

Geometrical nonlinearity analysis of a piano hammer using a finite element method

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1. Introduction

It is known that the nonlinearity of a piano hammer felt is approximated by $f = Ad^B$, where f is the force, d is the deformation of the felt, and A and B are constants [1,2]. This nonlinearity may be due to both geometrical and material nonlinearities. The aim of this paper is to find the contribution of the geometrical nonlinearity only. In other words, to find how big B is when only the geometrical nonlinearity is considered. A finite element method is used in this study. The material properties of the finite element model are assumed to be linear as well as isotropic.

This letter is a summary of a report prepared by the author when he was working for CBS Technology Center, Stamford, USA [3].

2. Finite element model

The finite element method software program used for the study is BOPACE. Only one quarter of the hammer felt is modeled as shown in Fig. 1 assuming that the hammer has a symmetrical shape. The felt is divided into 490 ($= 5 \times 7 \times 14$) elements. The number of elements is fairly small because the maximum number of elements is limited to 500 in the present program. Portions of the felt that are expected to undergo larger deformation when compressed are divided into smaller elements in order to increase the accuracy of analysis with the limited number of elements. The elements are categorized into five regions (I to V) and an identical Young's modulus is assigned to elements of each region. Region I indicates the outer most layer and region V indicates the two inner most layers. Nodes of each layer from the inner side (wood core) to the outer side follow elliptical curves. The wood core is assumed to be ideally rigid and acts as fixed constraints in the three directions for the nodes of elements of the inner most layer. Dimensions are given considering a base hammer. A hammer thickness ($2T$) of 10 mm and a string radius (R) of 2.5 mm are used for all calculations.

The deformation of the felt by the string is shown in Fig. 2. It is assumed that the contact width a is given by

$$a = \sqrt{Rd} \quad (1)$$

following the Hertz's law [4]. For a given d , nodes that have z -axis values smaller than a touch the string. Values of concentrated loads to nodes within a are found by a repetitive process so that the deformation of the felt takes the shape as

shown in Fig. 2. The force (f) is given by a summation of loads in the y -axis. The calculation was performed for four values of d : 0.0375, 0.075, 0.1125 and 0.15 cm.

Young's moduli and geometrical parameters are shown in Table 1 for 9 cases investigated in this study. The normalized value of 1 dyn/cm² is given to elements of the outermost layer (except for case 2). If the real values are larger than those in Table 1, for example, by 10^7 , the values of the force (f) should be multiplied by 10^7 .

In most of the 9 cases, 10 times larger Young's modulus is given to region V compared to region I because a preliminary experiment showed that the inner-most portion has a much larger stiffness than the outer-most portion. Case 1 is regarded as the reference case. In cases 2 to 4, Young's moduli of regions I to III are changed to see effects of moduli of those regions. In cases 5 and 6, the width and the height of the hammer are changed, respectively. Parameters of cases 7 to 9 are given to see mostly the effect of Young's modulus gradient along the y -axis ($x = z = 0$).

3. Results and discussion

Figure 3 shows f/d relations for cases 1 (○) and 2 (Y). The curves are those obtained by fitting the five points by the function $f = Ad^B$. The results show that the f/d characteristic is fitted very well by the exponential function.

Shapes of elements before (thin line) and after (thick line) the felt is deformed are shown in Fig. 4 for case 1 and $d = 0.15$ cm (only the upper part is shown). The top front elements are most severely deformed. The hammer swells slightly in thickness (z -axis) when it is compressed.

Values of A and B for cases 1 to 9 are shown in Table 2. The maximum value of B in Table 2 is 1.74, which is much smaller than a value of B (2.3 or larger) of a base hammer obtained from a measurement of f/d characteristic during the compression process. This indicates that the geometrical nonlinearity is much smaller than the measured nonlinearity. The remaining nonlinearity may be mostly due to the material nonlinearity.

It is natural to consider that regions that are more severely deformed are more influential to f/d characteristics. As Fig. 4 shows, those regions are the top portion of region I near the y -axis, region II, and partly region III. Smaller the Young's moduli of these regions are, the smaller the constant A is. And the larger the gradient of Young's modulus from the top portion of region I to region II is, the larger the value of B is (compare cases 1 to 4 with the identical geometry).

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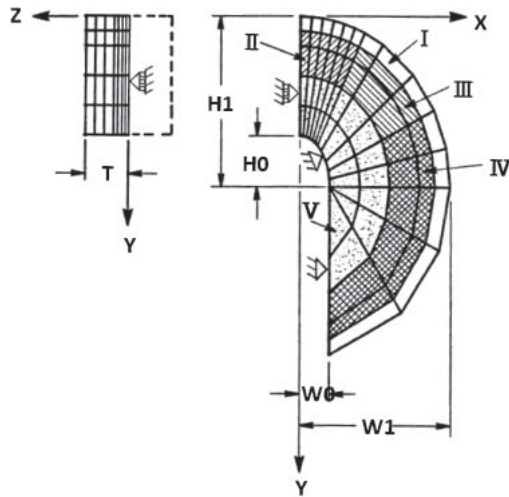


Fig. 1 Finite element model of a hammer.

