

PAPER

Humbucking pickup response excited by string vibration

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Abstract: As a factor to characterize the sound of an electric guitar, it is thought that a characteristic of the pickup contributes most. The pickups most often used are classified roughly into single-coil models and humbucking models. The single-coil pickup is made by winding the thin wires with several thousand turns of coils around six polarizing pole pieces each corresponding to a string of the guitar, and the change in the magnetic reluctance owing to the string vibration that causes the change in the magnetic flux is transformed into an electrical signal. The humbucking pickup is composed of one magnetic circuit with two single-coil pickups, and made to be in phase electrically and out of phase magnetically for the purpose of removing circumference magnetic noise. In this paper, the response of the humbucking pickup excited by a string vibration set up by a real commercial solid body electric guitar is analyzed, and a simulation result is shown to agree with an actual measured value with sufficient precision. In addition, the response of the humbucking pickup imitated with two single-coil pickups is compared with the single-coil pickup and some additional considerations in the characteristics have been gained through analysis.

Keywords: Electric guitar, Electromagnetic transducer, Humbucking pickup, String vibration, Negative stiffness

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1. INTRODUCTION

Approximately 80 years ago, the basic model of the electric guitar being used at present appeared. Since then, the basic structure of the electric guitar with a pickup in which the pole piece is wound with a coil has not changed, and the reduction of noise and the improvement of a peripheral device have been treated [1]. For the electric guitar with a solid body, the characteristics are summarized as follows. The decrement of the string vibration is relatively small because the body is made of the solid wood. Also, the volume can be adjusted arbitrarily using an amplifier because the string vibration is changed into an electrical signal by the pickup. In addition, various sound effects can be added to the electrical signal by using an effects unit. Although various components composing the electric guitar, including the strings, are considered to be factors that characterize the

tone, the characteristic of the pickup that converts the string vibration into an electrical signal is thought to have the strongest influence on it.

The pickup used for the electric guitar is the electromagnetic transducer and can be divided roughly into a single-coil type or a humbucking type. The former is wound with thin coils by several thousand turns around all pole pieces corresponding to the six strings. The latter is made in one magnetic circuit with two single-coil pickups and made to be in phase electrically by connecting the end of the two coils and out of phase magnetically for the purpose of removing circumference magnetism noise [2]. Generally, it is said that the single-coil pickup is susceptible to noise and feedback, but clean sound is provided, and that the humbucking pickup has a large output with low noise, and a warmer and more mellow tone is produced because the middle frequency range components are relatively rich.

It has also been sufficiently acknowledged by musicians and guitar lovers that Stratocaster of Fender Com-

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Fig. 1 Electric guitar with humbucking pickup.

pany, in which a single-coil pickup is used, and Les Paul of Gibson Corporation, in which the humbucking pickup is used, are representative guitars [3]. It is an established fact that these electric guitars had a great influence on guitar makers as the standard models up to now. However, there are few academic studies related to the electric guitar, and also, the development is carried out mainly through trial and error based on experience. Therefore, it is thought that the analysis of the response of the humbucking pickup acoustically and the comparison with that of the single-coil pickup are useful for designing future electric guitars.

In this paper, the response of the humbucking pickup mounted on a real commercial solid-body electric guitar and excited by string vibration is analyzed. It is shown that a simulation result agrees with the actual measured value with sufficient precision, and the characteristic of the imitated humbucking pickup constructed of two single-coil pickups is investigated by comparison with the response of one of two single-coil pickups.

2. THEORETICAL ANALYSIS

2.1. Analysis Model

The commercial electric guitar used for the analysis in this study is shown in Fig. 1. It is a copy product of the Les Paul model of Gibson Corporation, and the humbucking pickup is mounted on the front and the rear part. However, the rear pickup is excluded as the analysis is intended for only front pickup.

Figure 2 shows the electrical and magnetic principle of the humbucking pickup and the position relation with the string. Two single-coil pickups are located very close to each other, and a magnetic circuit is composed through the string by setting the magnet at the bottom center of both pickups [2]. Therefore, the magnetic flux directions of the two pickups against the string are opposite to each other. Then, by connecting the end terminals of the two coils, that are wound in same direction, and setting the start terminals to the output terminal, the electrical outputs of the two pickups become in phase. Thus, when the outside magnetic flux noise reaches the two pole pieces in phase, the electromotive forces are out of phase and will be canceled out.

