Nonlinear Analyses for the Permeability Coefficient of Frozen Soil

Jianjun Liu
Professor, School of Civil Engineering and Architecture, Southwest Petroleum University, China 610500; e-mail:315245690@qq.com

Jun Xie
School of Civil Engineering and Architecture, Southwest Petroleum University, China 610500; e-mail:ambitiousxjfeng@163.com

ABSTRACT

As a multi-phase porous medium, frozen soil property is influenced by the shape of soil particle, magnitude, soil temperature, stress and water content etc, therefore the confirmation of the permeability coefficient is extremely complex. According to the character that each component randomly disperses into the frozen soil, combining with the stochastic simulation method, a finite element numerical simulation software-COMSOL is applied to research the correlation between the permeability coefficient and influencing factors in this paper. Results illuminate that the permeability coefficient of frozen soil will increase with the increase of particle size, porosity, and unfrozen water content, in addition, the function relations are nonlinear ones; on the contrary, the permeability coefficient will decrease with the increase of the ice content and the moisture content, similarly the function relations are nonlinear ones. Research results correspond with the measured data and previous research, so it proves that such method is an effective and suitable means to research the frozen soil physical property.

KEYWORDS: frozen soil; Permeability coefficient; unfrozen water content; ice content; nonlinear law

INTRODUCTION

Permafrost refers to a kind of loose rock or soil containing water, which will be transformed to crystalline or weak bonding solid particle when atmosphere temperature brings down to zero centigrade or below. Permafrost is mainly composed of mineral grain, ice crystal, unfrozen water(film water and liquid water), gas(steam and air).As a kind of complicated and inhomogeneous multi-phase porous medium, its physical and mechanical property is apt to be influenced by temperature , water etc.

Permafrost property is primarily influenced by mineral grain, water content, ice content, porosity etc. Generally, the common way to obtain the physical parameters is making a serial of theoretical analysis and calculation, combining with the experimental data. Finally, the permafrost physical parameters can be given by fitting curve and making a bit of amendment. The research on permafrost physical, mechanical and thermal properties has developed for decades, the achievements are fruitful. Nevertheless, those achievements are obtained mainly from qualitative analysis, the quantitative analysis is seldom seen in the newspaper. Combining with theoretical analysis, experimental verification and numerical simulation, the paper researches the nonlinearity of permafrost physical parameters, in order to provide with theory and data support for permafrost engineering

The research on permafrost property began in the 1930s, modern frozen soil mechanics had been established several stages of development later. The earliest researcher is the former Soviet Union scientist-Н.А.ЦЫТОВИЧ, he revealed the physicochemical property of frozen soil from the mutual influence between electric molecular force field and the surface energy. Furthermore, influence of solid mineral particle on permafrost property not only depends on the shape, size, dispersity, but also is closely bound up with the physicochemical property of mineral particle surface. Therefore, permafrost property is controlled by mineral components and adsorbent cationic ingredients on the surface of soil particle[1].Jacob Bear established a porous medium model, which was composed of three groups of orthogonally capillary gaps and equal wide fracture groups, the N-S equation was applied to solve the average speed of water in the structure and researched the seepage rule for microcosmic pore structure[2].Lai Yuanming et al established a function relation between permeability coefficient and effective stress , water content to make a nonlinear analysis for the coupled problem of temperature, seepage, stress fields in cold regions tunnels[3].Zhao Jianjun, Xu Xuezu proposed a function relation between permeability coefficient and porosity, pore diameter according to the coupling theory of thermo-hydro in the freezing soil[4].Liu Jianjun expounded the correlation between macrocosmic research and microcosmic research on porous medium, and he further pointed out that it's significant to make the micro research, at last he compared the merits and demerits of the stochastic simulation method and the global rise scale method of the research on pore structure[5].Li Jushou , Liu Yingxi analysed the nonlinear relation between permeability coefficient and porosity, pore size by applying numerical simulation method[6].Deng Yousheng, He Ping made a research on saline permafrost, they pointed out that there was a linear relation between permeability coefficient and dry density in a certain range, but beyond that range the relation would be a power function form[7].Li Mingxiang, Hideo Kamiyama made a field study on moisture migration in loess aerated zone, they studied the moisture migration rule by monitoring the water content, the water potential distribution and their change, further, they studied the correlation between unsaturated permeability coefficient and water content by applying a method of moisture redistribution[8].

