1t} \equiv \max_{K_t} \{\Phi(K_t)dt - \underbrace{[(r_t + \sigma_1 s_{1t})K_t dt]}_{\text{Funding Cost}}\}$$

The first-order condition pins down the optimal investment level. $\Phi'(K_t) = r_t^* + \bar{\sigma} s_{1t}^*$. This condition does not depend on capital structure (i.e., debt-equity ratio) of the firm due to the Modigliani-Miller Theorem. Given the market price $\{r_t^*, s_{1t}^*\}$, the aggregate total investment is then

$$\mathbb{I}_t \equiv \underbrace{\Gamma}_{\text{Safe Asset Supply}} + \underbrace{\Phi'^{-1}(r_t^* + \bar{\sigma} s_{1t}^*)}_{\text{Risky Asset Supply}}$$

$$\mathbb{I}_{1t} \equiv \Phi'^{-1}(r_t^* + \bar{\sigma} s_{1t}^*)$$

both of which are downward-sloping: $\frac{\partial \mathbb{I}_t}{\partial (r_t + \sigma_1 s_{1t})} < 0$ and $\frac{\partial \mathbb{I}_{1t}}{\partial s_{1t}} < 0$. The market clearing conditions in a closed economy are the same as in the baseline model.

Consider two countries, US and EM. The only difference between the two economies is their exogenous endowment of safe assets $\Gamma^{US} > \Gamma^{EM}$. Let A_t be given. Since $\mathbb{S}_t = A_t$ and $\mathbb{S}_{1t} = \frac{s_{1t}}{\sigma_1}(A_t - \underline{a})$, the market clearing conditions yield

$$r_t^* + \sigma_1 s_{1t}^* = \Phi'(A_t - \Gamma^{EM}), \qquad s_{1t}^* = \bar{\sigma} \left(\frac{A_t - \Gamma^{EM}}{A_t - \underline{a}} \right)$$

It follows from these conditions that EM has a lower expected return on the risky asset, $r_t^* + \bar{\sigma} s_{1t}^*$ and a higher domestic Sharpe ratio, s_{1t}^*, than US. This leads to a lower r_t in EM. V_t^* is higher in EM, which is the only difference from Proposition 3. Intuitively speaking, given that EM and US have the same amount of household savings, a lower Γ^{EM} implies that more investment should be made in the risky sector. Due to the diminishing marginal return, EM ends up having a lower a $r_t^* + \bar{\sigma} s_{1t}^*$. Simultaneously, the share of investment in the domestic risky asset is higher in EM. This is equivalent to having a higher $\bar{\sigma}$ in the baseline model, so at the end the domestic Sharpe ratio should be higher to clear the market. It is also easy to prove that the results of Proposition 4 hold under this new microfoundation.

A.2 HJB Equations

In this section, I elaborate on more details about the Hamiltonian-Jacobian-Bellman equations that are used in various parts of the paper. I begin with a simple case in which financial prices are fixed and given as in Section 2.

A.2.1 Section 2. Exogenous Prices

Closed Economy

Define the value function associated with the maximization problem

$$J(a_{it}) = \max_{\{c_{it},\theta_{1it}\}} \mathbb{E}\left[\int_0^\infty e^{-(\delta+m)t}\log(c_{it}-\kappa)dt\right]$$

subject to the budget constraint (1.2). We can then restate a household's problem as

$$(\delta+m)J(a_{it}) = \max_{\{c_{it},\theta_{1it}\}}\left\{\log(c_{it}-\kappa) + J_a\{[r^* + \sigma_1 s_1^*\theta_{1it}]a_{it} - c_{it}\} + \frac{1}{2}J_{aa}\sigma_1^2\theta_{1it}^2 a_{it}^2\right\}$$

with the transversality condition

$$\lim_{t\to\infty} e^{-(\delta+m)t}J(a_{it}) = 0$$

The first order conditions are

$$c_{it} = (J_a)^{-1} + \kappa \tag{A.36}$$

$$\theta_{1it} = \frac{-s_1 J_a}{\sigma_1 a_{it} J_{aa}} \tag{A.37}$$

Plugging them into the value function, we obtain

$$(\delta+m)J(a_{it}) = -\log(J_a) + J_a(r^* a_{it} - \kappa) - 1 - \frac{1}{2}\frac{J_a^2 s_1^{*2}}{J_{aa}} \tag{A.38}$$

Pick $J = \frac{1}{\delta+m}\log(a_{it} - \frac{\kappa}{r^*}) + const.$ as a solution of (A.38) where $const. \equiv \frac{\log(\delta+m)}{\delta+m} + \frac{r^*+m}{(\delta+m)^2} - \frac{1}{\delta+m} - \frac{s_1^{*2}}{2(\delta+m)^2}$. Then, it is easy to show

$$J_a = \frac{1}{(\delta+m)(a_{it}-\frac{\kappa}{r^*})}, \quad J_{aa} = -\frac{1}{(\delta+m)\left(a_{it}-\frac{\kappa}{r^*}\right)^2}$$

Plugging these expressions into (A.38), we can verify that the right-hand side coincides with the left-hand side. That is,

$$\begin{aligned}
&\log\left(a_{it}-\frac{\kappa}{r^*}\right)+\log(\delta+m)+\frac{r^*+m}{\delta+m}-1+\frac{1}{2}\frac{s_1^{*2}}{(\delta+m)}\\
=&\log(\delta+m)+\log\left(a_{it}-\frac{\kappa}{r^*}\right)+\frac{r^*}{\delta+m}-1+\frac{1}{2}\frac{s_1^{*2}}{(\delta+m)}
\end{aligned}$$

Turning back to the first order conditions, (A.36) and (A.37), we can write the final solutions as

$$\begin{aligned}
c_{it}&=(\delta+m)\left(a_{it}-\frac{\kappa}{r^*}\right)+\kappa=(\delta+m)a_{it}+(r^*-m-\delta)\underline{a}\\
\theta_{1it}&=\frac{s_1^*}{\sigma_1}\left(1-\frac{\underline{a}}{a_{it}}\right)
\end{aligned}$$

where $\underline{a}\equiv\frac{\kappa}{r^*}$.

Open Economy

Now let me turn to open economy. Portfolio frontier is now given by $\{r,(s_1,\sigma_1),(s_2,\sigma_2)\}$ with ρ being the correlation between dz_{1t} and dz_{2t}. The results are summarized as follows

Lemma 2. *In open economy, household i's portfolio choices, $\theta_{it}=[\theta_{1it},\theta_{2it}]'$, are characterized by*

$$\theta_{it}=\Sigma^{-1}\begin{bmatrix}\sigma_1 s_1\\ \sigma_2 s_2\end{bmatrix}\left(1-\frac{\underline{a}}{a_{it}}\right)$$

where $\Sigma\equiv[\sigma_1^2,\rho\sigma_1\sigma_2;\rho\sigma_1\sigma_2,\sigma_2^2]$ is the variance-covariance matrix.

Proof. In open economy, the value function associated with households' problem can be written as

$$\begin{aligned}
(\delta+m)J(a_{it})=\max_{\{c_{it},\theta_{it}\}}\Big\{&\log(c_{it}-\kappa)+J_a\{[\sigma_1 s_1\theta_{1it}+\sigma_2 s_2\theta_{2it}+r]a_{it}-c_{it}\}\\
&+\frac{1}{2}J_{aa}\left((\sigma_1\theta_{1it})^2+(\sigma_2\theta_{2it})^2+2\rho\sigma_1\sigma_2\theta_{1it}\theta_{2it}\right)a_{it}^2\Big\}
\end{aligned}$$

The first order conditions with respect to θ_{1it}, θ_{2it} and c_{it} are

$$c_{it} = (J_a)^{-1} + \kappa$$
$$\theta_{it} = \Sigma^{-1} \begin{bmatrix} \sigma_1 s_1 \\ \sigma_2 s_2 \end{bmatrix} \left(-\frac{J_a}{a_{it} J_{aa}} \right)$$

Plugging them back to the value function, we have

$$(\delta + m) J(a_{it}) = -\log(J_a) + J_a(r a_{it} - \kappa) - 1 - \frac{1}{2} \left(\frac{s_1^2 + s_2^2 - 2\rho s_1 s_2}{1 - \rho^2} \right) \frac{J_a^2}{J_{aa}} \tag{A.39}$$

Pick $J = \frac{1}{\delta+m} \log(a_{it} - \frac{\kappa}{r}) + const.$ with $const. \equiv \frac{\log(\delta+m)}{\delta+m} + \frac{r+m}{(\delta+m)^2} - \frac{1}{\delta+m} - \frac{1}{2(\delta+m)^2} \left(\frac{s_1^2+s_2^2-2\rho s_1 s_2}{1-\rho^2} \right)$. As before, it is easy to verify that this is a solution to equation (A.39). So we have

$$\theta_{it} = \Sigma^{-1} \begin{bmatrix} \sigma_1 s_1 \\ \sigma_2 s_2 \end{bmatrix} \left(1 - \frac{\underline{a}}{a_{it}} \right) \tag{A.40}$$

