

# Segment opening calculation of shield tunnel based on double-sided elastic foundation beam when tunnel followed by well excavation

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## ABSTRACT

In view of the characteristics of longitudinal deformation and segment circumferential opening, tunnel is regarded as continuous long beam which locates in Winker foundation, vertical deflection differential equation was built when tunnel comes firstly and then working well is excavated based on double-side elastic foundation beam and corresponding solutions were calculated. And then the equivalent continuous theory was combined to obtain calculation equation of segment opening, which can provide basis for segment opening calculation when tunnel comes firstly and then working well is excavated. Comparing results of theory analysis with numerical results indicates that the calculation results calculated by double-side elastic foundation beam are consistent with numerical results and the method proposed in this work can effectively predict segment opening during the shield construction phase.

**KEYWORDS:** shield tunnel; segment opening; double-sided elastic foundation beam; numerical analysis; tunnels come firstly and then working well is excavated

## INTRODUCTION

In recent years, the method that tunnels come firstly and then foundation excavation has been widely used during the shield construction phase based on its advantages of less auxiliary measures, lower risks and shorter construction period, such as the Dong Shan Kou station of Guangzhou Metro Line 6, Wu Yang Cun station of Guangzhou Metro Line 5 and Guangzhou

Xijiang diversion project. Tunnels easily appear longitudinal deformation under subsequent foundation excavation when tunnels come firstly and then working wells are excavated. Comparing with lateral mechanical behaviors of shield tunnel, the longitudinal deformation characteristics is weaker; the segment opening is main components of longitudinal deformation [1], seepage or longitudinal tensile failure may occur because of excessive circumferential opening when tunnel vertical curvature reaches a certain value. So it is significant to explore the longitudinal deformation and segment opening of shield tunnel when tunnel comes firstly and then working well is excavated.

At present, many researchers have used a variety of approaches to explore the longitudinal deformation and segment opening of shield tunnel. He Chuan and Yang Chunshan etc [2-3] studied segment opening of shield tunnel by three-dimensional numerical models. Liu Yin and Zheng Yonglai etc [4-5] explored the longitudinal deformation and maximal opening of shield tunnel. Zhu Ling, Wang Zhiliang and Ye Fei etc [6-8] adopted the elastic foundation beam theory to build the tunnel calculation model and obtained the longitudinal deformation. Some shortages of the above researches are shown as follows: (1) The segment openings calculation are mainly based on numerical analysis method or theory combined with the software, which results are always great deviation from actual situation; (2) The longitudinal deformation calculation always consider one-side soil spring when the calculation is based on elastic foundation beam theory, whereas the effect of overlying soil can't be neglected when the buried depth of tunnel reaches a certain value.

Based on this, calculation method of segment opening was proposed when tunnel comes firstly and then working well is excavated based on the double-side elastic foundation beam theory and longitudinal equivalent continuous model, the method was used for typical engineering example calculation. Comparing with the numerical results shows that the method can accurately predict the segment opening of shield tunnel when tunnel comes firstly and then working well is excavated.

## CACULATION EQUATION OF SEGMENT OPENING

### Basic assumptions

The contact model between shield tunnel and foundation is shown in Fig.1. Some assumptions are proposed to deduce the longitudinal deformation when tunnel comes firstly and then foundation is excavated. Assumptions are as follows:

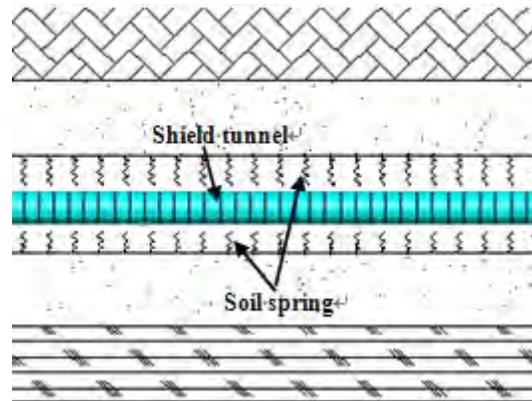
(1) The soil that shield tunnel locates in meets Winkler foundation model and the foundation surface settlement is proportional to the pressure at random points;

