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THE INFLUENCE OF CHEMICAL REACTION AND VISCOUS DISSIPATION ON UNSTEADY MHD FREE CONVECTION FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE SURFACE CONDITIONS

A numerical study is presented on the effects of chemical reaction and magnetic field on the unsteady free convection flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conducting fluid past an exponentially accelerated vertical plate by taking into account the heat due to viscous dissipation. The problem is governed by coupled non-linear partial differential equations. The dimensionless equations of the problem have been solved numerically by the implicit finite difference method of Crank-Nicolson's type. The effects of governing parameters on the flow variables are discussed quantitatively with the aid of graphs for the flow field, temperature field, concentration field, skin-friction, Nusselt number and Sherwood number. It is found that under the influence of chemical reaction, the flow velocity as well as concentration distributions reduce, while the viscous dissipation parameter leads to increase the temperature.

Keywords: MHD, free convection, viscous dissipation, implicit finite difference method, exponentially accelerated plate, variable temperature and chemical reaction.

The present investigation may be useful for the design of space ships, solar energy collectors, study of movement of oil or gas and water through the reservoir of an oil or gas field, underground water in river beds, filtration and water purification processes. This study of flow past a vertical surface can be utilized as the basis for many scientific and engineering applications, including earth science, nuclear engineering and metallurgy. In nuclear engineering, it finds its applications for the design of the blanket of a liquid metal around a thermonuclear fusion-fission hybrid reactor. In metallurgy, it can be applied during the solidification process. The results of the problem are also of great interest in geophysics, in the study of interaction of geomagnetic field with the fluid in the geothermal region.

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Free convection flow involving coupled heat and mass transfer occurs frequently in nature and in industrial processes. A few representative fields of interest in which combined heat and mass transfer plays an important role are designing chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing, and environmental pollution.

Pop and Soundalgekar [1] have investigated the free convection flow past an accelerated infinite plate. Raptis *et al.* [2] have studied the unsteady free convective flow through a porous medium adjacent to a semi-infinite vertical plate using finite difference scheme. Singh and Soundalgekar [3] have investigated the problem of transient free convection in cold water past an infinite vertical porous plate. Vajravelu and Sastry [4] studied free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. Free convection boundary layer flow of a non-Newtonian fluid along a vertical wavy wall was considered by Kumari *et al.* [5]. Chandran *et al.* [6] have discussed the unsteady free convection flow with heat flux and accelerated boundary motion.

The most common type of body force on a fluid is gravity, defined in magnitude and direction by the corresponding acceleration vector. Sometimes, electromagnetic effects are important. The electric and magnetic fields themselves obey a set of physical laws, which are expressed by Maxwell's equations. The solution of such problems requires the simultaneous solution of the equations of fluid mechanics and of electromagnetism. One special case of this type of coupling is the field known as magneto hydro dynamics (MHD).

Hydromagnetic flows and heat transfer have become more important in recent years because of their varied applications in agricultural engineering and petroleum industries. Recently, considerable attention has also been focused on new applications of magneto-hydrodynamics (MHD) and heat transfer, such as metallurgical processing. Melt refining involves magnetic field applications to control excessive heat transfer rate. Other applications of MHD heat transfer include MHD generators, plasma propulsion in astronautics, nuclear reactor thermal dynamics and ionized-geothermal energy systems. An excellent summary of applications can be found in Hughes and Young [7]. Sacheti *et al*. [8] obtained an exact solution for unsteady MHD free convection flow on an impulsively started vertical plate with constant heat flux. Shankar and Kishan [9] discussed the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate with variable temperature or constant heat flux. Takar *et al*. [10] analyzed the radiation effects on MHD free convection flow past a semi-infinite vertical plate using Runge-Kutta-Merson quadrature. Abd-El-Naby *et al*. [11] studied the radiation effects on MHD unsteady free convection flow over a vertical plate with variable surface temperature. Ramachandra Prasad *et al*. [12] have studied the transient radiative hydromagnetic free convection flow past an impulsively started vertical plate uniform heat and mass flux. Samria *et al.* [13] studied the hydromagnetic free convection laminar flow of an elastoviscous fluid past an infinite plate. Recently the natural convection flow of a conducting visco-elastic liquid between two heated vertical plates under the influence of transverse magnetic field has been studied by Sreehari Reddy *et al*. [14].

In all these investigations, the viscous dissipation is neglected. The viscous dissipation heat in the natural convective flow is important when the flow field is of extreme size or at low temperature or in high gravitational field. Such effects are also important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. A number of authors have considered viscous heating effects on Newtonian flows. Maharajan and Gebhart [15] reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Israel-Cookey *et al.* [16] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Zueco Jordan [17] used network simulation method (NSM) to study the effects of viscous dissipation and radiation on unsteady MHD free convection flow past a vertical porous plate. Suneetha *et al.* [18] have analyzed the effects of viscous dissipation and thermal radiation on hydromagnetic free convection flow past an impulsively started vertical plate. Recently, Suneetha *et al.* [19] studied the effects of thermal radiation on the natural convective heat and mass transfer of a viscous incompressible gray absorbing-emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Very recently Hitesh Kumar [20] has studied the boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field.

The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Possible applications of this type of flow can be found in many industries like power industry and chemical process industries.

The effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction have been studied by Das *et al*. [21]. Chamkha [22] obtained analytical solutions for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting and heat generating/absorbing fluid on a continuously moving vertical permeable surface in the presence of a magnetic field and first order chemical reaction. Kandasamy *et al.* [23,24] presented an approximate numerical solution of chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects and effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection. The influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces was included by Postelnicu [25] in porous media considering Soret and Dufour effects. Muthucumaraswamy and Valliammal [26] have presented the theoretical study of unsteady flow past an exponentially accelerated infinite isothermal vertical plate with variable mass diffusion in the presence of homogeneous chemical reaction of first order. Sharma *et al.* [27] have investigated the influence of chemical reaction and radiation on an unsteady magnetohydrodynamic free convective flow and mass transfer through viscous incompressible fluid past a heated vertical porous plate immersed in porous medium in the presence of uniform transverse magnetic field, oscillating free stream and heat source when viscous dissipation effect is also taken into account. Anand Rao and Shivaiah [28] have analyzed the effect of chemical reaction on an unsteady MHD free convective flow past an infinite vertical porous plate in the presence of constant suction and heat source. Approximate solutions have been derived for velocity, temperature, concentration profiles, skin friction, rate of heat transfer and rate of mass transfer using finite element method.

