

Boundary condition for finite-difference time-domain method using digital filters and efficient design of filter coefficients using equivalent mechanical system

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1. Introduction

When we analyze a sound field by the finite-difference time-domain (FDTD) method, we assume a locally reactive boundary condition and set real acoustic impedances that correspond to the normal incidence absorption coefficients. Under these conditions, we cannot account for the boundary frequency characteristics. To overcome this, Chiba *et al.* proposed a boundary condition taken from an equivalent electrical system [1] and Sakamoto *et al.* proposed one from an equivalent mechanical system [2]. Because these models are specialized for acoustic problems, they require little computational load and are efficient. However, each model requires a separate implementation. That is, we must modify the simulation program for these boundary conditions, and for another model, we also must modify the program. On the other hand, for the constrained interpolation profile method, we introduced a boundary condition using a digital filter and showed its effectiveness [3]. This boundary condition is general and does not require separate implementations. That is, we need not modify the simulation program to suit the model, and for another model, we have only to change the coefficients of a filter. However, it is not easy to design an efficient filter for acoustic problems.

To exploit both advantages, a boundary condition using digital filters is introduced to the FDTD method, and the relationship between the filter coefficients and the equivalent mechanical system [2] is clarified. These make both the use of an equivalent mechanical system and the design of filter coefficients easier and more efficient.

2. Introduction of IIR filters to boundary condition of FDTD method

Using a leapfrog scheme, where the time and space definition points of u and p are staggered, the locally reactive boundary condition of the FDTD method is defined as

$$u^{n+1} = \frac{p^{n+1/2}}{Z}, \quad (1)$$

where p , u , and Z are the sound pressure, particle velocity,

and acoustic impedance on the boundary, respectively. $()^n$ represents the value at time step n .

The boundary is considered to be a system with input p and output u . This system is modeled using a digital filter, as shown by

$$u^{n+1} = \sum_{i=0}^n b_i p^{(n+1/2)-i} - \sum_{i=1}^{n+1} a_i u^{(n+1)-i}. \quad (2)$$

Instead of giving the constant Z , optimal filter coefficients a_i and b_i are designed to realize the boundary frequency characteristics. Equation (1) is solved with $b_0 = 1/Z$ and a_i and other b_i assigned as zero.

3. Design of filter coefficients of IIR filter using equivalent mechanical systems

According to [2], when a boundary is considered as a mechanical system, as in Fig. 1, the equation of motion of a mass is defined as

$$pS = m \frac{\partial^2 x}{\partial t^2} + c \frac{\partial x}{\partial t} + kx, \quad (3)$$

where x and t are the displacement of a mass with area S and time, respectively. Furthermore, m , c , and k are the mass, the damping coefficient of the damper, and the stiffness of the spring, respectively.

Equation (3) is discretized and normalized by the area S , as shown in

$$p^{n+1/2} = m' \frac{x^{n+3/2} - 2x^{n+1/2} + x^{n-1/2}}{\Delta t^2} + c' \frac{x^{n+3/2} - x^{n+1/2}}{\Delta t} + k' x^{n+1/2}, \quad (4)$$

where Δt is the time step and m' , c' , and k' are the normalized m , c , and k , respectively. x is expressed by u , as shown in Eq. (5), because u is discretized as a time difference of x , as shown in Eq. (6).

$$x^{n+1/2} = \left(\frac{x^{n+1/2} - x^{n-1/2}}{\Delta t} + \frac{x^{n-1/2} - x^{n-3/2}}{\Delta t} + \dots + \frac{x^{1/2} - x^{-1/2}}{\Delta t} + \frac{x^{-1/2}}{\Delta t} \right) \Delta t = \Delta t \sum_{i=0}^n u^i, \quad (5)$$

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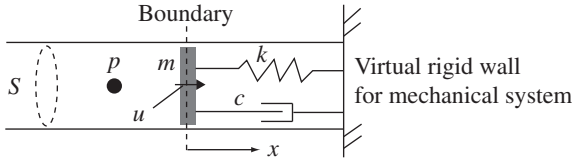


Fig. 1 Equivalent mechanical system for the boundary condition of the FDTD method.