(2) Groundwater seepage effects that caused by excavation of tunnel and foundation pit as well as the hydrostatic pressure caused by initial water level are not considered; the deformation compatibility condition is satisfied. The relative slip and dislocation between tunnel and foundation is not considered;

(3) The stress and displacement of soil are stable before the subsequent foundation excavation and after the tunnel excavation as well as the coefficient of subgrade reaction remains unchanged;

(4) Shield tunnel is simplified to an elastic beam and the lateral deformation of shield tunnel caused by working well excavation is not considered. According to the results of reference [3], the side of tunnel away from foundation pit can be considered to be fixed, which is little affected by working well excavation;

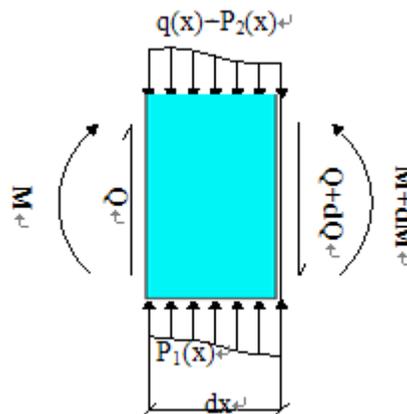
(5) The equation is deduced under the condition that the tunnel buried depth reaches a certain value [9-10] and the soil spring can bear tension with consideration of soil arch effect.



**Figure 1:** The contact model between tunnel and ground

### The equation of tunnel longitudinal deformation

The infinitesimal element of elastic beam is shown in Fig.2. The pressure between beam and overlaying soil, underlaying soil are  $p_2(x)$  and  $p_1(x)$  respectively under load  $q(x)$ . Taking displacement function  $y(x)$  as unknown variables and differential equation was deduced based on equilibrium.



**Figure 2:** The infinitesimal element of double-side elastic beam

The formula (1) to (4) can be obtained based on  $\sum F_{\text{vertical}}=0$  and  $\sum M_{\text{left}}=0$ .

$$Q + dQ + qdx - p_2 b dx - Q - p_1 b dx = 0 \quad (1)$$

$$\frac{dQ}{dx} = (p_1 + p_2)b - q \quad (2)$$

$$M + dM + \frac{p_1(dx)^2}{2} - \frac{(p_2+q)(dx)^2}{2} - M - (Q+dQ)dx = 0 \quad (3)$$

$$dM + \frac{p_1(dx)^2}{2} - \frac{(p_2+q)(dx)^2}{2} - Qdx - dQdx = 0 \quad (4)$$

The high-order term of formula (4) were neglected.

$$Q = \frac{dM}{dx} \quad (5)$$

So the formula (2) can be expressed as follow.

$$\frac{dM^2}{dx^2} = (p_1 + p_2)b - q \quad (6)$$

Flexural approximate differential equation was substituted into formula (6) to obtain formula (7).

$$-EI \frac{dy^4}{dx^4} = (p_1 + p_2)b - q \quad (7)$$

According to the assumption of Winkler foundation model, the relationship between foundation settlement  $y_i (i=1, 2)$  and pressure  $p_i (i=1, 2)$  can be expressed as formula (8).

$$p_i = k_i y_i \quad (i=1, 2) \quad (8)$$

where  $k_i$  is foundation coefficient on unit width. According to assumption (2),  $y_1$  is equal to  $y_2$ . So formula (7) can be expressed as follow.

$$\frac{dy^4}{dx^4} + \frac{(k_1 + k_2)}{4EI} 4y = \frac{q(x)}{EI} \quad (9)$$

where  $EI$  is equivalent longitudinal flexural stiffness,  $k_1$  and  $k_2$  are respectively coefficient of subgrade reaction overlaying soil and underlying soil.