The objective of the present work is to study the influence of chemical reaction on transient free convection flow of an incompressible viscous fluid past an exponentially accelerated vertical plate by taking into account viscous dissipative heat, under the influence of a uniform transverse magnetic field in the presence of variable surface temperature and concentration. We have extended the problem of Kishore *et al.* [29], which was an extension of Muthucumaraswamy *et al.* [30].

PROBLEM DEFINITION AND MATHEMATICAL ANALYSIS

The transient MHD free convection flow of an electrically conducting, viscous dissipative incompressible fluid past an exponentially accelerated vertical infinite plate with chemical reaction of first order has been presented. The flow configuration is shown in Figure 1.

The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. Since the plate is considered infinite in x'-direction, all flow quantities become self-similar away from the leading edge. Therefore, all the physical variables become functions of t' and y' only. At time $t' \le 0$, the plate and fluid are at the same temperature T'_∞ and concentration C'_∞ lower than the constant wall temperature T'_w and concentration C'_w respectively. At $t' > 0$, the plate is exponentially accelerated with a velocity $u' = u_0 \exp(a't')$ in its own plane and the plate temperature and concentration are raised linearly with time t'. Also, it is assumed that there is a homogeneous non-thermic (neither exothermic nor endothermic) chemical reaction of first order with rate constant k_l between the diffusing species and the fluid. The reaction is assumed to take place entirely in the stream. A uniform magnetic field of intensity, H_0, is applied in the y'-direction. Therefore, the velocity and the magnetic field are given by $\bar{q} = (u,v)$ and $\bar{H} = (0,H_0)$. The fluid being electrically conducting, the magnetic Reynolds number is much less than unity and hence the induced magnetic field can be neglected in comparison with the applied magnetic field in the absence of any

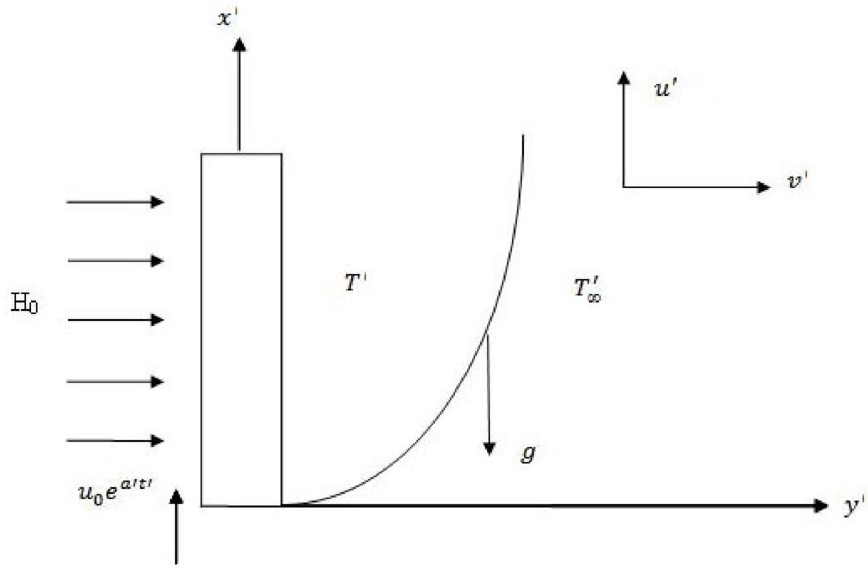

Figure 1. Flow configuration and coordinate system.

input electric field. Under the above assumptions, as well as Boussinesq's approximation, the equations of conservation of mass, momentum, energy and species governing the free convection boundary layer flow past an exponentially accelerated vertical plate can be expressed as:

$$\frac{\partial u'}{\partial y'}+\frac{\partial v'}{\partial y'}=0 \quad (1)$$

$$\frac{\partial u'}{\partial t'}=g\beta(T'-T'_{\infty})+g\beta^{*}(C'-C'_{\infty})+ +\upsilon\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma\mu_e^2H_0^2}{\rho}u' \quad (2)$$

$$\rho C_p\frac{\partial T'}{\partial t'}=k\frac{\partial^2 T'}{\partial y'^2}+\mu\left(\frac{\partial u'}{\partial y'}\right)^2 \quad (3)$$

$$\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial y'^2}-K_l(C'-C'_{\infty}) \quad (4)$$

with the following initial and boundary conditions:

$$\begin{aligned}&u'=0,\ T'=T'_{\infty},\ C'=C'_{\infty}\ \text{ for all }\ y',t'\le 0\\ &t'>0:\ u'=u_0\exp(a't'),\ T'=T'_{\infty}+(T'_w-T'_{\infty})At',\\ &C'=C'_{\infty}+(C'_w-C'_{\infty})At'\ \text{ at }\ y'=0\\ &u'\to 0,\ T'\to T'_{\infty},\ C'\to C'_{\infty}\ \text{ as }\ y'\to\infty\end{aligned} \quad (5)$$

where $A=u_0^2/\upsilon$, T'_w and C'_w are constants, not wall values.