If Ω is positive definite (i.e. $1 > \rho > 0$), the second order condition holds. □

A.2.2 Section 3: Endogenous Prices

Closed Economy

Let me begin with closed economy. Households maximize $\max_{\theta_{1it}, c_{it}} \mathbb{E}_0 \left[\int_0^\infty e^{-(\delta+m)t} \log(c_{it} - \kappa) dt \right]$ subject to the budget constraint $da_{it} = [(r_t^* + \sigma s_{1t}^* \theta_{1it}) a_{it} - c_{it}]dt + \sigma_1 \theta_{1it} a_{it}\, dz_{1t}$. The problem is equivalent to solving

$$\max_{\theta_{1it}, c_{it}} \mathbb{E}_0 \left[\int_0^\infty e^{-(\delta+m)t} \log c_{it} dt \right]$$

subject to

$$da_{it} = [(r_t^* + \sigma_1 s_{1t}^* \theta_{1it}) a_{it} - c_{it} - \kappa]dt + \sigma_1 \theta_{1it} a_{it}\, dz_{1t}$$

Recall that κ is assumed to be $\kappa = r_t^* \underline{a}$ where $\underline{a}$ is constant.

The key difference from Section 2 is that $r_t^* \equiv r^*(A_t)$ and $s_{1t}^* \equiv s_1^*(A_t)$ are now functions of the aggregate state variables A_t. We solve the household's problem by guessing and verifying an equilibrium. First, assume that households save according to

$$c_{it} = (\delta + m)a_{it} - (\delta + m)\bar{a}, \quad \theta_{1it} = \frac{s^*_{1t}}{\sigma_1}\left(1 - \frac{\underline{a}}{a_{it}}\right)$$

Substituting them into the households' budget constraint and integrating them with i, we can derive the evolution of wealth stock A_t as follows

$$\begin{aligned} dA_t &= \left[(r^*(A_t) + s^*_1(A_t)^2 - \delta - m)(A_t - \underline{a}) + m(A_t - A_0)\right] dt + s^*_1(A_t)A_t dz_{1t} \\ &\equiv \mu_A dt + \sigma_A dz_{1t} \end{aligned} \tag{A.41}$$

where $r^*(A_t)$ and $s^*_1(A_t)$ are given by (1.14) and (1.15) in the main text. Thus, the state variables for households' decision makings are A_t and a_{it}. Let $J(a_{it}, A_t)$ be the value function associated with the household's problem. We can then state the HJB equation as

$$\begin{aligned} (\delta + m)Jdt = \max_{c_{it},\theta_{1,it}} \Big\{ \log c_{it} + J_a\{(r^*_t + \sigma_1 s^*_{1t}\theta_{1,it})a_{it} - c_{it} - r^*_t\underline{a}\} + \frac{1}{2}J_{aa}\sigma_1^2\theta_{1it}^2 a_{it}^2 \\ + J_A\mu_A + \frac{1}{2}J_{AA}\sigma_A^2 + J_{Aa}\sigma_1\theta_{1it}a_{it}\sigma_A \Big\} dt \end{aligned} \tag{A.42}$$

where μ_A and σ_A come from (A.41). The transversality condition is given by $\lim_{t\to\infty} e^{-\delta t} J(a_{it}, A_t) \xrightarrow{p} 0$. Notice that the last three terms are added to the standard Merton's model. The first order conditions yield

$$\begin{aligned} c_{it} &= (J_a)^{-1} \\ \theta_{1it} &= \frac{-s^*_{1t}J_a}{\sigma_1 a_{it}J_{aa} + J_{Aa}\sigma_1 a_{it}\sigma_A} \end{aligned}$$

Let $J(a_{it}, A_t) \equiv \frac{1}{\delta+m}\log(a_{it} - \underline{a}) + \mathcal{C}(A_t)$ be a candidate value function where $\mathcal{C}(\cdot)$ is implicitly defined by an ordinary differential equation

$$(\delta + m)\mathcal{C}(A_t) = \log(\delta + m) + \frac{r^*(A_t) - \delta - m}{\delta + m} - \frac{1}{2}\frac{(s_1(A_t))^2}{\delta + m} + \mathcal{C}'(A_t)\mu_A + \frac{1}{2}\mathcal{C}''(A_t)\sigma_A^2$$

with suitable boundary conditions. This makes $\theta_{1it} = \frac{-s^*_{1t}J_a}{\sigma_1 a_{it}J_{aa}}$. Substituting c_{it}, θ_{1it} and J into the HJB equation, (A.42), we can verify that the left-hand side equals

$$\log(a_{it} - \underline{a}) + (\delta + m)\mathcal{C}(A_t)$$

and the right-hand side equals

$$\log(a_{it} - \underline{a}) + \log(\delta + m) + \frac{r_t^* - \delta - m}{\delta + m} - \frac{1}{2}\frac{s_{1t}^2}{\delta + m} + \mathcal{C}'(A_t)\mu_A + \frac{1}{2}\mathcal{C}''(A_t)\sigma_A^2$$

Therefore, we can confirm that the following solutions, along with the market clearing conditions, constitute an equilibrium.

$$\begin{aligned}
c_{it} &= (\delta + m)(a_{it} - \underline{a}) \\
\theta_{1it} &= \frac{s_{1t}^*}{\sigma_1}\left(1 - \frac{\underline{a}}{a_{it}}\right) \\
J(a_{it}, A_t) &= \frac{1}{\delta + m}\log(a_{it} - \underline{a}) + \mathcal{C}(A_t)
\end{aligned}$$

The transversality condition holds. The simplification comes from a property of log utility. It allows to decompose $J(a_{it}, A_t)$ into two additively separable terms.

Open Economy

We again solve for an equilibrium by guessing and verifying. First, assume that the solutions of the households problem are given by

$$c_{it} = (\delta + m)(a_{it} - \underline{a}) \tag{A.43}$$

$$\theta_{1it} = \frac{(s_{1t} - \rho s_{2t})}{\sigma_1(1 - \rho^2)}\left(1 - \frac{\underline{a}}{a_{it}}\right) \tag{A.44}$$

$$\theta_{2it} = \frac{(s_{2t} - \rho s_{1t})}{\sigma_2(1 - \rho^2)}\left(1 - \frac{\underline{a}}{a_{it}}\right) \tag{A.45}$$

Then we can write the market clearing conditions as

$$\begin{aligned}
\Phi'(K_{1t}) - \bar{\sigma}^{US} s_{1t} - \tau(1 - \lambda^{US}) &= r_t \\
\Phi'(K_{2t}) - \bar{\sigma}^{EM} s_{2t} - \tau(1 - \lambda^{EM}) &= r_t \\
\frac{s_{1t} - \rho s_{2t}}{\sigma_1(1 - \rho^2)}(A_{1t} + A_{2t} - 2\underline{a}) &= K_{1t} \\
\frac{s_{2t} - \rho s_{1t}}{\sigma_2(1 - \rho^2)}(A_{1t} + A_{2t} - 2\underline{a}) &= K_{2t} \\
K_{1t} + K_{2t} &= A_{1t} + A_{2t}
\end{aligned}$$

It follows from the market clearing conditions that the state variable in the economy is $\bar{A}_{1t} \equiv A_{1t} + A_{2t}$. Let $r(\bar{A}_t)$, $s_1(\bar{A}_t)$ and $s_2(\bar{A}_t)$ denote the market clearing prices pinned down by the above system of equations. Plug them into the budget constraints and integrate across households. We can then see that the aggregate state variable evolves according to

$$d\bar{A}_t = \mu_{\bar{A}} dt + \sigma_{\bar{A},1} dz_{1t} + \sigma_{\bar{A},2} dz_{2t}$$

where

$$\begin{aligned}
\mu_{\bar{A}} =& \left(\frac{\bar{A}_t - 2\underline{a}}{1-\rho^2}\right) \left[s_1(\bar{A}_t)^2 + s_2(\bar{A}_t)^2 - 2\rho s_1(\bar{A}_t) s_2(\bar{A}_t)) + r(\bar{A}_t) - \delta - m\right] \\
&+ m(A_t - A_0) \\
\sigma_{\bar{A},1} =& \frac{s_1(\bar{A}_t) - \rho s_2(\bar{A}_t)}{1-\rho^2} \bar{A}_t \\
\sigma_{\bar{A},2} =& \frac{s_2(\bar{A}_t) - \rho s_1(\bar{A}_t)}{1-\rho^2} \bar{A}_t
\end{aligned}$$