The longitudinal equivalent stiffness calculation is usually a simplified calculation formula [11], which results are always less than actual situation. Reference [12] was consulted to calculate equivalent stiffness in this work. The shield tunnel that consists of  $n$  segment rings was taken as research object. The circumferential compression and tensile stiffness, shear stiffness and flexural stiffness of shield tunnel are respectively  $E_c A_c$ ,  $G_c A_c$  and  $E_c I_c$ . The longitudinal joints are composed of compression and tensile springs, shear spring and flexural springs, which stiffness is  $k_n$ ,  $k_q$  and  $k_m$ . The shield tunnel can be equivalent to a homogeneous ring along the axial direction. The stiffness reductions caused by joints are expressed by stiffness reduction coefficient  $\eta_n$ ,  $\eta_q$  and  $\eta_m$ . The compression and tensile stiffness, flexural stiffness and shear stiffness are  $\eta_n E_c A_c$ ,  $\eta_q G_c A_c$  and  $\eta_m E_c I_c$  after reduction. The reduction coefficients are given as follows.

$$\eta_n = \frac{k_n}{k_n + \frac{E_c A_c}{l_s} \left(1 - \frac{1}{n}\right)} \quad (10)$$

$$\eta_q = \frac{k_q}{k_q + \frac{G_c A_c}{l_s} \left(1 - \frac{1}{n}\right)} \quad (11)$$

$$\eta_m = \frac{k_m}{k_m + \frac{E_c I_c}{l_s} \left(1 - \frac{1}{n}\right)} \quad (12)$$

where  $E_c$  is the elasticity modulus of segment (kPa),  $G_c$  is the shear modulus of segment (kPa),  $A_c$  is cross section area,  $l_s$  is the length of segment (m),  $I_c$  is segment sectional moment of inertia (m<sup>4</sup>), The radial coefficients of subgrade reaction are calculated reference [13].

$$k = \frac{3E}{(1+\nu)(5-6\nu)} \quad (13)$$

where  $k$  is radial reaction coefficient,  $E$  is elasticity modulus of soil,  $\nu$  is poisson ratio.

Basic differential equation (9) is a fourth-order linear nonhomogeneous differential equation including parameter  $\alpha = \sqrt[4]{(k_1 + k_2)/4EI}$ , which corresponds to the characteristics coefficient of elastic foundation beam theory and the corresponding characteristics length is  $l = 1/\alpha = \sqrt[4]{4EI/(k_1 + k_2)}$ . The formula can be written as

$$\frac{dy^4}{dx^4} + 4\alpha^4 y = \frac{q(x)}{EI} \quad (14)$$

The solution of basic differential equation (14) is composed of the general solution of homogeneous equation  $dy^4/dx^4 + 4\alpha^4 y = 0$  and the special solution of nonhomogeneous equation. The general solution to homogeneous equation above is as follow.

$$y = e^{\alpha x} (A \cos \alpha x + B \sin \alpha x) + e^{-\alpha x} (C \cos \alpha x + D \sin \alpha x) \quad (15)$$

where  $A$ ,  $B$ ,  $C$  and  $D$  are constants and calculated by boundary conditions.

Formula (14) indicates that formula (16) is the special solution of nonhomogeneous when polynomial  $q(x)$  is less quartic polynomial.

$$y_1(x) = \frac{q(x)}{k_1 + k_2} \quad (16)$$

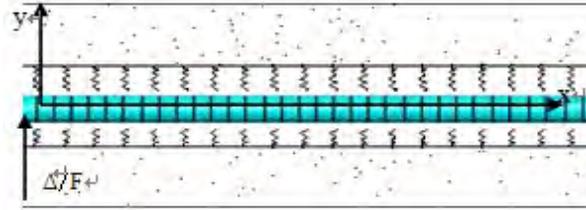
The general solution of basic differential equation of double-side beam on elastic foundation is obtained as

$$y = e^{\alpha x} (A \cos \alpha x + B \sin \alpha x) + e^{-\alpha x} (C \cos \alpha x + D \sin \alpha x) + \frac{q(x)}{k_1 + k_2} \quad (17)$$

### The equation of segment opening

Comparing with common construction method that working well excavates firstly and then shield tunnel is constructed, the tunnel will be affected by subsequent foundation pit excavation when tunnel comes firstly and then working well is excavated and then the segment longitudinal deformation appears. The effect of subsequent well excavation on tunnel is equivalent to displacement or concentrated force acting on tunnel side that is near the foundation pit.

According to basic assumptions (4) and (5), the upper spring is compression, while the lower spring is tension under displacement or concentrated load (Fig.3).



