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ACOUSTICAL LETTER

An indirect interpolation method for Head-Related Transfer Function pole-zero models

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1. Introduction

Head-related transfer function (HRTF), which describes the path between the sound source and the ear, plays an important role in 3D sound system [1]. The spatial perception using headphone can be controlled by binaural synthesis through HRTF. However, the system has to store the HRTFs of all directions to realize natural spatial perception, since the HRTF varies as a function of the sound direction. In fact, it is impossible to obtain the HRTFs of all directions through measurements because of the enormous amounts of time and physical loads of the listener. One solution to this problem is to obtain the HRTFs in arbitrary directions from a limited number of measured HRTFs.

Some interpolation methods have been developed which mainly focused on HRTF all-zero models, such as the linear interpolation method [2], the inter-positional transfer function (IPTF) method [3], etc. But pole-zero model is more preferred in binaural synthesis because of its low computation and memory cost. In [4], balanced model truncation method is employed to reduce the order of HRTFs. In [5], the IIR filter of HRTF is designed taking the log-magnitude errors into account. And in [6], the IIR filter is designed in warped frequency domain. Although pole-zero modeling of HRTF has obtained great progress, interpolation methods that are both automatic and effective for pole-zero models have not been developed so far. Generally, interpolation of pole-zero models may be realized by convex combinations of pole (zero) values from the neighboring HRTF models [7]. As an improvement of the convex combination method [7], records the pole (zero) tracks during the iterated approximation, and incorporates them to increase the interpolation accuracy. But the improved method in [7] is not applicable since extra information of the pole (zero) tracks has to be stored.

In this paper, an indirect interpolation method for HRTF pole-zero models is developed based on the all-zero interpolation methods. First, the method transforms the pole-zero models to all-zero models. Then, the target HRTF is interpolated from the reference all-zero models using the linear interpolation method. Last, the interpolated all-zero model is transformed back to the pole-zero model. The proposed method has better interpolation accuracy than conventional methods, and does not need calculating the pole (zero) positions.

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2. The indirect interpolation method

Suppose two reference pole-zero models $B_1(z)/A_1(z)$ and $B_2(z)/A_2(z)$, with the pole order n_a and the zero order n_b respectively. The target pole-zero function can be obtained through three steps as follows.

1) Transform the pole-zeros models $B_1(z)/A_1(z)$ and $B_2(z)/A_2(z)$ into the corresponding all-zero models $H_1(z)$ and $H_2(z)$ by calculating their impulse responses $h_1(n)$ and $h_2(n)$.

2) Calculate the target head-related impulse response (HRIR) $h(n)$ with the two reference HRIRs $h_1(n)$ and $h_2(n)$ by linear interpolation method, i.e.

$$h(n) = rh_1(n) + (1 - r)h_2(n) \tag{1}$$

where $h(n)$ is the target HRIR, $h_1(n)$ and $h_2(n)$ are two reference HRIRs, r is the dividing ration which relates to the distance of the desired point to the two reference point.

3) Transform $H(z)$, the Z-transform of $h(n)$, back to the pole-zero model $B(z)/A(z)$, with the same pole (zero) order as the reference one. The pole polynomial coefficients $a = [a(1), \cdots, a(n_a)]$, $(a(0) = 1)$, and zero polynomial coefficients $b = [b(0), \cdots, b(n_b)]$, can be calculated as follows. It is similar to the modified Yule-Walker method [8].

The equation error between $H(z)$ and $B(z)/A(z)$ is defined in Z-domain as

$$J_{ee} = H(z)A(z) - B(z) \tag{2}$$

Let $g(n) = a(n) * h(n)$, then the squared equation error in time domain is

$$\begin{aligned} J &= \sum_{n=0}^{\infty}(g(n) - b(n))^2 \\ &= \sum_{n=0}^{n_b}(g(n) - b(n))^2 + \sum_{n=n_b+1}^{\infty} g^2(n) \\ &= J_1 + J_2 \end{aligned} \tag{3}$$

where a is the pole polynomial coefficients, b is the zero polynomial coefficients, the symbol '$*$' denotes convolution operation, and

$$J_1 = \sum_{n=n_b+1}^{\infty} g^2(n) = \sum_{n=n_b+1}^{\infty}\left(h(n) + \sum_{l=1}^{n_a} a(l)h(n-l)\right)^2 \tag{4}$$

$$J_2 = \sum_{n=0}^{n_b}(g(n) - b(n))^2 \tag{5}$$

The coefficients a and b can be calculated by minimization of J_1 and J_2 respectively.

