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A Resonance Model of Quasi-Periodic Oscillations of Low-Mass X-Ray Binaries

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Abstract

We try to understand the quasi-periodic oscillations (QPOs) in low-mass neutron-star and black-hole X-ray binaries by a resonance model in warped disks with precession. Our main concern is high-frequency QPOs, hectohertz QPOs, and horizontal-branch QPOs in the z sources and the atoll sources, and the corresponding QPOs in black-hole X-ray binaries. Our resonance model can qualitatively, but systematically, explain these QPOs by regarding hectohertz QPOs as a precession of warp.

Key words: accretion, accrection disks — hectohertz QPOs — horizontal branch QPOs — kHz QPOs — relativity — resonance — warp — X-rays: stars

1. Introduction

Quasi-periodic oscillations (QPOs) have been observed in many low-mass X-ray binaries. They give important clues to understand disk structures as well as to evaluate the mass and spin of the central neutron stars or black holes. Although the mechanism of the QPOs is still under debate, recent observations suggest that they can be attributed to disk oscillations. Observations further suggest that some resonant processes are involved in the mechanism of the QPOs, and many disk oscillation models in this direction have been proposed since Abramowicz and Kluzniak (2001), e.g., Lamb and Miller (2003) and Kluzniak et al. (2004). In previous papers (Kato 2003, 2004a, 2004b) we have proposed that the QPOs are disk oscillations excited on a warped disk by a resonant process. The purpose of this paper is to examine how much the warp model is compatible with observations.

We consider a relativistic warped disk. On the disk we superpose disk oscillations. Then, some of these disk oscillations have resonant interactions with the disk at particular radii through non-linear coupling with the warp (Kato 2003, 2004a, 2004b). When the warp has no precession (this is the case considered in the above papers) the resonances occur at radii where one of the relations of $\kappa = (\sqrt{2}-1)\Omega$, $\kappa = \Omega/2$, or $\kappa = (\sqrt{3}-1)\Omega$ is satisfied, where $\kappa(r)$ is the epicyclic frequency and $\Omega(r)$ is the angular frequency of disk rotation, r being the radius from the disk center. In the case of relativistic Keplerian disks, these radii are, in turn, $r = 3.62\,r_{\rm g}$, $r = 4.0\,r_{\rm g}$, and $r = 6.46\,r_{\rm g}$ (Kato 2003, 2004a), where $r_{\rm g}$ is the Schwarzschild radius, defined by $r_{\rm g} = 2GM/c^2$, M being the mass of the central object. Kato (2004b) subsequently showed that among the resonant oscillations mentioned above, those at $\kappa = \Omega/2$ are excited spontaneously by the resonance process, itself. Kato (2004b) further showed that the high-frequency QPOs in black-hole X-ray binaries, which are usually a pair and have a frequency ratio close to 2 : 3, can be explained by this warped model.

In this paper we extend the warped disk model to the case where the warp has precession. We demonstrate that by this extension the main important characteristics of QPOs in X-ray binaries (neutron-star and black-hole X-ray binaries) can be qualitatively explained. Among three types of resonances, which tend, in the limit of no precession, to $\kappa = (\sqrt{2}-1)\Omega$, $\kappa = \Omega/2$, or $\kappa = (\sqrt{3}-1)\Omega$, we focus our attention in this paper on the middle one, since the resonances in this case spontaneously excite oscillations, as mentioned above.

2. Resonant Oscillations at $\kappa = (\Omega + \omega_{\rm p})/2$ on Warped Disks

Details of the resonance process on warped disks are presented by Kato (2003, 2004a, 2004b) in the case where the warp has no precession. The essence of the resonance processes is the same, even when a warp has precession. We thus present here only an outline. An overview of our nonlinear resonance model in the case where the warp has precession is sketched in figure 1 of Kato (2004c).