The research on permafrost is fruitful in many aspects all over the world, however, those researches mainly refer to the yield strength, creep deformation and so on, and the research on frozen soil physical property is reported rarely. In the past, the study on frozen soil permeability coefficient placed extra emphasis on pore size, porosity etc. But now, with the in-depth research on permafrost, many scholars find that the permafrost physical parameters are the extremely nonlinear ones, and the parameters are influenced by several of factors, such as soil particle size, porosity, density, water content, ice content etc. at present, research achievement on that is seldom seen in the newspaper. Taking influences of permafrost permeability coefficient: particle size, porosity, unfrozen water content and ice content into consideration, this paper establishes a microcosmic frozen soil mathematics model and makes a numerical simulation to study the nonlinear law of permafrost permeability coefficient.

FINITE ELEMENT MATHEMATICAL MODEL

As a multi-phase porous medium, the difference between frozen soil and common soil is their components, the frozen soil contains ice crystal, and the ice crystal has great effect on permafrost mechanical, thermal properties.Figure1 is the definition of each component and volume content, the volume of permafrost pore is be made up of gas volume, ice volume and unfrozen water volume. The moisture content consists of ice volume and unfrozen water volume.it can be expressed as following equation:

$$V_{\mathrm{w}} = V_{\mathrm{u}} + \frac{\rho_i}{\rho_w} V_i \qquad (1)$$

Where: ρ_i 、 ρ_w represent density of ice and water, respectively.

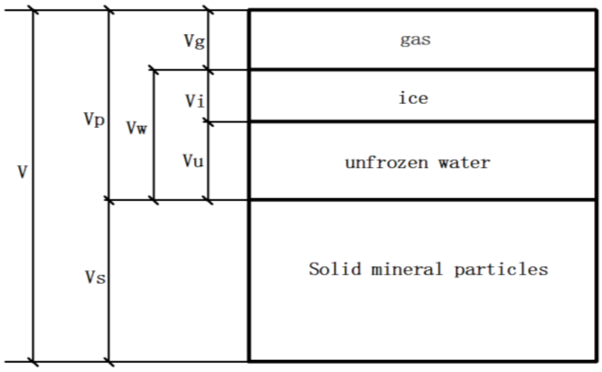

Figure1: Permafrost composition and volume content definition

FUNDAMENTAL ASSUMPTION

(1) The permeability coefficient is isotropic;

(2) Material is homogeneous;

(3) Ice is the impervious material;

(4) The size of each component is the same, that is, the pore size is almost equal to the soil particle size etc.

MATHEMATICAL GOVERNING DIFFERENTIAL EQUATION

Because the pore is very tiny, flow is extremely slow in the permafrost, so the flow is usually laminar flow. It is a scientific and practical method by simulating Darcy permeability test to study the rule of permafrost permeability coefficient change. The paper adopts the following Darcy equation as the governing differential equation to study frozen soil seepage problem.

$$\nabla \cdot \left(-\frac{k}{\eta} \nabla p \right) = 0 \qquad (2)$$

Where: k is permeability, m^2; η is the dynamic viscosity of water, Pa s; p is fluid pressure Pa.

As for frozen soil, at zero centigrade, η =1.974×10^{-3}Pa s, which is taken as the calculated value. In addition, the permeability coefficient of solid particle K_s=2×10^{-14}m/s [6].For such hypothesis that fluid is incompressible and underground water flow is laminar, and according to the basic principle of fluid mechanics, the permeability coefficient equation [9] for equal wide, straight pore is following.

$$K = \frac{\rho g b^2}{12\eta} \quad (3)$$

Where: K is the permeability coefficient of equal wide pore, m/s; ρ is the density of fluid, kg/m^3; g is the acceleration of gravity, m/s^2; b is the width of pore, m; η is the dynamic viscosity of water, Pa s.

THREE DIMENSIONAL PHYSICAL MODELS

The key of numerical simulation on permeability coefficient for multi-phase porous medium is optimizing the experimental data measured and transforming those mechanical, thermal parameters into an applicable and calculable parameters, which can be operated through numerical simulation, finally, the obtained results are correspond with reality.

Currently, lattice gas method and pore grid method are the main adopted numerical simulation methods to study multi-phase porous medium in microscopic scale, the two methods can effectively simulate the micro-seepage in the scale of pore. As a kind of shallow soil, the pore size of frozen soil is relatively large, so it's appropriate to adopt pore grid method to study the micro-seepage.