On introducing the following non-dimensional quantities:

$$\begin{aligned}&u=\frac{u'}{u_0},\ t=\frac{t'u_0^2}{\upsilon},\ y=\frac{y'u_0}{\upsilon},\ \theta=\frac{T'-T'_{\infty}}{T'_w-T'_{\infty}},M=\frac{\sigma\mu_e^2H_0^2\upsilon}{\rho u_0^2}\\ &Gr=\frac{g\beta\upsilon(T'_w-T'_{\infty})}{u_0^3},\Pr=\frac{\mu C_p}{k},E=\frac{u_0^2}{C_p(T'_w-T'_{\infty})},\\ &a=\frac{a'\upsilon}{u_0^2}\\ &Gc=\frac{g\beta^{*}\upsilon(C'_w-C'_{\infty})}{u_0^3},C=\frac{C'-C'_{\infty}}{C'_w-C'_{\infty}},K=\frac{\upsilon k_l}{u_0^2},Sc=\frac{\upsilon}{D}\end{aligned} \quad (6)$$

Equations (1)-(6) lead to:

$$\frac{\partial u}{\partial t}=Gr\theta+GcC+\frac{\partial^2 u}{\partial y^2}-Mu \quad (7)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+E\left(\frac{\partial u}{\partial y}\right)^2 \quad (8)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-KC \quad (9)$$

The initial and boundary conditions in non-dimensional quantities are:

$$\begin{aligned}&u=0,\ \theta=0,\ C=0\ \text{for all}\ y,t\le 0\\ &t'>0:\ u=\exp(at),\ \theta=t,\ C=t\ \text{at}\ y=0\\ &u'\to 0,\ \theta\to 0,\ C\to 0\ \text{as}\ y\to\infty\end{aligned} \quad (10)$$

The skin-friction, Nusselt number and Sherwood number are the important physical parameters for this type of boundary layer flow and are given in non-dimensional form, respectively, as:

$$\tau=-\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (11)$$

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0} \quad (12)$$

$$Sh=-\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (13)$$

NUMERICAL TECHNIQUE

In order to solve the unsteady, non-linear coupled Eqs. (7)-(9) under the initial and boundary conditions (10), an implicit finite difference scheme of Crank-Nicolson's type has been employed. The finite difference equation corresponding to Eqs. (7)-(9) are as follows:

$$\begin{aligned}\frac{u_{i,j+1}-u_{i,j}}{\Delta t}&=\frac{Gr}{2}(\theta_{i,j+1}+\theta_{i,j})+\frac{Gc}{2}(C_{i,j+1}+C_{i,j})+\\ &+\left(\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}+u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{2(\Delta y)^2}\right)-\\ &-\frac{M}{2}(u_{i,j+1}+u_{i,j})\end{aligned} \quad (14)$$

$$\begin{aligned}\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}&=\\ &=\frac{1}{\Pr}\left(\frac{\theta_{i-1,j+1}-2\theta_{i,j+1}+\theta_{i+1,j+1}+\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{2(\Delta y)^2}\right)+\\ &+E\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right)^2\end{aligned} \quad (15)$$

$$\begin{aligned}\frac{C_{i,j+1}-C_{i,j}}{\Delta t}&=\frac{1}{Sc}\times\\ &\times\left(\frac{C_{i-1,j+1}-2C_{i,j+1}+C_{i+1,j+1}+C_{i-1,j}-2C_{i,j}+C_{i+1,j}}{2(\Delta y)^2}\right)-\\ &-\frac{K}{2}(C_{i,j+1}+C_{i,j})\end{aligned} \quad (16)$$

Initial and boundary conditions take the following forms:

$$u_{i,0} = 0, \theta_{i,0} = 0, C_{i,0} = 0 \text{ for all } i \neq 0$$

$$u_{0,j} = \exp(aj\Delta t),\ \theta_{0,j} = j\Delta t,\ C_{0,j} = j\Delta t$$

$$u_{N,j} = 0,\ \theta_{N,j} = 0,\ C_{N,j} = 0 \tag{17}$$

where N corresponds to ∞.

Here the suffix "i" corresponds to y and "j" corresponds to t. Also, $\Delta t = t_{j+1} - t_j$ and $\Delta y = y_{i+1} - y_i$.

Here, we consider a rectangular grid with grid lines parallel to the coordinate axes with spacing Δy and Δt in space and time directions respectively. The grid points are given by $y_i = i\Delta y$, $i = 1,2,3,\ldots,N-1$ and $t_j = j\,\Delta t$, $j = 1,2,3,\ldots, P$. The spatial nodes on the j^{th} time grid constitute the j^{th} layer or level. The maximum value of y was chosen as 10 after some preliminary investigations, so that the boundary conditions of Eq. (16) are satisfied. After experimenting with few sets of mesh sizes, they have been fixed at the level Δy = = 0.05 and the time step Δt = 0.01, in this case, special mesh size is reduced by 50% and the results are compared. It is observed that when mesh size is reduced by 50% in y-direction, the results differ only in the fifth decimal place.

The complete solution of the discrete Eqs. (14)-(16) proceeds as follows:

1) Knowing the values of C, θ and u at a time $t = j$, calculate C and θ at time $t = j + 1$ using Eqs. (15) and (16) and solving the tri diagonal linear system of equations by using Thomas algorithm as discussed in Sastry [31].

2) Knowing C and θ at times $t = j$ and $t = j + 1$ and u at time $t = j$, solve Eq. (14) (*via* tri diagonal matrix inversion), to obtain u at time $t = j + 1$.

We can repeat steps 1 and 2 to proceed from $t = 0$ to the desired time value.

The implicit Crank-Nicolson method is a second order method ($O(\Delta t^2)$) in time and has no restrictions on space- and time-steps Δy and Δt, *i.e.*, the method is unconditionally stable (Jain *et al.* [32]). The derivatives involved in Eqs. (11)-(13) are evaluated using five point approximation formula.

The accuracy of the present model has been verified by comparing with the numerical solution of Kishore *et al.* [29] and the theoretical solution of Muthucumaraswamy *et al.* [30] through Table 1, as well as Figures 2 and 3, and the agreement between the results is excellent. This has established confidence in the numerical results reported in this paper.