We consider geometrically thin disks rotating with angular velocity $\Omega(r)$. The epicyclic frequency on the disk is denoted by $\kappa(r)$. The oscillations on geometrically thin disks are generally classified into g-mode and p-mode oscillations (see, e.g., Kato et al. 1998; Kato 2001). In simplified disks the oscillations are further classified by the set of (m, n), where $m = (0, 1, 2, \ldots)$ is the number of nodes in the azimuthal direction, and $n = (0, 1, 2, \ldots)$ is a number related to nodes in the vertical direction. That is, n represents the number of nodes that u_r (the radial component of velocity associated with oscillations) has in the vertical direction. It is noted, however, that u_z (the vertical component of velocity associated with oscillations) has $(n-1)$ nodes in the vertical direction, and $u_z = 0$ in the case of $n = 0$.

A warp is a global deformation of disks with $m = n = 1$. The warp is assumed to have a precession whose angular frequency is $\omega_{\rm p}$. On a disk deformed by the warp we superpose g-mode oscillations with arbitrary m and n. A g-mode oscillation with frequency ω and (m, n) has a relatively large amplitude, global pattern only around the radius where

$$(\omega - m\Omega)^2 - \kappa^2 = 0 \tag{1}$$

is satisfied. This can be understood if the dispersion relation for local perturbations is considered (e.g., Kato et al. 1998; Kato 2001). That is, the region of $(\omega - m\Omega)^2 - \kappa^2 > 0$ is a evanescent region of the oscillations. In the region where $(\omega - m\Omega)^2$ is smaller than κ^2, on the other hand, the oscillations have very short wavelengths in the radial direction in geometrically thin disks.

A non-linear interaction of this g-mode oscillation with the warp produces an oscillation with $\omega \pm \omega_{\rm p}$, $\tilde{m}$, and $\tilde{n}$, where $\tilde{m} = m \pm 1$ and $\tilde{n} = n \pm 1$ (these oscillations are called hereafter intermediate oscillations). These intermediate oscillations resonantly interact with the disk at the radius where the dispersion relation for these intermediate oscillations is satisfied (see Kato 2004b for detailed discussions). There are two types of resonances. One is resonances that occur through motions in the vertical direction, and the other is those through motions in the radial direction (see Kato 2004a, referred to Paper I). Here, we are interested in resonances in the horizontal direction. The horizontal resonances occur around the radius where

$$(\omega \pm \omega_{\rm p} - \tilde{m}\Omega)^2 - \kappa^2 \sim 0 \tag{2}$$

is satisfied. Combining equations (1) and (2), we find that the resonances occur at the radius of $\kappa = \Omega/2 \pm \omega_{\rm p}/2$ (cf. Paper I). After this resonance the intermediate oscillations feedback to the original oscillations, amplifying or dampening the original oscillations (Kato 2004b). Hereafter, we consider the case of $\kappa = \Omega/2 + \omega_{\rm p}/2$.

A detailed examination shows that when $\tilde{m}$ of the intermediate oscillation is $m - 1$, i.e., $\tilde{m} = m - 1$, the oscillations that resonantly interact with the disk at $\kappa = (\Omega + \omega_{\rm p})/2$ are those satisfying $\omega = m\Omega - \kappa$ at the resonant radius. On the other hand, the oscillations that resonantly interact with the disk at $\kappa = (\Omega + \omega_{\rm p})/2$ are those satisfying $\omega = m\Omega + \kappa$ there, when $\tilde{m} = m + 1$ (see Paper I).