First, there will produce 1000 random units by means of random function RAND (0, 1) ×1000 in EXCEL sheet. Second, the number in those units will be rearranged by means of function RANK and formed a serial of unduplicated and out-of-order data. Third, those data need to be edited by the software Ultra Edit in order to make those data met the format of the finite element software-COMSOL. Finally, COMSOL can randomly create the porosity 40% or 50% and the water content 25% or 30% etc finite element model.

Figure2 is the permafrost microscopic three-dimensional finite element model, which is composed of 10×10×10 units, and each unit represents one particle (solid, liquid, gas), each component distributes in the model randomly. Figure3~Figure5 represent the porosity 40%, unfrozen water content 25%, ice content 6% finite element model, respectively. In the following model, the front and the back boundaries are the impermeable boundaries, the upper and the lower boundaries are the known water head boundaries, and the upper boundary H_{up}=1.0m, the lower boundary H_{down}=1.0m+10b, b is the average size of particle, m.

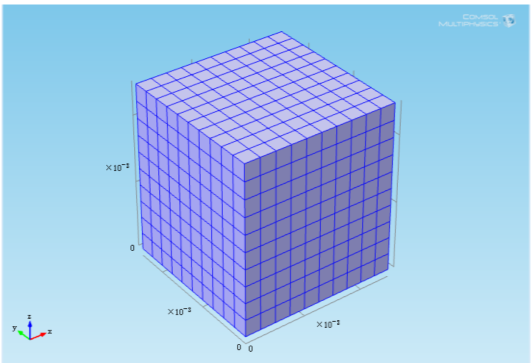

Figure2: Permafrost microscopic three-dimensional model

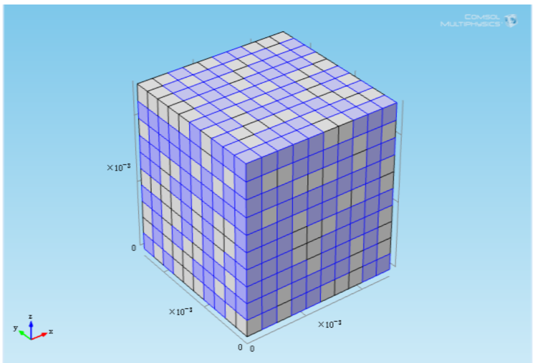

Figure3: Porosity 40% model (purple represents the solid particle)

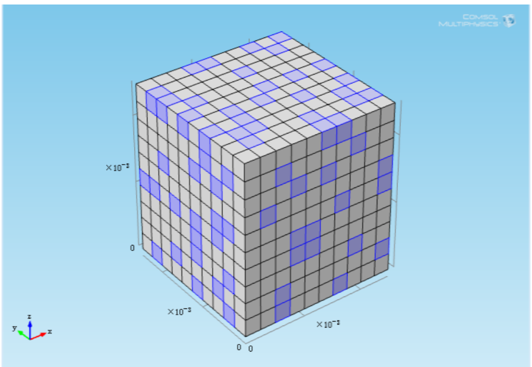

Figure4: Unfrozen water content 25% model

(purple represents unfrozen water)

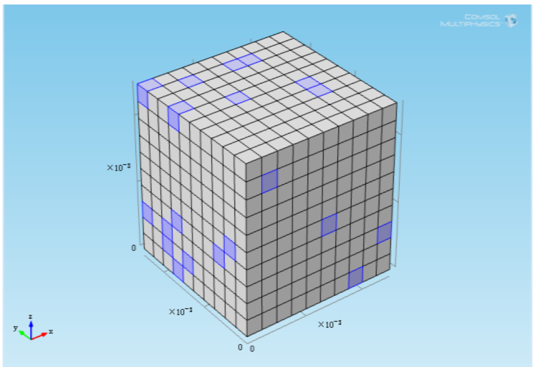

Figure5: Ice content 6% model

(purple represents ice)

NUMERICAL SIMULATION AND ANALYSIS

BASIC EXPERIMENTAL PARAMETERS

The permeability coefficient of frozen soil is influenced by many factors, in order to study the inherent law of the permeability coefficient. The paper considers such influencing factors: soil particle size, porosity, water content, unfrozen water content and ice content. Mo He County in Heilongjiang Province is located in the permafrost regions, it's very representative to make Mo He's frozen soil to be the research object. The geotechnical parameters and measured grain composition of frozen soil are showed in Table1.