**Figure 3:** Calculation model when tunnel come firstly and then well excavation

From above analysis, calculation model meets the boundary condition as follows. Both longitudinal displacement  $y$  and rotation angle  $\theta$  are 0 on the side that be far from coordinate origin, while the moment  $M$  is 0 and shear  $Q$  is  $-F$  as well as the longitudinal displacement  $y$  is  $\Delta$  at the origin. The boundary conditions can be converted into  $\theta = dy/dx = 0$ ,  $Q = d^3y/dx^3 = 0$  and  $M = -EI d^2y/dx^2 = 0$  based on differential relationship between internal force and displacement. According to the qualitative analysis of effects caused by subsequent well excavation on shield tunnel, the shield tunnel can be simplified to semi-infinite foundation beam.  $y(x)$  is 0 when  $x$  is  $\infty$ , and then substitute into formula (16).

$\infty * (A \cos \alpha x + B \sin \alpha x) = 0 \Rightarrow A = 0$  and  $B = 0$ , substituted into basic equation. There are no external loads on the beam, so  $q(x)$  is 0 and formula (17) can be written as

$$y = e^{-\alpha x} (C \cos \alpha x + D \sin \alpha x) \quad (18)$$

$d^2y/dx^2$  is 0 and  $y$  is  $\Delta$  when  $x$  is 0, Formula (18) was solved as

$$D = 0, C = \Delta$$

The final longitudinal deformation equation of tunnel when tunnel comes firstly and then well is excavated can be expressed as follow.

$$y = e^{-\alpha x} \Delta \cos \alpha x \quad (19)$$

In the similar way,  $Q = d^3y/dx^3$  is  $-F$  when the effect of subsequent excavation on tunnel is equivalent to concentrated force, and then the general solution of equation (18) can be expressed as follow.

$$y = e^{-\alpha x} \frac{F}{2EI\alpha^3} \cos \alpha x + \frac{F}{k_1 + k_2} \quad (20)$$

The deformation of segment is analyzed in length of one segment ring, which centre is in junction surface between two segments. It will appear the relative rotation angle  $\theta$  in circumferential seam and bending curvature  $\theta/l_s$  of tunnel when the segment is under bending moment  $M$ . According to the conditions that static equilibrium and deformation compatibility based on the equivalent continuous theory [14-15], the angle  $\varphi$  can meets formula (21).

$$\cot \varphi + \varphi = \pi \left( \frac{1}{2} + \frac{k_j l_s}{E_c A_c} \right) \quad (21)$$

$$k_{j1} = nE_j A / l \quad (22)$$

where  $k_{j1}$  is longitudinal joint elasticity stiffness;  $n$  and  $A$  are the number of longitudinal bolts and cross section area of one bolt respectively;  $l_s$  is the length of tunnel section (m);  $r$  is the segment radius (m);  $\varphi$  and  $x$  are respectively the position of segment bending neutral axis and angle, and  $x = r \sin \varphi$ .

The maximal segment opening of joint can be derived based on mechanical analysis [6].

$$\delta = \frac{M l_s}{E_c I_c} \frac{\pi \sin \varphi}{\cos^3 \varphi} (r + x) \quad (23)$$

According to the physical relationship between curvature and bending moment ( $K = M/EI$ ), formula  $EI = \eta_m E_c I_c$  and formula (12), the calculation equation of segment opening is shown as follow.

$$\delta = \frac{\pi \sin \varphi K l_s}{\cos^3 \varphi} \eta_m (r + x) \quad (24)$$

where  $M$  is the bending moment of shield tunnel (kN.m), The curvature of plain curve is  $K = \pm y'' / (1 + y'^2)^{3/2}$ . So the formula (24) can be expressed as

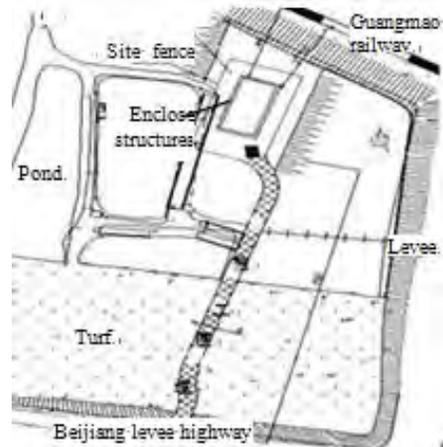
$$\delta = \frac{l_s \pi \sin \varphi}{\cos^3 \varphi} \frac{y''}{(1 + y'^2)^{3/2}} (\sin \varphi + 1) r \eta_m \quad (25)$$

The curvature is positive denotes positive segment opening and the segments relatively open, while relatively compressed when curvature is negative. The curvature is positive in this work because of the segment opening is positive during the foundation pit excavation phase.

