

ACOUSTICAL LETTER

Computational benchmark on sound transmission through an elastic plate: Comparison between frequency-domain and time-domain approaches

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1. Introduction

Recently various kinds of computational methods in acoustics have been developed and applied to many practical cases. Although numerical accuracy of each method is frequently validated, mutual comparison between different methods is rarely investigated. For the situation, a benchmark project named as AIJ-BPCA (Architectural Institute of Japan–Benchmark Problems for Computational Acoustics) has been open to the World Wide Web since 2004 [1]. This project provides a useful database for research and development, and also for users to choose an appropriate method [2,3]. In this paper, as a basic case of structural-acoustic coupling analysis, a benchmark problem on sound transmission through a thin elastic plate is proposed. Two kinds of numerical approaches are tested: one is frequency-domain solution combining the structural finite element method (FEM) and the acoustic boundary element method (BEM) [4], and the other is the finite-difference time-domain (FDTD) method [5]. Calculated results for the proposed problem are compared between the two approaches in computational accuracy and efficiency. Note that this study was carried out by the Subcommittee on Computational Methods for Environmental Acoustics in the Architectural Institute of Japan.

2. Benchmark problem

A sound transmission problem is supposed as shown in Fig. 1, where a plane wave with the amplitude of 1.0 Pa is perpendicularly incident upon an elastic plate that is flush mounted on an infinite rigid baffle. The plate has a surface of 1 meter square, and is simply supported along four edges. Supposing a 10 mm thick glass plate, the physical properties are given as shown in Table 1. Note that the surface $x = 0$ corresponds with the mid-surface of the plate, and if the thin plate theory is applied, the thickness of the baffle can be regarded as zero.

In the above problem, two tasks of calculating frequency-domain results are given as follows.

Task A: Sound pressure distribution

Calculate the sound pressure amplitude at 51 receivers located along the line R1, from (0.5, 0.0, 0.0) to (0.5, 1.0, 0.0) with 0.02 m intervals. The calculation frequencies are fixed at the center frequencies of 1/1 octave bands from 31.5 Hz to 2 kHz.

Task B: Frequency characteristics

Calculate the normal-incidence transmission loss of the plate at the frequencies from 50 Hz to 2 kHz with 1 Hz intervals.

Note that the above frequency ranges are limited up to 2 kHz for avoiding heavy computational load, which is not accordance with the direction of AIJ-BPCA.

3. Numerical approaches

3.1. Structural FE-acoustic BE coupled approach

In the frequency domain, the FEM is applied to the vibration field based on the Kirchhoff–Love thin plate theory, while the BEM is applied to the sound fields of the two sides. The vibration and sound fields are discretized by quadrilateral elements of which sizes are less than a 1/8 wavelength. For the loss factor of the plate, the specified value of 0.002 is constantly given at each frequency. Finally, these systems are coupled, and the global system is solved by the direct method with LU decomposition, which gives vibration velocities of the plate. In the post process, sound pressures are calculated from the obtained velocity distribution by the BEM, and then the transmission loss is calculated. The details of the approach are described in [4].

3.2. Structural-acoustic FDTD approach

The explicit FDTD method is applied to both the vibration and sound fields, and these fields are connected by the continuity of vibration and particle velocities. In the FDTD method, a high-order scheme on the staggered grid is applied [6], and the mesh sizes are set to be less than a 1/8 wavelength. The time intervals are set as $\Delta t = 2.5 \times 10^{-2}$ ms and $\Delta t = 1.25 \times 10^{-3}$ ms for sound and vibration fields, respectively. To simulate the semi-free sound fields of the two sides, each field is surrounded with perfectly matched layers [7] consisting of 15 layers. Regarding the loss factor, the constant value of 0.002 is approximated with two downward convex curves, dividing the frequency range in half in logarithmic scale [8], as shown in Fig. 2. For the two frequency ranges, calculations are separately performed in the

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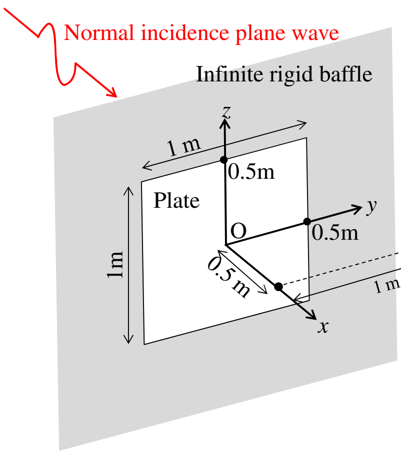

Fig. 1 Geometry of the sound transmission problem.

Table 1 Physical properties of the plate.

Thickness	0.01 m
Young's modulus	7.5×10^{10} N/m^2
Poisson's ratio	0.22
Density	2,500 kg/m^3
Loss factor	0.002

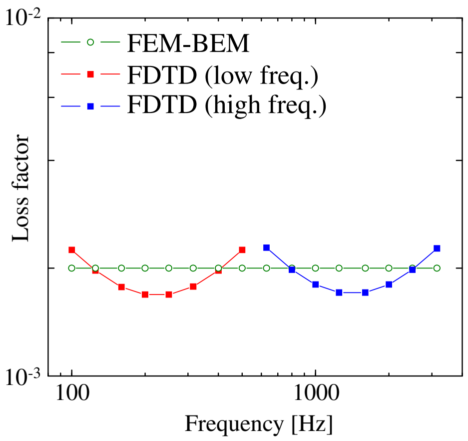

Fig. 2 Loss factors of the plate in the two approaches.

duration time of 2.4 s, (96,000 steps) to ensure sufficient convergence. The obtained transient responses are converted to the frequency domain by the FFT, and then the sound pressure distribution and the transmission loss are calculated.

