Dyna
ISSN: 0012-7353
dyna@unalmed.edu.co
Universidad Nacional de Colombia
Colombia

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Dyna, vol. 81, núm. 183, febrero, 2014, pp. 188-198
Universidad Nacional de Colombia
Medellín, Colombia

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ANÁLISIS DE PLANES DE MUESTREO CSP-1 CONSIDERANDO SISTEMAS DE INSPECCIÓN CON Y SIN ERROR DE MUESTREO

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Received for review April 10th, 2013, accepted July 29th, 2013, final version August, 21st, 2013

ABSTRACT: In this paper, we discuss the implementation of Continuous Sampling Plan (CSP)-1 under two scenarios: (i) infallible, and (ii) fallible inspection systems. For both cases, we develop an optimization model for designing a CSP-1 that minimizes the total expected cost. We use Markov theory to derive the expected results from the application of the CSP-1. A Bayesian approach is used to model the inspection system reliability. Based on the analyses for the two models, we offer a discussion on the adverse effects of disregarding inspection errors when implementing CSP-1.

Key words: CSP-1, quality, inspection sampling plan, Bayes, Markov, simulation, inspection error, optimization

RESUMEN: En el presente artículo, presentamos un análisis de las implicaciones relacionadas con ignorar errores de inspección cuando se implementa un plan de muestreo continuo del primer tipo (CSP-1 por sus siglas en inglés). Nuestro análisis cubre dos escenarios: (1) inspección perfecta o infalible, e (2) inspección imperfecta o falible. Para cada caso, presentamos un correspondiente modelo de optimización cuyo objetivo es el de minimizar el valor esperado del costo total. El comportamiento de los planes CSP-1 es modelado utilizando teoría Markoviana, mientras que la confiabilidad de los sistemas de inspección es modelada mediante un análisis Bayesiano. Las soluciones de ambos modelos son confrontadas para establecer comparaciones entre los dos escenarios.

Palabras clave: Calidad Saliente Promedio (CSP), Planes de Muestreo, Teorema de Bayes, Cadenas de Markov, Simulación, Error Muestral, optimización

1. INTRODUCTION

Quality is one of the most important factors considered by customers at the moment of selecting their suppliers. In an ideal world, customers would prefer suppliers whose products were absolutely perfect. However, in reality, customers usually agree to tolerate a certain proportion of defective units. Then, the task for the suppliers is to implement inspection policies that guarantee that the average outgoing quality level (AOQL) of their product, does not exceed a certain value, based on the customer's expectations regarding the acceptable proportion of defective units. An inspection policy can be defined as inspecting 100% of the products. However, due to financial limitations, a 100% inspection is not always viable. An alternative is to implement sampling inspection plans (SIPs), in which only a fraction of the total products is inspected. The parameters involved in SIPs are often selected in such a way that the total expected cost is minimized [1-4]. An important assumption when computing the expected cost of a SIP is that related to the efficiency or reliability of the inspection system. Such inspection systems can be human inspectors or machines (from now on we will use indifferently the terms "inspector" and "inspection system" to refer to both human and machine-based inspection systems). In both cases, the occurrence of inspection errors is inevitable. There are two types of inspection errors: Type I, which refers to classifying a defective unit as non-defective; and Type II, which refers to classifying a non-defective unit as defective. Disregarding these errors, i.e. assuming perfect performance of the inspector, is unrealistic and it can lead to inaccuracies in the computations of the expected SIP cost and performance.

In this paper, we use Markovian and Bayesian analysis to develop two optimization models for designing a

SIP of the type CSP-1 [5], which applies for products that are manufactured through a continuous process. The implementation of CSP-1 can be summarized as follows:

1. Initially, inspect 100% of the units until *i* consecutive units are found as non-defective.

2. Once *i* consecutive non-defective units have been inspected, discontinue the 100% inspection and start to systematically inspect only a fraction *f* of the units. The fractional inspection continues as long as the inspected units are non-defective. If a defective unit is found, reestablish 100% inspection, i.e. return to step 1.

In our first model, we consider the minimization of expected costs assuming perfect performance of the inspector. This initial model sets the path for developing a more realistic model in which our objective is to minimize the expected costs when inspection errors are taken into consideration. Based on the results obtained for such optimization models, we offer a discussion on the effects of disregarding inspection errors in CSP-1.

