

## Consideration of an absorbing boundary condition based on CIP method for finite-difference time-domain acoustic field analysis

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### 1. Introduction

Recently, many results on sound propagation using numerical analyses have been achieved by the finite-difference time-domain (FDTD) method [1–8]. For free-space simulation, treatment of the outer absorbing boundary is an important technical issue for the FDTD calculation of acoustic fields. Some absorbing boundary conditions (ABCs) have been proposed for FDTD analysis [9–13].

To date, Berenger's perfectly matched layer (PML) [11] is known to have the highest absorption of outgoing waves. However, this technique requires more memory and more complicated programming than other techniques. On the other hand, Mur's ABC [10] is useful when analyses do not require an boundary with extremely high absorption. Such analyses are suitable for a boundary treatment that requires little memory and simple programming.

In this study we presents an ABC based on the constrained interpolation profile (CIP) method [14–17]. This method was proposed recently by Yabe and coworkers [14,15] as a technique for application of the method of characteristics (MOC) [18,19] in the field of fluid dynamics [15,16]. A noticeable feature of the CIP method is that it uses both field values and their spatial derivatives on grid points, which is a distinctive attribute that distinguishes CIP from conventional methods.

Through this study, we investigate the application of CIP to an ABC for FDTD acoustic field analyses. We also report the performance results of the CIP-ABC.

### 2. Calculation

#### 2.1. FDTD analysis of acoustic fields

The linearized governing equations of acoustic fields are given in Eqs. (1) and (2).

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p \quad (1)$$

$$\nabla \cdot \mathbf{v} = -\frac{1}{K} \frac{\partial p}{\partial t} \quad (2)$$

In those equations,  $\rho$  denotes the density of the medium,  $K$  is the bulk modulus,  $p$  is the sound pressure, and  $\mathbf{v}$  is the particle velocity. Here, we assume that the calculation is for a lossless medium.

For simplicity, let us examine a one-dimensional (1-D) model. To analyze 1-D acoustic field propagation in the  $x$ -direction, we assume that  $\mathbf{v} = (v_x, 0, 0)$  and thereby obtain the following equations from Eqs. (1) and (2).

$$\frac{\partial v_x}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0, \quad (3)$$

$$\frac{\partial p}{\partial t} + K \frac{\partial v_x}{\partial x} = 0. \quad (4)$$

Therefore, by applying the FDTD algorithm to these two equations, Eqs. (5) and (6) are obtained using discretized components of sound pressure and particle velocity on grid points. Here, we use the second-order central difference approximation on the staggered grid depicted in Fig. 1.

$$v_x^{n+\frac{1}{2}}\left(i + \frac{1}{2}\right) = v_x^{n-\frac{1}{2}}\left(i + \frac{1}{2}\right) - \frac{\Delta t}{\rho} \frac{p^n(i+1) - p^n(i)}{\Delta x} \quad (5)$$

$$p^{n+1}(i) = p^n(i) + K \Delta t \frac{v_x^{n-\frac{1}{2}}\left(i + \frac{1}{2}\right) - v_x^{n-\frac{1}{2}}\left(i - \frac{1}{2}\right)}{\Delta x} \quad (6)$$

In these equations,  $\Delta x$  and  $\Delta t$  respectively denote the grid size and the time step; and  $f^n(i)$  represents  $f$  at time  $n\Delta t$  on grid point  $x = i$ .

#### 2.2. Absorbing boundary condition using the CIP method

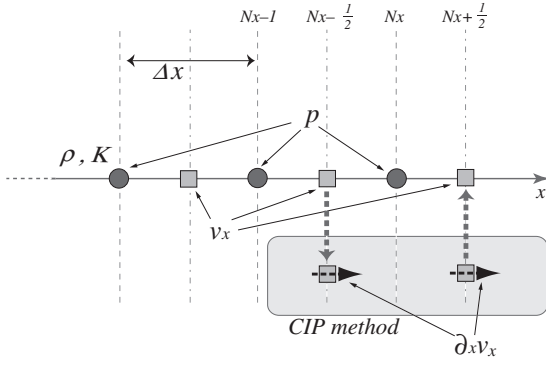
Figure 1 illustrates the staggered distribution of discrete fields and their spatial derivatives. In particular,  $v_x(1/2)$  and  $v_x(N_x + 1/2)$ , which are assumed as outer boundaries, require appropriate formulation.

