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SORET AND DUFOUR EFFECTS ON FREE CONVECTION FLOW OF A COUPLE STRESS FLUID IN A VERTICAL CHANNEL WITH CHEMICAL REACTION

The Soret and Dufour effects in the presence of chemical reaction on natural convection heat and mass transfer of a couple stress fluid in a vertical channel formed by two vertical parallel plates is presented. The governing non-linear partial differential equations are transformed into a system of ordinary differential equations using similarity transformations. The resulting equations are then solved using the Homotopy Analysis Method (HAM). Profiles of dimensionless velocity, temperature and concentration are shown graphically for various values of Dufour number, Soret number, Couple stress parameter and chemical reaction parameter.

Keywords: free convection, couple stress fluid, Soret and Dufour effect, chemical reaction, HAM.

The phenomena of natural convection heat and mass transfer fluid flow in vertical parallel plate channels have been the focus of extensive investigation for many decades due to their wide range of applications such as chemical engineering, environmental dynamics and architectural design, cooling of electronic equipment, solar collectors and passive solar heating, ventilation of buildings and heat removal in nuclear technology applicability. Studies on laminar natural convection between vertical parallel plates date back to 1942 when Elenbaas [1] did experimental and theoretical analysis on natural convection between isothermal parallel plates and presented an optimization of heat transfer rate. Bodoia and Osterle [2] numerically analyzed the problem of development of free convection boundary layer between parallel isothermal vertical plates for the case of symmetric heating using finite difference method and obtained results for variation in temperature, pressure and velocity throughout the flow field. Aung *et al.* [3] conducted experimental and numerical studies on developing laminar free convection between vertical flat

plates with asymmetric heating, for thermal boundary conditions of uniform heat flux and uniform wall temperature. Since then a number of studies have been reported in the literature focusing on the problem of free convection heat and mass transfer fluid flow between vertical parallel plates.

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. Chemical reaction can be described as either heterogeneous or homogeneous processes, which depends on whether it occurs at an interface or as a single-phase volume reaction. Research on combined heat and mass transfer with chemical reaction and thermophoresis effect can help to design for chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields as well as groves of fruit trees, damage of crops due to freezing, food processing and cooling towers. Cooling towers are the cheapest way to cool large quantities of water. In particular, the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. For example, formation of smog is a first-order homogeneous chemical reaction. Considering the emission of NO_2 from automobiles and other smoke-stacks, NO_2 reacts chemically in the atmosphere with unburned hydrocarbons

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(aided by sunlight) and produces peroxyacetyl nitrate, which forms an envelope, can be termed as photochemical smog. Recently, Pop *et al.* [4] investigated the steady fully developed mixed convection flow in a vertical channel with first-order chemical reaction. Shateyi *et al.* [5] considered the two-dimensional flow of an incompressible viscous fluid through a non-porous channel with heat generation and a chemical reaction.

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more integrate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (H_2 or He) and of medium molecular weight (N_2 or air). The Dufour effect was recently found to be of order of considerable magnitude such that it cannot be neglected (Eckeret and Drake, [6]). Kafoussias [7] presented the local similarity solution for combined free-forced convective and mass transfer flow past a semi-infinite vertical plate. Dursunkaya and Worek [8] studied diffusion thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [9] presented the same effects on mixed convective and mass transfer steady laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. Awad and Sibanda [10] studied the Dufour and Soret effects on heat and mass transfer in a micropolar fluid in a horizontal channel. Recently, free convection heat and mass transfer flow in a vertical channel with the Dufour effect was studied by Jha and Ajibade [11].

During recent years, the study of convection heat and mass transfer in non-Newtonian fluids has received much attention, because the traditional Newtonian fluids cannot precisely describe the characteristics of the real fluids. Ziabakhsh and Domairry [12] have obtained the solution for natural convection of the Rivlin-Ericksen fluid of grade three between two infinite parallel vertical flat plates. Sajid *et al.* [13] studied fully developed mixed convection flow of a viscoelastic fluid between permeable parallel vertical walls using HAM. In addition, progress has been considerably made in the study heat and mass transfer in magneto hydrodynamic flow of non-Newtonian fluids due to its application in many devices, like the MHD power generator, aerodynamics heating, electrostatic precipitation and Hall accelerator, etc.

Different models have been proposed to explain the behavior of non-Newtonian fluids. Among these, couple stress fluids introduced by Stokes [14] have distinct features, such as the presence of couple stresses, body couples and non-symmetric stress tensor. The main feature of couple stresses is to introduce a size dependent effect. Classical continuum mechanics neglects the size effect of material particles within the continua. This is consistent with ignoring the rotational interaction among particles, which results in symmetry of the force-stress tensor. However, in some important cases such as fluid flow with suspended particles, this cannot be true and a size dependent couple-stress theory is needed. These fluids are capable of describing various types of lubricants, blood, suspension fluids, etc. The study of couple-stress fluids has applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, and colloidal solutions, etc. Stokes [14] discussed the hydromagnetic steady flow of a fluid with couple stress effects. A review of couple stress (polar) fluid dynamics was reported by Stokes [15]. Recently, Srinvasacharya and Kaladhar [16] presented the analytical solution for free convective flow of couple stress fluid in an annulus with Hall and ion-slip effects.

The homotopy analysis method was first proposed by Liao in 1992 [17], is one of the most efficient methods in solving different types of nonlinear equations such as coupled, decoupled, homogeneous and non-homogeneous. Also, HAM provides us a great freedom to choose different base functions to express solutions of a nonlinear problem [18]. The application of the Homotopy Analysis Method (HAM) in engineering problems is highly considered by scientists, because HAM provides us with a convenient way to control the convergence of approximation series, which is a fundamental qualitative difference in analysis between HAM and other methods. Later, Liao [19] presented an optimal Homotopy Analysis approach for strongly nonlinear differential equations. HAM is used to get analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation [20]. Recent developments of

HAM, like convergence of HAM solution, Optimality of convergence control parameter discussed by Srinivasacharya and Kaladhar [21] for the couple stress fluid.