Table 1. Comparison of velocity with Muthucumaraswamy et al. [30] for Pr = 0.71, Gr = 5, t = 0.2

a	*y*	Muthucumaraswamy *et al.* [30]	Present results
0	0.0	1	1
	0.5	0.4492	0.4493
	1.0	0.1202	0.1207
2	0.0	1.4918	1.4918
	0.5	0.5574	0.5580
	1.0	0.1365	0.1373

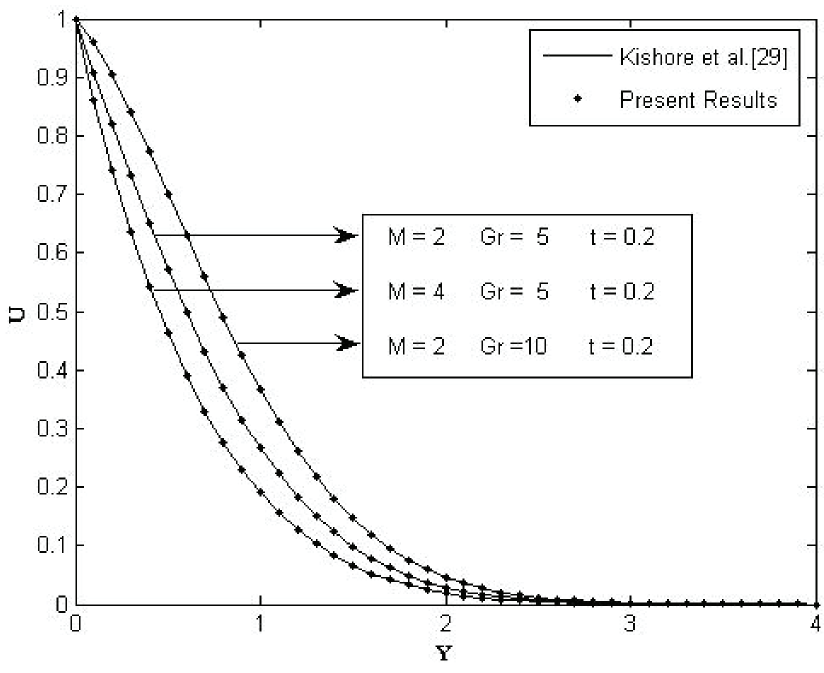

Figure 2. Velocity profile for different values of M and Gr when Pr = 0.71, E = 1 and a = 0.

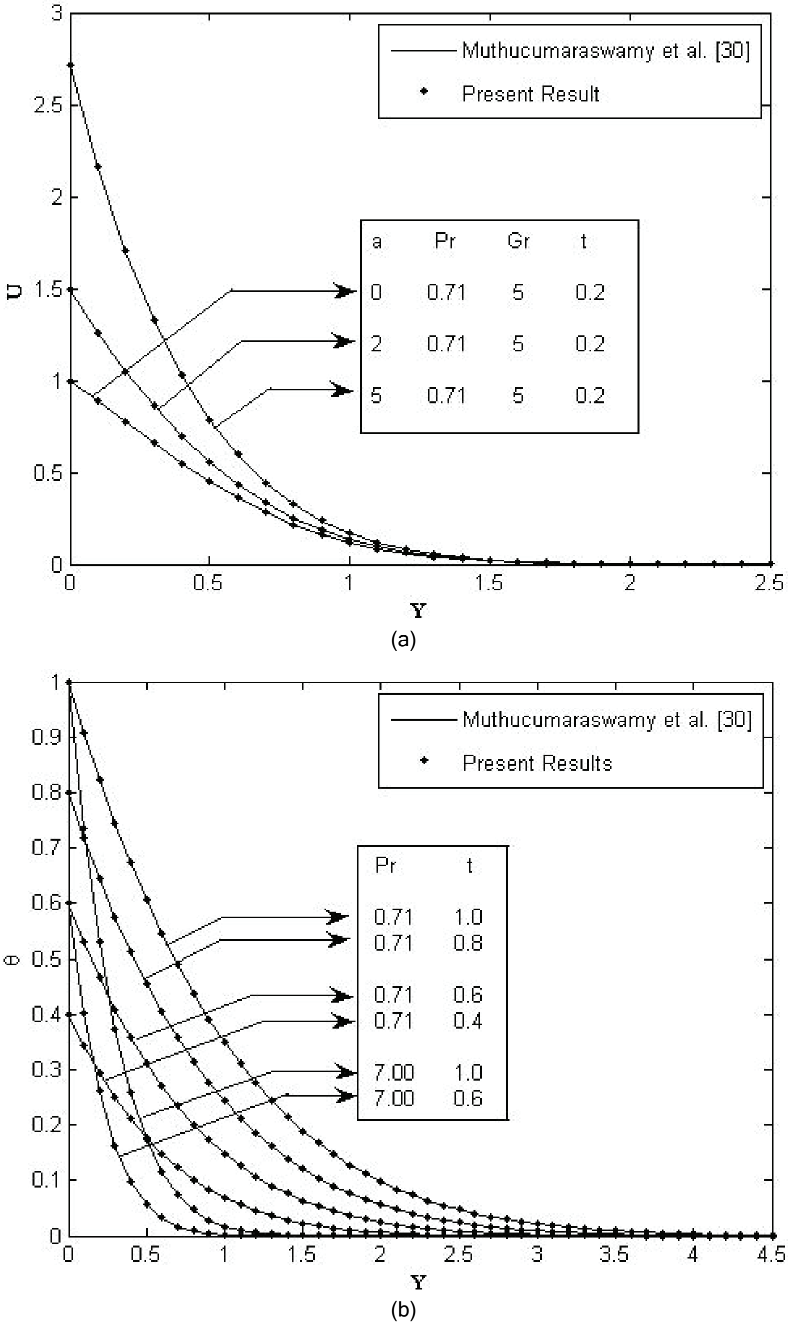

Figure 3. a) Velocity profile for different values of a and b) temperature profile for different values of Pr and t when M = 0, E = 0 and Gc = 0.

RESULTS AND DISCUSSION

In order to gain physical insight into the problem, we exhibit results to show how the material parameters of the problem affect the velocity, temperature and concentration profiles. Here we restricted our discussion to the aiding of favourable case only, for fluids with Prandtl number $Pr = 0.71$ which represent air at 20 °C at 1 atm. The diffusing chemical species of most common interest in air has a Schmidt number and is taken for hydrogen ($Sc = 0.22$), oxygen ($Sc = 0.66$) and carbon dioxide ($Sc = 0.94$).

The effects of governing parameters like magnetic field, thermal Grashof number as well as mass Grashof number, acceleration parameter and chemical reaction, on the transient velocity have been presented in the respective Figures 4-7 for both the cases of cooling and heating of the plate, and in the presence of foreign species ($Sc = 0.22$).