## ANALYSIS OF ENGINEERING EXAMPLE

### Brief introduction of engineering

Guangzhou xijiang water diversion project is the lifeline engineering and the crucial project for improving the quality of water in Guangzhou. Non-excavating methods of shield tunnel were adopted to across the major transportation hub in urban area. One of working wells was analyzed in this work. The well is located in Foshan city, 37.5m south of Guangmao Railway, 150m north of Beijiang Levee, about 130m from Beijiang levee highway and 61m from the east levee; The plane position is shown in figure.4. The receiving well is a rectangular foundation pit, and its clearance length, width and depth are 28m, 14m and 22.6m respectively. The supporting of foundation pit consists of 1.2 m diaphragm wall and 4 reinforced concrete internal bracing. By geological survey report and the related structure designs of the project, the parameters of soil and structures are listed in Table 1 to Table 3.



**Figure 4:** The schematic plan of receiving well

**Table 1:** Physico-mechanical parameters of soil

Geotechnical name	Density (g/cm <sup>3</sup> )	Consolidated quick shear		Compression modulus (MPa)	Deformation modulus E <sub>0</sub> (MPa)	Lateral pressure coefficient	Poisson ratio	Thickness of the soil /(m)
		Cohesion / (kPa)	Internal friction angle /(°)					
Artificial filled soil	1.90	10.0	18.0	4.7	10	0.49	0.33	1.5
Silty clay	1.99	19.2	15.2	5.86	12	0.43	0.30	4.5
Mud fine sand	2.00	5.0	25.0	5.93	18	0.39	0.28	1.9
Mud medium fine sand	2.01	0.0	30.0	6.52	20	0.35	0.26	5.2
Mud fine sand	1.90	4.0	29.6	6.89	20	0.39	0.28	10.7
Mud medium coarse sand	1.98	0.0	32.0	7.40	25	0.35	0.26	10.2
Mud sand gravel	1.90	--	36.0	8.14	30	0.30	0.23	4.8
Strong weathered tuff	2.00	--	--	--	120	--	--	21.2
Chemical churning pile strengthening	1.76	57.98	25.6					
Local strengthening of hole	1.6	500	35	200		0.3		
Railway subgrade	1.9	15	31	130				
Railway ballast	2.5			25500				

**Table 2:** Parameters of supporting structures

Type	Density (g/m <sup>3</sup> )	Length×width (mm×mm)	Thickness (mm)
Inner support 1	2.5	800×1000	
Inner support 2、3	2.5	1000×1200	
Inner support 4	2.5	800×1000	
Ring beam 1	2.5	1200×1000	
Ring beam 2、3	2.5	1200×1200	
Ring beam 4	2.5	1000×1000	
Support plate	2.5		1
Diaphragm wall	2.5		1.2
Base plate	2.5		0.8

Attention : the concrete grade of continuous wall and inner structures is C30

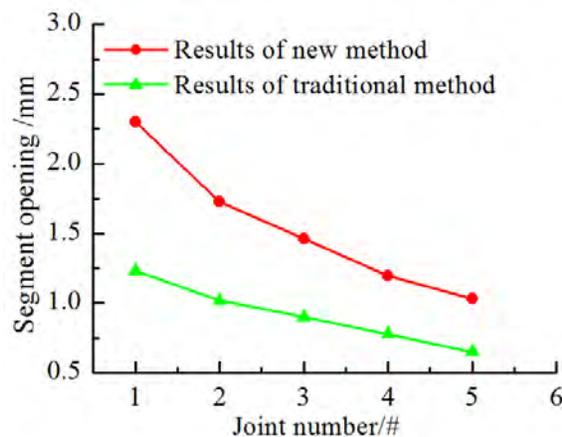
**Table 3:** Structural parameters of lining segment