In this paper, we have investigated the Soret and Dufour effects on steady free convective heat and mass transfer flow between two vertical parallel plates in couple stress fluid with chemical reaction. The Homotopy Analysis Method is employed to solve the governing nonlinear equations. Convergence of the derived series solution is analyzed. The behavior of emerging flow parameters on the velocity and temperature is discussed.

FORMULATION OF THE PROBLEM

Consider a steady, fully developed laminar free convection flow of a couple stress fluid between two permeable vertical plates distance $2d$ apart. Choose the coordinate system such that x-axis be taken along vertically upward direction through the central line of the channel, y is perpendicular to the plates and the two plates are infinitely extended in the direction of x. The plate $y = -d$ has the given uniform temperature T_1 and concentration C_1, while the plate $y = d$ is subjected to a uniform temperature T_2 and concentration C_2. Since the boundaries in the x direction are of infinite dimensions, without loss of generality, we assume that the physical quantities depend on y only. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. In addition, the Soret and Dufour effects with chemical reaction are considered. The flow is a free convection caused by buoyancy forces. The flow configuration and the coordinates system are shown in Figure 1. The fluid velocity vector, $\mathbf{v} = (u,v)$, is assumed to be parallel to the x-axis, so that only the x-component u of the velocity vector does not vanish but the transpiration cross-flow velocity v_0 remains constant, where $v_0 < 0$ is the velocity of suction and $v_0 > 0$ is the velocity of injection.
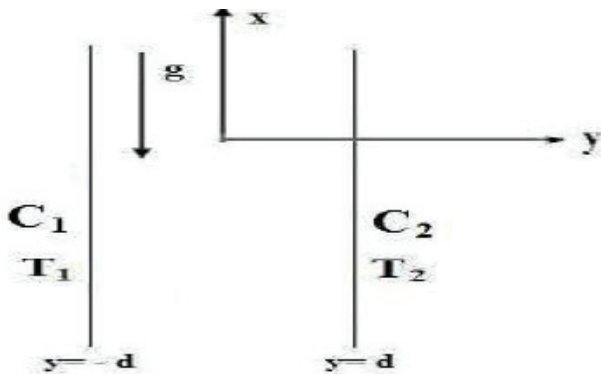

Figure 1. Geometrical configuration.

With the above assumptions and Boussinesq approximations with energy and concentration, the equations governing the steady flow of an incompressible couple stress fluid are:

$$v = v_0 = \text{constant} \tag{1}$$

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial^4 u}{\partial y^4} + \rho g \beta_T (T - T_1) + \\ + \rho g \beta_C (C - C_1) \tag{2}$$

$$v_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + 2 \frac{v}{C_P} \left(\frac{\partial u}{\partial y} \right)^2 + \\ + \frac{\eta_1}{\rho C_P} \left(\frac{\partial^2 u}{\partial y^2} \right)^2 + \frac{D K_T}{C_S C_P} \frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{D K_T}{T_m} \frac{\partial^2 T}{\partial y^2} - \\ - k_1 (C - C_1) \tag{4}$$

where u is the velocity component along x direction, ρ is the density, g is the acceleration due to gravity, p is the pressure, μ is the coefficient of viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansion, α is the thermal diffusivity, D is the mass diffusivity, C_P is the specific heat capacity, C_S is the concentration susceptibility, T_m is the mean fluid temperature, K_T is the thermal diffusion ratio, η_1 is the additional viscosity coefficient which specifies the character of couple-stresses in the fluid and k_1 is the rate of chemical reaction.

The boundary conditions are:

$$u=0 \text{ at } y = \pm d \tag{5a}$$

$$u_{yy} = 0 \text{ at } y = \pm d \tag{5b}$$

$$T=T_1, \; C=C_1 \text{ at } y = -d \tag{5c}$$

$$T = T_2, \; C = C_2 \text{ at } y = d \tag{5d}$$

The boundary condition (5a) corresponds to the classical no-slip condition from viscous fluid dynamics. The boundary condition (5b) implies that the couple stresses are zero at the plate surfaces.

Introducing the following similarity transformations:

$$y = \eta d, \; u = \frac{v \, Gr}{d^2} f, \; T - T_1 = (T_2 - T_1)\theta, \\ C - C_1 = (C_2 - C_1)\varphi \tag{6}$$

in Eqs. (2)-(4), we get the following nonlinear system of differential equations:

$$S^2 f^{(iv)} - f'' + \text{Re} \, f' - \theta - N\varphi = 0 \tag{7}$$

$$\theta'' - \mathrm{Re}\,\mathrm{Pr}\,\theta' + 2\,Br\,Gr^2(f')^2 + S^2 Br\,Gr^2(f'')^2 + \\ + D_f\,\mathrm{Pr}\,\varphi'' = 0 \tag{8}$$

$$\varphi'' - \mathrm{Re}\,Sc\,\varphi' + S_r Sc\,\theta'' - \\ - KSc\varphi = 0 \tag{9}$$

where primes denote differentiation with respect to η alone, $\mathrm{Re} = v_0 d / v$ is the Reynolds number, $Gr = g\beta_T(T_2 - T_1)d^3 / v^2$ is the Grashof number, $\mathrm{Pr} = \mu C_P / K_T$ is the Prandtl number, $Sc = v / D$ is the Schmidth number, $Br = \mu v^2 / K_T d^2 (T_2 - T_1)$ is the Brinkman number, $K = k_1 d^2 / v$ is the chemical re-action parameter, $S_r = DK_T(T_2 - T_1) / vT_m(C_2 - C_1)$ is the Soret number, $D_f = DK_T(C_2 - C_1) / vC_S C_P(T_2 - T_1)$ is the Dufour number, $N = \beta_C(C_2 - C_1) / \beta_T(T_2 - T_1)$ is the buoyancy ration, $S = 1/d\sqrt{\eta_1 / \mu}$ is the couple stress parameter, the effects of couple-stress are sig-nificant for large values of $S\,(= l/d)$, where $l = \sqrt{\eta_1 / \mu}$ is the material constant. If l is a function of the molecular dimensions of the liquid, it will vary greatly for different liquids. For example, the length of a polymer chain may be a million times the diameter of water molecule (Stokes [15]). Therefore, there are all the reasons to expect that couple-stresses appear in noticeable magnitudes in liquids with large mole-cules.