Here, we consider non-axisymmetric oscillations. Among them we are particularly interested in oscillations of a small number of m. Typical ones are $\omega = \Omega - \kappa$ ($m = 1, \tilde{m} = 0$), $\omega = \Omega + \kappa$ ($m = 1, \tilde{m} = 2$), and $\omega = 2\Omega - \kappa$ ($m = 2, \tilde{m} = 1$). As shown below, we identify these oscillations, in turn, to the horizontal branch QPOs, upper-frequency kHz QPOs, and lower-frequency kHz QPOs in the case of z sources. Considering this, we introduce the following notations:

$$\omega_{\rm H} = \Omega + \kappa, \quad \omega_{\rm L} = 2\Omega - \kappa, \quad \omega_{\rm HBO} = \Omega - \kappa. \tag{3}$$

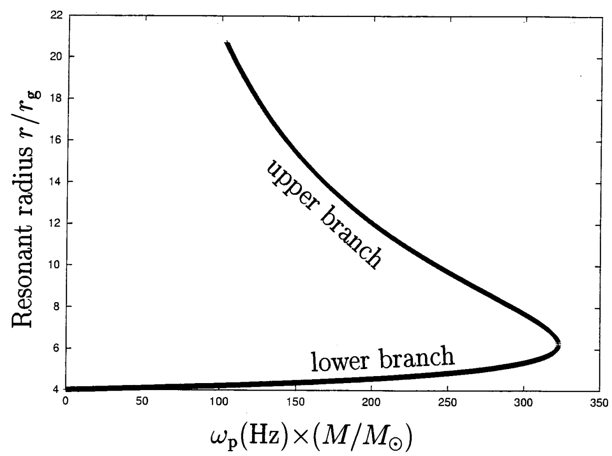

Fig. 1. The r–$\omega_{\rm p}$ relation giving the solution of the resonance condition $\kappa = (\Omega + \omega_{\rm p})/2$. The disk is Keplerian in the Schwarzschild metric.

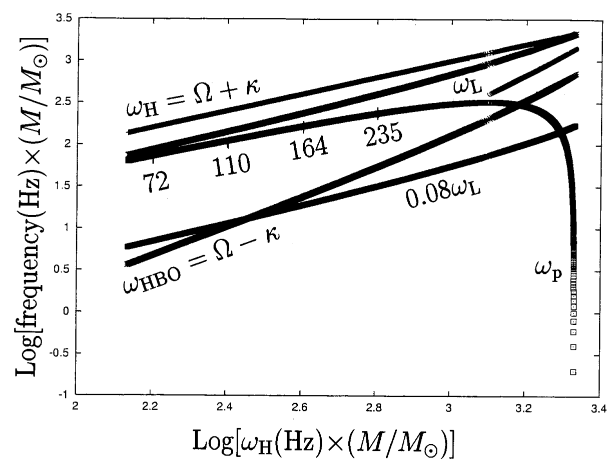

Fig. 2. The $\omega_{\rm L}$–$\omega_{\rm H}$, $\omega_{\rm HBO}$–$\omega_{\rm H}$, and $\omega_{\rm p}$–$\omega_{\rm H}$ relations obtained from the resonance condition $\kappa = (\Omega + \omega_{\rm p})/2$ for some frequency range of $\omega_{\rm H}$. For a comparison, the line of $\omega_{\rm H}$–$\omega_{\rm H}$ is shown and the value of $\omega_{\rm p}$ along the $\omega_{\rm p}$–$\omega_{\rm H}$ curve is shown at some points. For a comparison, the $0.08\,\omega_{\rm L}$–$\omega_{\rm H}$ relation is shown. The trancated curve on the upper-right corner represents the first harmonic of $\omega_{\rm HBO}$, i.e., it is the $2\omega_{\rm HBO}$–$\omega_{\rm H}$ relation. In order to avoid complexity, the curve is trancated. It is noted that the ratio $\omega_{\rm H}/\omega_{\rm L}$ decreases as $\omega_{\rm H}$ increases. The ratio is ~ 1.5 for $\omega_{\rm H}$ in the middle of the figure, and becomes 1.0 at the right end of the figure, where $\omega_{\rm H}$ is the maximum and $\omega_{\rm p} = 0$. At this right end, the ratio $\omega_{\rm H}$ ($= \omega_{\rm L}$) : $2\omega_{\rm HBO}$: $\omega_{\rm HBO}$ = 3 : 2 : 1. We regard the oscillations of $\omega_{\rm p}$ as hectohertz QPOs.