This study presents an ABC based on the CIP method [14,15]. Figure 1 shows the CIP method, in which both field values and their spatial derivatives on grid points are used to calculate the field values of the next time step. Using the derivatives of fields is a unique aspect of this method. The

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**Fig. 1** 1-D model of the CIP-ABC for the FDTD analysis of sound propagation.

formulation of the ABC using the present method is shown in Eqs. (7) and (8). First, we consider  $v_x$  on the grid point  $i = N_x + 1/2$ .

$$\begin{aligned} v_x^{n+1}\left(N_x + \frac{1}{2}\right) &= a\xi^3 + b\xi^2 \\ &+ v_x^n\left(N_x + \frac{1}{2}\right)\xi \\ &+ \partial_x v_x^n\left(N_x + \frac{1}{2}\right) \end{aligned} \quad (7) \quad \text{Here,}$$

$$\begin{aligned} \partial_x v_x^{n+1}\left(N_x + \frac{1}{2}\right) &= 3a\xi^2 + 2b\xi \\ &+ \partial_x v_x^n\left(N_x + \frac{1}{2}\right) \end{aligned} \quad (8)$$

Therein,

$$\begin{aligned} a &= \frac{\partial_x v_x^n\left(N_x + \frac{1}{2}\right) + \partial_x v_x^n\left(N_x - \frac{1}{2}\right)}{(\Delta x)^2} \\ &+ \frac{2\left\{v_x^n\left(N_x + \frac{1}{2}\right) - v_x^n\left(N_x - \frac{1}{2}\right)\right\}}{(-\Delta x)^3}, \end{aligned} \quad (9)$$

$$\begin{aligned} b &= \frac{3\left\{v_x^n\left(N_x - \frac{1}{2}\right) - v_x^n\left(N_x + \frac{1}{2}\right)\right\}}{(\Delta x)^2} \\ &- \frac{2\partial_x v_x^n\left(N_x + \frac{1}{2}\right) + \partial_x v_x^n\left(N_x - \frac{1}{2}\right)}{-\Delta x}, \end{aligned} \quad (10)$$

$$\xi = -c\Delta t. \quad (11)$$

In these equations,  $c$  represents the sound velocity, i.e.,  $c = \sqrt{K/\rho}$ . Also,  $\partial_x = \partial/\partial x$ . That is,  $\partial_x v_x$  represents the spatial derivative of  $v_x$  in the  $x$ -direction.

In addition,  $\partial_x v_x$  is given by

$$\partial_x v_x^n\left(N_x - \frac{1}{2}\right) = \frac{1}{4\Delta x} \left\{ v_x^n\left(N_x - \frac{5}{2}\right) \right.$$

$$\begin{aligned} &+ 3v_x^n\left(N_x - \frac{3}{2}\right) \\ &- 5v_x^n\left(N_x - \frac{1}{2}\right) \\ &\left. + v_x^n\left(N_x + \frac{1}{2}\right) \right\}. \end{aligned} \quad (12)$$

Because we should only consider waves propagating in the  $+x$ -direction on the outer boundary on the grid point  $i = N_x + 1/2$ ,  $\partial_x v_x^n(N_x + 1/2)$  is calculable using the CIP method.

We can similarly describe the boundary on grid point  $i = 1/2$ , as shown below.

$$\begin{aligned} v_x^{n+1}\left(\frac{1}{2}\right) &= a\xi^3 + b\xi^2 \\ &+ v_x^n\left(\frac{1}{2}\right)\xi \\ &+ \partial_x v_x^n\left(\frac{1}{2}\right). \end{aligned} \quad (13)$$

$$\begin{aligned} \partial_x v_x^{n+1}\left(\frac{1}{2}\right) &= 3a\xi^2 + 2b\xi \\ &+ \partial_x v_x^n\left(\frac{1}{2}\right). \end{aligned} \quad (14)$$

$$\begin{aligned} a &= \frac{\partial_x v_x^n\left(\frac{1}{2}\right) + \partial_x v_x^n\left(\frac{3}{2}\right)}{(\Delta x)^2} \\ &+ \frac{2\left\{v_x^n\left(\frac{1}{2}\right) - v_x^n\left(\frac{3}{2}\right)\right\}}{(-\Delta x)^3}, \end{aligned} \quad (15)$$

$$\begin{aligned} b &= \frac{3\left\{v_x^n\left(\frac{3}{2}\right) - v_x^n\left(\frac{1}{2}\right)\right\}}{(\Delta x)^2} \\ &- \frac{2\partial_x v_x^n\left(\frac{1}{2}\right) + \partial_x v_x^n\left(\frac{3}{2}\right)}{-\Delta x}, \end{aligned} \quad (16)$$

$$\xi = c\Delta t. \quad (17)$$

Moreover,  $\partial_x v_x(3/2)$  is given by

$$\begin{aligned} \partial_x v_x^n\left(\frac{3}{2}\right) &= -\frac{1}{4\Delta x} \left\{ v_x^n\left(\frac{7}{2}\right) \right. \\ &+ 3v_x^n\left(\frac{5}{2}\right) \\ &- 5v_x^n\left(\frac{3}{2}\right) \\ &\left. + v_x^n\left(\frac{1}{2}\right) \right\}. \end{aligned} \quad (18)$$

### 3. Numerical Results and Discussion

We show the numerical results obtained using the CIP-ABC, described in Sect. 2. One-dimensional sound propaga-

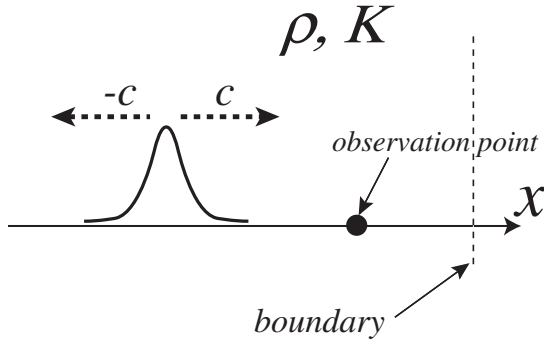


Fig. 2 1-D numerical model for CIP-ABC.