Boundary conditions (5) in terms of f, θ and ϕ become:

$$f = 0,\ f'' = 0,\ \theta = 0,\ \varphi = 0 \text{ at } \eta = -1 \\ f = 0,\ f'' = 0,\ \theta = 1,\ \varphi = 1 \text{ at } \eta = 1 \tag{10}$$

The HAM solution of the problem

For HAM solutions, we choose the initial appro-ximations of $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ as follows:

$$f_0(\eta) = 0,\ \theta_0(\eta) = \frac{1+\eta}{2},\ \varphi_0(\eta) = \frac{1+\eta}{2} \tag{11}$$

and choose the auxiliary linear operators:

$$L_1 = \frac{\partial^4}{\partial y^4},\ L_2 = \frac{\partial^2}{\partial y^2} \tag{12}$$

such that:

$$L_1(c_1 + c_2\eta + c_3\eta^2 + c_4\eta^3) = 0,\ L_2(c_5 + c_6\eta) = 0 \tag{13}$$

where $c_i (i = 1,2,...,6)$ are constants. Introducing non-zero auxiliary parameters h_1, h_2 and h_3, we develop the zeroth-order deformation problems as follow:

$$(1 - p)L_1[f(\eta; p) - f_0(\eta)] = \\ = ph_1 N_1[f(\eta; p), \theta(\eta; p), \varphi(\eta; p)] \tag{14}$$

$$(1 - p)L_2[\theta(\eta; p) - \theta_0(\eta)] = \\ = ph_2 N_2[f(\eta; p), \theta(\eta; p), \varphi(\eta; p)] \tag{15}$$

$$(1 - p)L_2[\varphi(\eta; p) - \varphi_0(\eta)] = \\ = ph_3 N_3[f(\eta; p), \theta(\eta; p), \varphi(\eta; p)] \tag{16}$$

subject to the boundary conditions:

$$f(-1; p) = 0,\ f(1; p) = 0,\ f''(-1; p) = 0,\ f''(1; p) = 0, \\ \theta(-1; p) = 0,\ \theta(1; p) = 1, \varphi(-1; p) = 0,\ \varphi(1; p) = 1 \tag{17}$$

where $p \in [0,1]$ is the embedding parameter and the non-linear operators N_1, N_2 and N_3 are defined as:

$$N_1[f(\eta; p), \theta(\eta; p), \varphi(\eta; p)] = \\ = S^2 f^{(iv)} - f'' + \mathrm{Re}\,f' - \theta - N\varphi \tag{18}$$

$$N_2[f(\eta; p), \theta(\eta; p), \varphi(\eta; p)] = \\ \theta'' - \mathrm{Re}\,\mathrm{Pr}\,\theta' + 2\,Br\,Gr^2(f')^2 + \\ + S^2 Br\,Gr^2(f'')^2 + D_f\,\mathrm{Pr}\,\varphi'' \tag{19}$$

$$N_3[f(\eta; p), \theta(\eta; p), \varphi(\eta; p)] = \\ \varphi'' - \mathrm{Re}\,Sc\,\varphi' + S_r Sc\,\theta'' - KSc\varphi \tag{20}$$

For $p = 0$ we have the initial guess approxima-tions:

$$f(\eta; 0) = f_0(\eta), \theta(\eta; 0) = \theta_0(\eta), \varphi(\eta; 0) = \varphi_0(\eta) \tag{21}$$

When $p = 1$, Eqs. (14)-(16) are same as (7)-(9) respectively, therefore at $p = 1$ we get the final solu-tions:

$$f(\eta; 1) = f(\eta), \theta(\eta; 1) = \theta(\eta), \varphi(\eta; 1) = \varphi(\eta) \tag{22}$$

Hence the process of giving an increment to p from 0 to 1 is the process of $f(\eta;p)$ varying conti-nuously from the initial guess $f_0(\eta)$ to the final solution $f(\eta)$ (similar for $\theta(\eta;p)$ and $\phi(\eta;p)$). This kind of conti-nuous variation is called deformation in topology so that we call system Eqs. (14)-(17), the zeroth-order deformation equation. Next, the mth-order deformation equations follow as:

$$L_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h_1 R_m^f(\eta) \tag{23}$$

$$L_2[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h_2 R_m^\theta(\eta) \tag{24}$$

$$L_2[\varphi_m(\eta) - \chi_m \varphi_{m-1}(\eta)] = h_3 R_m^\varphi(\eta) \tag{25}$$

with the boundary conditions:

$$f_m(-1) = 0,\ f_m(1) = 0, f_m''(-1) = 0,\ f_m''(1) = 0, \\ \theta_m(-1) = 0,\ \theta_m(1) = 0, \varphi_m(-1) = 0,\ \varphi_m(1) = 0 \tag{26}$$

where:

$$R_m^f = S^2 f^{(iv)} - f'' + \mathrm{Re}\,f' - \theta - N\varphi \tag{27}$$

48

$$R_m^\theta = \theta'' - \mathrm{Re}\,\mathrm{Pr}\,\theta' + 2\,Br\,Gr^2 \sum_{n=0}^{m-1} f'_{m-1-n} f'_n +$$
$$+ S^2 Br\,Gr^2 \sum_{n=0}^{m-1} f''_{m-1-n} f''_n + D_f\,\mathrm{Pr}\,\varphi'' \qquad (28)$$

$$R_m^\varphi = \varphi'' - \mathrm{Re}\,Sc\,\varphi' + S_r\,Sc\,\theta'' - K\,Sc\,\varphi \qquad (29)$$