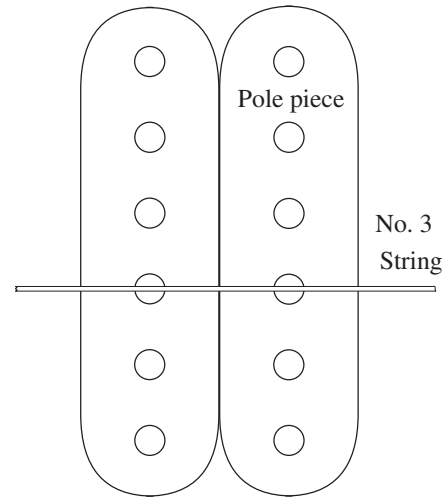
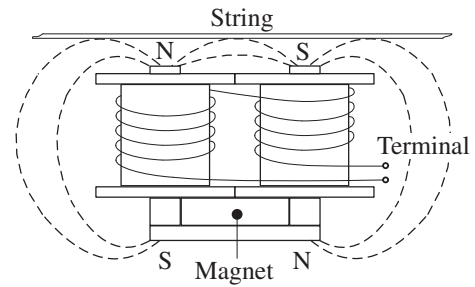


Fig. 2 Diagram of humbucking pickup.

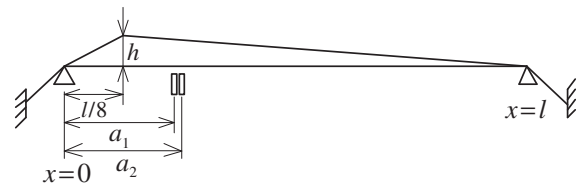


Fig. 3 String vibration model.

The analysis model is shown in Fig. 3. Let l be the length between a bridge and the nut and $x = a_1, a_2$ be the positions of the two pole pieces of the humbucking pickup with the origin set at the bridge. Also, it is supposed that the initial condition is set at the height h at the position $x = l/8$.

2.2. Vibration Equation

When the electromagnetic pickup is used to convert the string vibration into an electrical signal, it has been analyzed in detail that the attraction acts on the string vibration via the polarizing magnet of the pickup, and the negative stiffness component reduces the vibration frequency of the string depending on the strength and the attracting position [4]. It is indicated that the proper simulation result of the string vibration can be provided by linearizing the measured attraction to the static force and

the negative stiffness component. Also, when the plural negative stiffness existed discretely, it was shown that the inharmonicity expressing a nonharmonic property is almost the same as the total value of negative stiffnesses that each exist independently [5]. Although the flux at two magnetic poles of the humbucking pickup are antiphase, the two attractions of the magnet can be divided approximately into a static component F_0 and the negative stiffness component $-S_{ni}y(a_i, t)$, which is in inverse proportion to the string displacement, and the vibration equation of the model in Fig. 3 is presented as

$$\sigma \frac{\partial^2 y(x, t)}{\partial t^2} = T \frac{\partial^2 y(x, t)}{\partial x^2} - \sum_{i=1}^2 \delta(x - a_i) \{F_{0i} - s_{ni}y(a_i, t)\}, \quad (1)$$

where σ is a linear density and T is the tension of the string.

According to the analysis results [4], the static displacement affected by the static force is very small because the attraction is weak compared with the string tension. However, the negative stiffness reduces the vibration frequencies and, as a result, causes beats due to the interference between the partial tones whose amplitudes are comparatively large. Therefore, the vibration waveform and the amplitude change with time. Thus the vibration frequencies of the string with the humbucking pickup will be examined first.

2.3. Characteristic Equation

Equation (1) can be solved using the Laplace transform. Let $\mathcal{L}\{y(x, t)\} = Y(x, p)$ and $\mathcal{L}\{Y(x, p)\} = Y(\alpha, p)$ represent the Laplace transform for time variance t and position variance x , respectively. Setting the initial conditions $y(x, 0) = g(x)$ and $y'(x, 0) = 0$, the Laplace transform of Eq. (1) for t can be obtained as

$$p^2 Y(x, p) - pg(x) = c^2 Y_{xx}(x, p) - \sum_{i=1}^2 \delta(x - a_i) \left\{ \frac{r_i}{p} - \kappa_i Y(a_i, p) \right\}, \quad (2)$$

where $c^2 = T/\sigma$, $r_i = F_{0i}/\sigma$, and $\kappa_i = s_{ni}/\sigma$.

Next, setting $\mathcal{L}\{g(x)\} = G(\alpha)$, carrying the Laplace transform for x and setting the boundary condition $y(0, t) = Y(l, t) = 0$, the following equation is obtained:

$$Y(\alpha, p) = \frac{1}{\alpha^2 - \frac{p^2}{c^2}} \left[Y_x(0, p) - \frac{p}{c^2} G(\alpha) + \frac{1}{c^2} \sum_{i=1}^2 \left\{ \frac{r_i}{p} - \kappa_i Y(a_i, p) \right\} e^{-a_i \alpha} \right]. \quad (3)$$