The remaining of the paper is organized as follows: Section 2 contains our literature review. In section 3, we state the description of the problem, as well as the corresponding notation and assumptions. Section 4 presents an optimization model for CSP-1 when disregarding inspection errors, whereas section 5 contains the optimization model when taking into consideration such errors. In section 6, we discuss the economic impact of disregarding inspection errors, based on the results obtained on sections 4 and 5. In section 7 we extend our discussion by including the transition costs between total and partial inspection. Finally in section 8 we present some remarks, conclusions and future research directions.

2. LITERATURE REVIEW

In the literature, we can find some papers that analyze the effect of inspection errors in SIPs, such as that offered by [6], who use a Bayesian model to evaluate the efficiency of inspectors. Another interesting work that uses a Bayesian approach is that given by [7]. In this latter paper, the authors consider a type of SIP that accounts for the number of defects in each inspected product. The authors address the problem of analyzing the best *a priori* distribution to model the number of defects per unit under the presence of inspection errors. Another interesting paper that also considers inspection errors is that offered by [8] which establishes a set of results for matching Dodge-Romig single plans with Dodge-Romig plans under the presence of inspection errors.

As mentioned in the previous section, the design of SIPs is often subject to economic criteria such as in [9]. In that paper, the authors offer a mathematical model to design both 100% and single sampling plans considering potential inspection errors, while minimizing a loss function that accounts for deviations of quality characteristics from a certain target value. An earlier related work is offered by [10] in which the authors develop a model for locating inspection stations in an *n*-stage production system. The optimization criterion used in [10] is the cost per good unit accepted by the customer. Reference [11] also studies the impact of inspection error from an economical perspective. More specifically, in [11] the author develops a mathematical model and an algorithm for designing a SIP under inspection errors, while minimizing the expected associated cost. Reference [12] offers a model for minimizing inspection costs while imposing upper bounds on the inspection errors.

One fundamental difference between our paper and those mentioned above, is that they do not address CSP-1 in particular. Two papers that specifically investigate CSP-1 under inspection errors from an economic point of view are those given by [4], and [3]. In the first one, the authors present an optimal mixed policy of precise inspection and CSP-1 under the presence of inspection errors and return cost. In the second, the authors develop a model using a renewal reward process approach for selecting an economically optimal decision involving three alternatives: "do 100% inspection", "do not inspect" and "do a CSP-1 inspection". Their model accounts for both types of inspection errors. A key difference between the work given by [3] and ours is that we assume that as long as the customer's acceptable quality level (AQL) is satisfied, there is no penalization for defective units delivered to the customer. With this reasonable assumption, our optimization models are simplified to finding the optimal number of inspected units as the basis for designing an optimal CSP-1, instead on focusing on finding directly the parameters of the inspection plan (see sections 4 and 5).

Also, one of the main contributions of our work is that it highlights the impact of disregarding inspection errors when implementing CSP-1, which allows visualizing the importance of recognizing and measuring inspection errors. Similar analyses have been made for attribute SIPs, [13-17], but to our knowledge, this issue has not been yet addressed from the perspective of a supplier that implements CSP-1.

3. PROBLEM DESCRIPTION, NOTATION AND ASSUMPTIONS

In this section we present the description of our problem, as well as the corresponding notation and assumptions.

3.1. Problem Description

We focus our analysis on a supplier company that uses a continuous production scheme. The production process has an inherent defective fraction, which is greater than the AQL specified by the customer. In order to comply with the customer's AQL, the supplier implements an inspection plan of the type CSP-1 that guarantees that the AOQL, i.e. the expected proportion of defective units that are delivered to the customers, is lower than or equal to the customer's AQL. (The implementation of CSP-1 is as described in the Introduction).

The only two parameters involved in CSP-1 are *i* and *f*. The values for these two parameters are specified such that the resulting AOQL does not exceed the customer's AQL. Given the defective fraction of the process, different combinations of *i* and *f* can be used to achieve a desired AQL.

As mentioned before, the inspector participating in CSP-1 is often assumed to be infallible. However, in practice, inspection errors are likely to occur. In fact, according to the Second Law of Thermodynamics [18] and the Principle of Uncertainty of Heisenberg [19], it is not possible to have perfect inspectors, and if they were perfect, it would be impossible to prove it. Our objective is to perform an analysis of the implementation of CSP-1 under two scenarios: (1) assuming infallible inspection systems; and (2) considering the presence of inspection errors. We then study the impact of disregarding such inspection errors.