and, for m being integer:

$$\chi_m = 0 \text{ for } m \le 1$$
$$\quad = 1 \text{ for } m > 1 \qquad (30)$$

The initial guess approximations $f_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$, the linear operators L_1, L_2 and the auxiliary parameters h_1, h_2 and h_3 are assumed to be selected such that Eqs. (14)-(17) have solution at each point $p \in [0,1]$, and also with the help of Taylor's series and due to Eq. (21); $f(\eta;p)$, $\theta(\eta;p)$ and $\phi(\eta;p)$ can be expressed as:

$$f(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m \qquad (31)$$

$$\theta(\eta;p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m \qquad (32)$$

$$\varphi(\eta;p) = \varphi_0(\eta) + \sum_{m=1}^{\infty} \varphi_m(\eta) p^m \qquad (33)$$

in which h_1, h_2 and h_3 are chosen in such a way that the series (23)-(25) are convergent at $p = 1$. Therefore, we have from (22) that:

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \qquad (34)$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \qquad (35)$$

$$\varphi(\eta) = \varphi_0(\eta) + \sum_{m=1}^{\infty} \varphi_m(\eta) \qquad (36)$$

for which we presume that the initial guesses to f, θ and ϕ the auxiliary linear operators L and the non-zero auxiliary parameters h_1, h_2 and h_3 are so properly selected that the deformation $f(\eta;p)$, $\theta(\eta;p)$ and $\phi(\eta;p)$ are smooth enough and their m^{th}-order derivatives with respect to p in Eqs. (34)-(36) exist and are given respectively by:

$$f_m(\eta) = \frac{1}{m!}\left.\frac{\partial^m f(\eta;p)}{\partial p^m}\right|_{p=0} , \quad \theta_m(\eta) = \frac{1}{m!}\left.\frac{\partial^m \theta(\eta;p)}{\partial p^m}\right|_{p=0}$$

$$\varphi_m(\eta) = \frac{1}{m!}\left.\frac{\partial^m \varphi(\eta;p)}{\partial p^m}\right|_{p=0}$$

It is clear that the convergence of Taylor series at $p = 1$ is a prior assumption, whose justification is provided via a theorem [21], so that the system in (34)-(36) holds true. The formulae in (34)-(36) provide us with a direct relationship between the initial guesses and the exact solutions. All the effects of interaction of the chemical reaction as well as of the mass transfer, Soret and Dufour effects and couple stress flow field can be studied from the exact formulas (34)-(36). Moreover, special emphasis should be placed here that the mth-order deformation system (23)-(26) is a linear differential equation system with the auxiliary linear operators L whose fundamental solution is known.

Convergence of the HAM solution

The expressions for f, θ and ϕ contain the auxiliary parameters h_1, h_2 and h_3. As pointed out by Liao [17], the convergence and the rate of approximation for the HAM solution strongly depend on the values of auxiliary parameter h. For this purpose, h-curves are plotted by choosing h_1, h_2 and h_3 in such a manner that the solutions (31)-(33) ensure convergence [17]. Here to see the admissible values of h_1, h_2 and h_3, the h-curves are plotted for 15$^{\text{th}}$-order of approximation in Figures 2-4 by taking the values of the parameters $Pr = 0.71$, $Sc = 0.22$, $Br = 0.5$, $Re = 2$, $Gr = 0.5$, $N = 2$, $S = 0.5$, $D_f = 0.03$ and $S_r = 2.0$. It is clearly noted from Figure 2 that the range for the admissible values of h_1 is $-1.2 < h_1 < -0.4$. From Figure 3, it can be seen that the h-curve has a parallel line segment that corresponds to a region $-1.5 < h_2 < -0.4$. Figure 4 depicts that the admissible value of h_3 are $-1.5 < h_3 < -0.5$. A wide valid zone is evident in these figures ensuring convergence of the series. To choose optimal value of auxiliary parameter, the average residual errors (see Ref. [19] for more details) are defined as:

$$E_{f,m} = \frac{1}{2K} \sum_{i=-K}^{K} \left(N_1\left[\sum_{j=0}^{m} f_j(i\,\Delta t) \right] \right)^2 \qquad (37)$$

$$E_{f,\theta} = \frac{1}{2K} \sum_{i=-K}^{K} \left(N_2\left[\sum_{j=0}^{m} \theta_j(i\,\Delta t) \right] \right)^2 \qquad (38)$$

$$E_{f,\varphi} = \frac{1}{2K} \sum_{i=-K}^{K} \left(N_3\left[\sum_{j=0}^{m} \varphi_j(i\,\Delta t) \right] \right)^2 \qquad (39)$$

where $\Delta t = 1/K$ and $K = 5$. At different order of approximations (m), minimum of average residual errors are shown in Tables 1-3. It is clear from Table 1 that the average residual error for f is minimum at $h_1 = -0.9$. It can be seen from Table 2 that the minimum of average residual error for θ attains at $h_2 = -0.85$. Table 3 depicts that at $h_3 = -0.7$, E_ϕ attains minimum. Therefore, the optimum values of convergence control parameters are taken as $h_1 = -0.9$, $h_2 = -0.85$, $h_3 = -0.7$.

Figure 2. The h curve of $f(\eta)$ when $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$, $K = 0.1$.

The h curve of $\theta(\eta)$ when $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$, $K = 0.1$.