Then applying the Laplace inverse transform, the next result is provided in the region $a_{j-1} \leq x < a_j$ ($j = 1 \sim 3$):

$$Y(x, p) = Y_x(0, p) \frac{c}{p} \sinh \frac{p}{c} x - \frac{1}{c} \int_0^x g(\xi) \sinh \frac{p}{c} (x - \xi) d\xi + \sum_{i=0}^{j-1} \left\{ \frac{r_i}{p} - \kappa_i Y(a_i, p) \right\} \frac{1}{pc} \sinh \frac{p}{c} (x - a_i), \quad (4)$$

where $a_0 = 0$, $a_3 = l$, and $r_0 = \kappa_0 = 0$. Setting $j = 3$ and $x = l$, and the boundary condition $Y(l, p) = 0$, $Y_x(0, p)$ is obtained as

$$Y_x(0, p) = \frac{p}{c^2 \sinh \frac{p}{c} l} \int_0^l g(\xi) \sinh \frac{p}{c} (l - \xi) d\xi - \sum_{i=0}^2 \left\{ \frac{r_i}{p} - \kappa_i Y(a_i, p) \right\} \frac{\sinh \frac{p}{c} (l - a_i)}{c^2 \sinh \frac{p}{c} l}. \quad (5)$$

Substituting this into Eq. (4) with condition $i = j$, the following equation for $Y(x, p)$ is given:

$$Y(x, p) = \frac{\sinh \frac{p}{c} (l - x)}{c \sinh \frac{p}{c} l} \int_0^x g(\xi) \sinh \frac{p}{c} \xi d\xi + \frac{\sinh \frac{p}{c} x}{c \sinh \frac{p}{c} l} \int_x^l g(\xi) \sinh \frac{p}{c} (l - \xi) d\xi - \frac{\sinh \frac{p}{c} (l - x)}{pc \sinh \frac{p}{c} l} \times \sum_{i=0}^{j-1} \left\{ \frac{r_i}{p} - \kappa_i Y(a_i, p) \right\} \sinh \frac{p}{c} a_i - \frac{\sinh \frac{p}{c} x}{pc \sinh \frac{p}{c} l} \times \sum_{i=j}^2 \left\{ \frac{r_i}{p} - \kappa_i Y(a_i, p) \right\} \sinh \frac{p}{c} (l - a_i). \quad (6)$$

Namely, the solution is provided separately at three sections: $0 \leq x < a_1$, $a_1 \leq x < a_2$, and $a_2 \leq x \leq l$. Because $Y(x, p)$ is equal at $x = a_1$ in the regions $0 \leq x < a_1$ and $a_1 \leq x < a_2$, and at $x = a_2$ in the regions $a_1 \leq x < a_2$ and $a_2 \leq x \leq l$, $Y(a_1, p)$, $Y(a_2, p)$ are then provided:

$$\left. \begin{aligned} Y(a_1, p) &= \frac{1}{W(p)} \left[pD_1(p)H_2(p) - \frac{R_1(p)H_2(p)}{p} + \kappa_2 \left\{ pD_2(p) - \frac{R_2(p)}{p} \right\} \times \sinh \frac{p}{c} a_1 \sinh \frac{p}{c} (l - a_2) \right] \\ Y(a_2, p) &= \frac{1}{W(p)} \left[pD_2(p)H_1(p) - \frac{R_2(p)H_1(p)}{p} + \kappa_1 \left\{ pD_1(p) - \frac{R_1(p)}{p} \right\} \times \sinh \frac{p}{c} a_1 \sinh \frac{p}{c} (l - a_2) \right] \end{aligned} \right\}, \quad (7)$$

where

$$W(p) = H_1(p)H_2(p) - \kappa_1 \kappa_2 \sinh^2 \frac{p}{c} a_1 \sinh^2 \frac{p}{c} (l - a_2). \quad (8)$$

For $i = 1$ and 2 , $H_i(p)$, $D_i(p)$, and $R_i(p)$ are represented as

$$H_i(p) = pc \sinh \frac{p}{c} l - \kappa_i \sinh \frac{p}{c} (l - a_i) \sinh \frac{p}{c} a_i \quad (9)$$

$$D_i(p) = \sinh \frac{p}{c} (l - a_i) \int_0^{a_i} g(\xi) \sinh \frac{p}{c} \xi d\xi + \sinh \frac{p}{c} a_i \int_{a_i}^l g(\xi) \sinh \frac{p}{c} (l - \xi) d\xi \quad (10)$$

$$R_i(p) = r_1 \sinh \frac{p}{c} a_1 \sinh \frac{p}{c} (l - a_i) - r_2 \sinh \frac{p}{c} (l - a_2) \sinh \frac{p}{c} a_i. \quad (11)$$

Substituting $Y(a_1, p)$ and $Y(a_2, p)$ into Eq. (6), the final image equation can be obtained.

Namely, Eq. (8) is the characteristic equation with the humbucking pickup, and also, $H_1(p)$ and $H_2(p)$ are characteristic equations respectively when the two single-coil pickup are each set up alone.