3.2. Assumptions

Our assumptions can be summarized as follows:

- The value of the defective fraction of the production process is deterministic and known.
- Throughout CSP-1, every rejected unit must be replaced by an acceptable one, according to the inspector criteria.
- Additional units produced to replace rejected ones, are also inspected.
- As long as the AOQL delivered by the company is at most equal to the customer's AQL, there is no penalization for defective units received by customers.
- The purpose of the supplier company is to design a CSP-1 that guarantees an AOQL lower than or equal to the customer's AQL.
- For simplicity, we perform our computations for a shipment of Q units delivered to the customer.

3.3. Notation

- θ_2 : event of a unit being defective
- θ_1 : event of a unit being non-defective
- $P(\theta_j)$: probability of occurrence of event θ_j, for $j = 1,2$. Note that $P(\theta_2)$ is the fraction defective of the process, and that $P(\theta_1) = 1 - P(\theta_2)$
- AQL : acceptable quality level specified by the customer. We assume that $AQL < P(\theta_2)$. Otherwise, there would be no need to implement CSP-1.
- $AOQL_A$: average outgoing quality level for a CSP-1 under infallible inspection systems
- $AOQL_B$: average outgoing quality level for a CSP-1 under inspection errors
- Q : size of the production batch to be analyzed

- S_1 : event of the inspection system classifying a unit as non-defective
- S_2 : event of the inspection system classifying a unit as defective
- $P(S_k)$: probability of occurrence of event S_k for $j=1,2$. Note that $P(S_1)=1-P(S_2)$
- U_A : expected number of inspected units for a CSP-1 under infallible inspection systems
- U_B : expected number of inspected units for a CSP-1 under fallible inspection systems
- c_s : cost of inspecting one unit
- c_r : cost of rejecting one defective unit

4. ECONOMIC ANALYSIS FOR CSP-1 UNDER IDEAL INSPECTION PROCEDURES

In this section we present a discussion on the optimal expected cost of implementing CSP-1 under ideal conditions, i.e. infallible inspection systems.

As mentioned before, for CSP-1 to be useful, the AOQL must be at most equal to the consumer's AQL. A way to assign values to the parameters involved in the CSP-1 is by using Markovian analysis, in which CSP1- procedure is considered as an ergodic Markov chain. Two Markovian states are identified: (1) 100% inspection and (2) systematic inspection [18]. When analyzing CSP-1 as a Markov chain, the transition matrix would be as depicted in Table 1 [20]. Based on Table 1, we can define the steady-state probabilities $x^*_{100\%}$ and x^*_f respectively for the states of 100% and fractional inspection [20]:

$$x^*_{100\%}=\frac{P(\theta_2)}{P(\theta_2)+P(\theta_1)^i} \tag{1}$$

$$x^*_f=\frac{P(\theta_1)^i}{P(\theta_2)+P(\theta_1)^i} \tag{2}$$

Recall that Q represents the amount of products being inspected. Then, the expected number of inspected units in CSP-1 is given by $Q\left[x^*_{100\%}+fx^*_f\right]$ [20], which can be expressed as follows:

$$U_A=Q\frac{P(\theta_2)+fP(\theta_1)^i}{P(\theta_2)+P(\theta_1)^i}, \tag{3}$$

Table 1. Transition Matrix for CSP-1

	100% Inspection	Fractional Inspection
100% Inspection	$1-P(\theta_1)^i$	$P(\theta_1)^i$
Fractional Inspection	$P(\theta_2)$	$P(\theta_1)$

The total expected number of rejected units would be equal to the number of detected defective units, i.e. $U_AP(\theta_2)$. The AOQL would be equal to proportion of the undetected defective units, which can be expressed as a function of U_A as follows [20]:

$$AOQL_A(U_A)=\frac{[Q-U_A]P(\theta_2)}{Q} \tag{4}$$

Two types of costs must be considered for the implementation of CSP-1: inspection cost, and defective units cost. The expected inspection cost would be c_sU_A, where U_A is given by expression (3).

On the other hand, the cost due to defective units involves the replacement of defective detected units by non-defective ones. This implies producing and inspecting additional units in order to replace the defective ones. Note that due to the defective fraction of our process, for producing an amount of X non-defective units, the expected number of units to be produced would be $\frac{X}{P(\theta_1)}$, since we should expect $XP(\theta_2)$ units to be defective. Let us denote c_p as the cost of manufacturing one unit of product. Then, the expected cost due to defective units would be $\left[c_p+c_s\right]\frac{U_AP(\theta_2)}{P(\theta_1)}$. Additionally, we need to include the cost of rejecting each defective unit, denoted by c_r.