External diameter D/m	Inner diameter d/m	Ring width $l_s$ /m	Elastic modulus of concrete $E_c$ /kPa	Diameter of bolts d/mm	Length of bolts $l$ mm	Number of bolts	Elastic modulus of bolts $E_b$ /kPa
6	5.4	1.6	$3.45 \times 10^7$	30	400	11	$2.06 \times 10^8$

### The calculation of segment opening

The elasticity stiffness  $k_{j1}$  is calculated ( $k_{j1}=192284.7\text{kN/m}$ ) based on Table 3 and formula (22). The rotation angle  $\varphi$  is 1.18 at neutral axis by calculating. According to formula (13),  $k_1$  and  $k_2$  are 14.1MPa/m, so the characteristics length of double-side elastic foundation beam  $l=1/\alpha$  is 17.5m and characteristics coefficient  $\alpha$  is 0.057; and the corresponding characteristics coefficient of elastic foundation beam is 0.04 and the characteristics length is 25m. 20 segment rings that length is 30 m were taken as research object; there are 19 joints corresponding to the segments and number for 1 to 19. The one near the foundation pit is the first joint. One distances of which are from load action point to both ends is less than characteristic length, whereas the other is greater than characteristic length. The parameters of table 3 and the segment number substituted into formula (12) to obtain the  $\eta_m=0.0114$ .

The tendency of segment opening from the 1st joint to the 19th joint presents gradually increases reference [3]. 6 segment rings and 5 joints that next to the foundation pit were calculated in this work based on the consideration of the opening number is a great many. The vertical displacement of tunnel is 28mm by field measurement and curvature is calculated (1/1984). The related parameters were substituted into formula (25) to calculate the segment opening is 2.3 mm., Segment opening of the other 4 joints can be calculated (Fig. 5) in the same way.

**Figure 5:** The openings of six segments near the foundation pit

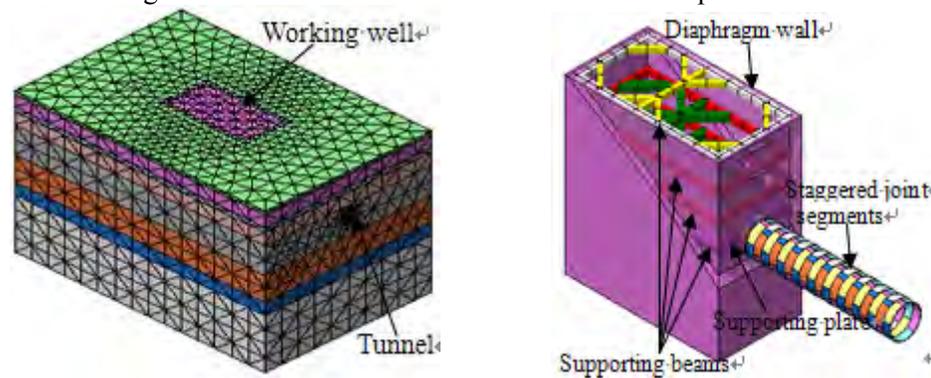
It has significant differences between the results of traditional method and the new method that was proposed in this work. The maximal opening of traditional method is 1.23mm, which is the 0.535 times of results calculated by new method, while the longitudinal deformation of traditional method is greater than that calculated by new method; The main reason is that the traditional elastic foundation beam only considers one-dimensional soil compression springs. On one hand, the constraints of the tunnel were weakened and the longitudinal displacement of tunnel is

greater; on the other hand, the tunnel deformation is more uniform and longitudinal deformation curvature is smaller without considering soil tension springs.