Figure 4 illustrates the influence of M in cases of cooling and heating of the plate. It is found that the

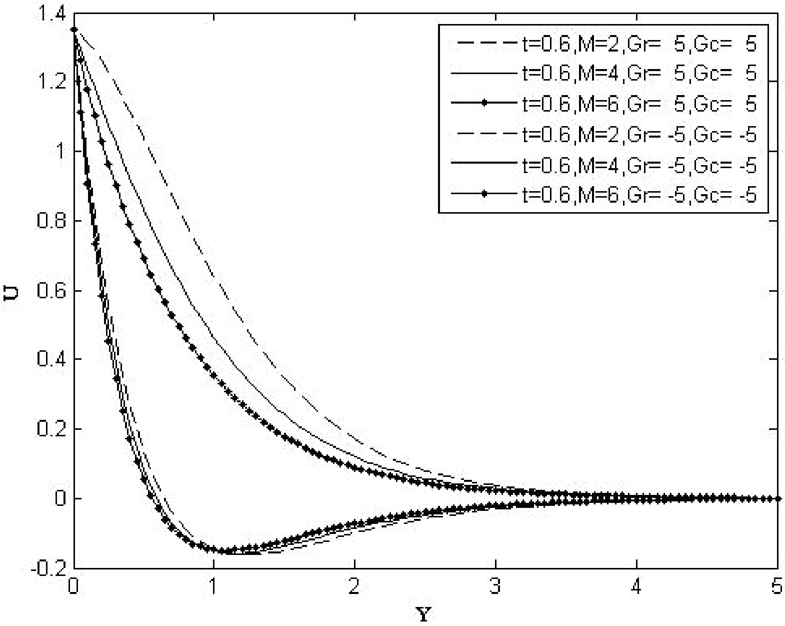

Figure 4. Velocity profile for different values of M when Pr = 0.71, a = 0.5, E = 0.5, Sc = 0.22, K = 1.

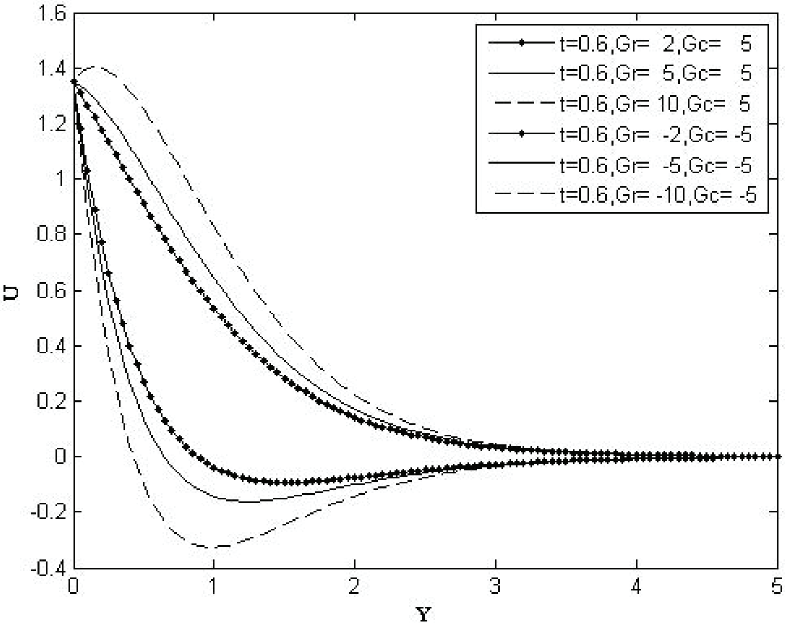

Figure 5. Velocity profile for different values of Gr when Pr = 0.71, a = 0.5, E = 0.5, Sc = 0.22, K = 1.

velocity decreases with increasing magnetic parameter for air (Pr = 0.71) in presence of hydrogen. The presence of a transverse magnetic field produces a resistive force on the fluid flow. This force is called the Lorentz force, which leads to slow down the motion of an electrically conducting fluid. Also, it is interesting to see that an increase in M leads to a decrease in the flow velocity in the interval $0 \leq y < 1$, while this behavior is reversed for $y \geq 1$ in case of heating of the plate (Gr < 0).

It is observed from Figures 5 and 6 that greater cooling of the surface (an increase in Gr) and increase in Gc results in an increase in the velocity for air. This is due to the fact an increase in the values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow. The reverse effect is observed in case of heating of the plate (Gr < 0).

It is seen from Figure 7 that under the influence of chemical reaction, the flow velocity reduces in air

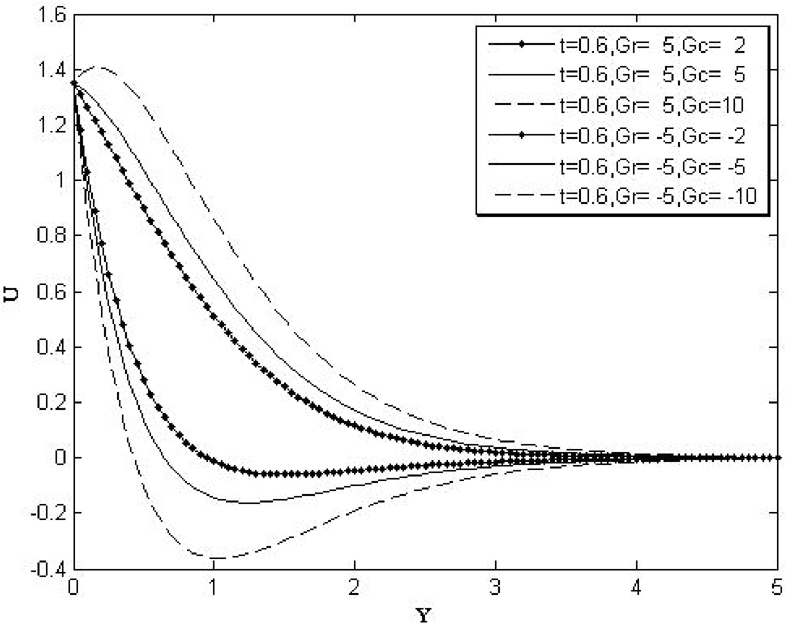

Figure 6. Velocity profile for different values of Gc when Pr = 0.71, a = 0.5, E = 0.5, Sc = 0.22, K = 1.