4. Results

4.1. Computational accuracy

The calculated results for Task A are shown in Fig. 3. Comparing between the two approaches, the sound pressure amplitude distributions are in good agreement below 1 kHz, however a considerable discrepancy is observed at 2 kHz. A major reason should be that the FDTD method involves a numerical dispersion error at higher frequencies, especially in the vibration field.

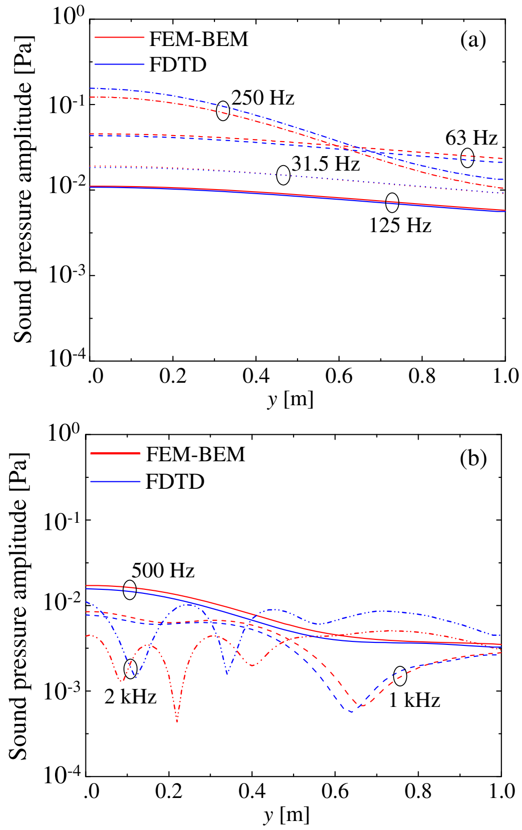

Fig. 3 Distributions of sound pressure amplitude on the line R1, calculated by the two approaches: (a) 63 Hz to 250 Hz, (b) 500 Hz to 2 kHz.

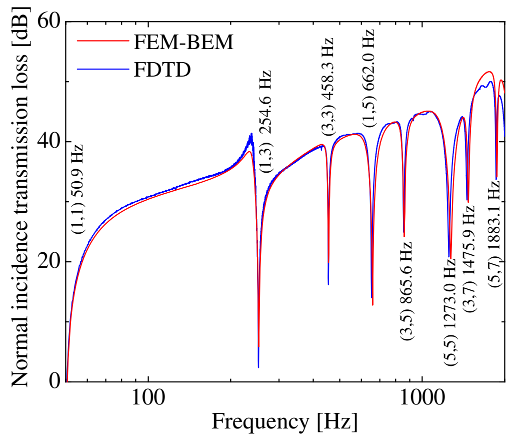

Fig. 4 Transmission loss of the plate calculated by the two approaches.

Next, the results for Task B are shown in Fig. 4, with indicating the theoretical natural frequencies of plate vibration. Generally, the frequency characteristics of transmission loss are similar with each other by the two approaches. However, the resonant frequencies with the FDTD method are slightly lower than those with the FEM-BEM at higher frequencies. The greatest discrepancy between the two approaches is observed at the (5, 5) mode, where the errors

Table 2 Computational specifications of the two approaches.

Method	Frequency/Time steps	Mesh size	DOFs	Used memory	Calculation time	CPU
FEM-BEM	951 steps (50–1,000 Hz)	3.70 cm	2,916	129 MB	4.84 s/step	Intel Core i7 960 (3.2 GHz)
	1,001 steps (1,001–2,000 Hz)	2.00 cm	10,404	1,612 MB	258.58 s/step	
FDTD	96,000 steps (2.4 s)	1.96 cm	16,881,050	128 MB	0.68 s/step	Intel Core i7 920 (2.67 GHz)

to the theoretical frequency are 0.16% for the FEM-BEM, and 1.65% for the FDTD method. As mentioned above, it should be also caused by numerical dispersion.

4.2. Computational efficiency

Table 2 shows the computational specifications of the two approaches. The calculations are performed by different PCs using a single processor, and computational times are measured per frequency and time steps for the two approaches, respectively. It is noted that the following comparisons between the two approaches are made under the condition with comparable mesh sizes. Regarding memory storage, the FEM-BEM used about 1.6 GB for the high frequency range, while the FDTD used about 130 MB, less than 1/10 of the former. Regarding computational time, it took 73.1 hours in total with the FEM-BEM, for 1951 frequency steps. With the FDTD method, it took 36.3 hours for the two calculations for low and high frequency ranges, each of which has 96,000 time steps. In this case, the FDTD method spent about 1/2 of the computational time with the FEM-BEM, although computational accuracy is a little deteriorated at high frequencies. In addition, the total computational time with the FDTD method increases in proportion to the number of frequency ranges to approximate the frequency characteristics of loss factor.

5. Conclusion

A basic benchmark problem for structural-acoustic simulation is proposed, where considering a sound transmission though an elastic plate mounted on an infinite rigid baffle. As a comparison between frequency-domain and time-domain approaches, the structural FE-acoustic BE coupled approach and the structural-acoustic FDTD approach are tested. As a result, the two numerical approaches were generally validated in computational accuracy. In addition, it was demonstrated that, if comparable mesh sizes are used, the frequency-domain approach has relatively high accuracy, whereas the time-domain approach has an advantage of obtaining wideband frequency responses with fast computation time and small memory storage. Finally, this benchmark problem and the presented computational results will be useful for validation of other structural-acoustic computational methods.

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