3. ANALYSIS AND MEASUREMENT RESULTS

3.1. Numerical Data and Inharmonicity

The inharmonicity δ_n is defined as

$$\delta_n = 1200 \log_2 \frac{f_n}{nf_1} \quad [\text{cent}], \quad (12)$$

where f_1 is the fundamental frequency and f_n is the n th partial tone frequency. The inharmonicity of the vibration frequencies calculated by setting Eq. (8) to zero is indicated in Fig. 4. Because the solution of the characteristic equation becomes a purely imaginary number, by setting $p = j\omega$, the relation with the n th partial tone

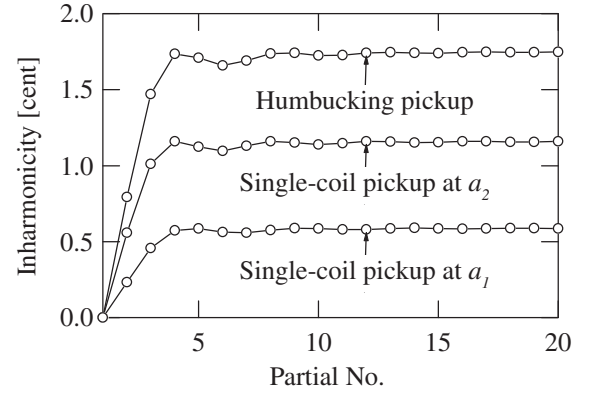


Fig. 4 Inharmonicity of a string affected by humbucking pickup.

frequency can be shown as $\omega_n = 2\pi f_n$. The string used here is the 3rd string of the G_3 tone with a basic frequency of 195.997718 Hz. The data is given as follows

String length	$l = 0.6305$	[m]
Linear Density	$\sigma = 0.9721 \times 10^{-3}$	[kg/m]
Tension	$T = 59.36$	[N/m]
Initial height at $l/8$	$h = 1 \times 10^{-3}$	[m]
Positions of pickup	$a_1 = 0.1365$	[m]
	$a_2 = 0.1535$	[m]
Negative stiffness	$s_{n1} = 0.8$	[N/m]
	$s_{n2} = 1.3$	[N/m].

Here, the ratio of two negative stiffnesses are determined from the measured flux densities at the pole piece of two pickups, because the attraction is proportional to the square of the magnetic flux, and both negative stiffnesses are estimated by referring to the calculated nonharmonic frequencies and measured beat frequency.

3.2. Vibration Displacement

The influence of the negative stiffness on the amplitude of the string is very small [4]. Because it can be thought that the influence of damping on the vibration frequency is also small, the vibration displacement of the string can be assumed to be

$$y(x, t) = \sum_{n=1}^{\infty} A_n e^{-\gamma_n t} \sin \frac{n\pi}{l} x \cdot \cos \omega_n t, \quad (13)$$

where A_n is the amplitude of the n th partial tone when the initial displacement of the string is $g(x)$, and is approximately given as follows [6]:

$$A_n = \frac{2}{l} \int_0^l g(x) \sin \frac{2\pi n f_1}{c} dx. \quad (14)$$

In addition, γ_n is the damping coefficient and shall be determined using experimentally obtained values as

$$\gamma_n = \gamma_0 + \gamma_s \omega_n + \gamma_v \omega_n^2, \quad (15)$$

where $\gamma_0 = 0.6295$, $\gamma_s = 0.296 \times 10^{-5}$, and $\gamma_v = 2.585 \times 10^{-8}$. However, it was assumed that $\gamma_2 = 0.35889$, because it greatly deviated from the Eq. (15) near the 2nd partial tone. In addition, the vibration frequency generally changes with damping, but such change shall be ignored because the amount of change here is extremely small.

3.3. Electromotive Force of Pickup

The response of the single-coil pickup excited by a string is analyzed in detail as a case of the effect of the single attraction described above [6,7]. Here, for each single-coil pickup constructing the humbucking pickup, assuming the magnetomotive force U_i ($i = 1, 2$), the compound reluctance $\mathcal{R}_i$, the turn number of the coil N_i , the magnetic flux Φ_i , the current of the coil I_i , the voltage added to the coil E_i and the load impedance Z_i , the fundamental equation of each single-coil pickup is represented as

$$\left. \begin{aligned} U_i &= \mathcal{R}_i \Phi_i + N_i I_i \\ E_i &= Z_i I_i - N_i \frac{d\Phi_i}{dt} \end{aligned} \right\}. \quad (16)$$

Also, assuming that the voltage source and the load are not connected, and that only electromotive force will be treated, only the first term shall be a target. The reluctance is divided into the static component and the dynamical component, that is, $\mathcal{R}_i = \mathcal{R}_{Si} + \eta_i y(x_i, t)$ [6]. Thus, because of $\Phi_i = U_i / \mathcal{R}_i$, the electromotive force E_{mi} can be obtained as

$$E_{mi} = -N_i \frac{d\Phi_i}{dt} = \frac{N_i U_i \eta}{\{\mathcal{R}_{Si} + \eta y(x, t)\}^2} \cdot \frac{\partial y(x, t)}{\partial t}. \quad (17)$$

Then the output of the humbucking pickup is given as the sum of the two single-coil pickup outputs that calculated from Eq. (17) for $i = 1, 2$.