Let $J(a_{it}, \bar{A}_t)$ denote the value function of households' problem. In open economy, the HJB equation is given by

$$\begin{aligned}
(\delta + m) J dt = \max_{c_{it}, \theta_{it}} \Big\{ & \log c_{it} + J_a \{[\sigma_1 s_{1t} \theta_{1,it} + \sigma_2 s_{2t} \theta_{2it} + r_t] a_{it} - c_{it} - r_t \underline{a}\} \\
& + \frac{1}{2} J_{aa} ((\sigma_1 \theta_{1,it})^2 + (\sigma_2 \theta_{2it})^2 + 2\rho \sigma_1 \sigma_2 \theta_{1it} \theta_{2it}) a_{it}^2 \\
& + J_{\bar{A}} \mu_{\bar{A}} + \frac{1}{2} J_{\bar{A}\bar{A}} (\sigma_{\bar{A},1}^2 + \sigma_{\bar{A},2}^2 + 2\rho \sigma_{\bar{A},1} \sigma_{\bar{A},2}) \\
& + J_{\bar{A}a} (\sigma_1 \theta_{1it} a_{it} (\sigma_{\bar{A},1} + \rho \sigma_{\bar{A},2}) + \sigma_2 \theta_{2it} a_{it} (\sigma_{\bar{A},2} + \rho \sigma_{\bar{A},1})) \Big\} dt
\end{aligned}$$

Pick

$$J(a_{it}, \bar{A}_t) \equiv \frac{1}{\delta + m} \log(a_{it} - \underline{a}) + \mathcal{C}(\bar{A}_t)$$

where $\mathcal{C}(\bar{A}_t)$ is a solution of the following differential equation

$$(\delta + m)\mathcal{C}(A_t) = \log(\delta + m) + \frac{r_t - \delta - m}{\delta + m} - \frac{1}{2} \frac{s_1^2 + s_2^2 - 2\rho s_1 s_2}{(\delta + m)(1-\rho^2)} + \mathcal{C}'(\bar{A}_t) \mu_{\bar{A}} + \frac{1}{2} \mathcal{C}''(\bar{A}_t) \sigma_{\bar{A}}^2$$

with suitable boundary conditions. Note that the value function is consistent with consumption and portfolio choices given by (A.43), (A.44) and (A.45). Substituting them into the HJB equation, the left-hand side equals

$$\log(a_{it} - \underline{a}) + (\delta + m)\mathcal{C}(\bar{A}_t)$$

while the right-hand side equals

$$\log(a_{it} - \underline{a}) + \log(\delta + m) + \frac{r_t - \delta - m}{\delta + m} - \frac{1}{2}\frac{s_1^2 + s_2^2 - 2\rho s_1 s_2}{(\delta + m)(1 - \rho^2)} + \mathcal{C}'(\bar{A}_t)\mu_{\bar{A}} + \frac{1}{2}\mathcal{C}''(\bar{A}_t)\sigma_{\bar{A}}^2$$

The transverality condition also holds. Therefore, the proposed solutions and market clearing conditions constitute an equilibrium.

A.3 Additional Details

A.3.1 Kolmogorov Forward Equation (Section 2)

We have seen that the wealth of individual i, conditional on being alive, evolves according to an Itô diffusion process

$$da_t = [(r^* + s_1^{*2} - \delta - m)(a_{it} - \underline{a})]dt + s_1^* a_t dz_{1t} \tag{A.46}$$

in the case of autarky. Subscript i is repressed. Let $(\Omega, \mathcal{F}, \mathbb{P})$ represent a probability space on which the above diffusion process is defined. Note that (A.46) can be expressed as an integral form

$$a_t(\omega) = a_0 + \int_0^t [(r^* + s_1^{*2} - \delta - m)(a_\tau(\omega) - \underline{a})]d\tau + \int_0^t s a_\tau(\omega) dz_{1\tau}(\omega) \tag{A.47}$$

for $\omega \in \mathcal{F}$. The last term on the right-hand side expression is the Ito integral defined by

$$\int_0^t s_1^* a_\tau(\omega) dz_{1\tau}(\omega) = \lim_{n\to\infty} \int \zeta_n(t, w) dz_{1t}(\omega)$$

where $\{\zeta_n\}$ is a sequence of elementary functions

$$\zeta_n(t,\omega) = \sum_{j=0}^{n-1} s_1^* a_{t_j}(\omega)\mathbb{I}_{[t_j,t_{j+1})}(t)$$

$$\mathbb{I}_{[t_j,t_{j+1})}(t) = \begin{cases} 1 & \text{if } t \in [t_j, t_{j+1}) \\ 0 & \text{otherwise} \end{cases}$$

over evenly spaced intervals i.e. $t_j = tj/n$. Pick any $\bar{\omega} \in \mathcal{F}$ such that $z_{1t_1}(\bar{\omega}) = z_{1t_2}(\bar{\omega}) = ... = z_{1t_k}(\bar{\omega}) = z_0$ for all $t_1, ..., t_k$ and any k. Such a trajectory always exists because a random vector $(z_{1t_1}, ..., z_{1t_k})$ is Gaussian. In this case, we can see that

$$\zeta_n(t,\bar{w}) = \sum_{j=0}^{n-1} s_1^* a_{t_j}(\bar{\omega})(z_{1t_j} - z_{1t_{j+1}}) = 0$$

for all n. Thus, along this trajectory, we have

$$a_t(\bar{\omega}) = a_0 + \int_0^t [(r^* + s_1^{*2} - \delta - m)(a_\tau(\bar{\omega}) - \underline{a})]d\tau \tag{A.48}$$

From this point on, we denote by $a_t \equiv a_t(\bar{\omega})$ the solution of the differential equation (A.48). Reviving the subscript i, we can write its dynamics as $da_{it} = [(r^* + s_1^{*2} - \delta - m)(a_{it} - \underline{a})]dt$

Along this trajectory, we can define a cross-sectional wealth distribution $G_t(a)$ in period t as $G_t(a) = \int_{[0,1]} \mathbb{I}_{\{i\in[0,1]:a_{it}\le a\}}(i)di$. I denote by $g_t(a) = \frac{\partial G_t(a)}{\partial a}$ its density distribution. Take a small interval $[t, t+dt)$. A mdt measure of households drop out and replaced with the newborn households. Those who remain accumulate (or deccumulate) their wealth from a_{it} to $a_{i,t+dt} = a_{it} + [(r^* + s_1^{*2} - \delta - m)(a_{it} - \underline{a})]dt$. In view of this dynamics, the period $t + dt$ wealth distribution can be written as

$$G_{t+dt}(a) = (1 - mdt)G_t\left(a - [(r + s^2 - \delta - m)(a - \underline{a})]dt\right) + mdtG_0(a)$$

which leads to

$$\frac{G_{t+dt}(a) - G_t(a)}{dt} = -mG_t(a) + mG_0(a) + (1 - mdt)\frac{G_t\left(a - [(r + s^2 - \delta)(a - \kappa)]dt\right) - G_t(a)}{dt}$$

Taking $dt \to 0$, we have

$$\frac{dG_t(a)}{dt} = -mG_t(a) + mG_0(a) - [(r^* + s^{*2} - \delta - m)(a - \underline{a})]g_t(a)$$

Differentiating the both sides with respect to a, we can derive

$$\frac{d}{dt}g_t(a) = -mg_t(a) + mg_0(a) - \frac{d}{da}[(r^* + \sigma_1 s_1^* \theta_1(a))a - c(a)]g_t(a)$$

where $\theta_1(a) = \frac{s_1^*}{\sigma_1}(1 - \frac{\underline{a}}{a_{it}})$ and $c(a) = (\delta + m)a + (r^* - \delta - m)\underline{a}$. We can analogously derive the case for the open economy.

A.3.2 Convergence of the Wealth Distribution

In autarky, the stationary wealth distribution is the unique solution of the following ordinary differential equation

$$0 = -mg(a) + mg_0(a) - \frac{\partial[(r^* + s_1^{*2} - \delta - m)(a - \underline{a})]g(a)}{\partial a}$$

subject to the condition $\int_{\underline{a}}^{\infty} g(a) = 1$. The aim of this subsection is to show that

$$\int_{\kappa}^{\infty} |g_t(a) - g(a)| da \leq e^{-mt}$$

I follow a similar strategy to Gabaix *et al.* (2016) to prove this inequality.

Lemma 3. *For any twice continuously differentiable function $q(a,t)$, the following inequality holds*

$$\frac{\partial |q(a,t)|}{\partial t} \leq -m|q(a,t)| + m|g_0(a)| - (r^* + s_1^{*2} - \delta - m)\frac{\partial |(a - \underline{a})q(a,t)|}{\partial a}$$

Proof. Let $z(q(a,t)) = \sqrt{\epsilon^2 + q(a,t)^2}$. This mapping has property: $\lim_{\epsilon \to 0} z(q(a,t)) = |q(a,t)|$.