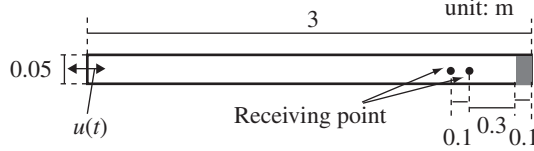


Fig. 2 Geometry of acoustic tube.

$$u^{n+1} = \frac{x^{n+3/2} - x^{n+1/2}}{\Delta t}, \quad (6)$$

where x^n and u^n ($n \leq 0$) are zero. We substitute Eq. (5) into Eq. (4), and obtain the z -transform of Eq. (4), as shown in

$$P(z) = \left(\alpha_0 - \frac{m'}{\Delta t} z^{-1} + k' \Delta t \sum_{i=0}^n z^{-(1+i)} \right) U(z), \quad (7)$$

$$\alpha_0 = \frac{m'}{\Delta t} + c',$$

where $P(z)$ and $U(z)$ are the z -transforms of p and u , and z is a delay operator. Here the transfer function $H(z) = U(z)/P(z)$ is obtained as

$$\begin{aligned} H(z) &= \frac{(1 - z^{-1})\Delta t}{\alpha_0 \Delta t + (-2m' - c' \Delta t + k' \Delta t^2)z^{-1} + m' z^{-2}}, \\ &= \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}. \end{aligned} \quad (8)$$

This shows that an equivalent mechanical system is a second-order IIR filter. $H(z)$ has one zero (unity) and two poles. Although additionally required variables in [2] are $x^{n+1/2}$ and $x^{n-1/2}$, those in the proposed method are u^n , u^{n-1} , and $p^{n-1/2}$. The latter is larger than the former. However, in a three-dimensional field, because the required variables for both boundary conditions are proportional to $O(N^2)$, these are much smaller than those for the FDTD method, which is proportional to $O(N^3)$. Here, N is the mesh number on each axis. The complete set of required variables is almost the same for both boundary conditions.

4. Validation with acoustic tube problem

Referring to [4], Δx and Δt were set to 0.0025 m and 2.576 μ s, respectively. Here, Δx is the space step. To validate the effectiveness of the aforementioned IIR filter, the acoustic tube problem was analyzed. It had a 0.1 m-thick layer of absorptive material for which R was 15,000 Ns/m⁴ on the right wall, as shown in Fig. 2. At the left boundary of the tube, the particle velocity was given by

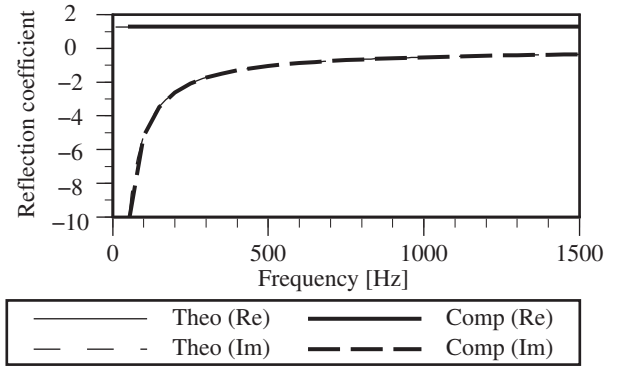


Fig. 3 Complex reflection coefficient calculated by the FDTD method (comp) and theory (theo).

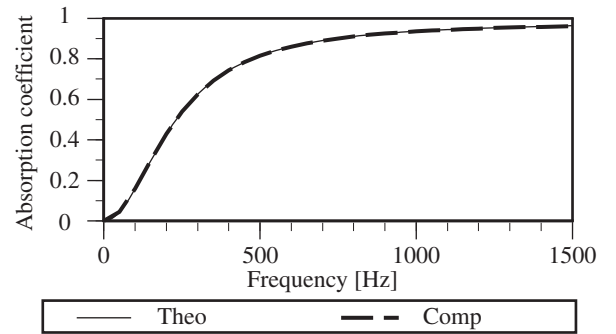


Fig. 4 Absorption coefficient calculated by the FDTD method and theory.

$$u(t) = 4A^2 B(t - \tau)(3 - 2A(t - \tau)^2) \exp(-A(t - \tau)^2), \quad (9)$$

where τ , A , and B were set to 3.57 ms, 4×10^6 , and 1.3×10^{-12} , respectively [3]. This source included the components from 100 to 1,800 Hz at a point -20 dB from the peak [3].

The coefficients of the equivalent mechanical model were designed with the least squares error criterion as follows: $m' = -4.71 \times 10^{-7}$ kg/m², $c' = 5.24 \times 10^2$ kg/m² s, and $k' = 3.26 \times 10^3$ N/m³.

The calculated complex reflection coefficient and absorption coefficient with their theoretical counterparts are shown in Figs. 3 and 4. These were calculated by the transfer function method. These results are in agreement even when using the second-order IIR filter.

5. Conclusions

A boundary condition using digital filters was introduced to the FDTD method in order to account for the boundary frequency characteristics. This boundary condition does not require a separate implementation for each new system. In addition, the relationship between the filter coefficients and the equivalent mechanical system was clarified. This relationship permits easy and efficient design of the filter.

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