Because the electric guitar treated here is a product already on the market, the number of turns of the coil as well as the magneto-motive force of the humbucking pickup are unknown, and it also is difficult to establish the value of the reluctance. Then, those values are obtained by reverse calculation using the simulation result as follows:

$$\begin{aligned} N_1 &= N_2 = 8000 \\ U_1 &= U_2 = 355.442 \quad [\text{A}] \\ \mathcal{R}_{S1} &= \mathcal{R}_{S2} = 1.72903 \times 10^{10} \quad [\text{A/Wb}] \\ \eta_1 &= \eta_2 = 1.97752 \times 10^{12} \quad [\text{A/Wb/m}]. \end{aligned}$$

3.4. Comparisons with Experimental Results

On the basis of the above-mentioned analysis, the calculated outputs of the humbucking pickup are shown in Fig. 5 with the measured result. To confirm the long

response (0–4.05 s), the 50 ms-long waveform for every 1 s is shown, which approximately corresponds to ten periods. The low-pass filter that cuts off the frequency of 6.25 kHz has been applied because there was high-frequency noise in the measuring signal. It is confirmed that, in addition to the decrease of amplitude due to damping, the waveforms vary with time because of the existing inharmonicity indicated in Fig. 4.

The data was saved by confirming that the same wave pattern was provided through several trials because the initial condition was given by picking the string up as height h at $x = l/8$ in two fingers. Although the initial condition has an angle with prominent sharpness at $x = l/8$ theoretically, it is confirmed that roundness appears to the wave pattern in the first stage, because the realization is difficult and the high-frequency components are lacked. However, subsequently, it is recognized that the simulation results are in good agreement with the measured results.

Figure 6 shows the envelopes of the two results by narrowing a time scale from 0 to 4 s. The beats of a long period are generated by the interference between the partial tones because the inharmonicity occurs under the influence of negative stiffness; it is recognized that the calculation results are in good agreement with the measurement results.

4. CHARACTERISTICS OF HUMBUCKING PICKUP

4.1. Response of Imitated Humbucking Pickup

In this section, the characteristic of the response of the imitated humbucking pickup, constructed of two single-coil pickups on the guitar mentioned in the previous section, is compared with the one of two single-coil pickups to clarify the property of the humbucking pickup.

The imitated humbucking pickup is shown in Fig. 7. The setting condition is the same as for the original humbucking type described in chapter 3, and the single-coil pickup is set at midway between the two pickups, that is $x = 0.145$. The negative stiffnesses of both of the two single-coil pickups used here are 2.1 N/m.

The vibration frequencies and the inharmonicity of both single-coil and imitated humbucking pickups are shown in Table 1 and Fig. 8, respectively. It is confirmed that inharmonicity of the humbucking pickup becomes approximately 2 times that of the single-coil pickup, as described above.

The measured responses of the two types of pickup are shown in Fig. 9. The amplitude ratio of both pickups is about 1:2, and the beats occur strongly in the humbucking pickup compared with the single-coil pickup. It can be explained that strong beats are generated by the interference between the low partial tones because the partial tone frequencies of the humbucking pickup having two negative

