

A time-domain finite element model of permeable membrane absorbers

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1. Introduction

Sound absorbers play an important role to create better acoustics in built environments. Recently, the development of a permeable membrane (PM) absorber for room acoustics applications is an area of active research [1–3] because a PM absorber has superior material properties such as light transmissibility, flexibility and lightweight properties. Various theoretical and computational models of PM absorber have been proposed [1–4] for designing a new PM absorber and for room acoustics simulations in a frequency domain.

Meanwhile, the development of efficient time-domain wave-based numerical methods for room acoustics simulations is also an active area of research [5] because a time domain analysis is attractive from the perspective of room acoustics evaluation such as the visualization, auralization of sound field and the calculation of room acoustical parameters. The finite-difference time-domain method, the time-domain finite-element method and the constrained interpolation profile method are the examples of methods. However, compared to a frequency domain analysis, a time domain analysis has a difficulty in the modeling of frequency dependence of the absorption characteristics of various absorbers, and it is one of the research topics in the time domain analysis to increase the applicability and the prediction accuracy. The numerical modeling of PM absorber in a time domain is also important for full utilization of the absorption performance of various PM absorbers in room acoustical design using the time domain analysis.

This paper proposes a simple time-domain finite element (FE) model of a permeable membrane based on the formulation of the limp membrane FEs [4] in a frequency domain. The verification is performed using impedance tube problems for measuring the normal incidence absorption characteristics of absorbers, in which the absorption characteristics obtained by the time domain FE model are compared with those obtained by theory and a frequency domain FE model.

2. FE modeling of PM absorber in time domain

2.1. Boundary condition of PM surfaces

According to the formulation of limp membrane FEs in a frequency domain proposed by Sakuma *et al.* [4], we present the time domain FE model of permeable membranes, in which

it is assumed that membranes have no tension. Figure 1 shows an FE model of a PM with the surface density M and the flow resistance R , in which $\Omega_{e,air}$ and $\Gamma_{e,M}$ respectively represent the air element and the PM element derived with contributions from both boundary surfaces of PM: $\Gamma_{e,Ma}$ and $\Gamma_{e,Mb}$. p_a and p_b represent the sound pressures at the two sides of PM, respectively. v_f and v_m respectively represent the particle velocity near and inside a material, and the vibration velocity of PM. n_a and n_b are the normal vectors at a boundary.

In this formulation, the vibration equation in a time domain is given as

$$M\dot{v}_m = p_a - p_b, \quad (1)$$

where $\dot{\cdot}$ signify the first-order derivative with respect to time. As for the permeability of PM, the flow resistance in a time domain can be defined as [1]

$$R = \frac{\dot{p}_a - \dot{p}_b}{\dot{v}_f - \dot{v}_m}, \quad (2)$$

with assuming the time factor $\exp(i\omega t)$ where i and ω represent the imaginary unit and the angular frequency. By combining the Eq. (1) and Eq. (2), the first-order derivative value of particle velocity v_f is given as

$$\dot{v}_f = \frac{1}{R}(\dot{p}_a - \dot{p}_b) + \frac{1}{M}(p_a - p_b), \quad (3)$$

With the Eq. (3), the PM can be modeled using the following vibration boundary conditions on both boundary surfaces.

$$\frac{\partial p}{\partial n} = \begin{cases} -\rho_0 \dot{v}_f & \text{on } \Gamma_{e,Ma} \\ \rho_0 \dot{v}_f & \text{on } \Gamma_{e,Mb} \end{cases} \quad (4)$$

Here, ρ_0 represents the air density.