Next, one can show that

$$
\begin{aligned}
&\frac{\partial z(q(a,t))}{\partial t} - \left(-mz(q(a,t)) + mg_0(a) - \frac{\partial[(r^* + s_1^{*2} - \delta - m)(a - \underline{a})]z(q(a,t))}{\partial a} \right) \\
=&z'(q) \left(\frac{\partial q(a,t)}{\partial t} + \frac{\partial q(a,t)}{\partial a}(r^* + s_1^{*2} - \delta - m)(a - \underline{a}) \right) + mz(q(a,t)) - mg_0(a) \\
=&z'(q)(-mq(a,t) + mg_0(a)) + mz(q(a,t)) - mg_0(a) \\
=&\underbrace{m(z'(q)q(a,t) + z(q(a,t)))}_{\text{Term (i)}} + \underbrace{mg_0(a)(z'(q) - 1)}_{\text{Term (ii)}}
\end{aligned}
$$

Term (i) and Term (ii) vanish to zero as $\epsilon \to 0$, because

$$
\begin{aligned}
\text{Term (i)} &= m \left(-\frac{q^2}{\sqrt{\epsilon^2 + q^2}} + \sqrt{\epsilon^2 + q^2} \right) = m \left(\frac{\epsilon^2}{\sqrt{\epsilon^2 + q^2}} \right) \\
\text{Term (ii)} &= mg_0(a) \left(\frac{q}{\sqrt{\epsilon^2 + q^2}} - 1 \right)
\end{aligned}
$$

On the other hand, the left-hand side becomes

$$
\frac{\partial |q(a,t)|}{\partial t} - \left(-m|q(a,t)| + m|g_0(a)| - (r^* + s^{*2} - \delta - m) \frac{\partial |(a - \underline{a})q(a,t)|}{\partial a} \right)
$$

since $\lim_{\epsilon \to 0} z(q(a,t)) = |q(a,t)|$. This completes the proof. $\square$

Using this lemma and substituting $q(a,t) = g_t(a) - g(a)$, it is straightforward to show that

$$
\begin{aligned}
\int_{\kappa}^{\infty} \frac{\partial |g_t(a) - g(a)|}{\partial t} da &\leq \int_{\kappa}^{\infty} \left\{ -m|g_t(a) - g(a)| - (r^* + s_1^{*2} - \delta - m) \frac{\partial |(a - \underline{a})q(a,t)|}{\partial a} \right\} da \\
&= \int_{\kappa}^{\infty} \{ -m|g_t(a) - g(a)| \} \, da
\end{aligned}
$$

Then, Gronwell's lemma leads to

$$
\int_{\kappa}^{\infty} |g_t(a) - g(a)| da \leq e^{-mt}
$$

So the wealth distribution converges to $g(a)$ as $t \to \infty$

A.3.3 Discrete Time Model

In this subsection, I follow Merton (1992) to construct a diffusion process in the households' problem. I use the standard O notations to describe asymptotic properties. That is, $f_1(h) =$

$O[f_2(h)]$ if $\lim_{h\to 0} f_1(h)/f_2(h)$ is bounded and$f_1(h) = o[f_2(h)]$ if $\lim_{h\to 0} f_1(h)/f_2(h) = 0$. Also, $f_1(h) \sim f_2(h)$ if $f_1(h) = O[f_2(h)]$ but $f_1(h) \neq o[f_2(h)]$.

Consider a finite time interval $[0, T)$ prior to a structural change in period $T \equiv nh$. Financial markets are cleared at time $0, h, 2h, ...$ and nh respectively. Here, h denotes the minimum length of time between the successive clearings of markets. By investing the K_t units of goods in period $t \equiv kh$, the representative firm generates new goods

$$\Phi(K_t)h + \bar{\sigma} K_t \epsilon_{t+h}$$

in period $t + h$ where ϵ_{t+h} is the unanticipated productivity change between period t and period $t + h$. The following assumptions are made on ϵ_{t+h}.

(A1) ϵ_{t+h} can take on any one of n_ϵ distinct values. For $k = 1, ..., n_\epsilon$, let $\epsilon(k)$ denote one of its values and $p(k)$ represent probability that $\epsilon_{t+h} = \epsilon(k)$ occurs conditional on all information in period t. Assume that $\epsilon(k)$ is a sufficiently well behaved function of h such that $\epsilon(k) \sim h^{1/2}$ and $p(k) = O(1)$

(A2) $\mathbb{E}_t[\epsilon_{t+h}] = 0$ and $\lim_{h\to 0} \sum_{k=1}^{n_\epsilon} p(k)\epsilon(k)^2/h = 1$

(A3) $\{\epsilon_{kh}\}_{k=1}^{n}$ are independent and identically distributed across times $k = 1, 2, ..., n$

A market equilibrium price is given by $\{r(\mathcal{S}_t), (s(\mathcal{S}_t), \sigma)\}$, which is a function of the aggregate state variables $\mathcal{S}_t$. The firm's problem can be stated as maximizing

$$V_t h \equiv \max_{K_t, D_t, E_t} \{\Phi(K_t) - r^*(\mathcal{S}_t)D_t - (r^*(\mathcal{S}_t) + \sigma_1 s_1^*(\mathcal{S}_t) + \tau)E_t\}h$$

subject to the following constraints:

$$K_t = D_t + E_t, \quad \sigma_1 = \frac{\bar{\sigma} K_t}{E_t}, \quad D_t \leq \lambda K_t$$

The equilibrium value of $r^*(\mathcal{S}_t)$ and $s_1^*(\mathcal{S}_t)$ are pinned down by the market clearing conditions that I will specify later.

Turning back to the budget constraint, household i comes into period t with wealth a_{it}. We can write a_{it} as

$$a_{it} = n_{it} p_t + n_{it}^D \tag{A.49}$$

where n_{it} is the number of stock shares, p_t is the stock price and n_{it}^D is the number of deposits owned by individual i in period t. The portfolio weight on risky assets is $\theta_{1it} = n_{it} p_t / a_{it}$. Each deposit pays $r_t h$ in period $t+h$ with no uncertainty. Each equity share pays $x(\mathcal{S}_t)h + \sigma_1 p_t \epsilon_{t+h}$ in period $t+h$ and it also pledges future payoffs $x(\mathcal{S}_{t+kh})h + \sigma_1 p_{t+kh} \epsilon_{t+(k+1)h}$ for all future periods $k = 1, 2, ...$ Ex post return of equity shares, $\frac{x(\mathcal{S}_t)h}{p_t} + \sigma_1 \epsilon_{t+h}$, depends on the realization of ϵ_{t+h}. After the dice are rolled up, ϵ_{t+h} is determined, interest rates are paid and households rebalance their portfolio. All trades are made at known current prices. Households also receive annuity mh for their wealth holdings. We can then obtain

$$\begin{aligned}
&(n_{it}\pi(\mathcal{S}_t) + n_{it}^D r(\mathcal{S}_t) - c_{i,t+h})h + n_{it} p_t \sigma \epsilon_{t+h} \\
=&(n_{i,t+h} - n_{it}) p_{t+h} + (n_{i,t+h}^D - n_{it}^D)
\end{aligned} \tag{A.50}$$

Merging (A.49) and (A.50), we have

$$\begin{aligned}
a_{i,t+h} - a_{it} &= n_{i,t+h} p_{t+h} - n_{it} p_t + n_{i,t+h}^D - n_{it}^D \\
&= (n_{i,t+h} - n_{it}) p_{t+h} + n_{it}(p_{t+h} - p_t) + n_{i,t+h}^D - n_{it}^D \\
&= \left(\left(r^*(\mathcal{S}_t) + \theta_{1it} \left(\frac{x(\mathcal{S}_t)}{p_t} - r^*(\mathcal{S}_t) \right) \right) a_t - c_{t+h} \right) h + \frac{p_{t+h} - p_t}{p_t} a_{it} + \sigma_1 \theta_{1it} a_{it} \epsilon_{t+h}
\end{aligned}$$

Finally, the equilibrium value of $r^*(\mathcal{S}_t)$ and $s^*(\mathcal{S}_t)$ are pinned down by the market clearing conditions.

$$E_t = \int_{i \in [0,1]} p_t n_{it} di \quad \text{and} \quad D_t = \int_{i \in [0,1]} n_{it}^D di$$

In equilibrium, the financial intermediary equalizes

$$\frac{p_{t+h} - p_t}{p_t} + \frac{x(\mathcal{S}_t)h}{p_t} + \sigma \epsilon_{t+h} = \left(r(\mathcal{S}_t) + \sigma s(\mathcal{S}_t) \right) h + \sigma \epsilon_{t+h}$$

since, otherwise, they can make profits by taking long-short strategies. I assume that $x(\mathcal{S}_t) = r^*(\mathcal{S}_t) + \sigma_1 s_1^*(\mathcal{S}_t)$, so have $p_t = 1$ for all t as long as there is no unanticipated change. Let

$r_t^* \equiv r^*(\mathcal{S}_t)$ and $s_t \equiv s_1(\mathcal{S}_t)$. Plugging this into the household's budget constraint and taking a limit $h \to 0$, we can see that $a_{i,t}$ follows a diffusion process

$$da_{it} = ((r_t^* + \sigma s_{1t}^* \theta_{it})a_{it} - c_{it})\, dt + \sigma_1 \theta_{1it} a_{it} dz_{1t}$$

in view of (A1), (A2), and (A3).