Then the total expected cost for CSP-1, can be expressed in terms of U_A as:

$$C_A\left(U_A\right)=U_A \left[c_s+\frac{c_p P\left(\theta_2\right)}{P\left(\theta_1\right)}+\frac{c_s}{P\left(\theta_1\right)}+c_r P\left(\theta_2\right)\right] \tag{5}$$

The expected cost for CSP-1 under infallible inspection systems, given by equation (5), has U_A as its only variable. The minimization of (5) is restricted by the fact that the AOQL given in (4) must be at most equal to the customer's AQL. Then our optimization model can be expressed as:

Minimize (5) subject to:

$$\left[1-\frac{U_A}{Q}\right]P\left(\theta_2\right)\leq AQL \tag{6}$$

We have that the AOQL is a decrement function of the proportion of inspected units. On the other hand, by inspecting (5) we notice that the expected cost is an increasing function of U_A. Therefore, the optimal, value of U_A, i.e. U_A^*, is the minimum U_A that allows complying the constraint given by (6). Any value lower than U_A^* would imply an AOQL greater than the customer's AQL, whereas a value greater than U_A^* would not be economically optimal. Then, U_A^* can be computed as follows:

$$U_A^*=U_A \mid \left[1-\frac{U_A}{Q}\right]P\left(\theta_2\right)=AQL \tag{7}$$

When solving (7) we obtain:

$$U_A^*=\left[P\left(\theta_2\right)-AQL\right]\frac{Q}{P\left(\theta_2\right)} \tag{8}$$

Having determined the value of U_A^*, it is possible to compute the parameters of CSP-1 using equation (3) which relates $U_A, P\left(\theta_2\right), f, i$ and Q, where $U_A, P\left(\theta_2\right)$ and Q would be given. We can specify the value of one of the parameters and then solve for the other, based on the desired value of the AOQL. In this case, it is simpler to select a value of i and then solve for f, as follows:

$$f=1-\frac{AOQL\left[P\left(\theta_2\right)+P\left(\theta_1\right)^i\right]}{P\left(\theta_2\right)P\left(\theta_1\right)^i} \tag{9}$$

At this point, we would like to highlight two properties regarding the optimal expected cost and the optimal expected number of inspected units for CSP-1 under infallible inspection. First we have that expression (8) can be stated as $U_A^*=Q-\frac{\left[AQL\right]Q}{P\left(\theta_2\right)}$. Therefore, we can conclude that U_A^* increases with $P\left(\theta_2\right)$. This is a reasonable result since, intuitively, we would expect to have to inspect more units to guarantee a certain AQL for greater defective fractions. Second, by inspecting expression (5) we can notice that the optimal expected cost increases with $P\left(\theta_2\right)$. Again, this finding is also reasonable since the inspection, replacement and rejection costs increases with the defective fraction of the process.

5. EXPECTED COST FOR CSP-1 UNDER INSPECTION ERRORS

In the ideal case discussed in the previous section, an underlying assumption is that the inspection procedure is infallible. Therefore, it is assumed that whenever an inspected unit is classified as non-defective, such a unit is actually non-defective (an analogous analysis applies for units that are classified as defective). However, in practice we would expect inspection systems not to be infallible

When we drop the infallibility assumption, the transition matrix given in Table 1 does no longer apply for CSP-1, since the probability of an inspected unit being classified as defective is not equal to the actual defective fraction of the process. Also, recall that S_2 is the event of the inspection system classifying a unit as defective, and S_1 as non-defective. Anytime an event S_1 occurs, it means that the inspected unit has been accepted, whereas S_2 implies the rejection of the inspected unit. In our model we consider two types of conditional probabilities:

- Validity: $P\left(S_i \mid \theta_j\right) for\, i=1,2, j=1,2$ gives the probability that the inspector makes a decision S_i given that the actual status of the inspected unit is θ_j. Note that such conditional probabilities are inherent to the inspector.
- Prediction: $P\left(\theta_j \mid S_k\right) for\, j=1,2, k=1,2$ gives the probability that an inspected unit is θ_j, given that the inspector has classified it as S_k.