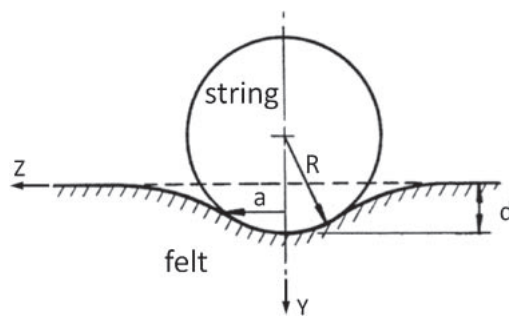


Fig. 2 Sketch of the contact surface between the string and the hammer felt.

Table 1 Young's modulus and geometry of the hammer models for cases 1 to 9.

| case | Young's modulus (dyn/cm ²) | | | | | Geometry (cm) | | | |
|------|--|------|------|------|------|---------------|------|----|------|
| | I | II | III | IV | V | W0 | W1 | H0 | H1 |
| 1 | 1 | 3.16 | 3.16 | 3.16 | 10 | 3.5 | 17.5 | 6 | 20 |
| 2 | 0.5 | 3.16 | 3.16 | 3.16 | 10 | 3.5 | 17.5 | 6 | 20 |
| 3 | 1 | 1.78 | 3.16 | 3.16 | 10 | 3.5 | 17.5 | 6 | 20 |
| 4 | 1 | 3.16 | 1.78 | 3.16 | 10 | 3.5 | 17.5 | 6 | 20 |
| 5 | 1 | 3.16 | 3.16 | 3.16 | 10 | 3.5 | 20 | 6 | 20 |
| 6 | 1 | 3.16 | 3.16 | 3.16 | 10 | 3.5 | 17.5 | 6 | 17.5 |
| 7 | 1 | 2 | 4 | 7 | 10 | 3.5 | 17.5 | 6 | 20 |
| 8 | 1 | 1.41 | 2 | 2.65 | 3.16 | 3.5 | 17.5 | 6 | 20 |
| 9 | 1 | 1 | 1 | 1 | 1 | 3.5 | 17.5 | 6 | 20 |

A hammer with a larger width (case 5) or with a smaller height (H1–H0) from the top of the hammer to the top of the wood piece (case 6) has a larger value of A . The increase of A caused by the larger width is due to the increase of the contact area with the string. The decrease of the height causes the increases of both A and B , indicating that it is a very influential parameter to the f/d characteristics.

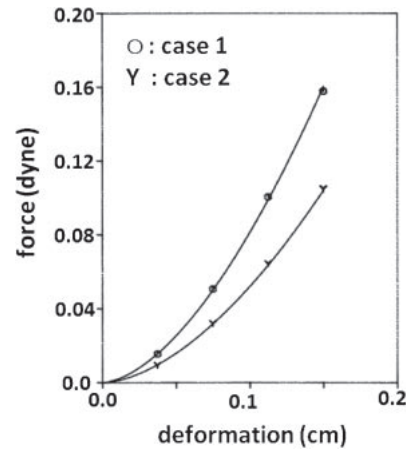


Fig. 3 Force vs deformation characteristics for cases 1 and 2.

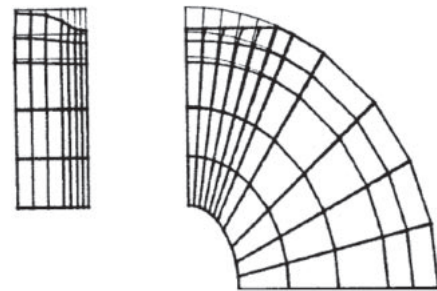


Fig. 4 Shapes of elements before (thin line) and after (thick line) the felt is deformed for case 1 and $d = 0.15$ cm.

Table 2 Values of A and B for cases 1 to 9.

| | case | | | | | | | | |
|---|------|------|------|------|------|------|------|------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 3.79 | 2.89 | 2.88 | 3.72 | 4.08 | 4.86 | 3.38 | 2.03 | 0.84 |
| B | 1.67 | 1.74 | 1.65 | 1.67 | 1.66 | 1.70 | 1.65 | 1.59 | 1.43 |

A hammer with a larger gradient of Young's modulus from the top of the hammer into the inward direction has a larger degree of nonlinearity (compare cases 1, 7, 8, 9). Case 6, which has the second largest B among 9 cases, also shows this trend possibly because it has a larger effective gradient than case 1 due to the smaller height (H1–H0).

References

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