Appendix B

Appendix to Chapter 2

B.0.1 Proof of Proposition 1

The objective function of entrepreneurs can be simply transformed to $\mathbb{E}[C_{m1}] - \frac{\gamma}{2}\mathbb{V}[C_{m1}]$. We can state an entrepreneur's problem as

$$\max_{\{\alpha_i,\beta_i\}} \left\{ \left[\sum_{n\in X_i} \alpha_{i,n} \left(\pi_i + \pi_n\right) - \sum_{n\in X_i} \beta_{i,n}(1+r_n) \right] - \frac{\gamma}{2}[\alpha_i', \beta_i']\Omega_0 \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \right\} - \sum_{n\in X_i\setminus\{1\}} \tau_i f_n$$

where $\alpha_i = [\alpha_{i,1}, ...\alpha_{i,N}]$ and $\beta_i = [\beta_{i,2}, ...\beta_{i,n}]$. The budget constraints are

$$\sum_{i\in X_k} q_i\alpha_{k,i} = e_m + \sum_{i\in X_k} \beta_{k,i} \tag{B.1}$$

$$\alpha_{k,i} \geq 0, \beta_{k,i} \geq 0 \text{ for all } i \tag{B.2}$$

so we can write $\beta_{k,1} = \sum_{i\in X_m} q_i\alpha_{k,i} - e_m - \sum_{i\in X_m\setminus\{1\}} \beta_{k,i}$. The objective function can then be written as

$$\sum_{i\in X_k} \alpha_{k,i}q_i \left(\frac{\pi_i + \pi_k}{q_i} - r_1\right) - \sum_{i\in X_k\setminus\{1\}} \beta_{k,i}(r_i - r_1) + r_1 e_m \\ - \frac{\gamma}{2}[\alpha_k', \beta_k']\Omega \begin{bmatrix} \alpha_k \\ \beta_k \end{bmatrix} - \sum_{i\in X_m\setminus\{1\}} \tau_i f_i$$

From the first-order conditions, we can write the solutions as

$$\begin{bmatrix} \alpha_{k,1} q_1 \\ \ldots \\ \beta_k \end{bmatrix} = \frac{\Omega^{-1}}{\gamma} \begin{bmatrix} s_{k,1}\sigma_1 - r_1 \\ \ldots \\ r_1 - r_I \end{bmatrix}$$

$$\beta_{k,1} = \sum_{i \in X_k} q_i \alpha_{k,i} - e_m - \sum_{i \in X_k \setminus \{1\}} \beta_{k,i}$$

where $s_{i,n} = \frac{\pi_i + \pi_n}{\sigma_n q_n}$. X_k is chosen such that the objective function is maximized. Next, turning to the spread, notice that for firm i we have

$$Roa_i = \frac{\sum_{n \in X_i} \pi_n \alpha_{i,n}}{\sum_{n \in X_i} q_n \alpha_{i,n}}, \quad \text{and} \quad Int_i = \frac{\sum_{n \in X_i} r_n \beta_{i,n}}{\sum_{n \in X_i} \beta_{i,n}}$$

We can then write

$$\begin{aligned} \mathbb{E}[\mathcal{S}_i] &= \frac{\sum_{n \in X_i} \pi_n \alpha_{i,n}}{\sum_{n \in X_i} q_n \alpha_{i,n}} - \frac{\sum_{n \in X_i} r_n \beta_{i,n}}{\sum_{n \in X_i} \beta_{i,n}} \\ &= \sum_{n \in X_i} \frac{\pi_n \omega_{\alpha,n}}{q_n} - \sum_{n \in X_i} r_n \omega_{\beta,n} \\ &= \sum_{n \in X_i} s_{i,n} \sigma_n \omega_{\alpha,n} - \sum_{n \in X_i} r_n \omega_{\beta,n} \end{aligned}$$

where $s_n = \frac{\pi_n}{q_n \sigma_n}$. Turning to the gap between $\mathbb{E}[\mathcal{S}_m]$ and $\mathbb{E}[\mathcal{S}_d]$, note that $\omega_{\alpha,1} = 1$ and $\omega_{\beta,1} = 1$ for domestic firms. Thus, we can write

$$\begin{aligned} \mathbb{E}[\mathcal{S}_m] - \mathbb{E}[\mathcal{S}_d] &= \sum_{n \in X_m} s_n \sigma_n \omega_{\alpha,n} - \sum_{n \in X_m} r_n \omega_{\beta,n} - s_1 \sigma_1 + r_1 \\ &= \Big\{ \sum_{n \in X_m} s_n \sigma_n \omega_{\alpha,n} - s_1 \bar{\sigma}_m \Big\} - \Big\{ \sum_{n \in X_m} r_n \omega_{\beta,i} - r_1 \Big\} + \{ s_1 \bar{\sigma}_m - s_1 \sigma_1 \} \end{aligned}$$

where $\bar{\sigma}_m \equiv \sqrt{[\alpha'_m, \beta'_m]\Omega[\alpha'_m, \beta'_m]'}$ is the average volatility faced by firm m.

B.0.2 Proof of Proposition 2

The only difference from the baseline case is the followings:

$$\begin{aligned}\mathbb{E}[\mathcal{S}_i] &= \frac{\sum_{n\in X_i}(\pi_i+\pi_n)\alpha_{i,n}}{\sum_{n\in X_i} q_n\alpha_{i,n}} - \frac{\sum_{n\in X_i} r_n\beta_{i,n}}{\sum_{n\in X_i}\beta_{i,n}} \\ &= \sum_{n\in X_i}\frac{(\pi_n+\pi_i)\omega_{\alpha,n}}{q_n} - \sum_{n\in X_i} r_n\omega_{\beta,n}\end{aligned}$$

which leads to

$$\begin{aligned}\mathbb{E}_0[\mathcal{S}_m]-\mathbb{E}_0[\mathcal{S}_d] = &\sum_{n\in X_m}\pi_m\omega_{\alpha,n}/q_m + \Big\{\sum_{n\in X_m} s_n\sigma_n\omega_{\alpha,n} - s_1\bar{\sigma}_m\Big\} \\ &-\Big\{\sum_{n\in X_m} r_i\omega_{\beta,n} - r_1\Big\} + \{s_1\bar{\sigma}_m - s_1\sigma_1\}\end{aligned}$$

The first term is added, which we interpret as excess profits from intangible assets.

Appendix C

Appendix to Chapter 3

C.1 Details on Algorithms

In this section, I elaborate on more details about numerical simulations presented in Section 3.4.

C.1.1 Closed Economy

(i) Households' Problem The utility function is given by $u(c_{it}) = \frac{c_{it}^{1-\gamma}}{1-\gamma}$. Households' problem is now replaced by

$$\max_{c_{it}} \ \mathbb{E}_0 \left[\int_0^\infty e^{-(\delta+m)t} u(c_{it}) dt \right]$$

subject to

$$\begin{aligned} da_{it} &= [(r_t^* + \sigma_1 s_{1t}^* \theta_{1it}) a_{it} + w_{1t}^* l_{it} + r^h h_{it} - c_{it} - \kappa] dt + \sigma_1 \theta_{1it} a_{it} \, dz_{1t} \\ \theta_{1it} &= \max \left\{ \frac{s_{1t}^*}{\chi_1 \sigma_1} \left(1 - \frac{\chi_2}{a_{it}} - \frac{\chi_3}{a_{it} l_{it}} \right), 0 \right\} \end{aligned} \tag{C.1}$$

Unlike the baseline model, the portfolio choice function is taken as given. The functional form in (C.1) is an approximation to the actual solution in a sense that $\chi_1 = \gamma$, $\chi_2 = \kappa$, and $\chi_3 = 0$ would arise if the model dispensed with labor income and housing assets.[1]

(ii) Market Clearing Conditions One key advantage of Assumption 1 is that the market clearing conditions are now characterized by a finite number of aggregate variables. Let me first

[1]One can confirm this by applying the method in Appendix A.2.2 with $u(c_{it}) = \frac{c_{it}^{1-\gamma}}{1-\gamma}$.

turn to the market clearing conditions in a closed economy.

$$\begin{aligned} r_t^* &= \alpha Z A_{1t}^{\alpha-1} L^{1-\alpha} - \bar{\sigma}^{US} s_{1t}^* - \tau(1-\lambda^{US}) \\ (1-\lambda)A_{1t} &= \int_i a_{it}\theta_{1it} di \\ w_{1t}^* &= (1-\alpha) Z A_{1t}^{\alpha} L^{-\alpha} \end{aligned}$$