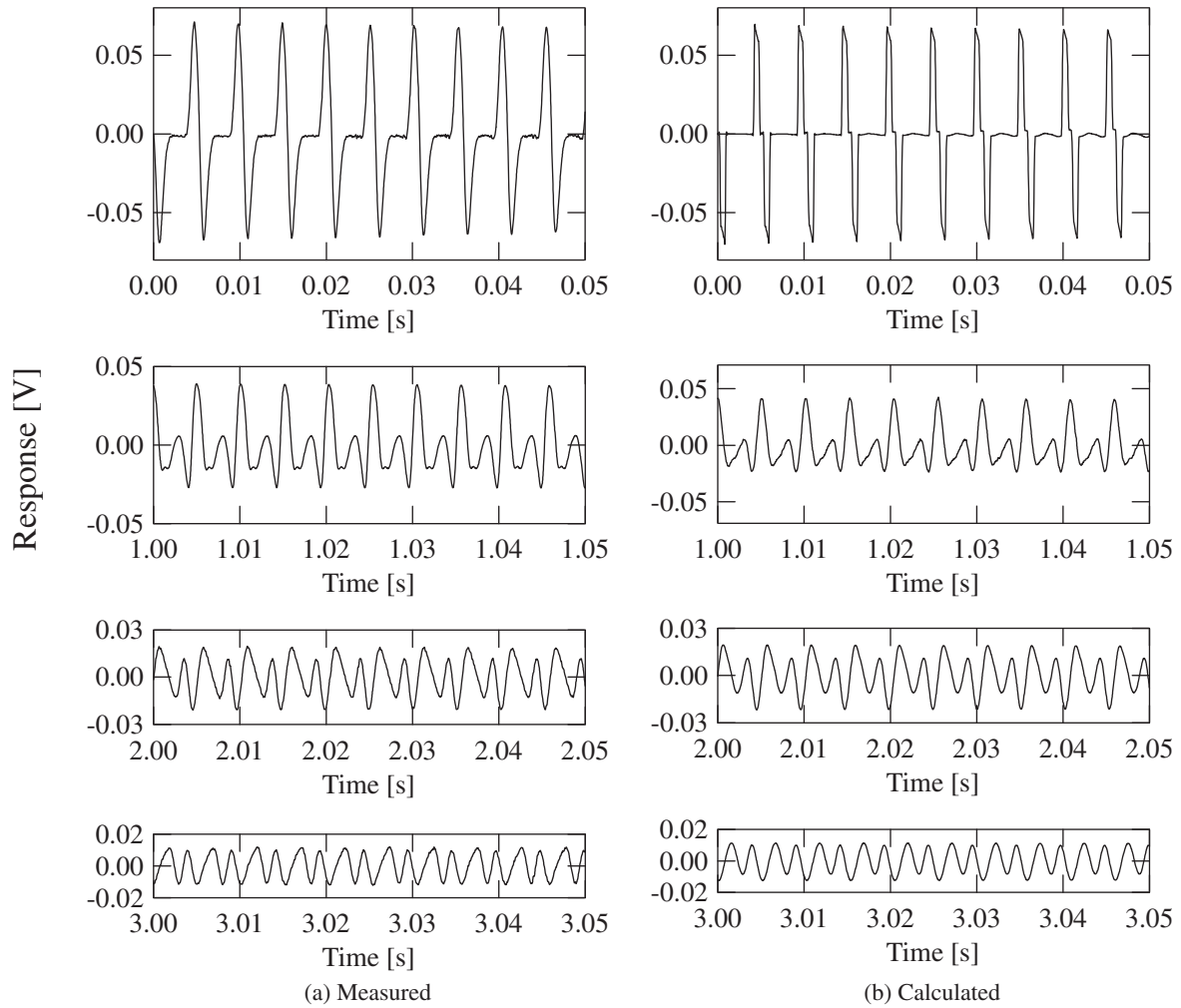


Fig. 5 Calculated and measured humbucking pickup responses excited by string vibration.

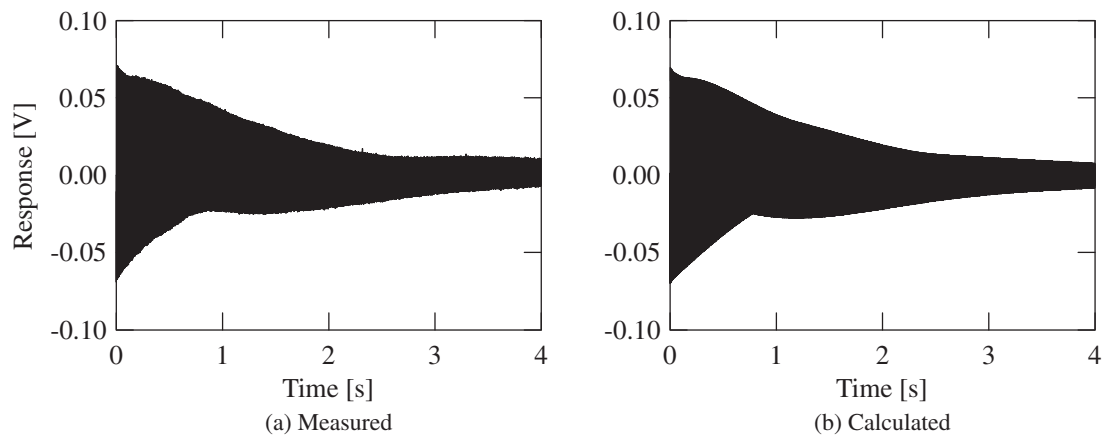


Fig. 6 Calculated and measured humbucking pickup responses from 0 to 4 s.

stiffnesses are reduced much more than that of the single-coil pickup.

4.2. Partial Tones Constitution of Pickup

The spectrograms of the responses of both the single-

coil and humbucking pickups are shown in Fig. 10. The partial tone amplitudes determined by the initial condition expressed in Eq. (14) are indicated in Fig. 11. It is understood that, when paying attention to the low partial tones, the output of the second partial tone is the largest,



Fig. 7 Imitated humbucking pickup.

Table 1 Partial frequencies of a string affected by negative stiffness.