Step 1: let $\partial J_1/\partial a(i) = 0$, $(i = 1, \cdots, n_a)$, we yield the following equation set

$$\sum_{n=n_b+1}^{\infty}\left(h(n) + \sum_{l=1}^{n_a} a(l)h(n-l)\right)h(n-i)$$
$$= \sum_{n=n_b+1}^{\infty} h(n)h(n-i) + \sum_{l=1}^{n} a(l) \sum_{n=n_b+1}^{\infty} h(n-l)h(n-i)$$
$$= 0, \quad i = 1, \cdots, n_a \tag{6}$$

Let

$$c(k, j) = \sum_{n=n_b+1}^{\infty} h(n-k)h(n-j) \tag{7}$$

The equation set (6) can be expressed in matrix form

$$CX = c_0 \tag{8}$$

where

$$X = [a(1), \cdots, a(n_a)]^{\mathrm{T}}, \quad c_0 = [c(0, 1), \cdots, c(0, n_a)]^{\mathrm{T}},$$
$$C = \begin{bmatrix} c(1,1) & c(2,1) & \cdots & c(n_a,1) \\ c(1,2) & c(2,2) & \cdots & c(n_a,2) \\ \vdots & \vdots & \ddots & \vdots \\ c(1,n_a) & c(2,n_a) & \cdots & c(n_a,n_a) \end{bmatrix}$$

Because $c(k, j) = c(j, k)$, matrix C is generally a positive-definite symmetric matrix, thus Cholesky decomposition algorithm [8] may be employed to accelerate the solution of (8).

Step 2: after finding the pole polynomial coefficients a with (8), the zero polynomial coefficients b may be found by minimization of J_2. It can be solved by Pade approximation [9], which is expressed as

$$b(n) = g(n), \quad n = 0, \cdots, n_b \tag{9}$$

Thus with (8) and (9), the all-zero model can be transformed to the pole-zero model.

In a similar way, the proposed interpolation method may be generalized to more reference points, and may also use other all-zero interpolation methods.

The proposed method has several advantages. First, its interpolation performance is independent of the pole-zero approximation method, thus it is a general interpolation method. Second, it needn't calculating the pole (zero) positions, which is computation consuming. Third, it can be easily generalized to more reference points and can employ other all-zero interpolation methods besides the linear interpolation method.

3. Experiment and analysis

3.1. Pole-zero model approximation

The balanced model truncation method is employed in this paper to approximate the HRTFs with pole-zero models. The minimum-phase characteristic of HRTF is assumed during the approximation because it is beneficial to the approximation and can not cause special perception reduction in synthesis [10].

The KEMAR HRTF database is used [11]. There are 72 measured positions in the horizontal plane with an equal interval of 5°. After equalization, minimum-phase reconstruction, and truncation, the resulted HRIR is 128 taps long. Balanced model truncation is applied to the HRIR to get the pole-zero model, whose pole order and zero order are all set 20. The obtained pole-zero models are used as the original HRTFs in the experiment.

3.2. Experiment results

Four interpolation methods are compared in the experiment: the proposed method, the convex combination method, the impulse response linear interpolation, and the frequency magnitude linear interpolation [7]. For the convenience of comparison, all the four methods use the original pole-zero models obtained in Section 3.1 as the reference HRTF.

The HRTFs are interpolated at 4 interval conditions respectively: 10°, 20°, 30°, and 40°. The interpolation is performed so that the total number of original and interpolated HRTFs is 72 with an interval of 5°. The performance of the interpolation is evaluated by the spectral distortion (SD), which is defined as

$$SD = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left(20\log_{10}\left(\frac{|H(\omega)|}{|\hat{H}(\omega)|}\right)\right)^2 d\omega} \text{ (dB)} \tag{10}$$

where $H(\omega)$ and $\hat{H}(\omega)$ are the original and interpolated magnitude of the pole-zero models respectively. The smaller the SD, the more accurate the interpolation is.

