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ACOUSTICAL LETTER

# Expression for noninvasive measurement of internal map of local Joule heat consumption using acousto-electric effect

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## 1. Introduction

For living biological tissues, we have been developing noninvasive electromagnetic techniques for reconstructing internal maps of current flow and electric properties such as electric conductivity and permittivity [1–3]. In our approach, on the basis of magnetic vector measurements performed in the vicinity of the target using a superconducting quantum interference device (SQUID), a flux meter etc., an internal current density vector map is reconstructed (the inverse problem on Biot-Savert law [4]), from which the electric properties of the tissue are subsequently reconstructed. The problem of electric property reconstructions is a linear-differential-type inverse problem. Our approach enables reconstructions *in vivo*/*in situ*. That is, a target in which a current normally exists (i.e., inherent current flow) can also be dealt with simply by placing a pickup coil close to the target. Thus, our techniques enable us to evaluate the electric conductive paths of normal and cultured nerves as well as the electric properties of tissues (brain, heart, muscle, etc.). Similarly to other approaches [5–11], injected, excited and evoked currents can also be dealt with.

In such other approaches, various current dipole reconstruction techniques for the heart (e.g., [5,6]) and brain (e.g., [7]) using paired electrodes have been developed. Moreover, for the reconstruction of conductivity, in refs. [8,9], paired excitation and detection electrodes are used; in ref. [10], paired excitation coils and detection electrodes are used; and in ref. [11], paired excitation and detection coils are used. In all these conductivity reconstructions, a sensitivity theorem [8,12,13] is used; thus, the reconstruction problems are nonlinear-integration-type inverse problems.

Recently, the combination of a sensitivity theorem [8,12,13] and the acousto-electric (AE) effect [14–16] first described in [17] has been used to map internal currents [18,19] and dipoles [20]. That is, the relation between the AE voltage signal detected using paired electrodes and the ultrasound (US) pulsed echo signal is used (for instance, the amplitude of the AE signal is proportional to that of the US signal [15], in which the proportional coefficient is about $0.1\%\,\mathrm{MPa}^{-1}$ in a 0.9% NaCl solution used [16]) for monitoring cardiac [18,20] and nerve [19] activity. However, for the reconstructions, a major assumption is required, i.e., the homogeneity of conductivity. Alternatively, the AE technique is also used for visualizing an inhomogeneous conductivity map [21]. Although the above-mentioned electromagnetic techniques for reconstructing current density and conductivity maps generally have lower spatial resolutions than other useful methods of CT (e.g., X-rays), in future these AE techniques may become more effective than the electromagnetic techniques mentioned above. That is, the spatial resolution of the image is determined by the point spread function (PSF) of the US pulse wave used.

In this report, we propose a new, accurate AE imaging technique for visualizing electromagnetic phenomena and electric properties using fewer assumptions. That is, we theoretically derive an expression for the noninvasive measurement of internal local Joule heat consumption generated by the current flow (i.e., inherent, evoked, injected or excited current), which is useful regardless of the homogeneity of the conductivity. Briefly, the local Joule heat consumption can be measured by temporal measurements of the US pulsed echo signals and electric power using paired electrodes attached onto the target.

## 2. Derivation of local Joule heat consumption

In refs. [12,13], the measured transfer impedance change $\Delta Z$ between two pairs of electrodes (A and B) and (C and D) caused by a change in the distribution of conductivity $\Delta\sigma(x,y,z)$ in a conductive target with volume $V$ (see Fig. 1) is expressed as

$$\Delta Z = \int_V \Delta\sigma(x,y,z)\frac{\nabla\phi(\sigma(x,y,z))}{I_\phi} \cdot \frac{\nabla\psi(\sigma(x,y,z)+\Delta\sigma(x,y,z))}{I_\psi}dV = \int_V \Delta\sigma(x,y,z)(\boldsymbol{L}\cdot\boldsymbol{L}')dV, \quad (1)$$

where $\phi(\sigma(x,y,z))$ is the potential distribution generated in the target with the conductivity distribution $\sigma(x,y,z)$ when a current $I_\phi$ is injected using the electrode pair (A and B); $\psi(\sigma(x,y,z)+\Delta\sigma(x,y,z))$ is the potential distribution generated in the target with the conductivity distribution $\sigma(x,y,z)+\Delta\sigma(x,y,z)$ when a current $I_\psi$ is injected using the electrode pair (C and D); $\boldsymbol{L}$ and $\boldsymbol{L}'$ are referred to as the lead fields when the conductivity distributions are $\sigma(x,y,z)$ and