2.2. Spatial FE discretization for sound field analysis with PM

We consider a closed sound field Ω with a rigid boundary Γ_0 , a vibration boundary Γ_v , an impedance boundary Γ_z and a boundary Γ_M related to a PM governed by the wave equation. By introducing the FE approximations to a sound pressure and a weight function in the weak form derived from the wave equation, which is the standard FE procedure, the discretized matrix equation in a time domain is obtained as

$$\sum_e^{n_e} \left[\int_{\Omega_{e,air}} N^T N dV \ddot{p}_e + c_0^2 \int_{\Omega_{e,air}} \nabla N^T \nabla N dV p_e \right]$$

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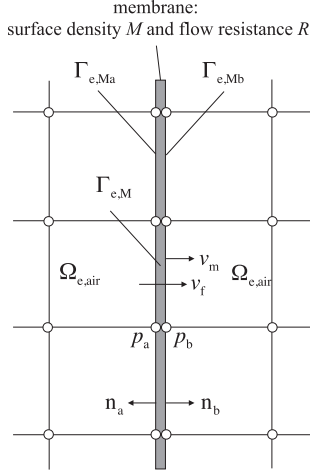


Fig. 1 An FE model of a PM with the surface density M and the flow resistance R .

$$-c_0^2 \int_{\Gamma_{e,v} + \Gamma_{e,z} + \Gamma_{e,M}} N^T \frac{\partial p}{\partial n} dA = 0, \quad (5)$$

where N , p_e , n_e and \dots respectively represent the shape function, the nodal sound pressure vector, the number of FEs and the second-order derivative with respect to time.

As shown in the procedure in the frequency domain formulation [4], the contributions from both boundary surfaces of PM can be considered by substituting the derived boundary conditions, Eq. (4), into the third term in the left-hand side of Eq. (5), which is presented below.

$$\int_{\Gamma_{e,M}} N^T \frac{\partial p}{\partial n} dA = \frac{\rho_0}{R} S_e \dot{p}_e + \frac{\rho_0}{M} S_e p_e, \quad (6)$$

where

$$S_e = \int_{\Gamma_{e,M_a}} N_a^T (N_a - N_b) dA - \int_{\Gamma_{e,M_b}} N_b^T (N_a - N_b) dA. \quad (7)$$

Here, N_a and N_b represent the shape functions at nodes on Γ_{e,M_a} and Γ_{e,M_b} in which pair of nodes has same function form. Considering the remaining boundary conditions in term of Γ_0 , Γ_v and Γ_z , the semi-discretized matrix equation for the sound field Ω is represented as

$$M\ddot{p} + c_0^2 \left(K + \frac{\rho_0}{M} S \right) p + c_0 \left(C + \frac{\rho_0 c_0}{R} S \right) \dot{p} = f, \quad (8)$$

where M , K , C and S respectively represent the global mass matrix, the global stiffness matrix, the global dissipation matrix and the global matrix related to the contribution of PM. p and f are the sound pressure vector and the external force vector, respectively. In Eq. (8), $(\rho_0/M)S$ and $(\rho_0 c_0/R)S$ are newly added terms for modeling PM in a time domain.

2.3. Time discretization using direct time integration method

As a direct time integration method, Newmark β method is applied to solve the Eq. (8). The resulting implicit time marching scheme, which includes the presented matrix related to the contribution of PM, is expressed as

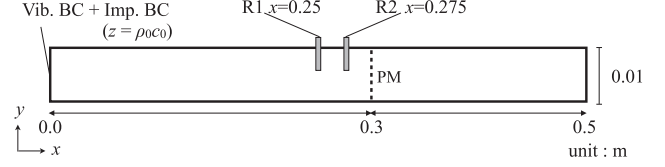


Fig. 2 Two-dimensional tube model for calculating absorption characteristics of a single PM absorber with the surface density of 0.25 kg/m^2 and the flow resistance of $1,000 \text{ Ns/m}^3$.