Partial No.	Single-coil	Humbucking
1	195.997718	195.997718
2	392.165539	392.340553
3	588.474262	588.959882
4	784.759971	785.526836
5	980.940317	981.89154
6	1,177.086507	1,178.198492
7	1,373.285987	1,374.602086
8	1,569.520792	1,571.059840
9	1,765.723032	1,767.461602
10	1,961.888136	1,963.811866

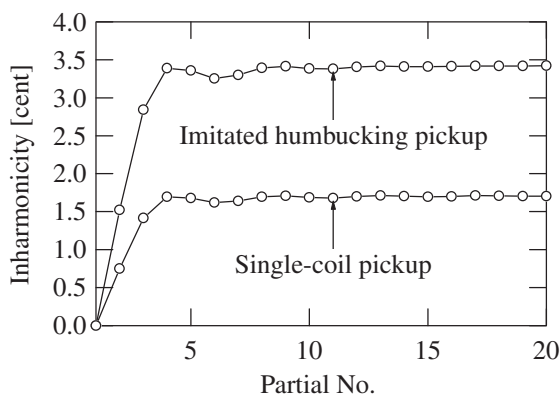


Fig. 8 Inharmonicity of a string affected by imitated humbucking pickup and single-coil pickup.

because the loop position is consistent with the positions of both pickup types, and the output of the pickup is proportional to the velocity indicated in Eq. (17). In addition, for the humbucking pickup, the output shows a large value at from the 1st to 3rd partial tones because the responses of both pickups are added. Also, when paying attention to the partial tone constitution for approximately 1 s in the initial stage, the difference from the case of the single-coil is marked because some partial tones shall be canceled out, being in inverse phase at two pole piece of

the humbucking pickup, and only the 15th partial tone is recognized from among higher partial tones than 12th.

For example, in the 13th partial tone, because there is a nodal point in the center of the humbucking pickup, the output is not at all seen in the single-coil pickup or in the humbucking pickup. It has been reported in detail that the response of the pickup had a nonlinear property of against the displacement of the string, and that many distortion components were generated [7]. The 12th partial tone is not seen in the output, although there is no nodal point. It is thought that the component has been canceled out with the distortion components regenerated in out of phase by the low partial tones of relatively large amplitudes. On the contrary, the 7th partial tone that is originally a small displacement is generated for a relatively long time in the humbucking pickup response. Such differences are why the sound of the single-coil pickup is clean, or why the sound of humbucking pickup has warmer and more mellow tone because the middle frequency range is relatively emphasized.

When watching the 5th partial tone of the humbucking pickup, it is confirmed that the beats occur because the weak points are seen about every 0.7 s. It is considered analytically that the various frequency components, such as $f_2 + f_3$, $2f_1 + f_3$, and $3f_1 + f_2$, are generated as intermodulation distortion of the low partial tones with comparatively large amplitudes. They are slightly different from the value of f_5 , and as a result, they generate the beats as interference between the partial tone. The values of $f_5 - 2f_1 - f_3$ and $f_5 - 3f_1 - f_2$ become 1.25 and 1.597 Hz by calculating each frequency using Eq. (8), and it is presumed that the beats seen in measurement results occurred because plural beats acted at the same time.

4.3. Examination of SNR of Pickup

Figure 12 illustrates the signal-noise ratios of both single-coil and imitated humbucking pickups, that are calculated from the RMS value for 0.05 s at every moment, where the signal and the noise are the low-pass-filtered and high-pass-filtered pickup responses with the cut-off frequency 6.25 kHz, respectively. Although, the difference of the SNR at the early parts between the single-coil pickup and the humbucking pickup is only 2.8 dB because the numerous highly partial tones exist in the single-coil pickup, and subsequently, the SNR of the humbucking pickup becomes greater than that of the single-coil pickup nearly 5 dB.

5. CONCLUSIONS

The responses of the humbucking pickup mounted on a commercial electric guitar were analyzed by linearizing the attraction to the static component and the negative stiffness component. It has been shown that the analytical solution

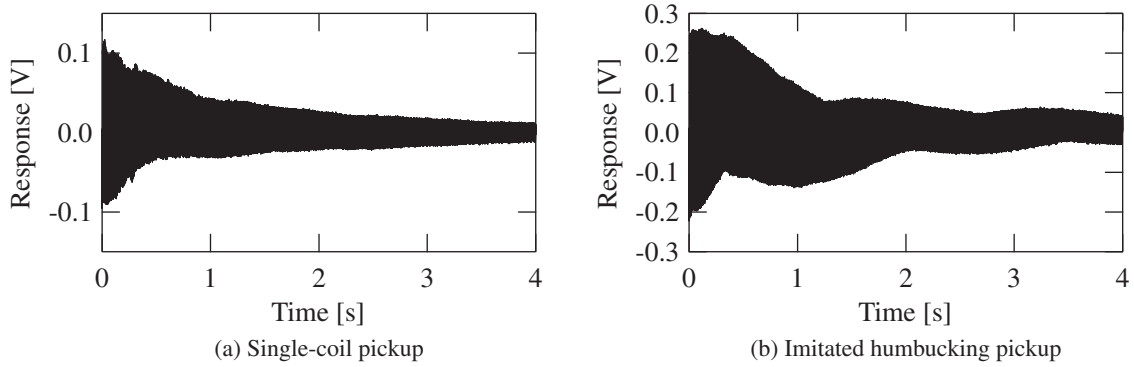


Fig. 9 Envelopes of pickup responses from 0 to 4 s.