## NUMERICAL CALCULATIONS

### Numerical model

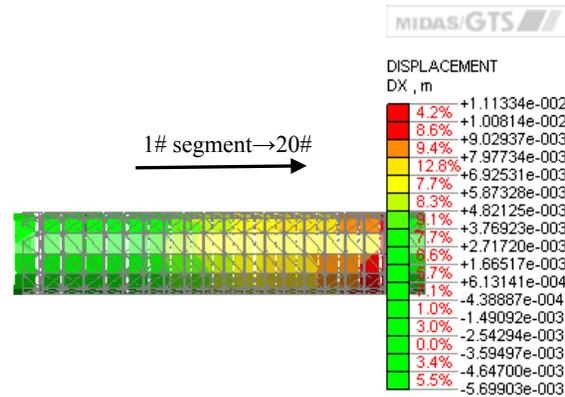
The geometrical size of the model containing length, width and height of the model are 90m, 60m and 60m. Three-dimensional solid elements are applied to simulate soil, segment, segment connection, grouting and diaphragm wall of the model, and beam elements were used to simulate supporting structures. Supporting plate and shield were stimulated with shell elements. The soil and structures respectively used Mohr-Coulomb model and elastic model. Numerical modeling, supporting structures and tunnel were shown in Fig.6. The model includes 20 segment rings that number for 1 to 20 and the one near the foundation pit is No.20. The 20th segment has a certain distance away from working well and it is slightly affected by foundation excavation, so the boundary of the 20th segment includes lateral and vertical direction displacement constraints.



**Figure 6:** General model and supporting structures

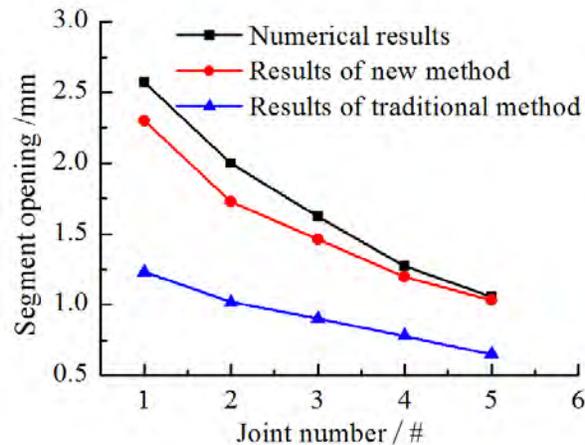
By the experience, the equivalent Young's modulus of straight bolt is  $5.4 \times 10^7$  kPa and the segment circular seam bears 1600kN/m pressure along circumferential direction in construction stage. In present work, jacking forces were simplified to the pressure acting on a circular pad, and the equivalent pressure is 5400kPa [17]. The grouting pressures from vault to arch bottom is 0.3Mpa to 0.45Mpa and linear variation. According to the soil situation, the excavation face pressure is 120kPa.

## Results and analysis



**Figure 7:** Displacement of segments

The displacement nephogram of lining Segments is shown in Fig.7. Dotted line denotes before deformation and the nephogram denotes after deformation. The segment openings of different conditions are shown in Fig.8.



**Figure 8:** Segment opening of different conditions

The distribution laws of segment opening of three conditions are similar. It presents obvious decreasing tendency from foundation-side to border. The results of new method proposed in this work are more consistent with numerical results based on distribution and maximum of segment opening, so it's more reasonable to calculate segment opening. The maximum of numerical results is 2.57mm, which is 10.5% difference from that of new method (2.3mm) and 52.1% difference from that of traditional method (1.23mm).

Fig.8 also indicates that influencing scope of subsequent foundation excavation on segment opening is mainly within 5 segment rings and 4 joints near the diaphragm wall based on double-side elastic foundation beam when tunnel comes firstly and then working well is excavated because the deformation of the 5th joint is in the elastic state, which is consistent with the results of reference [3].

## CONCLUSIONS

Vertical deflection differential equation of double-side elastic foundation beam was built, which is suitable for construction method that tunnel comes firstly and then working well is excavated. The calculating equation of segment circumferential opening was proposed and applied to a typical project example. Some conclusions are obtained based on results comparison of the different methods.

(1) The longitudinal deformation and segment opening of shield tunnel can effectively be calculated by double-side elastic foundation beam theory when tunnel comes firstly and then working well is excavated.

(2) Comparing with the traditional elastic foundation beam theory, the maximum and influencing scope of segment opening can be more accurately predicted by the method proposed in this work.

(3) The influencing range of subsequent foundation excavation on segment opening is mainly within 5 segment rings and 4 joints near the diaphragm wall by theory calculation.

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