The first and second lines imply that financial markets for the domestic safe and risky assets are cleared. The last line is associated with the labor market clearing condition. Note that the second line can be written as

$$\begin{aligned} (1-\lambda)A_{1t} &= \int_{a_{it} \geq \chi_2 + \chi_3/l_i} a_{it} \frac{s_{1t}}{\sigma_1 \chi_1} \left(1 - \frac{\chi_2}{a_{it}} - \frac{\chi_3}{a_{it}\ell_i}\right) di \\ &= \frac{s_{1t}}{\bar{\sigma}\chi_1}(A_{1t} - F_{1t}^a - F_i) \end{aligned}$$

where $F_{1t}^a = \int_{a_{it} < \chi_2 + \chi_3/l_i} a_{it} di$ and $F_i = \int_{a_{it} \geq \chi_2 + \chi_3/\ell_i} (\chi_2 + \chi_3/\ell_i) di$. Here, F_i is a time-invariant variable as, due to the setting of this model, the measure of households whose wealth levels are below the threshold $\chi_2 + \chi_3/l_i$ is constant. This is because when their wealth levels are close to this threshold they only invest in safe assets to retain their wealth. In actual simulations, it is convenient to express these market clearing conditions as

$$\begin{aligned} r_t^* \equiv r^*(A_{1t}, F_{1t}^a) &= \alpha Z A_{1t}^{\alpha-1} L^{1-\alpha} - \bar{\sigma}^{US} s_1^*(A_{1t}) - \tau(1-\lambda^{US}) \\ s_{1t}^* \equiv s_1^*(A_{1t}, F_{1t}^a) &= \frac{\bar{\sigma}^{US}\chi_1 A_{1t}}{A_{1t} - F_{1t}^a - F_i} \\ w_{1t}^* \equiv w_1^*(A_{1t}, F_{1t}^a) &= (1-\alpha) Z A_{1t}^{\alpha} L^{-\alpha} \end{aligned}$$

(iii) Simulation Algorithm Numerical simulations for closed economy proceed in four steps. First, I begin with a guess for the law of motion for the state variables. In the model where all households retain wealth $\chi_2 + \chi_3/\ell_i$, A_{1t} acts as a sufficient statistics for current prices. The following law of motion fits the model well:

$$\frac{dA_{1t}}{A_{1t}} = ((\psi_2 - 1)\log A_{1t} + \psi_1)\, dt + \bar{\sigma}^{US} dz_{1t} \tag{C.2}$$

where ψ_1 and ψ_2 are constants. It is worth noting that (C.2) corresponds to $\log A_{1,t+1} = \psi_2 \log A_{1,t} + \psi_1 + \bar{\sigma}^{US}\epsilon_t$ in the discrete-time setting. This functional form is identical to the one used by Krusell and Smith (1998). On the other hand, in the model where some households are indebted, one may also consider the law of motion for F^a_{1t}. In practice, $dF^a_{1t} = 0$ worked well around the stationary state.

Second, given an initial guess for ψ_1 and ψ_2, I solve differential equations that characterize saving decisions of households and evolution of the wealth distribution. Let $J_t \equiv J(a, \ell, \varepsilon, A_t)$ denote the value function associated with households' optimization problem. $g_t(a, \ell, \epsilon)$ represents probability density distribution across households at time t. They are pinned down by

$$(\delta + m)J_t dt = \max_{c,\theta_1} \Big\{ u(c) + \frac{\partial J_t}{\partial a} v_t(a, \ell, \epsilon) + \frac{1}{2}\frac{\partial^2 J_t}{\partial a^2}(\sigma_1\theta_1 a)^2 + \frac{\partial J_t}{\partial \epsilon}(-\beta\epsilon) + \zeta \int_{-\infty}^{\infty} (J_t(a, \ell, x) - J_t(a, \ell, \epsilon))\phi(x)dx + \frac{1}{dt}\mathbb{E}_t[dJ_t] \Big\} dt \quad \text{(C.3)}$$

$$\frac{d}{dt} g_t(a, \ell, \epsilon) = -mg_t(a, \ell,\epsilon) + mg_0(a, \ell, \varepsilon) - \frac{d}{da}[v_t(a, \ell, \epsilon)g_t(a, \ell, \epsilon)] - \zeta g_t(a, \ell, \epsilon) + \zeta\phi(\epsilon) \int_{-\infty}^{\infty} g_t(a, \ell, x)dxd\ell \quad \text{(C.4)}$$

where $v_t(a, l, \epsilon) \equiv [(r^*_t + \sigma_1 s^*_{1t}\theta_1)a + w^*_{1t} l + r^h h - c - \kappa]$ indicate individual savings. To find a numerical approximation to the solution, I turn to a finite difference method called "Upwind Scheme". More details are referred to the next subsection.

The third step is to check if the initial guess for ψ_1 and ψ_2 is consistent with the ones derived from the Kolmogorov Forward Equation. Set the simulation period, say $t = 0$ to $t = T$. Starting from g_0, I compute $g_1, ..., g_T$ sequentially by applying the Kolmogorov Forward Equation. It is then easy to compute the mean of each distribution, $A_{1,0}, ..., A_{1,T}$. The model-implied estimates, $\hat{\psi}_1$ and $\hat{\psi}_2$, are obtained by running ordinary least squares over the series. If the distance between (ψ_1, ψ_2) and $(\hat{\psi}_1, \hat{\psi}_2)$ is sufficiently small, terminate the process. Otherwise, start with another guess for ψ_1 and ψ_2 and repeat the above steps.[2]

Finally, once the model converges, I compute the fit of the model to the observed data. I

[2] In actual practice, one can recursively update ψ^k_1 and ψ^k_2 to find a fixed point faster. The initial guess is denoted by ψ^0_1 and ψ^0_1. I can then derive $\hat{\psi}_1$ and $\hat{\psi}_2$ from the Kolmogorov Forward Equation. Update $\psi^1_1 = \hat{\psi}_1$ and $\psi^1_2 = \hat{\psi}_2$ and solve the model again. Repeat this process until the distance between (ψ^k_1, ψ^k_2) and $(\psi^{k+1}_1, \psi^{k+1}_2)$ becomes sufficiently small. This method appears to work well in simulation although there is no established result for this.

use 1989 as the benchmark year due to data availability of the Survey of Consumer Finances. I calibrate parameter values such that the stationary wealth distribution implied by the model fits the actual wealth distribution in the data.

(cf) Unwind Scheme Here, I briefly summarize the core idea to approximately solve the HJB equation and Kolmogorov Forward Equation. Achdou *et al.* (2017) provides a nice introduction to this method and applications. I follow their notation and exposition throughout this chapter. Let a_i, ℓ_j and A_k denote i'th, j'th and k'th coordinates of each variable; there are I, J and K discrete points along each dimension of the space. Let's first consider the case $\epsilon = 0$ for all t. With this apparatus in place, it is natural to express other variables as

$$\theta_{1,i,j,k} = \max\left\{ \frac{s_1^*(A_k)}{\chi_1\sigma_1}\left(1 - \frac{\chi_2}{a_i} - \frac{\chi_3}{a_i\ell_j}\right), 0 \right\}$$

$$I_{i,j,k} = (r^*(A_k) + \sigma_1 s_1^*(A_k)\theta_{1,i,j,k})a_i + \ell_j w_1^*(A_k) + r^h h_{i,j,k} - \kappa$$

where $I_{i,j,k}$ is gross income of individual households and $h_{i,j,k}$ is the holding of housing assets corresponding to $a_{i,j,k}$

The value function is defined over these points so I shall use the short-hand notation $J_{i,j,k} \equiv J(a_i, \ell_j, A_k)$. Starting with an initial guess of $J^0_{i,j,k}$, the aim of this exercise is to iteratively update $J^n_{i,j,k}$ until it converges to a certain function. A natural initial guess is $J^0_{i,j,k} \equiv \frac{u(I_{i,j,k})}{\delta+m}$. Let n denote a current iteration. The second step of this exercise is to compute a first difference, $(J^n_{i,j,k})'$, using

$$s^{n,F}_{i,j,k} \equiv I_{i,j,k} - (u')^{-1}\left(\frac{J^n_{i+1,j,k} - J^n_{i,j,k}}{\Delta a}\right) \tag{C.5}$$

$$s^{n,B}_{i,j,k} \equiv I_{i,j,k} - (u')^{-1}\left(\frac{J^n_{i,j,k} - J^n_{i-1,j,k}}{\Delta a}\right) \tag{C.6}$$

$$(J^n_{i,j,k})' \equiv \left(\frac{J^n_{i+1,j,k} - J^n_{i,j,k}}{\Delta a}\right)\mathbb{I}_{s^{n,F}_{i,j,k}>0} + \left(\frac{J^n_{i,j,k} - J^n_{i-1,j,k}}{\Delta a}\right)\mathbb{I}_{s^{n,F}_{i,j,k}<0} + u'(I_{i,j,k})\mathbb{I}_{s^{n,F}_{i,j,k}\leq 0 \leq s^{n,B}_{i,j,k}}$$