To see the accuracy of the solutions, the residual errors are defined for the system as:

$$RE_f = S^2 f_n^{(iv)} - f_n'' + \operatorname{Re} f_n' - \theta_n - N\varphi_n \qquad (40)$$

$$RE_\theta = \theta_n'' - \operatorname{Re}\operatorname{Pr}\theta_n' + 2\,Br\,Gr^2(f_n')^2 + \\ + S^2 Br\,Gr^2(f_n'')^2 + D_f\operatorname{Pr}\varphi_n'' \qquad (41)$$
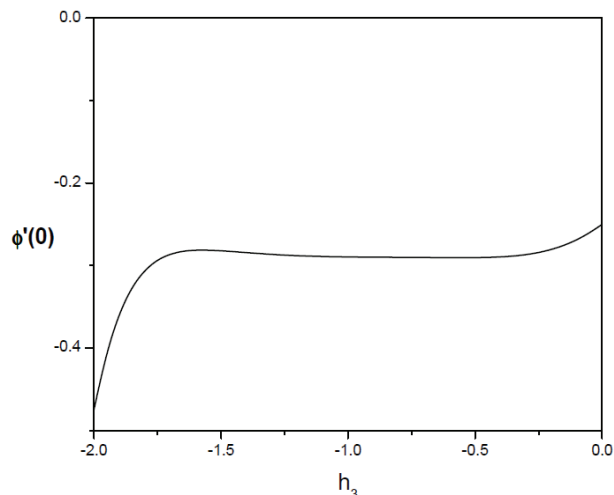

Figure 4. The h curve of $\phi(\eta)$ when $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$, $K = 0.1$.

Table 1. Optimal value of h_1 at different order of approximations

Order (m)	Optimal of h_1	Minimum of E_m
10	-0.89	9.45×10^{-6}
15	-0.9	5.42×10^{-7}
20	-0.9	2.46×10^{-8}

Table 2. Optimal value of h_2 at different order of approximations

Order (m)	Optimal of h_2	Minimum of E_m
10	-0.84	2.14×10^{-6}
15	-0.85	1.74×10^{-7}
20	-0.85	6.61×10^{-8}

Table 3. Optimal value of h_3 at different order of approximations

Order (m)	Optimal of h_3	Minimum of E_m
10	-0.705	5.72×10^{-5}
15	-0.7	2.61×10^{-6}
20	-0.7	5.30×10^{-8}

$$RE_\varphi = \varphi_n'' - \operatorname{Re}Sc\,\varphi_n' + S_r Sc\,\theta_n'' - KSc\varphi_n \qquad (42)$$

where $f_n(\eta)$, $\theta_n(\eta)$ and $\phi_n(\eta)$ are the HAM solution for $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$. For optimality of the convergence control parameters, residual error (Rashidi *et al.*, 2011) for different values of h in the convergence region is displayed in Figures 5-7. We examine that $h_1 = -0.9$, $h_2 = -0.85$, $h_3 = -0.7$ gives a better solution. Table 4 establishes the convergence of the obtained series solution. It is found from the above observations that the series given by Eqs. (31)-(33) converge in the whole region of η when $h_1 = -0.9$, $h_2 = -0.85$, $h_3 = -0.7$.

In order to pursue the convergence of the HAM solutions to the exact ones, the graphs for the ratio (following the recent work of Srinivasacharya and Kaladhar [21]):

$$\beta_f = \left| \frac{f_m(h)}{f_{m-1}(h)} \right|, \; \beta_\theta = \left| \frac{\theta_m(h)}{\theta_{m-1}(h)} \right|, \; \beta_\varphi = \left| \frac{\varphi_m(h)}{\varphi_{m-1}(h)} \right| \qquad (43)$$
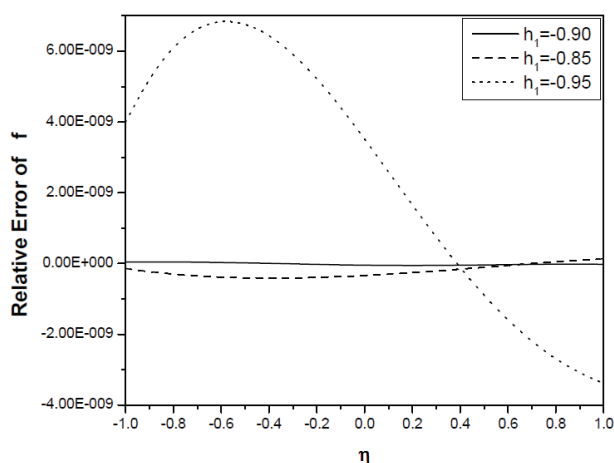
perature and concentration solutions seem to converge in an oscillatory manner requiring more terms in the homotopy series. Thus, the convergence to the exact solution is assured by the HAM method.
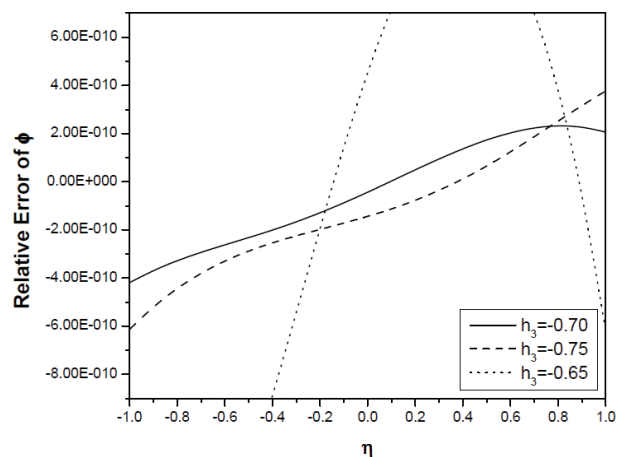

Figure 5. Relative Error of f(η) when $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$, $K = 0.1$.

Figure 7. Relative Error of ϕ(η) when $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$, $K = 0.1$.

Figure 6. Relative Error of θ(η) when $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$, $K = 0.1$.

