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Consonances and arrangement of the Marquis Yǐ set-bells

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1. Introduction

The tomb of Marquis Yǐ 乙 of a small dukedom state, Zēng 曾, in the early Warring States period in the southern region of China was excavated in 1978 at Suíxiàn 隨縣 in Húběi 湖北 province. That year, 65 bells (set-bells) were unearthed. Each bell can produce two tones corresponding to two-type, front and diagonal, striking points. The set-bells were hung on a wooden rack in three tiers. There were three blocks of bells, 6+6+7, in the top tiers, three blocks of bells, 11+12+10, in the middle tiers, and two blocks of bells, 3+10, in the bottom tiers. The entire set of bells covers a range of more than five octaves. Since two striking rods and six wooden mallets were discovered, it is conjectured that two players stand in front of the bottom-tier bells, and three players, center, right and left, stand behind the bells to play the middle- and top-tier bells. A large number of studies have been made on the acoustics of these set-bells [1–7]. It has been found that the bells have perfect fifth [3] and major third [4,5] consonances. However, little is known about the spatial arrangement of the consonant chords of the bells. The purpose of this work is to study the bell arrangement.

Funatsu and Okamoto introduced a classification method of musical notes [8], which is based on 4-dimensional (4D) vector space on integer (lattice space). This method is an extension of Oettingen's, Ellis' and Tanaka's methods [9] based on 2-dimensional (2D) lattice space, and of Sato's method [10] based on 3-dimensional (3D) lattice space, where both the spaces are generated from prime numbers greater than 2 as $a/b = 2^{x_0} \cdot 3^{x_1} \cdot 5^{x_2} \cdot 7^{x_3} \cdot 11^{x_4} \cdots =: (x_1, x_2, x_3, x_4, \cdots)$. This method was applied to Śruti theory in Indian music [8], well-tempered tuning in Western music [11], Japanese traditional music [12] and three sets of two-tone set-bells, the Marquis Yǐ bells, the Gào Yǒng Zhōng 王孫誥 bells and that unearthed in Xìnyáng 信陽, Hénán 河南 province in 1957 [7]. As a result, the frequencies of the Marquis Yǐ set-bells were predicted to be such that the number of consonant intervals is maximum [7]. In this study, on the basis of the above result, all possible three- or four-note consonant chords and their configurations are obtained. Moreover, we find almost all the arrangements can easily be struck by five players.

2. Arrangements of consonances in Marquis Yǐ set-bells

The ratios of predicted frequencies to that of the Zhèng-Gǔ 正鼓 note of the middle (M)-2-12 bell, which is the 12th bell, counted from the left, of the center block in the middle-tier, can be factored as $a/b = 2^{x_0} \cdot 3^{x_1} \cdot 5^{x_2} \cdot 7^{x_3} =: (x_1, x_2, x_3)$. This expression indicates that the note space of the set-bells is a 3D lattice space. When we describe the 3D lattice space, there is the problem that we must decide the direction of the three vectors forming the bases corresponding to the perfect fifth (3:2) and major third (5:4) notes and the note of ratio 7:4. In the case of a 2D lattice space, a common point of Oettingen's, Ellis' and Tanaka's methods is that all harmonic notes are placed in the first quadrant. The reason for this is that harmonic notes are considered acoustically important, and the first quadrant is easy to see. Consequently, in the case of the 3D lattice space, we should select the vectors such that the harmonic notes fall in the front of the space. However, the three vectors of Sato's method [10] are not in such a configuration. Thus we should use the reflection of Sato's basis of the major third note reflected at the plane determined by his bases of the perfect fifth note and the 7:4 note. In addition, we should configurate all notes on triangular-lattce points or tetrahedral-lattice points in the 2D or 3D space, respectively, because a three-note consonant chord is expressed as a triangle, which is the reflection of the triangle of the major or minor chord in Tanaka's method [9,11,12] in the 2D triangular lattice, reflected at the line determined by the basis of the perfect fifth note.

Using the above-described method, we visualise the notes of the Marquis Yǐ set-bells, as shown in Fig. 1. Several examples of three- or four-note consonant chords are presented in Fig. 2. In example (a), a major chord can be sounded when the center mallet-player strikes the Zhèng-Gǔ 正鼓 note of the M-2-4 bell and the right mallet-player strikes the Cè-Gǔ 側鼓 notes of the M-2-12 and M-3-1 bells, which are at the right end of the center block and at the left end of the right block in the middle tier, respectively. Next, in example (b) of a minor chord and example (c) of a major chord, the notes in each tetrahedron which has four triangles, i.e. four types of three-note chords, are consonant with one another. All four-note configurations of the same shapes as the above two tetrahedrons are also four-note consonant chords. The numbers of type (b) and (c) set notes are 4 and 6, respectively. Four players simply strike four bells simultaneously for all but one of the above 10 (=4+6) four-note consonant chords, and for the case of the above exception three players can easily strike all three-bell combinations. Moreover, we find that there are 26 major chords, 16 minor chords and 31 three-note consonant chords which have the intervals of ratio 7:4, 7:5 and 7:6, by examining the shapes in Fig. 1. All but