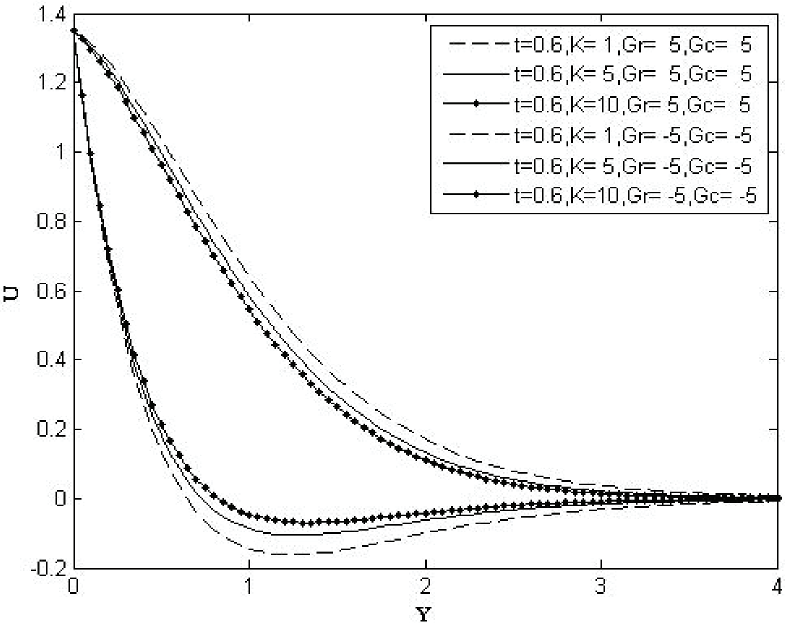

Figure 7. Velocity profile for different values of K when Pr = 0.71, a= 0.5, E = 0.5, Sc = 0.22.

for cooling of the plate (*Gr* > 0). The hydrodynamic boundary layer becomes thinner as the chemical reaction parameter increases. The reverse effect is noticed in the case of heating of the plate (*Gr* < 0).

It is marked from Figure 8 that the increasing values of the viscous dissipation parameter enhancing the flow temperature for the cases of air and water. This is due to the fact that the Eckert number is the ratio of the kinetic energy of the flow to the boundary layer enthalpy difference. The effect of viscous dissipation on flow field is to increase the energy, yielding a greater fluid temperature. However, significantly, it is observed that the temperature decreases with increasing *Pr*.

A decrease in concentration with increasing *Sc* as well as *K* is observed from Figures 9 and 10. Also, it is noted that the concentration boundary layer becomes thin as the Schmidt number as well as chemical reaction parameter increases.

It is noticed from Figure 11 that the skin friction increases with an increase in magnetic field, acceleration parameter and chemical reaction, while it de-

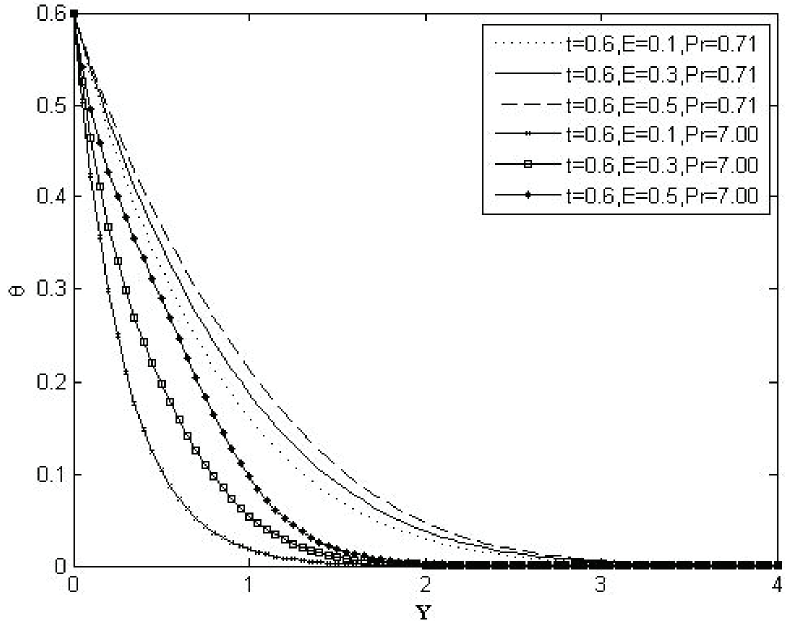

Figure 8. Temperature profile for different values of E and Pr when M = 2, Gr = 5, Gc = 5, a = 0.5, Sc = 0.22, K = 1.

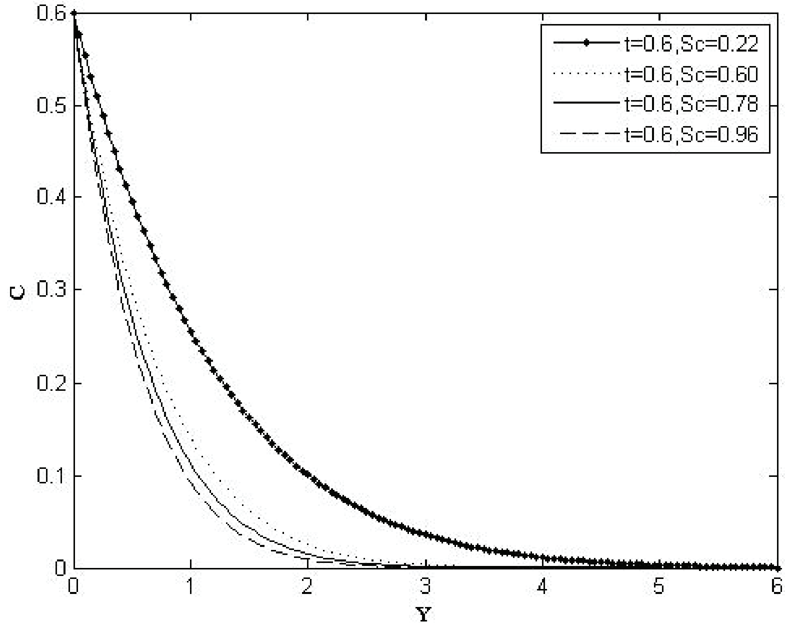

Figure 9. Concentration profile for different values of 'Sc' when M = 2, Gr = 5, Gc = 5, Pr = 0.71, a = 0.5, E = 0.5, K = 1.

creases with an increase in the thermal Grashof number for air.