Table 1: Geotechnical Parameters and Measured Grain Composition of Frozen Soil

Soil	Water content (%)	Soil grain composition (%)					Average particle diameter (mm)	Permeability coefficient (m/s)
		>0.05 (mm)	0.05~ 0.01 (mm)	0.01~ 0.005 (mm)	0.005~ 0.002 (mm)	<0.002 (mm)		
Clay A	45.6	7.4	13.7	22.8	20.4	35.7	0.010	1.34e-07
Clay B	43.9	8.9	20.4	23.5	15.9	31.3	0.013	1.90e-07
Silty clay	40.5	22.5	18.7	16.8	19.4	22.6	0.019	2.98e-07
Loam	35.8	39.2	17.6	19.9	10.9	12.4	0.026	6.04e-07
Silty sand	30.7	42.8	23.6	20.1	7.8	5.7	0.030	7.05e-07

RESULTS AND ANALYSIS

The permeability coefficient of frozen soil is obtained by numerical simulation, the attribute of each unit is randomly defined in the computation model, and the Darcy law is applied to be the governing equation to make microscopic numerical simulation. First step is calculating the average flow velocity V. Second step is the unit time seepage flow Q, formula(4), which can be obtained by making a surface integral for some section(the underside of computation model).Finally, the equivalent permeability coefficient of frozen soil is obtained by the Darcy formula(5)[10], as follow:

$$Q = \int V dA \qquad (4)$$

$$K = \frac{Q}{Ai} \qquad (5)$$

Where: K is the equivalent permeability coefficient of frozen soil, m/s; A is the area of the model calculated section, m^2; i is the hydraulic gradient, 1.

(1) Soil particle diameter. The calculated value and measured value of permeability coefficient is showed in Table 2, and illustration6 reflects the correlation between particle average diameter and the equivalent permeability coefficient .The results show that the permeability coefficient of frozen soil will increase with the increase of soil particle diameter, the function relation between the permeability coefficient of frozen soil and soil particle diameter is nonlinear, by fitting the obtained data, the correlation can be given.

The formula of the calculated value:

$$K = 1.0e^{-4}(-0.806b^2 + 0.2715b - 0.0016) \qquad (6)$$

The formula of the measured value:

$$K = 1.0e^{-4}(-0.807b^2 + 0.2707b - 0.0016) \qquad (7)$$

Where: K is the equivalent permeability coefficient of frozen soil, m/s; b is the soil particle average diameter, m.

Table 2: Comparison between the Calculated Value and the Measured Value of the Permeability Coefficient

Model	Average particle diameter (mm)	Porosity (%)	Unfrozen water content (%)	Ice content (%)	Pore permeability coefficient (m/s)	Permeability coefficient(m/s) The calculated value	Permeability coefficient(m/s) The measured value	Relative error (%)
1	0.010	0.50	0.35	0.06	4.14e-05	1.26e-07	1.34e-07	6.75%
2	0.013	0.50	0.35	0.06	6.99e-05	1.68e-07	1.75e-07	3.87%
3	0.019	0.50	0.35	0.06	1.49e-04	2.95e-07	2.78e-07	-5.85%
4	0.026	0.50	0.35	0.06	2.80e-04	5.12e-07	5.24e-07	2.32%
5	0.030	0.50	0.35	0.06	3.72e-04	7.32e-07	6.97e-07	-4.73%

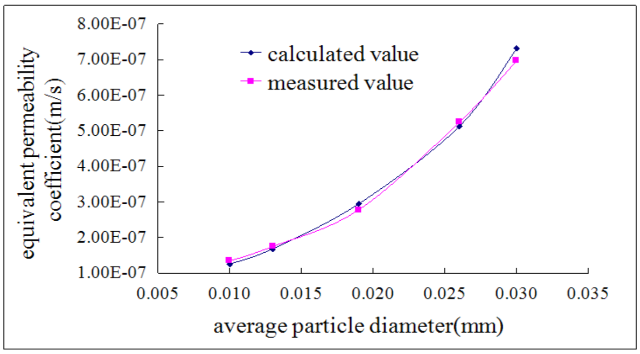

Figure 6: Correlation of the Average Particle Diameter and the Equivalent Permeability Coefficient

(2) Soil porosity. The correlation between equivalent permeability coefficient and porosity for pore size 0.010mm and 0.013mm is showed in picture7.From picture7, it can draw a

conclusion that the permeability coefficient of frozen soil gradually will increase with the increase of the porosity. What's more, when the porosity is very tiny, the correlation between them is nonlinear, when the porosity beyond a certain range, it may be linear, the conclusion is consistent with the article's experimental results[12].Numerical calculation results show that the frozen soil is tighter, the permeability coefficient of frozen soil is smaller. Compared to the silt and sand, the clay's porosity has more influence on the permeability coefficient of frozen soil.