Against the number of terms m in the homotopy series is presented in Figures 8-10. The figures strongly indicate that a finite limit of β will be attained in the limit of $m \rightarrow \infty$, which will remain less than unity (actually, the figures imply a limit of 0.72, 0.60, 0.61 for f, θ and ϕ respectively). The velocity, tem-
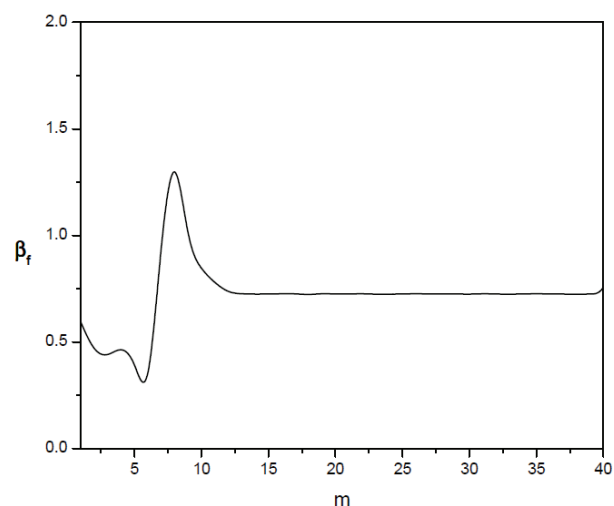

Figure 8. The ratio βf for the convergence of the HAM solutions.

RESULTS AND DISCUSSION

In order to study the effects of couple stress fluid parameter S, Soret number S_r, Dufour number D_f and

Table 4. Convergence of HAM solutions for different order of approximations

Order	f(0)	θ(0)	φ(0)
5	0.3445079570369	0.9387526590286	0.714341787183
10	0.3469289973158	0.9391186279252	0.7185190052065
15	0.3469028512745	0.9391200829032	0.7185007723456
20	0.3469028443006	0.9391200548421	0.7185004129811
30	0.3469028442964	0.9391200546142	0.7185004129787
40	0.3469028442924	0.9391200546125	0.7185004129522
50	0.3469028442924	0.9391200546125	0.7185004129523

chemical reaction parameter K explicitly, computations were carried out by taking $Pr = 0.71$, $Sc = 0.22$, $Br = 0.5$, $Re = 2$, $Gr = 0.5$, $N = 2$. The values of Soret number S_r and Dufour number D_f are chosen in such a way that their product is constant according to their definition provided that the mean temperature T_m is kept constant [9]. These values are used throughout the computations, unless otherwise indicated.
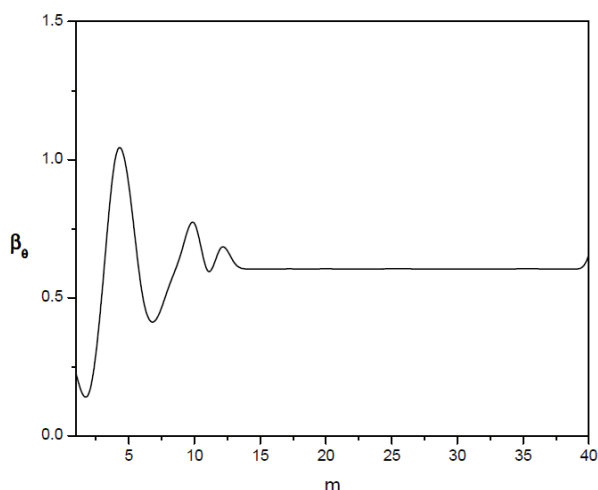

Figure 9. The ratio β_θ for the convergence of the HAM solutions.

Figure 10. The ratio β_ϕ for the convergence of the HAM solutions.

Figure 11 displays the non-dimensional velocity for different values of Soret number S_r and Dufour number D_f, with $S = 1.0$ and $K = 0.1$. It can be observed from this figure that the velocity of the fluid decreases with the decrease of Dufour number (or increase of Soret number). The dimensionless temperature for different values of Soret number S_r and Dufour number D_f with $S = 1.0$ and $K = 0.1$ is shown in Figure 12. It is clear that the temperature of the fluid

decreases with the decrease of Dufour number (or increase of Soret number). Figure 13 demonstrates the dimensionless concentration for different values of Soret number S_r and Dufour number D_f with $S = 1.0$ and $K = 0.1$. It is seen that the concentration of the fluid decreases with increase of Dufour number (or decrease of Soret number).
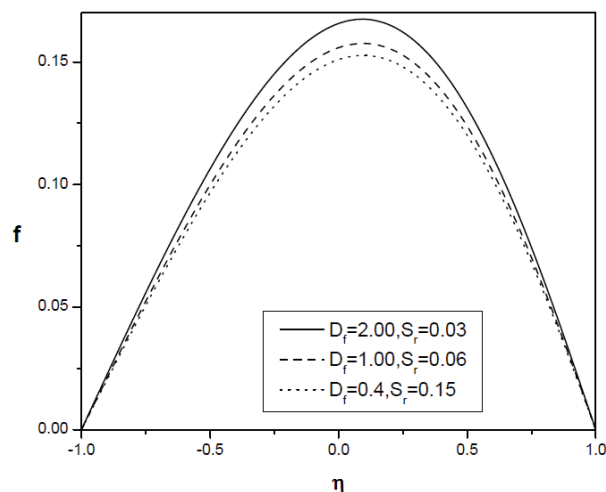

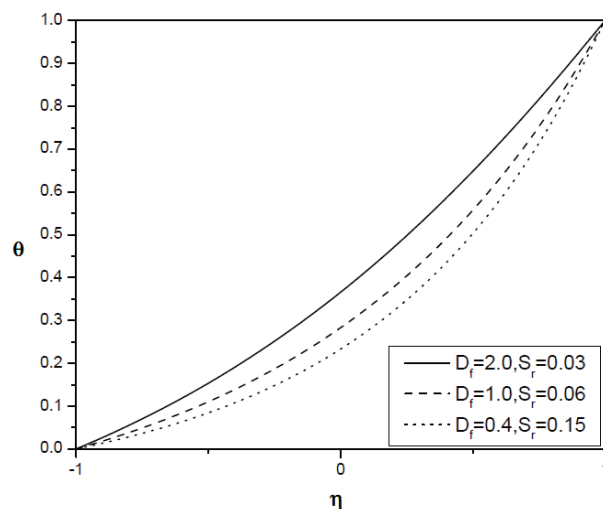Figure 11. Velocity profile for different values of D_f, S_r at $S = 1.0$, $K = 0.1$.