The two types of inspection errors discussed in the Introduction section can be now defined in terms of the conditional probabilities stated above, as follows:

$$P_I = P(Error\,Type\,I) = P(S_2 \mid \theta_1) \tag{10}$$

$$P_{II} = P(Error\,Type\,II) = P(S_1 \mid \theta_2) \tag{11}$$

By applying Bayes theorem and using the conditional probabilities discussed above, we can compute the probability of a unit being classified as defective, $P(S_2)$, as follows:

$$P(S_2) = P(S_2 \mid \theta_2)P(\theta_2) + P(S_2 \mid \theta_1)P(\theta_1) \tag{12}$$

In a similar way, we can compute $P(S_1)$. Then, the transition matrix for a CSP-1, when considering inspection errors, would be as shown in Table 1, but replacing θ_j by S_j. We can perform a statistical analysis similar to that offered in section 3.1 to compute an expression for the expected number of inspected units (U_B), and the corresponding AOQL ($AOQL_B$). For U_B we have:

$$U_B = \frac{P(S_2) + fP(S_1)^i}{P(S_2) + P(S_1)^i} Q \tag{13}$$

To find the AOQL, we need to compute the expected proportion of defective units that would be delivered to the customers. First, we have the expected number of defective units that are inspected, and that are accepted due to error type II, i.e. $U_B P(\theta_2|S_1)P(S_1)$. Also, when performing the inspection, some units will be rejected and additional units would be produced and inspected to replace the rejected ones, which would be equal to $\frac{U_B P(S_2)}{P(S_1)}$. Among these additional units, we will also have defective units that are erroneously classified as non-defective and are sent to the customers, i.e. $\frac{U_B P(S_2)}{P(S_1)} P(\theta_2|S_1)P(S_1)$ or simply, $U_B P(S_2)P(\theta_2|S_1)$. Additionally, there will be some defective units delivered to the customers which come from those units that are never inspected during the fractional inspection. Therefore, the AOQL in this case can be expressed as:

$$AOQL_B(U_B) = \frac{U_B P(\theta_2|S_1)P(S_1) + U_B P(S_2)P(\theta_2|S_1)}{Q} + \frac{(Q - U_B)P(\theta_2)}{Q} \tag{14}$$

which can be simplified as follows:

$$AOQL_B(U_B) = \frac{QP(\theta_2) + U_{i_2}\left[P(\theta_2|S_1) - P(\theta_2)\right]}{Q} \tag{15}$$

Regarding the expected cost for CSP-1 under inspection errors, we have four different components: (1) cost of inspecting units, (2) cost of replacing rejected units, (3) cost due to rejecting defective units, and (4) opportunity cost due to inspection error Type I. The first cost is simply $c_s U_B$. For the expected cost of replacing rejected units we have that, as discussed in section 3.1, we need to produce and inspect additional units to replace those that have been rejected. Note that a proportion equal to $P(S_2)$ of such additional units would be also rejected. Then, the expected number of additional units that we must produce and inspect to replace $U_B P(S_2)$ rejected units would be $\frac{U_B P(S_2)}{1 - P(S_2)}$.

Therefore, the second component of the expected cost for CSP-1 under inspection errors would be $\left[c_s + c_p\right]\frac{U_B P(S_2)}{P(S_1)}$. The expected costs related to rejecting defective units would be simply $c_r BP(\theta_2|S_2)P(S_2)$.

Finally, let us denote as c_I the cost of erroneously classifying a non-defective unit as defective (Type I error). Then, the expected cost due to a Type I error would be $U_B c_I P(\theta_1|S_2)P(S_2)$. Recall that we have assumed that defective units sent to the clients do not generate an extra cost as long as the AOQL is lower than or equal to the customer's AQL. Therefore, we do not introduce any cost due to errors Type II. Then, the total expected cost of implementing CSP-1 under inspection errors is given by equation (16).

$$C_B\left(U_B\right)=U_B \left[\begin{array}{l} c_s+c_s\dfrac{P\left(S_2\right)}{P\left(S_1\right)}+c_p\dfrac{P\left(S_2\right)}{P\left(S_1\right)} \\ +c_I P\left(\theta_1|S_2\right)P\left(S_2\right)+c_r P\left(S_2\right) \end{array}\right] \tag{16}$$

The expected cost for CSP-1 given by equation (16), has U_B as its only variable. The minimization of (16) is restricted by the fact that the AOQL given in (14) must be at most equal to the customer's AQL. Then our optimization model can be expressed as:

Minimize (16) subject to:

$$\frac{QP\left(\theta_2\right)+U_B\left[P\left(\theta_2|S_1\right)-P\left(\theta_2\right)\right]}{Q}\leq AQL \tag{17}$$