$$\begin{aligned} & \left[M + \beta \Delta t^2 c_0^2 \left(K + \frac{\rho_0}{M} S \right) + \frac{c_0 \Delta t}{2} \left(C + \frac{\rho_0 c_0}{R} S \right) \right] \dot{p}^{(n+1)} \\ & = f^{(n+1)} - c_0 \left(C + \frac{\rho_0 c_0}{R} S \right) p - c_0^2 \left(K + \frac{\rho_0}{M} S \right) Q, \end{aligned} \quad (9)$$

with

$$P = \dot{p}^n + \frac{\Delta t}{2} \ddot{p}^n, \quad (10)$$

$$Q = p^n + \Delta t \dot{p}^n + \left(\frac{1}{2} - \beta \right) \Delta t^2 \ddot{p}^n, \quad (11)$$

where n , β and Δt respectively represent the time step, the parameter related to the accuracy and stability of the Newmark β method and the time interval. The linear system of equations of Eq. (9) can be solved using a direct method or an iterative method. Sound pressure $p^{(n+1)}$ at $t = (n+1)\Delta t$ and its first-order derivative value $\dot{p}^{(n+1)}$ can be calculated as

$$p^{(n+1)} = p^n + \Delta t \dot{p}^n + \Delta t^2 \left(\frac{1}{2} - \beta \right) \ddot{p}^n + \Delta t^2 \beta \ddot{p}^{(n+1)}, \quad (12)$$

$$\dot{p}^{(n+1)} = \dot{p}^n + \frac{1}{2} \Delta t (\ddot{p}^n + \ddot{p}^{(n+1)}). \quad (13)$$

In this paper, we use Fox-Goodwin method as the direct time integration method, which is one of the Newmark methods with $\beta = 1/12$. The performance of the presented scheme in room acoustics simulations without installing PM absorbers can be found in Refs. 6–8, in which 8-node hexahedral FEs and 27-node hexahedral FEs are used for a spatial discretization.

3. Verification with impedance tube problem

To verify the presented time domain FE model of PM, we performed numerical experiments based on the impedance tube method for two PM absorbers. Normal incidence absorption characteristics of a single PM absorber and a PM space absorber without a rigid backing were respectively calculated, and the calculated absorption characteristics were compared with those calculated by theory and FE analyses in a frequency domain.

3.1. Single PM absorber

Figure 2 shows an impedance tube model for a two dimensional FE analysis, where a PM with the surface density of 0.25 kg/m^2 and the flow resistance of $1,000 \text{ Ns/m}^3$ was placed in front of a rigid wall with an air cavity of 0.2 m thickness. According to the transfer function method using two microphones, the sound pressures at two receiving points R1 and R2 were calculated using the presented time

domain formulation to further compute the surface impedance z_0 and the normal incidence absorption coefficient α_0 . The electro-acoustical equivalent circuit theory was used for the verification.

In the theoretical and FE analyses, c_0 and ρ_0 were respectively assumed to be 343.7 m/s and 1.205 kg/m³. The sound source used here is a modulated Gaussian pulse, which was given as vibration acceleration waveform on the surface in the left-hand side of the tube end. Here, the surface also has the characteristic impedance of the air $\rho_0 c_0$. The upper limit of frequency was assumed as 4 kHz. Meanwhile, vibration velocity of 1.0 m/s was given for the same surface in the frequency domain analysis. The remaining boundaries were assumed to be rigid. Acoustic FEs used here were four-node quadrilateral elements with modified integration rules to reduce the dispersion error [9,10], in both frequency and time domains. We also used four-node quadrilateral PM elements constructed using the same shape function as air elements. An FE mesh was created by elements with the length of 0.005 m. The analyzed time length is 0.1 s and the time interval was set to 1/137,480 s.

Figures 3(a) and 3(b) respectively show comparisons of z_0 and α_0 calculated using the time domain formulation, the frequency domain formulation and the theory, where the z_0 and α_0 by time domain formulation are in good agreement with those by the frequency domain formulation and the theory.