3. Precession of Warps

We consider a relativistic Keplerian disk with the Schwarzschild metric, and examine the radii where the resonance condition, $\kappa = (\Omega + \omega_{\rm p})/2$, is satisfied as functions of $\omega_{\rm p}$. The condition is satisfied at two different radii when $\omega_{\rm p} > 0$. (In the case of $\omega_{\rm p} = 0$, we have only one radius, i.e., $4.0\,r_{\rm g}$. The other one is ∞.) As $\omega_{\rm p}$ increases, the inner radius becomes larger than $4.0\,r_{\rm g}$, while the other one decreases from infinity. At a certain critical value of $\omega_{\rm p}$, both radii coincide and above the critical value of $\omega_{\rm p}$, there is no solution of $\kappa = (\Omega + \omega_{\rm p})/2$. The results are shown in figure 1. The unit of the abscissa is $\omega_{\rm p}(M/M_\odot)$. The critical value of $\omega_{\rm p}$ is $\sim 325\,$Hz when $M/M_\odot = 1.0$, while $\sim 162\,$Hz when $M/M_\odot = 2.0$. The branch of larger value of r on the r–$\omega_{\rm p}$ plane is hereafter called the upper branch and that of the lower one the lower branch. The frequencies ($\omega_{\rm H}$, $\omega_{\rm L}$, $\omega_{\rm HBO}$) at radii of the lower branch decrease as $\omega_{\rm p}$ increases, since the resonance radius moves outward. On the other hand, the frequencies at radii of the upper branch increase as $\omega_{\rm p}$ increases. To compare with observations, the $\omega_{\rm L}$–$\omega_{\rm H}$, $\omega_{\rm HBO}$–$\omega_{\rm H}$, and $\omega_{\rm p}$–$\omega_{\rm H}$ relations are shown in figure 2, which is free from precession. For a comparison, the straight line of the $\omega_{\rm H}$–$\omega_{\rm H}$ relation is also added. Values of

$\omega_\mathrm{p}(M/M_\odot)$ are shown, for convenience, at some points on the ω_p–ω_H curve. The trancated curve on the upper-right corner is the first harmonic of ω_HBO, i.e., it represents the $2\omega_\mathrm{HBO}$–ω_H relation. The curve extends to the left, but is cut in order to avoid complexity of the figure. It is useful to compare this figure with figure 2.9 of van der Klis (2004). We take the standpoint that the variation of the precession is the cause of time variations of QPO frequencies in a single object.

As the precession frequency changes, the frequencies ω_H, ω_L, and ω_HBO vary along the curves in figure 2. Observations show that the changes of the horizontal branch QPOs and lower-frequency kHz QPOs are correlated so that the former frequencies are ~ 0.08-times the latter ones (Psaltis et al. 1999). Hence, for a comparison, the curve of the $0.08\,\omega_\mathrm{L}$–ω_H relation is shown in figure 2. Figure 2 shows that the curve of the $0.08\,\omega_\mathrm{L}$–ω_H relation crosses the curve of the ω_HBO–ω_H relation at $\omega_\mathrm{H} \sim 320\,(M/M_\odot)^{-1}\,\mathrm{Hz}$, which corresponds to $\omega_\mathrm{p} \sim 115\,(M/M_\odot)^{-1}\,\mathrm{Hz}$.

It is noted that the curves shown in figure 2 are mass-independent, since the axes are normalized by $M/M_\odot$. That is, the results hold in a wide range of frequency by changing the mass.