It is found from Figure 12 that the rate of heat transfer falls with increasing *E*. The Nusselt number for *Pr* = 7 is higher than that of *Pr* = 0.71. The reason for this is that smaller values of *Pr* are equivalent to increasing thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than higher values of Prandtl number. Hence, the rate of heat transfer is enhanced.

It is marked from Figure 13 that the rate of concentration transfer increases with increasing values of chemical reaction parameter *K* and Schmidt number.

CONCLUSIONS

The effects of chemical reaction, magnetic field and viscous dissipation on free convection flow past an exponentially accelerated vertical plate with variable surface temperature and concentration have been

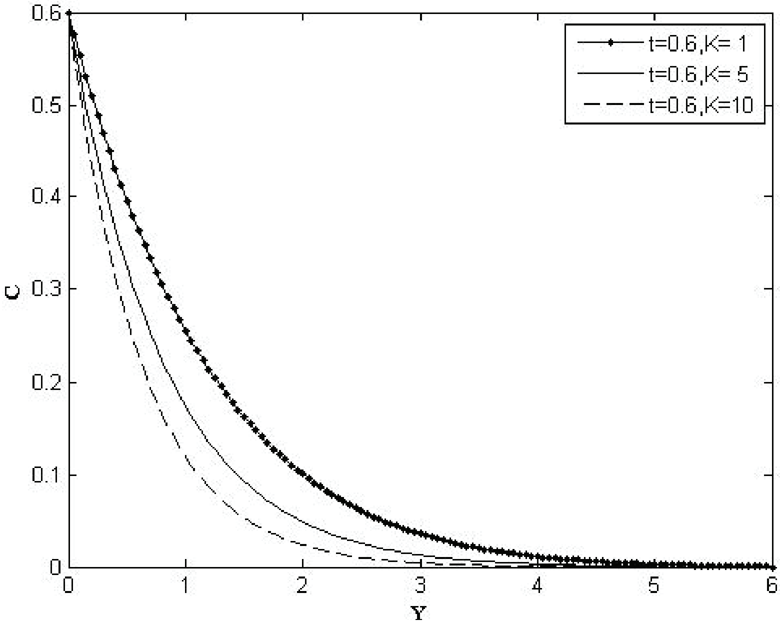

Figure 10. Concentration profile for different values of 'K' when M = 2, Gr = 5, Gc = 5, Pr = 0.71, a = 0.5, E = 0.5, Sc = 0.22.

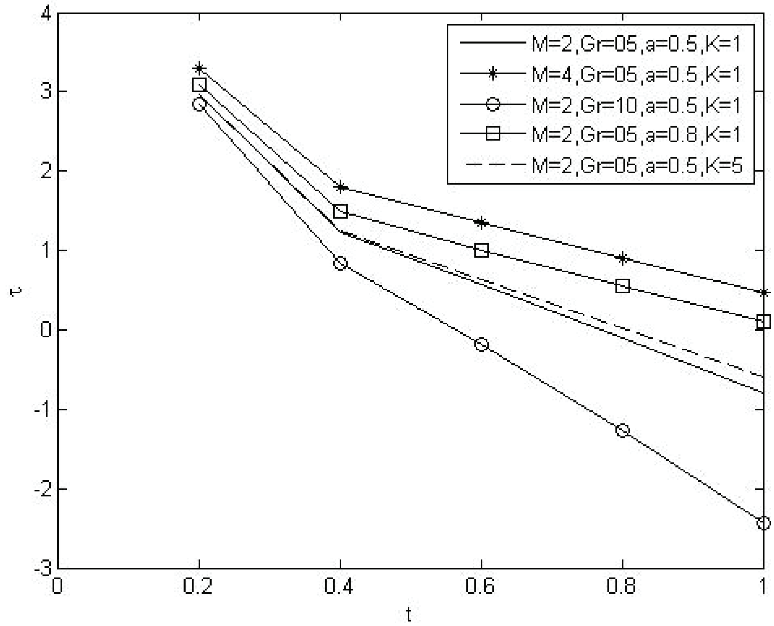

Figure 11. Skin friction profile when Gc = 5, Pr = 0.71, E = 0.5, Sc = 0.22.

studied numerically. From the present numerical investigation, following conclusions have been drawn:

- It is found that the velocity decreases with increasing magnetic parameter (M). Also, it is interesting to see that an increase in M leads to a decrease in the flow velocity in the vicinity of the plate, while this behavior is reversed for away from the plate in case of heating of the plate ($Gr < 0$).
- Under the influence of chemical reaction, the air velocity reduces for cooling of the plate ($Gr > 0$). Also, the reverse effect is noticed in the case of heating of the plate ($Gr < 0$).
- The increase of the viscous dissipation enhances the fluid temperature, and the temperature decreases with increasing Pr.
- A decrease in concentration with increasing Schmidt number as well as chemical reaction parameter is observed.
- Skin friction increases with an increase in magnetic field, acceleration parameter and chemical