Figure 12. Temperature profile for different values of D_f, S_r at $S = 1.0$, $K = 0.1$.

In Figures 14-16, the effects of the couple stress parameter S on the dimensionless velocity, temperature and concentration profiles are presented for fixed values of $S_r = 2.0$, $D_f = 0.03$ and $K = 0.1$. As S increases, it can be observed from Figure 14 that the maximum velocity decreases in amplitude. This happens because of the rotational field of the velocity generated in couple stress fluid. It is clear from Figure 15 that the temperature decreases with the increase

of couple stress fluid parameter S. It can be seen from Figure 16 that the concentration of the fluid decreases with the increase of couple stress fluid parameter S.
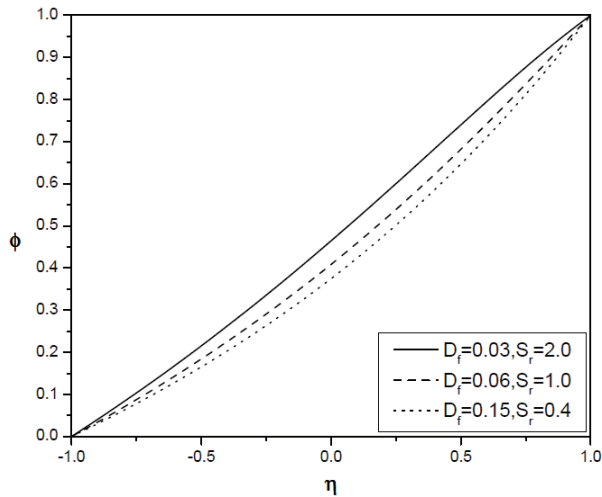

Figure 13. Concentration profile for different values of D_f, S_r at $S = 1.0$, $K = 0.1$.
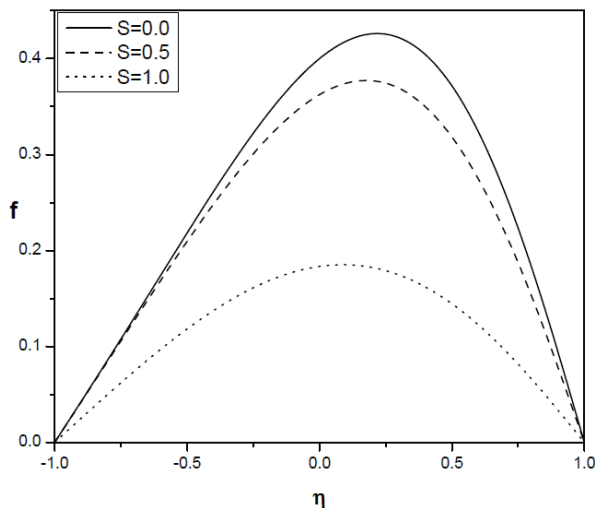

Figure 14. Velocity profile for different values of S at $D_f = 0.03$, $S_r = 2.0$, $K = 0.1$.

Figures 17-19 represent the effect of chemical reaction K on $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$. It can be seen from these figures that the velocity $f(\eta)$ decrease with an increase in the parameter K. The dimensionless temperature decreases as K increases. The concentration $\phi(\eta)$ decreases with an increase in the parameter K. Higher values of K amount to a fall in the chemical molecular diffusivity, *i.e.*, less diffusion. Therefore, they are obtained by species transfer. An increase in K will suppress species concentration. The concentration distribution decreases at all points of the flow field with the increase in the reaction parameter. This shows that heavier diffusing species have greater

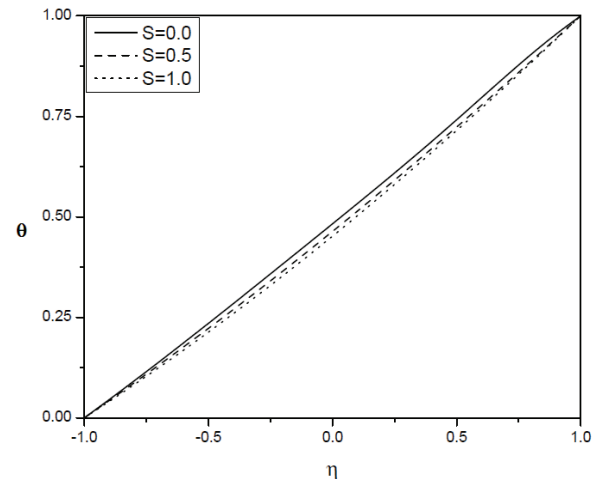retarding effect on the concentration distribution of the flow field.

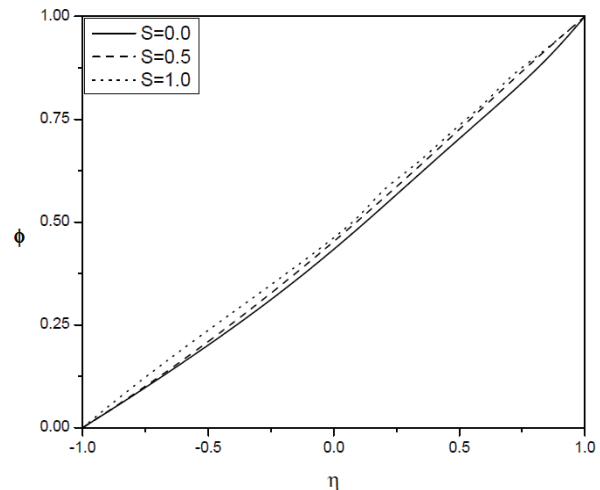Figure 15. Temperature profile for different values of S at $D_f = 0.03$, $S_r = 2.0$, $K = 0.1$.