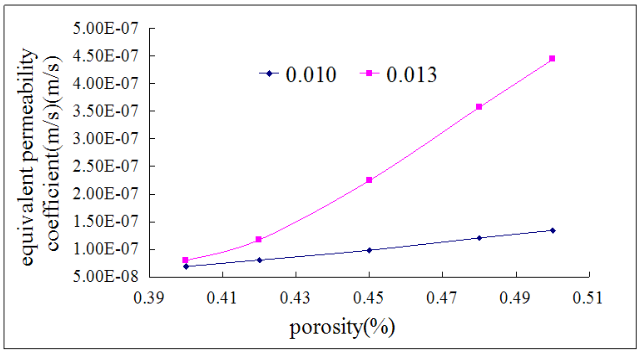

Figure 7: Correlation of the Porosity and the Equivalent Permeability Coefficient

(3)Unfrozen water content. Generally, the unfrozen water content is lower, the permeability coefficient of frozen soil is smaller. Because low water content frozen soil belongs to the low saturation soil, in which there will exist rather too much bubble which will decrease the flow area, even block the tiny pore; meanwhile, the increase of ice content will also decrease the flow area.Picture8 shows the correlation of the unfrozen water content and the equivalent permeability coefficient, where the average particle diameters are 0.010mm and 0.013mm, respectfully. The curves reveal that the permeability coefficient of frozen soil has a trend of monotonic increase with the increase of unfrozen water content; what's more, the function between them is a parabolic function approximately.

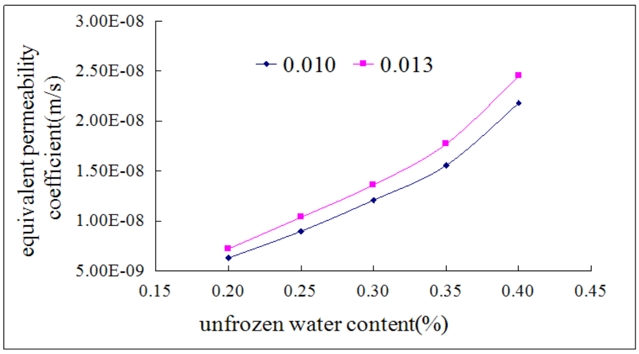

Figure 8: Correlation of the Unfrozen Water Content and the Equivalent Permeability Coefficient

(4) The ice content. Ice content is one of the most important factors which affect the permeability coefficient of frozen soil. The correlation of the ice content and the equivalent permeability coefficient is showed in picture9, where the average particle diameters are 0.010mm and 0.013mm, respectfully. From picture9, it can draw a conclusion that the permeability coefficient sharply decreases with the increase of the ice content, and when the ice content comes to some ratio , the change of the permeability coefficient of frozen soil is rather small. The dominating reason is that the increase of ice content will lead to the decrease of the effective

porosity, in addition, the unfrozen water content will reduce and the original pore size will diminish, so the reasons given above result in a sharp decline of the permeability coefficient of frozen soil.

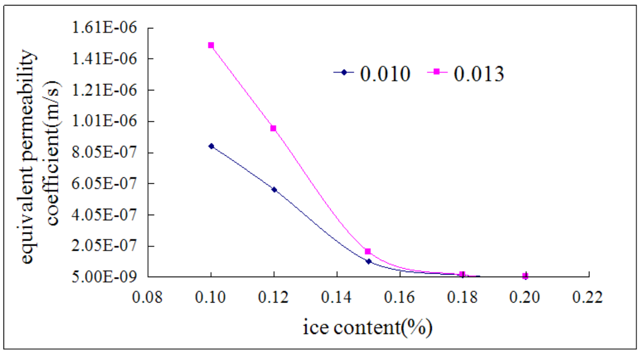

Figure 9: Correlation of the Ice Content and the Equivalent Permeability Coefficient