4. Summary and Discussion

In luminous neutron-star low-mass X-ray binaries (the z sources), we typically have four distinct types of QPOs. These are the $\sim$ 5–20 Hz normal branch oscillation (NBO), the 15–60 Hz horizontal branch oscillation (HBO), and the $\sim$ 200–1200 Hz kilohertz QPOs that typically occur in pairs. The QPOs in the atoll sources (less luminous neutron-star low-mass X-ray binaries) can also be classified into similar types of oscillations. In addition, in the atoll sources the hectohertz QPOs are observed in the frequency range of 100 Hz–200 Hz. In the z sources, however, the presence of hectohertz QPOs is uncertain, or ambiguous. In black-hole X-ray binaries, we have high-frequency QPOs in the range of 100 Hz to 450 Hz, which usually appear in pairs. A comprehensive review on QPOs is presented by van der Klis (2004).

One of important characteristics of QPOs in the z sources is the presence of a strong correlation between kilohertz QPOs and HBOs. That is, the frequency of the lower kHz QPO and that of HBO are correlated in each object so that the former is ~ 0.08-times the latter. This correlation extends from neutron-star to black-hole X-ray binaries in nearly three orders of magnitude in frequency (Psaltis et al. 1999, see figure 2 of their paper). In our resonance model this correlation can be explained if variation of precession frequency occurs around $115\,(M/M_\odot)^{-1}$ Hz, say, $70 \times (M/M_\odot)^{-1}\,\mathrm{Hz} \sim 170\,(M/M_\odot)^{-1}\,\mathrm{Hz}$. A question is whether such a precession has been observed. In relation to this issue, it it interesting to note that the observed hectohertz QPOs are roughly in the frequency range mentioned above. This suggests that the hectohertz QPOs might be a manifestation of the warp. One of the characteristics of the hectohertz QPOs is that their frequencies are nearly constant (e.g., van der Klis 2004). In our resonance model, the relation of $\omega_\mathrm{HBO} \sim 0.08\,\omega_\mathrm{L}$ is realized over in a wide range of ω_H without much changing the value of ω_p, as shown in figure 2. This is consistent with the idea that the warp represents the hectohertz QPOs. The precession of disks in X-ray binaries is theoretically expected, since the radiation force from central star gives torques on warped disks (Pringle 1996; Maloney et al. 1996). It is not clear, however, whether such a high-frequency precession as required here is generally expected. We suppose that the frequency of the precession is related to rotation of the central star. A non-axisymmetric pattern on a rotating stellar surface will give rise to a precession of the disk through the effects of a radiative force or a magnetic field.

Another important characteristic of QPOs in neutron-star X-ray binaries is that the frequency ratio of the pair kHz QPOs is close to 3 : 2, but changes with time so that the ratio decreases with an increase of the frequency. This observational trend is realized in our model. In our model the ratio $\omega_\mathrm{H}/\omega_\mathrm{L}$ is 1.5 at $\omega_\mathrm{H} \sim 700\,(M/M_\odot)^{-1}\,\mathrm{Hz}$, and becomes larger than 1.5 for smaller ω_H. For larger ω_H the ratio tends to unity as the resonant radii approaches to $4.0\,r_\mathrm{g}$ (i.e., $\omega_\mathrm{p} = 0$).

Next, let us consider black-hole X-ray binaries. The pair of high-frequency QPOs in these objects changes little their frequencies, keeping the ratio close to 3 : 2. This is different from the case of neutron-star X-ray binaries. Our resonance model qualitatively explains this. Considering their mass and the observed frequencies of QPOs, we think that in the case of black-hole X-ray binaries the observed QPOs are those resulting from the lower branch of the r–ω_p relation (see figure 1). That is, the resonance radius is close to $4.0\,r_\mathrm{g}$. This case corresponds to the upper-right corner of figure 2. In the limit of $\omega_\mathrm{p} = 0$, the resonance occurs at $r = 4.0\,r_\mathrm{g}$ and the frequency ratio of ω_L and the first harmonics of ω_HBO is just 3 : 2. (It is noted that the first harmonic of ω_HBO has been also observed in the z sources.) These QPOs change little their frequencies for a change of precession frequency, since as shown in figure 1 the resonance radius remains close to $4.0\,r_\mathrm{g}$ for a large change of ω_p. Another explanation of little change of frequencies of high-frequency QPOs in black-hole binaries is that the precession is really small in the case of black holes, since the radiation force from the central object is absent. It is noted that in our model the modes of the pair QPOs in the black-hole binaries are different from those in neutron-star binaries. That is, the pairs in the former are ω_H (= ω_L) and $2\omega_\mathrm{HBO}$, while those in the latter are ω_H and ω_L.