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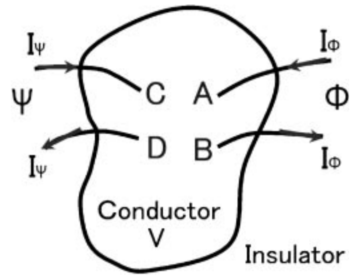


**Fig. 1** Schematic of measurement system for explaining eq. (1).

$\sigma(x,y,z) + \Delta\sigma(x,y,z)$, respectively. This equation, which expresses the compensation theory using sensitivity, is used for electrocardiographic impedance plethysmography [12,13] and electric impedance computed tomography (EICT) [8–11], in which $\boldsymbol{L}$ is used as an approximation instead of $\boldsymbol{L}'$.

Thus, when using only one pair of electrodes, the impedance change $\Delta Z$ detected using a measured current $I_\phi$ and potential $\phi$ is expressed as

$$\Delta Z = \int_V \Delta\sigma(x,y,z) \left| \frac{\nabla\phi(\sigma(x,y,z))}{I_\phi} \right|^2 dV. \tag{2}$$

For an electric and echoic medium, using the AE effect [14–17], the temporal change $\Delta\sigma(x,y,z,t)$ in conductivity $\sigma(x,y,z)$ due to a pressure wave $p(x,y,z,t)$ can be written as

$$\Delta\sigma(x,y,z,t) = \sigma(x,y,z) K_{\mathrm{I}} p(x,y,z,t), \tag{3}$$

where $K_{\mathrm{I}}$ is an interaction constant [15,16].

Thus, the temporal impedance change $\Delta Z(t)$ due to a pressure pulse wave [15] can be written as

$$\Delta Z(t) = \int_V \sigma(x,y,z) \left| \frac{\nabla\phi(\sigma(x,y,z))}{I_\phi} \right|^2 K_{\mathrm{I}} p(x,y,z,t) dV. \tag{4}$$

In [18,19], for an electric and echoic US medium (see Fig. 2), for instance, a heart, brain, or nerve, by measuring the AE signal generated by a US pulse wave $p(x,y,z,t)$, which is expressed as

$$\begin{aligned} v_{\mathrm{AE}}(t) &= I_\phi \Delta Z(t) \\ &= \int_V \boldsymbol{J}(x,y,z,t) \cdot \boldsymbol{L}(x,y,z) K_{\mathrm{I}} p(x,y,z,t) dV, \end{aligned} \tag{5}$$

the temporal current density map $\boldsymbol{J}(x,y,z,t) \cdot \boldsymbol{L}(x,y,z)$ $[= \sigma(-\nabla\phi) \cdot \boldsymbol{L}]$ can be obtained under the assumptions that

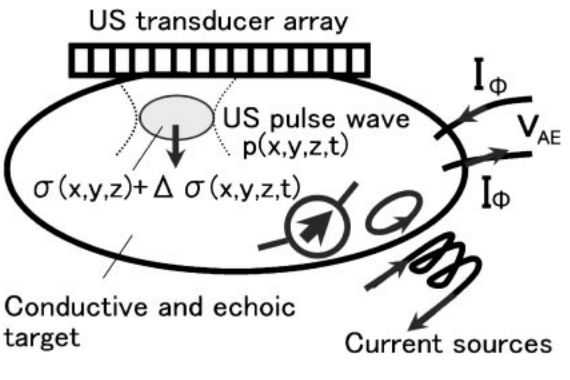


**Fig. 2** Schematic of measurement system of AE voltage signal $v_{\mathrm{AE}}$.

the conductivity $\sigma(x,y,z)$ is spatially homogeneous for the calculation of $\boldsymbol{L}(x,y,z)$ and that $K_{\mathrm{I}}$ is also spatially homogeneous. That is, the amplitude of the AE signal $v_{\mathrm{AE}}(t)$ is proportional to that of the US echo signal. Moreover, $v_{\mathrm{AE}}(t)$ has the same frequency and phase as those of the US echo signal. The spatial resolution of the current density map is determined by the PSF $p(x,y,z,t)$ of the US pulse wave used. In ref. [20], to map a 2D current density vector $\boldsymbol{J}(x,y)$, simultaneous equations are derived by performing multiple measurements of $v_{\mathrm{AE}}(t)$ using different paired electrodes, on which an inversion is carried out. However, under the same assumption, all the corresponding lead fields must be calculated by applying numerical methods (e.g., the finite element method) to the whole body of the target. In ref. [21], $v_{\mathrm{AE}}(t)$ is also used to image conductivity maps for an electrically inhomogeneous medium using an injected current. In the near future, the limitations of the AE techniques will be clarified.