Note that by assuming that $P\left(\theta_2\right)>P\left(\theta_2|S_1\right)$, the AOQL is a decrement function of the proportion of inspected units. This is a reasonable assumption, since the role of the inspector is to reduce the number of defective units delivered to the customers (otherwise, it would be better not to use the inspector). On the other hand, the expected cost is an increasing function of U_B. Therefore, we can proceed as before to find the optimal value of U_B, i.e. U_B^*, which would be the minimum U_B that allows complying the constraint given by (17). Then, U_B^* can be computed as follows:

$$U_B^*=U_B \left| \frac{QP\left(\theta_2\right)+U_B\left[P\left(\theta_2|S_1\right)-P\left(\theta_2\right)\right]}{Q} = AQL \right. \tag{18}$$

Then, we have:

$$U_B^*=\frac{\left[P\left(\theta_2\right)-AQL\right]Q}{P\left(\theta_2\right)-P\left(\theta_2|\ S_1\right)} \tag{19}$$

Having determined the value of U_B^*, it is possible to compute the parameters of CSP-1 using equation (13). To do so, we can specify the value of one of the parameters and then solve for the other, based on the desired value of the AOQL. As mentioned in the previous section, it is simpler to select a value of i and then solve for f, as follows:

$$f=\frac{\left[P\left(\theta_2\right)-AQL\right]\left[P\left(S_2\right)P\left(S_1\right)^i\right]}{P\left(S_1\right)^i\left[P\left(\theta_2\right)-P\left(\theta_2|\ S_1\right)\right]}-\frac{P\left(S_2\right)}{P\left(S_1\right)^i} \tag{20}$$

By inspecting expression (20) we have the following properties, which will be stated without proof due to their simplicity:

Property 1: the optimal expected number of inspected units, i.e. U_B^*, is directly proportional to the difference between the fraction defective of the process and the customer's AQL.

Property 2: the optimal expected number of inspected units, i.e. U_B^*, is inversely proportional to the difference $\left[P\left(\theta_2\right)-P\left(\theta_2|S_1\right)\right]$. Notice that, given that a unit has been delivered to the customer, if we did not implement an inspection plan at all, the probability of such a unit resulting defective is $P\left(\theta_2\right)$; whereas if we implemented a CSP-1, such probability would decrease to $P\left(\theta_2|S_1\right)$. Then, this property states that U_B^* is inversely proportional to the improvement obtained for having an inspection system, regarding the chances of delivering a defective unit to the customer.

Regarding the optimal expected cost, we have that it increases with the proportion of rejected units and with the probability of rejecting a non-defective unit. As it depends on U_B^*, it also increases with $\left[P\left(\theta_2\right)-AQL\right]$ and decreases with $\left[P\left(\theta_2\right)-P\left(\theta_2|S_1\right)\right]$.

6. CSP-1 UNDER INFALLIBLE INSPECTION VS. CSP-1 UNDER INSPECTION ERRORS

Let us consider the case in which we are interested in implementing a CSP-1 while minimizing the total expected cost. If we took into consideration the presence of inspection errors, we would compute the optimal value for U_B^* as discussed in section 5. However, if we neglected such inspection errors by assuming infallible inspection systems, we would compute a suboptimal value U_A^* based on the analysis provided in section 4. The difference between U_B^* and U_A^* can be stated as:

$$U_A^* - U_B^* = Q\left[P(\theta_2) - AQL\right] \left[\frac{1}{P(\theta_2)} - \frac{1}{P(\theta_2) - P(\theta_2|S_1)}\right] \quad (21)$$

Expression (21) is always lower than or equal to zero, by our assumptions that $P(\theta_2) > P(\theta_2|\ S_1)$ and $P(\theta_2) > AQL$. Therefore, by implementing a CSP-1 with the expected number of inspected units equal to $U_A^* < U_B^*$, we should expect that in the long run we will not be able to comply with the customer's AQL. By inspecting expression (21) we can formulate the following properties:

Property 3: The difference $\left[U_A^* - U_B^*\right]$ is directly proportional to the difference between the fraction defective and the customer's AQL. This means that even though both U_B^* and U_A^* increases with $\left[P(\theta_2) - AQL\right]$, the rate at which such an increment occurs for U_B^* is greater than for U_A^*. This can be verified by examining expressions (8) and (19), from which we obtain that the rate at which U_B^* changes with $\left[P(\theta_2) - AQL\right]$ is $\dfrac{Q}{P(\theta_2) - P(\theta_2 \mid S_1)}$. This rate is greater than that for U_A^*, which is equal to $\dfrac{1}{P(\theta_2)}$.