We have restricted our attention only to the resonances at $\kappa = (\Omega + \omega_\mathrm{p})/2$, since the analyses of growth rate of resonant oscillations suggest that the resonances at $\kappa = (\sqrt{2} - 1)\Omega + \omega_\mathrm{p}$ and $\kappa = (\sqrt{3} - 1)\Omega + \omega_\mathrm{p}$ are not spontaneously excited (Kato 2004b). However, examinations of these cases are worth-while. As an example, some results in the case of $\kappa = (\sqrt{2} - 1)\Omega + \omega_\mathrm{p}$ are shown in figure 3. For simplicity, in this figure, only the resonant oscillations resulting from the upper branch of the r–ω_p curve are shown. The frequency of precession required to obtain $\omega_\mathrm{HBO} \sim 0.08\,\omega_\mathrm{L}$ is around $50\,(M/M_\odot)^{-1}\,\mathrm{Hz}$–$100\,(M/M_\odot)^{-1}\,\mathrm{Hz}$.

Our model predicts that some QPOs that are not yet observed are present in neutron-star and black-hole X-ray binaries. In our model the observed QPOs in neutron-star X-ray binaries are the resonance oscillations at resonant radii belonging to the upper branch of the r–ω_p diagram (figure 1). Resonances

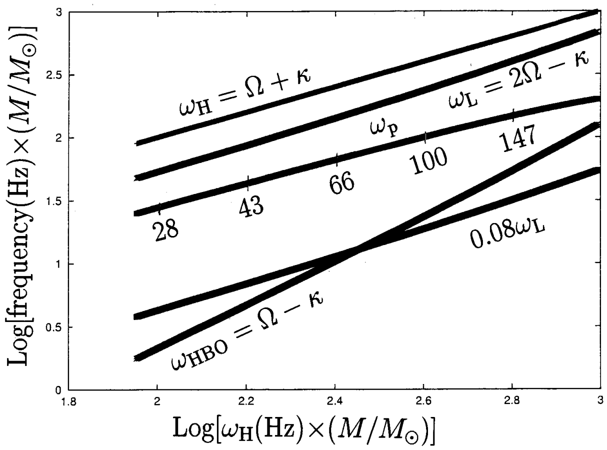

Fig. 3. Same as figure 2, except for $\kappa = (\sqrt{2} - 1)\Omega + \omega_p$. In this figure, for simplicity, only the resonant oscillations that occur on the upper branch of the r–ω_p plane are shown.

coming from a resonance radius of the lower branch are also expected. They have higher frequencies compared with the observed ones. The observed QPOs in black-hole X-ray binaries, on the other hand, are interpreted to be resonant oscillations belonging to the lower branch of the r–ω_p relation. Resonant oscillations resulting from radii of the upper branch are also expected. These oscillations have lower frequencies compared with the observed ones.

Note added on April 30:

In the present paper we have considered horizontal resonances of g-mode oscillations. In this model, the precession required is retrograde. Furthermore, time variation of the precession is required in order to explain time variation of QPO frequencies. In the case of vertical resonances of g-mode oscillations, however, the observed QPO frequencies and their time variations can be explained as a result of time change of disk structure in the vertical direction, without appealing to precession. This is discussed in a subsequent paper.

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