The third step is to convert $c^n_{i,j,k} = (u')^{-1}((J^n_{i,j,k})')$. The forth step is to update $J^n_{i,j,k}$, $n = 1, 2, ...$

according to

$$\begin{aligned}
&\frac{J^{n+1}_{i,j,k}-J^{n}_{i,j,k}}{\Delta}+(\delta+m)J^{n+1}_{i,j,k}\\
=&u(c^{n}_{i,j,k})+\frac{J^{n+1}_{i+1,j,k}-J^{n+1}_{i,j,k}}{\Delta a}v^{+}_{i,j,k}+\frac{J^{n+1}_{i,j,k}-J^{n+1}_{i-1,j,k}}{\Delta a}v^{-}_{i,j,k}\\
&+\frac{(\sigma_1\theta_{1,i,j,k}a_i)^2}{2}\frac{J_{i,j,k+1}-2J_{i,j,k}-J_{i,j,k-1}}{\Delta a^2}+\frac{\bar{\sigma}_1^2}{2}\frac{J_{i,j,k+1}-2J_{i,j,k}-J_{i,j,k-1}}{\Delta A^2}\\
&+(\sigma_1\theta_{1,i,j,k}a_i)\bar{\sigma}^{US}\frac{J_{i+1,j,k+1}-J_{i,j,k+1}-J_{i+1,j,k}+J_{i,j,k}}{\Delta a\Delta A}
\end{aligned}$$

where Δ is the iteration step size, Δa is the gap between two points in asset grid and ΔA is the gap in the state variable grid. $v^{+}_{i,jk}$ and $v^{-}_{i,j,k}$ represent saving functions given by

$$v^{+}_{i,j,k}\equiv\max\{0,I_{i,j,k}-c^{n,F}_{i,j,k}\}\quad\text{and}\quad v^{-}_{i,j,k}\equiv\min\{0,I_{i,j,k}-c^{n,B}_{i,j,k}\}$$

where

$$\begin{aligned}
c^{n,F}_{i,j,k}&\equiv(u')^{-1}\left(\frac{J_{i+1,j,k}-J_{i,j,k}}{\Delta a}\right)\\
c^{n,B}_{i,j,k}&\equiv(u')^{-1}\left(\frac{J_{i,j,k}-J_{i-1,j,k}}{\Delta a}\right)
\end{aligned}$$

$J^{n}_{i,j,k}$ is updated until $||J^{n}_{i,j,k}-J^{n-1}_{i,j,k}||$ becomes sufficiently small. See the discussion in Section 5 of Achdou *et al.* (2017) for the convergence property. The method is called 'Upwind Scheme' because it uses a forward difference approximation whenever the drift of the state variable is positive; a backward difference is used whenever the drift is negative.

A byproduct of this exercise is a numerical approximation to the Kolmogorov Forward Equation. Let $t_1,...,t_n$ denote grid points over evenly-spaced time intervals. Let $g^{n}_{i,j}\equiv g_{t_n}(a_i,\ell_j)$ be wealth-labor distribution over grid points. For every n, one has to find the nearest k such that $A_k\approx\sum_{i,j}g^{n}_{i,j}a_i$. Starting from $g^{0}_{i,j}$, one has to update $g^{n}_{i,j}$ iteratively according to a difference equation:

$$\frac{g^{n+1}_{i,j}-g^{n}_{i,j}}{\Delta t}=-\frac{(s^{n,F}_{i,j,k})^{+}g^{n}_{i,j}-(s^{n,F}_{i-1,j,k})^{+}g^{n}_{i-1,j}}{\Delta a}-\frac{(s^{n,B}_{i+1,j,k})^{-}g^{n}_{i+1,j}-(s^{n,B}_{i,j,k})^{-}g^{n}_{i,j}}{\Delta a}$$

where $s^{n,F}_{i,j,k}$ and $s^{n,F}_{i,j,k}$ are the values computed from (C.5) and (C.6). Here, I use $(x)^{+}\equiv\max\{x,0\}$ and $(x)^{-}\equiv\min\{x,0\}$ to simplify notations. I use this updating process to compute the series

$A_{1,1}, A_{1,2}$ and $A_{1,n}$ in autarky. The stationary wealth distribution can be computed by replacing the left-hand side with zero.

C.1.2 Open Economy

(i) Households' Problem In open economy, the budget constraint and portfolio choice functions are now replaced by

$$\begin{aligned}
da_{it} =& [(r_t + \sigma_1 s_{1t}\theta_{1it} + \sigma_2 s_{2t}\theta_{2it})a_{it} + w_{1t}l_{it} + r^h h_{it} - c_{it} - \kappa]dt \\
&+ \sigma_1\theta_{1it}a_{it}dz_{1t} + \sigma_2\theta_{2it}a_{it}\,dz_{2t} \\
\theta_{1it} =& \max\left\{ \frac{s_{1t} - \rho s_{2t}}{\chi_1\sigma_1(1-\rho^2)}\left(1 - \frac{\chi_2}{a_{it}} - \frac{\chi_3}{a_{it}l_{it}}\right), 0 \right\} \\
\theta_{2it} =& \max\left\{ \frac{s_{2t} - \rho s_{1t}}{\chi_1\sigma_1(1-\rho^2)}\left(1 - \frac{\chi_2}{a_{it}} - \frac{\chi_3}{a_{it}l_{it}}\right), 0 \right\}
\end{aligned}$$

from time T onward.

(ii) Market Clearing Conditions Under this setting, the market clearing conditions can be stated as

$$\alpha Z K_{1t}^{\alpha-1} L_1^{1-\alpha} - \bar{\sigma}^{US} s_{1t} - \tau(1-\lambda^{US}) = r_t \tag{C.7}$$

$$\alpha Z K_{2t}^{\alpha-1} L_2^{1-\alpha} - \bar{\sigma}^{EM} s_{2t} - \tau(1-\lambda^{EM}) = r_t \tag{C.8}$$

$$\frac{s_{1t} - \rho s_{2t}}{\bar{\sigma}^{US}(1-\rho^2)\chi_1}\left(A_{1t} + A_{2t} - F_{1t}^a - F_{2t}^a - 2F_i\right) = K_{1t} \tag{C.9}$$

$$\frac{s_{2t} - \rho s_{1t}}{\bar{\sigma}^{EM}(1-\rho^2)\chi_1}\left(A_{1t} + A_{2t} - F_{1t}^a - F_{2t}^a - 2F_i\right) = K_{2t} \tag{C.10}$$

$$K_{1t} + K_{2t} = A_{1t} + A_{2t} \tag{C.11}$$

$$(1-\alpha) Z K_{1t}^{\alpha} L_1^{-\alpha} = w_{1t} \tag{C.12}$$

when A_{1t} and A_{2t} are given. Let $\bar{A}_t \equiv A_{1t} + A_{2t}$ and $\zeta(\bar{A}_t) \equiv A_{1t} + A_{2t} - 2F^a_{1t} - 2F_i$ to simplify notation. We can merge some of these equations and write them as

$$\left(\frac{s_{1t} - \rho s_{2t}}{\bar{\sigma}^{US}(1-\rho^2)\chi_1} + \frac{s_{2t} - \rho s_{1t}}{\bar{\sigma}^{EM}(1-\rho^2)\chi_1}\right)\zeta(\bar{A}_t) = \bar{A}_t \tag{C.13}$$

$$\begin{aligned}&\alpha Z\left(\frac{s_{1t} - \rho s_{2t}}{\bar{\sigma}^{US}(1-\rho^2)\chi_1}\zeta(\bar{A}_t)\right)^{\alpha-1} L_1^{1-\alpha} - \bar{\sigma}^{US}s_{1t} - \tau(1-\lambda^{US})\\ =&\alpha Z\left(\frac{s_{2t} - \rho s_{1t}}{\bar{\sigma}^{US}(1-\rho^2)\chi_1}\zeta(\bar{A}_t)\right)^{\alpha-1} L_2^{1-\alpha} - \bar{\sigma}^{EM}s_{2t} - \tau(1-\lambda^{EM})\end{aligned} \tag{C.14}$$

where the first equation follows from (C.9), (C.10) and (C.11), and the second equation results from (C.7), (C.8), (C.9) and (C.10). It follows from these equations that s_{1t} and s_{2t} are functions of $\bar{A}_t$ so I denote them by $s_{1t} \equiv s_1(\bar{A}_t)$ and $s_{2t} \equiv s_2(\bar{A}_t)$. Substituting them back into (C.7) and (C.12), we can express r_t and w_{1t} as

$$r_t \equiv r(\bar{A}_t) = \alpha Z\left(\frac{s_1(\bar{A}_t) - \rho s_2(\bar{A}_t)}{\bar{\sigma}^{US}(1-\rho^2)\chi_1}\zeta(\bar{A}_t)\right)^{\alpha-1} L_1^{1-\alpha} - \bar{\sigma}^{US}s_1(\bar{A}_t) - \tau(1-\lambda^{US}) \tag{C.15}$$

$$w_{1t} \equiv w_1(\bar{A}_t) = (1-\alpha)Z\left(\frac{s_1(\bar{A}_t) - \rho s_2(\bar{A}_t)}{\bar{\sigma}^{US}(1-\rho^2)\chi_1}\zeta(\bar{A}_t)\right)^{\alpha} L_1^{-\alpha} \tag{C.16}$$

Therefore, we can regard $\bar{A}_t$ as the sole state variable associated with the market clearing conditions.