Property 4: The magnitude of the difference $\left[U_A^* - U_B^*\right]$ decreases with the improvement obtained for having an inspection system, defined as $\left[P(\theta_2) - P(\theta_2|S_1)\right]$. Moreover, in the ideal case in which an accepted unit never happens to be defective, i.e. $P(\theta_2|S_1) = 0$, we would have $U_A^* = U_B^*$.

A reasonable assumption is that if a customer receives a shipment of units whose AOQL is greater than the customer's AQL, all units in such a shipment would be rejected and returned to the supplier. Therefore, if we design our CSP-1 disregarding inspection errors, in the long run we should expect the customer to return all of our shipments! Hence, we can conclude that using U_A^* as the optimal policy in the presence of inspection error, is not economically suboptimal, but simply unviable.

7. EXTENSION CONSIDERING TRANSITION COSTS BETWEEN TOTAL AND PARTIAL INSPECTIONS

So far we have only considered the costs related to inspection, rejection and classification errors. This approach is valid only when the cost due to the transition between total and partial inspection is insignificant. However, let us assume that we we would need to stop the production process and reprogram the inspection system when shifting from one type of inspection to the other. In that case, the assumption that the transition costs between total and partial inspections can be neglected would no longer apply. Let us first focus in the ideal case in which there are no inspection errors. Let us also introduce the following additional notation:

- c_1: cost for shifting from 100% to systematic inspection
- c_2: cost for shifting from systematic to 100% inspection
- U_1: number of expected units to be inspected during 100% inspection
- U_2: number of expected units to be inspected during systematic inspection

The expected number of times that we would shift from 100% to systematic inspection is given by the steady-state probability $x_{100\%}^*$ times the total number of units, Q, times the probability of shifting from 100% to systematic inspection which is $P(\theta_1)^i$, i.e. $Qx_{100\%}^* P(\theta_1)^i$. We also have that $U_1 = Qx_{100\%}^*$. Then, we have that the expected number of times that we would shift from 100% to systematic inspection would be given by $U_1 p(\theta_1)^i$. Similarly, the expected number of times in which we would shift from systematic to 100% inspection would be given by $U_2 p(\theta_2)$, where

$U_2 = Qfx_f^*$. Then the total expected cost under ideal inspection would be given by:

$$C_A\left(U_A\right) = U_A \left[c_s + \frac{c_p P\left(\theta_2\right)}{P\left(\theta_1\right)} + \frac{c_s}{P\left(\theta_1\right)} + c_r P\left(\theta_2\right)\right] + c_1 U_1 p\left(\theta_1\right)^i + c_2 U_2 p\left(\theta_2\right) \qquad (22)$$

As mentioned before, the minimization of the cost given by (22) is subject to (6).

Note that from expression (22) we have the following properties:

- $U_1 + U_2 = U_A$.
- Our findings presented in section 4 regarding the optimal number of units to be inspected still apply. This means that the optimal number of units to be inspected is $U_A^* = U_A \mid \left[1 - \frac{U_A}{Q}\right] P\left(\theta_2\right) = AQL$.

  To see why, let us first consider the case in which we inspected less than U_A^*. In that case we would violate the constraint given by (6) which is not allowed. For the other case, let us consider that we inspect $U_A^{'}$ units where $U_A^{'} > U_A^*$. Then, clearly the first term of (22) would be greater for $U_A^{'}$ than for U_A^*. Also, $C_A\left(U_A\right)$ is an increasing function of both U_1 and U_2. So if there is a combination of values $U_1 + U_2$ that allows complying with the constraint, such a combination would dominate any other that yields to a greater values of $U_1 + U_2$. Therefore, U_A^* is always a better solution than $U_A^{'}$.

$C_A\left(U_A\right)$ can be expressed as:

$$C_A\left(U_A\right) = U_A \left[c_s + \frac{c_p P\left(\theta_2\right)}{P\left(\theta_1\right)} + \frac{c_s}{P\left(\theta_1\right)} + c_r P\left(\theta_2\right)\right] + c_1 U_1 p\left(\theta_1\right)^i + c_2\left(U_A - U_1\right) p\left(\theta_2\right) \qquad (23)$$

By realizing that $U_A^* = U_A \mid \left[1 - \frac{U_A}{Q}\right] P\left(\theta_2\right) = AQL$,

we just need to focus on optimizing $C_A\left(U_A\right)$ in terms of U_1. Therefore, our optimization problem becomes:

$$Min\, Z\left(U_1\right) = c_1 U_1 p\left(\theta_1\right)^i + c_2\left(U_A^* - U_1\right) p\left(\theta_2\right) \qquad (24)$$