However, in our case, Eq. (4) is written as follows to enable the evaluation of the electromagnetic phenomena and electric properties in an electric and echoic target with fewer assumptions. That is, by multiplying Eq. (4) by $I_\phi^2$, we obtain the expression that the internal local Joule heat consumption $w(x,y,z,t)[= \sigma(x,y,z)| - \nabla\phi(x,y,z,t)|^2]$ generated by the current flow $J(x,y,z,t)$ resulting from the injection of current from a pair of electrodes can be estimated by measuring the electric power $W(t)$ and echo signals regardless of the homogeneity of the conductivity, i.e.,

$$\begin{aligned} W(t) &= I_\phi^2 \Delta Z(t) \\ &= \int_V w(x,y,z,t) K_{\mathrm{I}} p(x,y,z,t) dV, \end{aligned} \tag{6}$$

where only the assumption of the spatial homogeneity of $K_{\mathrm{I}}$ is used. That is, by measuring the time sequences of the US echo signals $p(x,y,z,t)$ and the electric power $W(t)$ modulated by the US frequency, a temporal map of the local Joule heat consumption $w(x,y,z,t)$ can be obtained. The modulated $W(t)$ can be estimated from the product of the measured voltage and current, or the square of the voltage or current.

However, when the internal local Joule heat consumption $w(x,y,z,t)$ generated due to an inherent, evoked or excited current is estimated, $\boldsymbol{L}(x,y,z)$ in Eq. (5) must be properly realized with respect to the points of interest determined by $p(x,y,z,t)$ such that the directions of $\boldsymbol{L}(x,y,z)$ approximately correspond to those of $\boldsymbol{J}(x,y,z,t)$ at the points. That is, the paired electrodes must be properly configured using *a priori* knowledge about the generated current source such that $\sigma(x,y,z)| - \nabla\phi(x,y,z,t)|^2]$ is approximately realized as in Eq. (6). Multiple configurations of paired electrodes should also be used. In such a case, the measured electric power $W(t)$ can also be superimposed to map $w(x,y,z,t)$ approximately.

## 3. Conclusions

Owing to the use of fewer assumptions such as the homogeneity of conductivity, our proposed AE technique yields a more accurate map of local Joule heat consumption than other AE techniques that yield a map of current flow or conductivity. However, although a higher spatial resolution of the Joule heat consumption map will be obtained by our AE technique, the respective current vector density components cannot be evaluated similarly to in other AE techniques except for that reported in ref. [20]. However, in contrast to the technique reported in ref. [20], our AE technique does not require the inversion of equations. Generally, such an inversion leads to a significant decrease in the stability of estimation. Furthermore, the calculation of lead fields is not required. That is, neither *a priori* knowledge of the internal conductivity distribution nor the shape of the whole body of the target is required. In addition, knowledge of the position of the pair of electrodes is also not required, although the proper position must be realized when the current is not injected but is inherent, evoked or excited. Thus, as special equipment for our AE technique, only US equipment, a simple electric circuit and a synchronizing circuit for data acquisition are used. Real-time mapping can also be realized more simply by exploiting the advantage of conventional US equipment using an array transducer. Compared with electromagnetic techniques using a SQUID, the cost of our AE technique should be low.

Thus, our AE technique is expected to be useful for visualizing electric paths, current flow and electric property. Our AE technique may also be used for the nondestructive evaluations of electric and echoic structures and materials. Here one may note that the increased tissue temperature caused by US exposure affects the conductivity [22]. For noninvasive or nondestructive evaluation, the increased temperature is an error source in the mapping of local Joule heat consumption. However, for living tissues, a perfusion may mitigate such a problem. The feasibility of our AE technique will be reported elsewhere in the near future.

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