Figure 16. Concentration profile for different values of S at $D_f = 0.03$, $S_r = 2.0$ $K=0.1$.
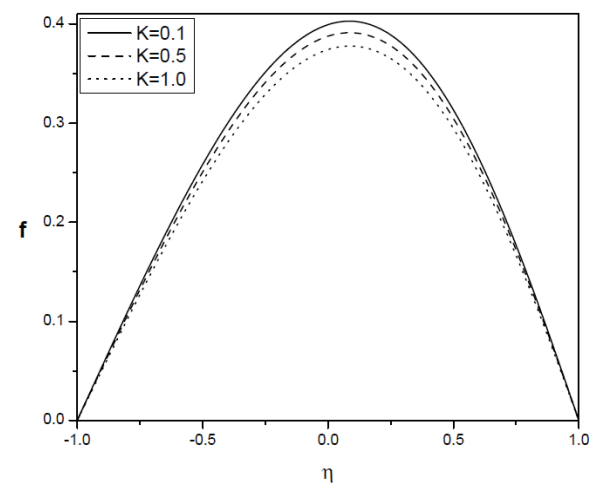

Figure 17. Velocity profile for different values of K at $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$.
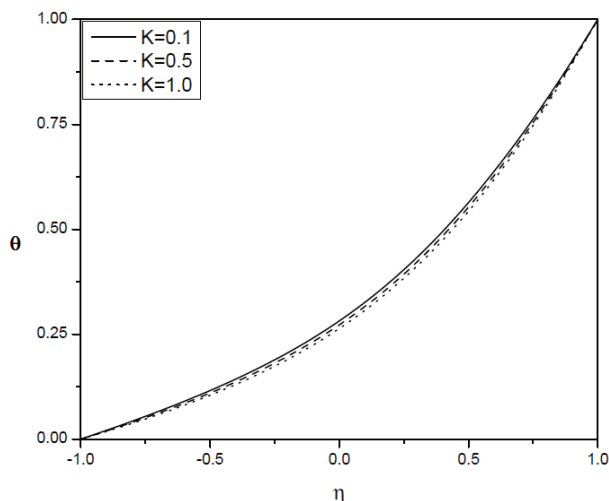
Figure 18. Temperature profile for different values of K at $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$.
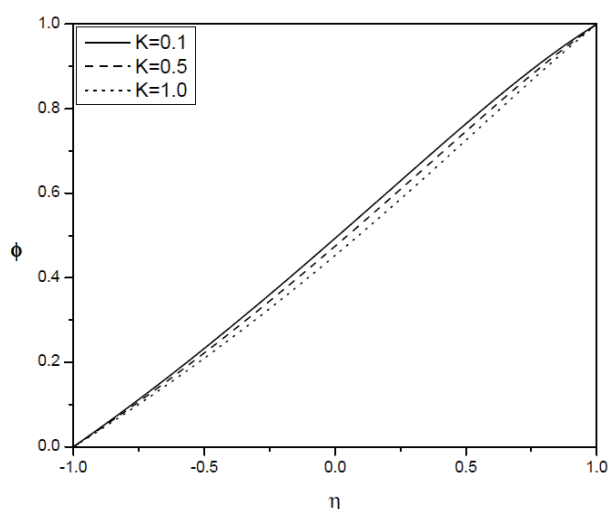

Figure 19. Concentration profile for different values of K at $D_f = 0.03$, $S_r = 2.0$, $S = 1.0$.

CONCLUSIONS

In this paper, the Dufour and Soret effects on steady free convection of a couple stress fluid flowing through a vertical channel with heat and mass transfer has been studied. Using similarity transformations, the governing equations have been transformed into non-linear ordinary differential equations. The approximate analytical series solutions are obtained applying homotopy analysis method (HAM). From the present study we observe that:

1. The velocity and the dimensionless temperature of the fluid decreases with the decrease of Dufour number (or increase of Soret number) and with increase of Dufour number (or decrease of Soret number) the concentration of the fluid decreases.

2. The presence of couple stresses in the fluid decreases the velocity and temperature and increases the concentration.

3. The velocity, temperature and concentration decrease with the increase in the reaction parameter.

Nomenclature

Br	Brinkman number
C	Concentration
C_p	Specific heat at constant pressure
C_S	Concentration susceptibility
D_f	Dufour number
D	Solutal diffusivity
f	Reduced stream function
g	Acceleration due to gravity
Gr	Grashof number
k_1	Rate of chemical reaction
K	Chemical reaction parameter
K_T	Thermal diffusion ratio
Pr	Prandtl number
Re	Reynolds number
S	Couple stress parameter
Sc	Schmidt number
S_r	Soret number
T	Temperature
T_m	Mean fluid temperature
u, v	Velocity components in the x and y directions, respectively
x, y	Cartesian coordinates along the plate and normal to it.

Greek letters

α	Thermal diffusivity
β_T, β_C	Coefficients of thermal and solutal expansion
η	Similarity variable
η_1	The coupling material constant
θ	Dimensionless temperature
ϕ	Dimensionless concentration
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Density of the fluid.

Superscript

$'$	Differentiation with respect to.

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NAUČNI RAD

SORETOVI I DUFOUROVI UTICAJI NA PRIRODNO KONVEKTIVNO STRUJANJE FLUIDA SA NAPONSKIM SPREGOM U VERTIKALNOM KANALU SA HEMIJSKOM REAKCIJOM

Prikazani su Soretovi i Dufourovi uticaji na prenos toplote i mase prirodnom konvekcijom u prisustvu hemijske reakcije u vertikalnom kanalu koji formiraju dve vertikalne paralelne ploče za fluid sa naponskim spregom. Odgovarajuće nelinearne parcijalne diferencijalne jednačine su transformisane u sistem običnih diferencijalnih jednačina primenom transformacija sličnosti. Rezultujuće jednačine su zatim rešene pomoću metode homotopne analize (HAM). Profili bezdimenzionalnih brzine temperature i koncentracije su prikazani grafički za različite vrednosti Fufourovog broja, Sretovog broja, parametra naposnkog sprega i parametra hemijske reakcije.

Ključne reči: prirodna konvekcija, fluid sa naponskim spregom, Soretov i Dufourov efekat, hemijska reakcija, HAM.