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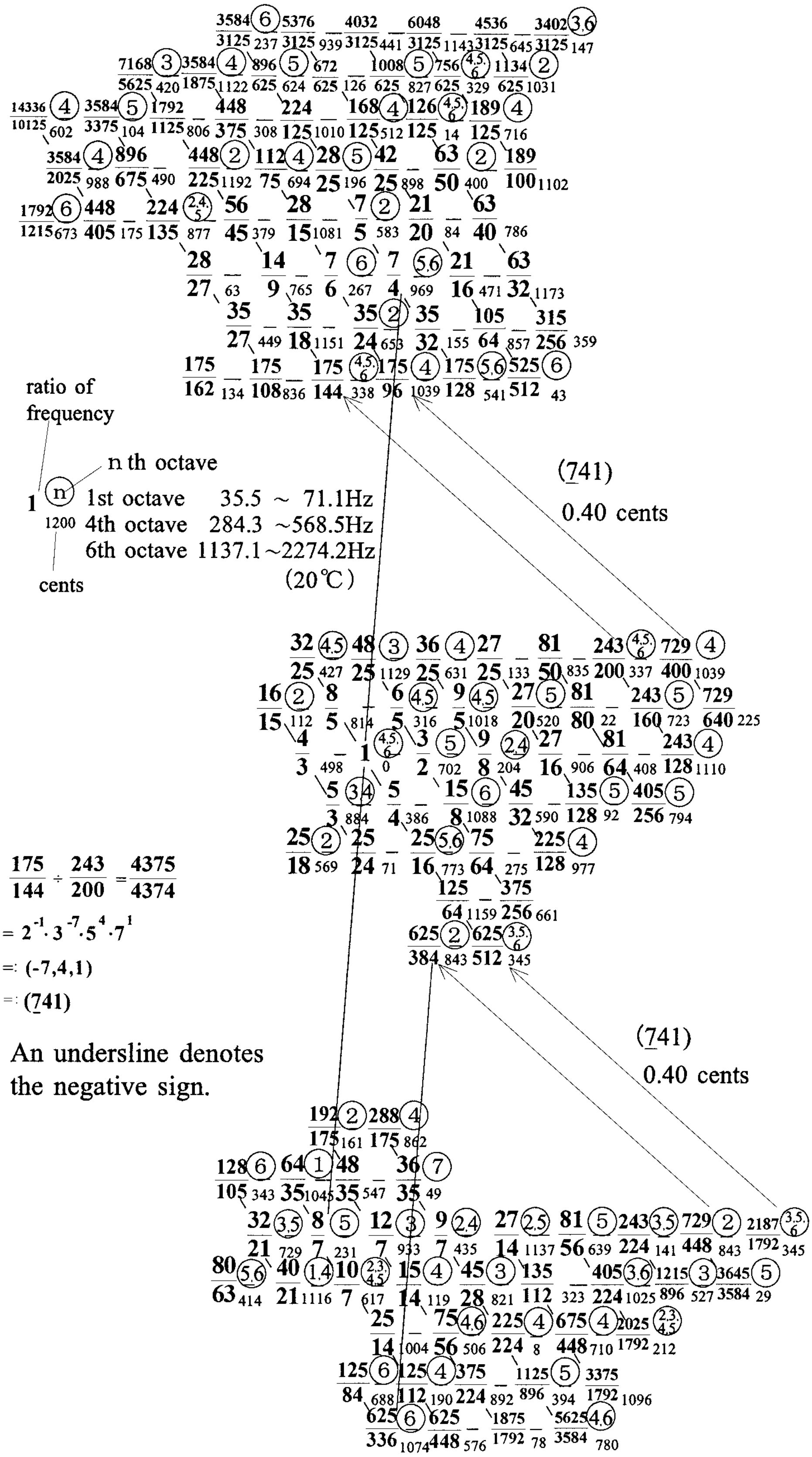

Fig. 1 The mathematical structure of the frequencies of the Marquis Yǐ set-bells. Circles indicate the notes of the set-bells. The circled number indicates the n-th octave note, provided the Zhèng-Gǔ 正鼓 note of the middle (M)-2-12 bell is a standard pitch with a frequency of 284.27 Hz at 20°C.

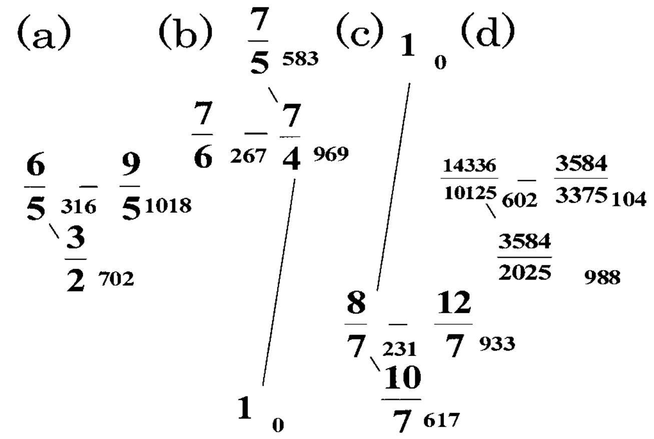

Fig. 2 Examples of the chords of the Marquis Yǐ set-bells.

one of the 73 (=26+16+31) three-notes consonant-chords can be struck simply by three players. We do not believe that such a arrangement was produced by chance. It is thought that immense efforts was expended to produce bells that peal consonant chords. Finally, for example (d), a player must simultaneously strike the Cè-Gǔ 側鼓 notes of the M-2-7 and M-2-10 bells, the distanse between which is about 70 cm. It is also thought that the M-2-10 bell and the M-2-11 bell were interchanged [2]. If these two bells are not interchanged, the distanse between them is about 100 cm. Most people would find it difficult to strike these bells simultaneously. Thus there is a possibility that these bells were interchanged on purpose.

3. Conclusions

It should be concluded that the arrengement of the Marquis Yǐ set-bells were carefully selected such that the above 72 three-note consonant chords could easily be struck by five players. Namely, there was a distribution of 78 notes in a one-octave conversion. If pieces of old Chinese music or new compositions using the above chords are available, the Marquis Yǐ set-bells may be played consonantly as in the days of ancient China.

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