By inspecting (24) we have that the optimal value of U_1, namely U_1^* is given by:

- $U_1^* = U_A^*$ if $c_1 p\left(\theta_1\right)^i - c_2 p\left(\theta_2\right) \le 0$. This means that we would remain in 100% inspection and we would avoid shifting to systematic inspection. This can be achieved by making $i = U_A^*$ and f = 0.
- $U_1^* = 0$ if $c_1 p\left(\theta_1\right)^i - c_2 p\left(\theta_2\right) \ge 0$. This would imply that we never enter 100% inspection. This can be achieved by starting directly the systematic inspection, making $i = 0$. The value of f must be such that:

$$U_2^* = Qfx_f^* = Qf \frac{p\left(\theta_1\right)^i}{p\left(\theta_1\right)^i + p\left(\theta_2\right)} = \left[P\left(\theta_2\right) - AQL\right] \frac{Q}{P\left(\theta_2\right)} \qquad (25)$$

Then, the optimal value for f would be given by:

$$f = \left(\frac{\left[P\left(\theta_2\right) - AQL\right]}{P\left(\theta_2\right)}\right)\left(\frac{p\left(\theta_1\right)^i + p\left(\theta_2\right)}{p\left(\theta_1\right)^i}\right) \qquad (26)$$

Now let us discuss the case in which we consider inspection errors. By a similar analysis to that applied for the case without inspection errors, we obtain that:

$$C_B\left(U_B\right) = U_B \left[c_s + c_s \frac{P\left(S_2\right)}{P\left(S_1\right)} + c_p \frac{P\left(S_2\right)}{P\left(S_1\right)} + c_I P\left(\theta_1 | S_2\right) P\left(S_2\right) + c_r P\left(S_2\right)\right] + c_1 U_1 p\left(S_1\right)^i + c_2 U_2 p\left(S_2\right) \qquad (27)$$

It can be easily shown that, as in the previous case, our findings for section 5 still apply and therefore $U_B^* = \frac{\left[P(\theta_2) - AQL\right]Q}{P(\theta_2) - P(\theta_2 | S_1)}$. Then, our optimization problem becomes:

$$Min\, Z(U_1) = c_1 U_1 p(S_1)^i + c_2 \left(U_B^* - U_1\right) p(S_2) \tag{28}$$

By inspecting (28) we arrive to similar conclusions as those exposed for the previous case:

- $U_1^* = U_B^*$ if $c_1 p(S_1)^i - c_2 p(S_2) \le 0$. Therefore $i = U_B^*$ and f = 0.
- $U_1^* = 0$ if $c_1 p(S_1)^i - c_2 p(S_2) \ge 0$. This can be achieved by starting directly the systematic inspection, making $i = 0$. The value of f must be such that:

$$U_2^* = Qfx_f^* = Qf \frac{p(S_1)^i}{p(S_1)^i + p(S_2)} = \frac{\left[P(\theta_2) - AQL\right]Q}{P(\theta_2) - P(\theta_2 | S_1)} \tag{29}$$

Then, the optimal value for f would be given by:

$$f = \left(\frac{\left[P(\theta_2) - AQL\right]}{P(\theta_2) - P(\theta_2 | S_1)}\right)\left(\frac{p(S_1)^i + p(S_2)}{p(S_1)^i}\right) \tag{30}$$

8. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

In this paper we have presented two models for designing optimal CSP-1 from an economic perspective, under two scenarios: infallible and fallible inspection systems. Our work combines Markovian and Bayesian analysis to model the uncertain parameters of the system. We found that for both models, the optimal decision is to implement a CSP-1 whose expected number of inspected units is the minimum required to guarantee that the AOQL does not exceed the customer's AQL. From our findings, we can conclude that implementing the model for infallible inspection systems when actual inspection errors are present, results in a violation of the customer's AQL. We have also extended our analysis for the cases in which the costs for shifting between 100% and systematic inspections cannot be neglected. Our results show that the optimal answer in this case is either to remain whether in 100% or in systematic inspection, but never switch between the two styles of inspection within the same process .

As future research direction we recommend to consider the defective fraction as stochastic rather than deterministic. In this regard, the probability function of the fraction defective can be modeled using the Normal approximation to the Binomial distribution. Also, our analysis can be extended to other SIPs, such as CSP-2, CSP-3 and other related ones.

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