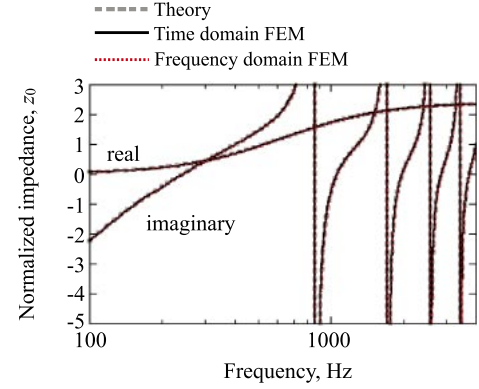
3.2. PM space absorber

Figure 4 shows an impedance tube model for a two dimensional FE analysis, where a PM with the surface density of 0.25 kg/m² and the flow resistance of 1,000 Ns/m³ was placed without a rigid backing. The absorptivity $\alpha_0 - \tau_0$ of the PM absorber, which is the actual dissipated energy inside the absorber, was calculated according to the impedance tube method with four microphones, where the τ_0 is the normal incidence transmission coefficient. In the FE analyses, sound pressures at receiving points R1~R4 were calculated. The sound sources used here were the same as previous section in the frequency and time domains. As for the boundary conditions, the surfaces of tube end have the characteristic impedance of the air $\rho_0 c_0$ and the remaining boundaries are rigid. FEs used for the spatial discretization is also the same as previous section. The theory based on the Helmholtz integral formula [1] is used for the verification.

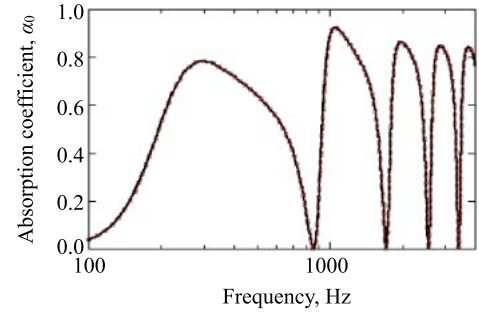
Figure 5 shows the absorptivity $\alpha_0 - \tau_0$ of the PM absorber calculated using the time domain formulation, the frequency domain formulation and the theory, in which all results are overlapped well. From the results of the numerical experiments, it can be said that the presented time domain formulation can analyze the absorption characteristics of the PM absorbers.

4. Conclusions

A time domain FE model of a PM absorber, which is based on the limp membrane FEs in frequency domain, was proposed, and an implicit time domain finite element formulation for sound field analyses with PM absorbers was presented. The validity was presented through the numerical experiment based on the impedance tube problem with two



(a) normalized impedance



(b) absorption coefficient

Fig. 3 Comparison of absorption characteristics ((a) z_0 and (b) α_0) of a single PM absorber calculated using the time domain formulation, the frequency domain formulation and the electro-acoustical equivalent circuit theory. The all results are mostly overlapped in both z_0 and α_0 .

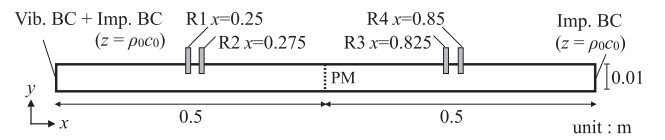


Fig. 4 Two-dimensional tube model for calculating $\alpha_0 - \tau_0$ of a PM space absorber with the surface density of 0.25 kg/m² and the flow resistance of 1,000 Ns/m³.

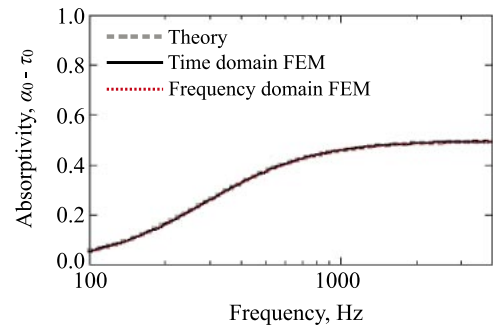


Fig. 5 Comparison of $\alpha_0 - \tau_0$ s of a PM space absorber calculated using the time domain formulation, the frequency domain formulation and the theory based on the Helmholtz integral formula. The all results are mostly overlapped.

PM absorbers, and the results showed that the time domain formulation works well for modeling the absorption of PM absorbers. Application of the presented formulation to room acoustics simulations is the subject of the future research.

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