(iii) Simulation Algorithm Let me describe a simulation algorithm for open economy. For expository purposes, I first consider the case that the initial state of the economy is open while setting aside transition from autarky to open economy. The simulation begins by guessing the law of motion for the state variables. In the simple case where all households retain $a_{it} \geq \chi_2 + \chi_3/\ell_i$, the following functional form works well in simulations:

$$\frac{d\bar{A}_t}{\bar{A}_{1t}} = ((\psi_2 - 1)\log \bar{A}_t + \psi_1)\,dt + \sigma_{\bar{A},1}dz_{1t} + \sigma_{\bar{A},1}dz_{1t} \tag{C.17}$$

where

$$\begin{aligned}\sigma_{\bar{A},1} =& \frac{s_1(\bar{A}_t) - \rho s_2(\bar{A}_t)}{1-\rho^2}\bar{A}_t\\ \sigma_{\bar{A},2} =& \frac{s_2(\bar{A}_t) - \rho s_1(\bar{A}_t)}{1-\rho^2}\bar{A}_t\end{aligned}$$

$s_1(\bar{A}_t)$ and $s_2(\bar{A}_t)$ are solutions of the system of equations, (C.13) and (C.14). Households take this motion into account when they make investment decisions. Again, in the model where some households are indebted, one may also consider the law of motion for F^a_{1t} and . In practice, $dF^a_{1t} = 0$ worked well around the stationary state.

Second, given an initial guess for ψ_1 and ψ_2, I solve differential equations that characterize saving decisions of households and evolution of the wealth distribution. This step is similar to that of closed economy. Let $J_t \equiv J(a, \ell, \varepsilon, A_t)$ denote the value function associated with households' optimization problem. $g_t(a, \ell, \epsilon)$ represents probability density distribution across households at time t. They are characterized by the HJB equation and the Kolmogorov Forward Equation shown in (C.3) and (C.4), but now the saving function, v_t, is replaced by

$$v_t(a, l, \epsilon) \equiv [(r_t + \sigma_1 s_{1t}\theta_1 + \sigma_2 s_{2t}\theta_2)a_{it} + w_{1t}l + r^h h - c - \kappa]$$

The market clearing conditions are (C.13), (C.14), (C.15) and (C.16). To find a numerical approximation to the solution, I again turn to a finite difference method ("Upwind Scheme") as in closed economy.

The third step is to check if the initial guess for ψ_1 and ψ_2 is consistent with the ones derived from the Kolmogorov Forward Equation. I again set the simulation period, say $t = T$ to $t = 2T$. The procedure is similar to that of closed economy. Starting from g_T, I compute $g_{T+1}, ..., g_{2T}$ sequentially by applying the Kolmogorov Forward Equation. It is then easy to compute the mean of each distribution, $A_{1,0}, ..., A_{1,T}$. The model-implied estimates, $\hat{\psi}_1$ and $\hat{\psi}_2$, are obtained by running ordinary least squares over the series. If the distance between (ψ_1, ψ_2) and $(\hat{\psi}_1, \hat{\psi}_2)$ is sufficiently small, terminate the process. Otherwise, start with another guess for ψ_1 and ψ_2 and repeat the above steps.

(iv) Transition to open economy Once the model converges, I turn back to transitional dynamics from autarky to open economy. The first thing to consider is to incorporate capital gains stemming from unanticipated changes in the economy. Let p_t denote the price of a unit of equity for taking a $\sigma_1 dz_{1t}$. No arbitrage condition implies that, immediately after the shock, the price movement is dictated by

$$\frac{dp_t}{p_t} + \frac{(r_t^* + \sigma_1 s_{1t}^*)dt}{p_t} = (r_t + \sigma_1 s_{1t})dt$$

In the numerical simulation, I approximate the path of p_t by turning to a discrete-time version

$$\mathbb{E}_t \left[\frac{p_{t+1} - p_t + r_t^* + \sigma_1 s_{1t}^*}{p_t} \right] = r_t + \sigma_1 s_{1t}$$

Since $p_t = 1$ for all $t < T$, we have

$$\begin{aligned} p_T^{new} &\approx \mathbb{E}_T \left[\sum_{t=T}^{\infty} \frac{(r_t^* + \sigma s_t^*)}{(1 + r_t + \sigma s_{1t})^t} \right] \\ &\equiv \mathbb{E}_T \left[\sum_{t=T}^{\infty} \frac{(r^*(A_{1t}) + \sigma s_1^*(A_{1t}))}{(1 + r(A_{1t} + A_{2t}) + \sigma s_1(A_{1t} + A_{2t}))^t} \right] \end{aligned}$$

I then simulate the stochastic processes of A_{1t} and A_{2t} according to (C.17) in discrete time. I run Monte Carlo simulations to compute the expected value numerically.

Let $g_{i,j,k}^n$ denote the wealth distribution prior to financial globalization, and $g_{i,j,k}^{n+1}$ denote the wealth distribution immediately after capital gains are realized. $g_{i,j,k}^{n+1}$ is updated according to a difference equation

$$\frac{g_{i,j}^{n+1} - g_{i,j}^n}{\Delta t} = -\frac{(s_{i,j,k}^{n,F})^+ g_{i,j}^n - (s_{i-1,j,k}^{n,F})^+ g_{i-1,j}^n}{\Delta a} - \frac{(s_{i+1,j,k}^{n,B})^- g_{i+1,j}^n - (s_{i,j,k}^{n,B})^- g_{i,j}^n}{\Delta a}$$

But now $I_{i,j,k}$ in $s_{i,j,k}^{n,F}$ and $s_{i,j,k}^{n,B}$, which are computed from (C.5) and (C.6), is replaced by

$$(r^*(A_k) + \sigma_1 s_1^*(A_k)\theta_{1,i,j,k} + p_T^{new})a_i + \ell_j w_1^*(A_k) + r^h h_{i,j,k} - \kappa$$

Essentially, households receive unanticipated capital gains and these incomes show up in the budget constraint in period $T = t_n$. Starting from $g_{i,j,k}^{n+1}$, I again use the difference equation without capital gains to compute $g_{i,j,k}^{n+2}$ ...., $g_{i,j,k}^{2n}$.

C.1.3 Estimation of the Risk Premium

To illustrate the point, let d_{t+1} denote the dividend for year $t+1$, P_t denote the price at the end of year t and R_{t+1} denote the return for year $t+1$. The return is then measured as the dividend yield $\frac{d_{t+1}}{P_t}$ plus the rate of capital gains.

$$R_{t+1} = \frac{d_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t} \tag{C.18}$$

What we aim to estimate here is $\mathbb{E}_t[R_{t+1}]$. The key identifying assumption is the stationarity of the valuation ratios. Fama and French (2002) proposes that the dividend growth rate $(d_{t+1} - d_t)/d_t$ can be an estimate of the expected capital gains under the assumption that the dividend-price ratio d_t/P_t is stationary i.e. mean reverting. In the same manner, the earning growth rate $(E_{t+1} - E_t)/E_t$ can be an alternative estimate of the expected capital gains if the earning-price ratio is a stationary process. Furthermore, Campbell and Thompson (2008) combines this formula with the steady-state relation between dividend growth and accounting return on equity. The return for year $t+1$ can be expressed as

$$R_{t+1} = \frac{d_{t+1}}{e_{t+1}} \frac{e_{t+1}}{P_t} + \left(1 - \frac{d_{t+1}}{e_{t+1}}\right) \frac{e_{t+1}}{B_t} \tag{C.19}$$

where B_t is the book value of equity. Campbell (2008) then uses three-year smoothed return on equity, dividend yields, and payout ratios to estimate the time varying equity premium. In the analysis that follows, I consider these three approaches and denote them by FF Dividends, FF Earnings and CT RoE respectively. I use five-year smoothed dividend growth rates and earnings growth rates for the first two cases. These methods are well suited to judging whether the average realized return is high or low relative to the expected return implied by fundamentals.