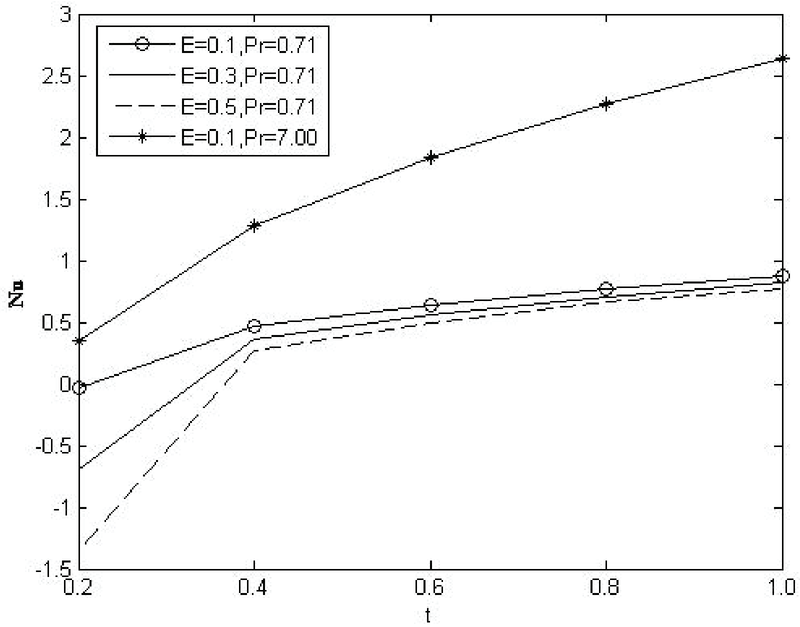

Figure 12. Nusselt number profile when M = 2, Gr = 5, Gc = 5, a = 0.5, K = 1, Sc = 0.22.

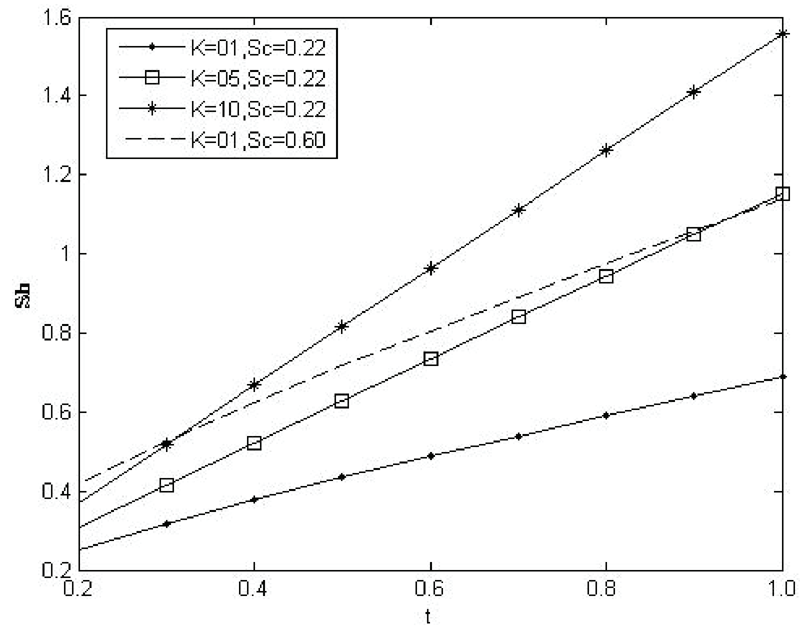

Figure 13. Sherwood number profile when M = 2, Gr = 5, Gc = 5, a = 0.5, Pr = 0.71, E = 0.5.

reaction, while it decreases with an increase in thermal Grashof number.

It is found that the rate of heat transfer falls with increasing Eckert number, while it increases with an increase in Prandtl number.

Nomenclature

A, a, a' Constants
C_p Specific heat at constant pressure, J kg^{-1} K^{-1}
C' Species concentration, kg m-3
C Dimensionless concentration
D Mass Diffusion coefficient, m^2 s^{-1}
E Eckert number
Gr Thermal Grashof number
Gc Mass Grashof number
g Acceleration due to gravity, m s^{-2}
H_0 Magnetic field intensity, A m^{-1}
k Thermal conductivity, W m^{-1} K^{-1}
K_l Chemical reaction coefficient, s^{-1}
K Dimensionless chemical reaction parameter
M Magnetic parameter
Nu Nusselt number

Pr Prandtl number
Sc Schmidt number
T' Temperature of the fluid near the plate, K
T Dimensionless temperature of the fluid near the plate
t' Time, s
t Dimensionless time
u' Velocity of the fluid in the x'-direction, m s^{-1}
u_0 Velocity of the plate, m s^{-1}
u Dimensionless velocity
y' Coordinate axis normal to the plate, m
y Dimensionless coordinate axis normal to the plate

Greek symbols

β Volumetric coefficient of thermal expansion, K^{-1}
β^* Volumetric coefficient of thermal expansion with concentration, K^{-1}
θ Dimensionless temperature
μ Coefficient of viscosity, Pa s
μ_e Magnetic permeability, H m-1
ν Kinematic viscosity, m^2 s^{-1}
ρ Density of the fluid, kg m^{-3}
σ Electrical conductivity of the fluid, V A^{-1} m^{-1}
τ Dimensionless shear stress

Subscripts

w Conditions at the wall
∞ Conditions in the free stream.

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NAUČNI RAD

UTICAJ HEMIJSKE REAKCIJE I VISKOZNE DISIPACIJE NA NESTACIONARNO MHD PRIRODNO KONVEKTIVNO STRUJANJE PORED EKSPONENCIJALNO UBRZANE VERTIKALNE PLOČE SA PROMENLJIVIM USLOVIMA POVRŠINE

Prikazano je numeričko proučavanje uticaja hemijske reakcije i magnetnog polja na karakteristike prenosa toplote i mase, u slučaju nestacionarnog prirodno-konvektivnog strujanja viskoznog, nestišljivog i elektro-provodnog fluida pored eksponencijalno ubrzane vertikalne ploče uzimajući u obzir i toplotu usled viskozne disipacije. Ovaj problem je opisan sistemom nelinearnih parcijalnih diferencijalnih jednačina. Bezdimenzionalne jednačine su rešene numerički primenom implicitnog metoda konačnih razlika Crank-Nicolson-ovog tipa. Uticaji određujućih parametara na promenljive strujanja su diskutovani na osnovu grafičkih prikaza strujnog, temperaturnog i koncentracijskog polja, trenja, Nusselt-ovog broja i Sherwood-ovog broja. Utvrđeno je da se pod uticajem hemijske reakcije, brzina strujanja i raspodele koncentracije smanjuju, dok viskozna disipacija izaziva povećanje temperature.

Ključne reči: MHD, prirodna konvekcija, viskozna disipacija, implicitna metoda konačnih razlika, eksponencijalno ubrzana ploča.