(5) The moisture content. The average particle diameters are 0.010mm and 0.013mm, respectfully, the correlation of the ice content and the equivalent permeability coefficient is showed in picture10.Calculation results show that the permeability coefficient continuously decreases with the increase of the moisture content, accordingly the slope of the curve decreases. Because of the increase of the moisture content, the effective pore will be filled with unfrozen water and ice, which leads to the decrease of effective porosity, therefore the macro performance, is that the seepage velocity will lower, and the permeability coefficient will decrease. The volume ratio of unfrozen water content and moisture content is also an important factor, where the correlation of this volume ratio and the equivalent permeability coefficient is showed in picture11.On the one hand, when the moisture content is some constant value, the permeability coefficient will increase sharply with the increase of the unfrozen water; on the other hand, when the unfrozen water content and ice content is some constant value, the permeability coefficient will decrease with the increase of the moisture content. The formula of the volume ratio and the permeability coefficient fits power function, which is given as follow:

$$K = 2.08 \times 10^{-4} \times \left(\frac{W_u}{W} \right)^{15.0915} \qquad (8)$$

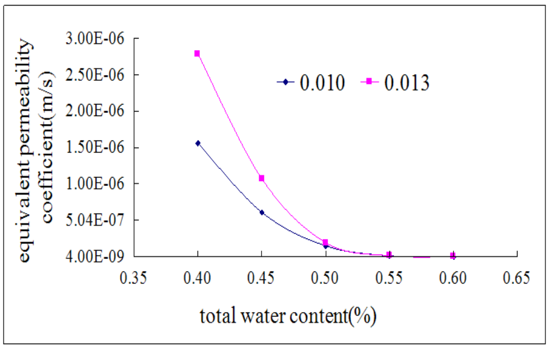

Figure 10: Correlation of the Moisture Content and the Equivalent Permeability Coefficient

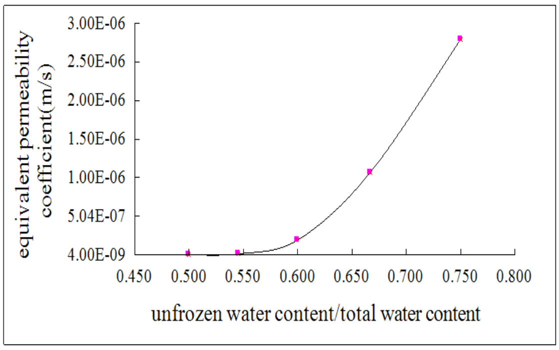

Figure 11: Correlation of the Volume Ratio and the Equivalent Permeability Coefficient

CONCLUSION

1) Permafrost is a kind of multi-phase porous medium, which is influenced by temperature, stress and water etc, and these influencing factors interact. Proceeding from the angle of each component of the permafrost and its content, and combining with the Darcy law to simulate the permafrost, the paper studies the influences on the permeability coefficient of frozen soil and the inherent variation.

2) Simulation results show that the permeability coefficient of permafrost is influenced by multiple factors, including the porosity, particle diameter, grain composition, water content, ice content etc. Because the water content and ice content is controlled by temperature. Ultimately, the permeability coefficient of permafrost is influenced by temperature and the property of mineral particles. As a matter of fact, the soil density and salt content etc [7] should be considered; therefore, the future research should consider comprehensively all kinds of influences.

3) The permeability coefficient of permafrost is a complex physical parameter. Research results demonstrate that the permeability coefficient increases with the increase of mineral particle diameter, porosity and unfrozen water content, and the slope of those curves also increases. The correlation between them is nonlinear one, which coincides with the actual experimental data, the results can provide references for the future study.

4) The results of the study show that the increase of the ice content and the moisture content will lead to the decrease of permeability coefficient of permafrost. The curves show that there is the nonlinear relationship between them, when the ice content and the moisture content reaches to some value, the permeability coefficient of permafrost changes very little. The results also show that there is a power function relation between the volume ratio of unfrozen water content and the moisture content and permeability coefficient of permafrost, which is accorded with the existing research results [11].

5) On the basis of the character that each component randomly distributes in permafrost, and combining with the stochastic simulation method, the paper applies the finite element numerical simulation software -COMSOL Multiphysics to study the correlation between the permeability coefficient of permafrost and influencing factors. The results of the study show that the relative error between the obtained data and the measured data is within the permitted range, which provides a practical method to study the physical parameters of permafrost. However, the computation model is an ideal model, and the influence of the stress on the permeability coefficient of permafrost isn't considered in this paper, so the calculated results have some discrepancy compared with the actual value.

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