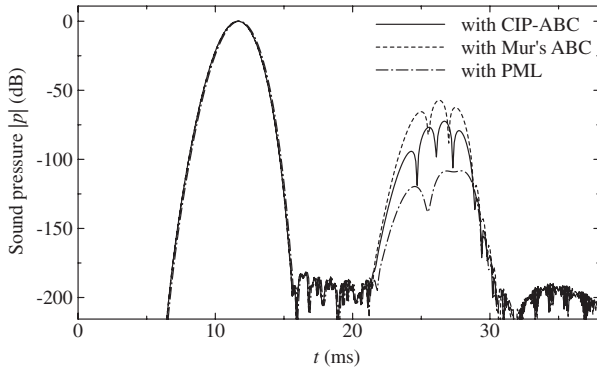


Fig. 3 Magnitude of sound pressure ( $|p|$ ) waveform (at  $x = 22.5$  m).

tion is demonstrated in the analysis domain using the CIP-ABC. Figure 2 shows the numerical model.

Other calculation parameters are as follows: the direction of acoustic field propagation,  $\pm x$  (1-D analysis); grid size,  $\Delta x = 0.05$  m; time step,  $\Delta t = 1.0 \times 10^{-4}$  s; number of grid points,  $N_x = 500$ ; and length of analysis region, 25 m.

Figure 3 shows the sound pressure waveform that is calculated by FDTD analysis using the CIP-ABC at  $x = 22.5$  m, where  $\rho = 1.21 \text{ kg/m}^3$  and  $K = 1.4236 \times 10^5 \text{ N/m}^2$  (i.e., the medium is air [20–22]). In Fig. 3, the wave reflected from the boundary can be confirmed at  $t = 25$  ms. Here, the initial pressure at  $t = 0$  is given as

$$p = e^{-\alpha(x-x_0)^2} [\text{N/m}^2]. \quad (17 \leq x \leq 20), \quad (19)$$

where  $\alpha = 1/48\Delta x^2$  and  $x_0 = 18.5$ .

The results obtained using Mur's first-order ABC are also illustrated in Fig. 3, together with those obtained using PML (eight layers). All parameters in the calculations ( $\Delta x$ ,  $\Delta t$ , medium, etc.) are identical. This figure shows that the magnitude of the waves reflected from CIP-ABC is smaller than that from Mur's ABC, although it is larger than PML.

Figure 4 shows the reflection coefficient of each ABC as a function of frequency, where these results are obtained using the results shown in Fig. 3. Here, 686 Hz coincides with 10 points per wavelength (PPW).

A comparison of the three results shown in Fig. 4 clarifies the following: The reflection coefficient obtained using the CIP-ABC is less than that obtained from Mur's ABC. For example, the reflection coefficient for the CIP-ABC is  $-50$  dB

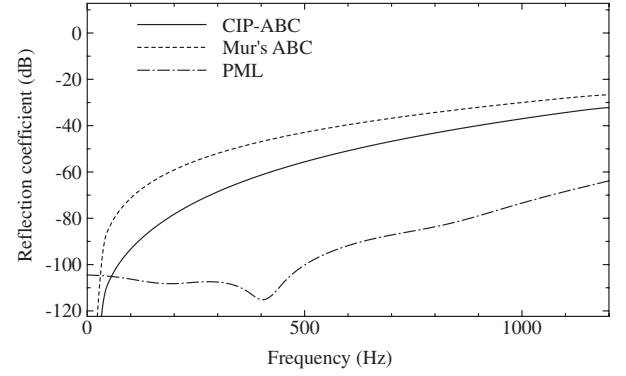


Fig. 4 Comparison of reflection coefficients of CIP-ABC, Mur's ABC, and PML.

at  $\text{PPW} = 10$ , whereas that for Mur's ABC is  $-40$  dB. On the other hand, Berenger's PML clearly has a higher performance than the CIP-ABC with respect to absorption. However, the CIP-ABC uses less memory, and, moreover, the programming of the CIP-ABC is almost as simple as that of Mur's ABC, as described in Sect. 2. Therefore, if the analysis does not require a boundary with extremely high absorption performance, then use of the CIP-ABC is recommended.

#### 4. Conclusion

In this study we proposed a new ABC, which we examined the ABC using the CIP method for the FDTD analysis of acoustic fields.

The analysis of 1-D sound propagation was carried out using the CIP-ABC was implemented. The results obtained in this study clarified that the CIP-ABC has better absorption performance than Mur's first-order ABC. That is, the CIP-ABC maintains a reflection coefficient that is lower than that for Mur's ABC at all frequencies.

Although this ABC has lower performance than PML in terms of absorption, it requires less memory and uses less complicated programming.

On the basis of these results, we intend to investigate the application of the CIP-ABC to 2-D or 3-D analyses in the near future.

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