Figure 1 shows the average SD of four interpolation methods at different interpolation intervals. For all the methods, the interpolation performance decreases with the interpolation interval. The performance of the magnitude interpolation method is the best, and the convex combination method is the worst among the four ones. The proposed method shows slightly worse accuracy than the impulse response interpolation method and the magnitude interpolation method, and obvious better accuracy than the convex combination method. Figure 2 depicts the direction change of the SD in the horizon plane for four methods respectively, when the interpolation interval is 10°. The proposed method also shows similar accuracy as two linear interpolation methods, and better accuracy than the convex combination method.

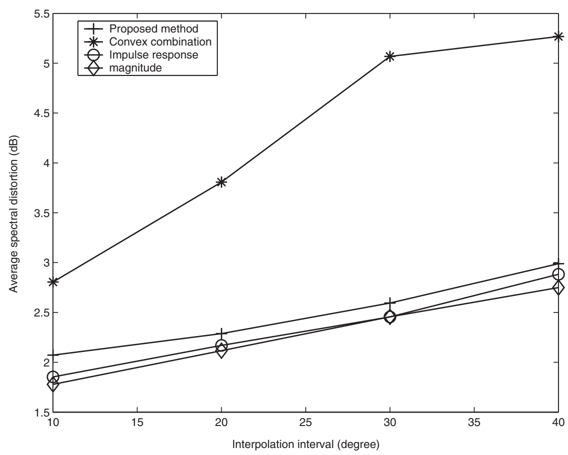

Fig. 1 Average SD across the horizontal plane as a function of interval angle.

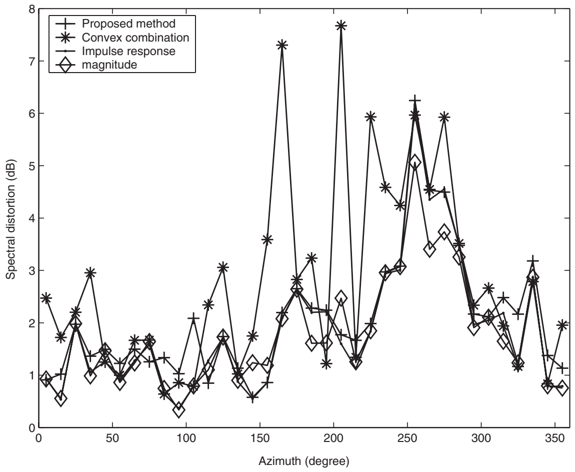

Fig. 2 SD for each direction in the horizontal plane at the interpolation interval of 10°.

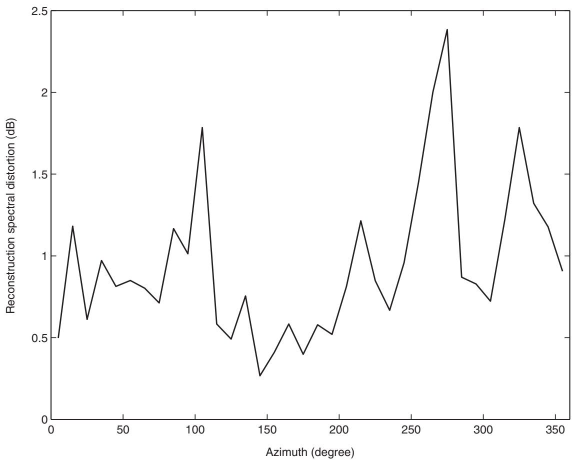

Fig. 3 Reconstruction SD for each direction in the horizontal plane at the interpolation interval of 10°.

In essence, the interpolation procedure of the proposed method is similar to that of the impulse response interpolation method, except that the proposed method has one additional FIR to IIR reconstruction process. The reconstruction SD from FIR to IIR in the proposed method is calculated when the interpolation interval of 10°, and is shown in Fig. 3 as a function of direction in the horizontal plane. The average reconstruction SD across the horizontal plane is about 0.95 dB, which is small enough to guarantee the performance of the proposed method. Thus besides the linear method, the proposed method can use other all-zero interpolation methods with low distortion.

It can be concluded from the experiments that, the proposed method has similar interpolation accuracy as the linear all-zero interpolation method, and has better interpolation accuracy than the convex combination method. It can be generalized to other all-zero interpolation methods with low distortion.

4. Conclusion

The indirect interpolation method interpolates the target pole-zero model based on the sophisticated all-zero interpolation method, and has similar interpolation accuracy as the linear interpolation method. The proposed method shows better performance than the conventional method, meanwhile it does not need calculating the pole (zero) positions during the interpolation. The proposed method gives a promising way for HRTF interpolation.

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