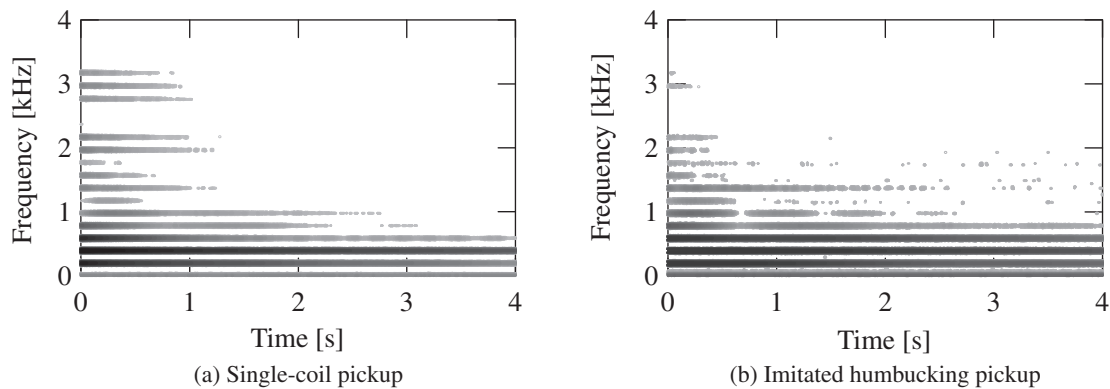
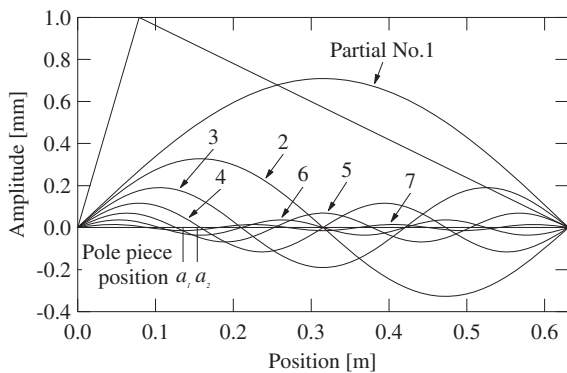
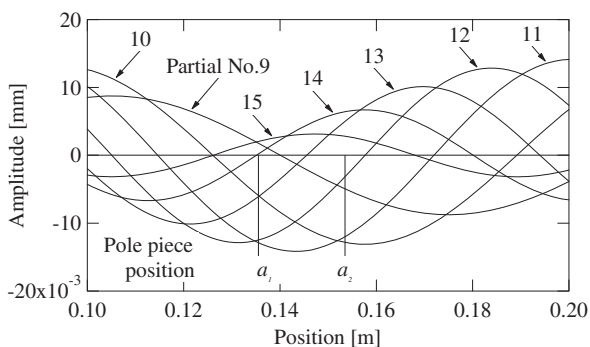


Fig. 10 Spectrograms of pickup responses from 0 to 4 s.



(a) Amplitudes of partial tones from 1st to 7th



(b) Amplitudes of partial tones from 9th to 15th

Fig. 11 Initial condition and amplitudes of partials.

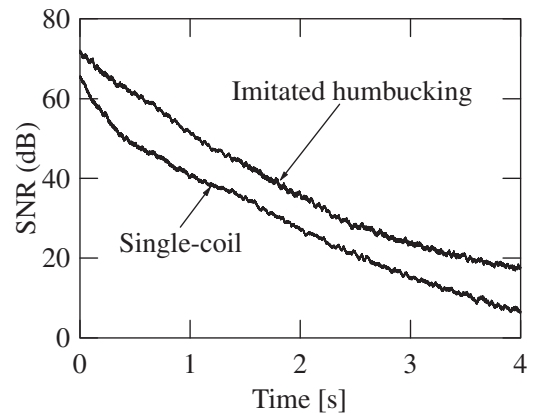


Fig. 12 SNR of single coil and Imitated humbucking pickup response.

was provided by solving the vibration equation using the Laplace transform and dividing the reluctance of the magnetic circuit of the pickup and the string into static and dynamical components, and that the result was in good agreement with the measurement results. Additionally, the characteristics of both pickup types expressed subjectively were indicated objectively using the spectrum structure, by analyzing the responses of the single-coil pickup and the

imitated humbucking pickup composing two single-coil pickups. Also, the generating the beats in the response was clarified by analyzing the partial tone frequencies. Furthermore, the SNR at all moments for both pickups were calculated, and it was found that the humbucking pickup clearly contributes to noise reduction.

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