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INVITED REVIEW

Lord Rayleigh: A master of theory and experiment in acoustics

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1. INTRODUCTION

One of the ways in which the scientific community honours a major contribution to knowledge is to name the effect or principle in question after its discoverer. The British physicist Lord Rayleigh is outstanding in this respect: his name is associated in the scientific literature with a remarkable number of important theoretical ideas and practical techniques, including Rayleigh's Principle, Rayleigh scattering, Rayleigh waves, Rayleigh streaming, the Rayleigh Criterion and the Rayleigh disc.

In a long and very active career, Rayleigh contributed to many different scientific disciplines, but he retained a strong interest in acoustics throughout his life. One of his first papers was on acoustic resonators [1], and his final paper, published posthumously, treated the reflection of sound from a perforated wall [2]. His great two-volume treatise on *The Theory of Sound* [3], although published in the nineteenth century, remains an essential textbook and work of reference for acousticians of the present day. The collected edition of his scientific papers [4], prepared by Rayleigh himself and published in six volumes between 1899 and 1920, includes many papers which have become foundation stones of modern acoustics.

After a brief outline of Rayleigh's biography, the present paper describes some of the major advances which Rayleigh made in the field of acoustics.

2. BRIEF BIOGRAPHY OF RAYLEIGH

The definitive biography of Rayleigh was written by his son, R. J. Strutt, fourth Baron Rayleigh [5]. A useful brief life is provided in R. B. Lindsay's *Lord Rayleigh: The Man and His Work* [6], which also includes a review of Rayleigh's contributions to science and selections from 23 of his scientific papers. The present short summary is largely indebted to these two sources.

John William Strutt was born on 12th November 1842, the eldest son of an English nobleman. After a childhood and adolescence marked by periods of ill health, he studied mathematics at the University of Cambridge, graduating with distinction in 1865. In 1866 he was elected a Fellow of Trinity College, Cambridge. On the death of his father in 1873 he succeeded to the title, becoming the third Baron Rayleigh. During his studies at Cambridge he had become keenly interested in experimental physics, and he set up a laboratory at the family home, Terling Place in Essex.

In 1874 the Cavendish Laboratory of Physics was opened at the University of Cambridge. The first Cavendish Professor was James Clerk Maxwell, and after Maxwell's untimely death in 1879 Rayleigh agreed to take up the Cavendish chair. He held this post for five years, during which he built up the laboratory as a major teaching resource, and carried out important experiments on electrical standards. In 1884 he returned to his laboratory at Terling Place, where most of his subsequent research was carried out. Among the outstanding results to emerge from this laboratory was the discovery of argon, for which Rayleigh was awarded the Nobel Prize for Physics in 1904.

Rayleigh contributed greatly to the promotion and dissemination of science. He was Secretary of the Royal Society for eleven years from 1885, and its President from 1905 to 1908. He served as President of the British Association for the Advancement of Science in 1884. From 1882 until 1905 he was Professor of Natural Philosophy at the Royal Institution in London, during which period he gave many popular lectures on scientific topics. From 1908 until his death on 30th June 1919 he was Chancellor of the University of Cambridge.

3. RAYLEIGH'S FIRST PAPER IN ACOUSTICS

On 24th November 1870 Rayleigh's first paper on an acoustical subject, "On the Theory of Resonance," was read to the Royal Society in London; the following year it was published in the Society's *Philosophical Transactions* [1]. Rayleigh had carefully studied the work of Helmholtz on resonant acoustical cavities, and in this paper he presented an alternative and more general approach to the theory of Helmholtz resonators.

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This early paper already demonstrates clearly several features which remained characteristic of Rayleigh's approach and writing style. The tone is relatively informal: he describes the numerous experimental observations awaiting adequate theoretical explanation as

'a sort of dead weight on the scientific stomach, ... which must remain undigested until theory supplies a more powerful solvent than any now at our command.'

The intentions of the paper are clearly laid out in a lengthy introduction, and each step in the mathematical treatment is carefully explained. He later returned to develop the material in more didactic form in *The Theory of Sound* [7].

Two features of Rayleigh's work on resonators are worth commenting on. Firstly, the theory is based on calculations of kinetic and potential energy, and a concentration on energetic methods was to be an important feature of much of Rayleigh's subsequent acoustical work. Secondly, he makes extensive use of an electrical analogy, in which acoustic volume velocity is taken as the analogue of electric current. This analogy, further developed by Webster [8], has been immensely fruitful for subsequent work in acoustics.

Pursuing the electrical analogy, Rayleigh defines in this paper an important quantity, the "conductivity" c of an aperture. In Rayleigh's treatment, the acoustic flow through the aperture is characterised by the velocity potential ϕ; the acoustic particle velocity $\boldsymbol{v}$ and acoustic pressure p are given by

$$\boldsymbol{v} = \nabla\phi$$

$$p = -\rho_0 \frac{\partial\phi}{\partial t}.$$

The conductivity is defined by the equation

$$\dot{X} = c(\phi_1 - \phi_2),$$

where $\dot{X}$ is the volume flow rate through the aperture, and ϕ_1 and ϕ_2 are the (assumed constant) velocity potentials some distance on either side of the aperture. Rayleigh then derives the resonance frequency N of a gas-filled cavity of volume S with an aperture of conductivity c as

$$N = \frac{a}{2\pi}\sqrt{\frac{c}{S}},$$

where a is the speed of sound in the gas. Rayleigh presents solutions for c for elliptical apertures, making use of mathematical results previously obtained in the determination of the electrostatic charge on an ellipsoidal conductor. He also obtains approximate solutions for the conductivity of a number of other channels, and deals with the problem of coupled resonators.

It is also very characteristic of Rayleigh's comprehensive approach to the problems which he tackled that he ends the paper by describing a number of careful experiments which he devised and carried out in order to verify his theoretical results.

In this first contribution to the literature of acoustics, Rayleigh already identified several important paths for future investigators. His lumped-parameter treatment of apertures and orifices has remained standard, although the parameter commonly used to describe an orifice in modern treatments is the acoustic inertance (the pressure per unit volume acceleration), related to Rayleigh's conductivity c by $M_A = \rho_0/c$ [9].

4. THE RAYLEIGH DISC

In June 1880, shortly after taking up the Cavendish chair at the University of Cambridge, Rayleigh decided to carry out a redetermination of the standard unit of electrical resistance, the ohm. A committee of the British Association had been set up in 1861 to determine this unit, using an apparatus designed by William Thomson (later Lord Kelvin). The apparatus consisted of a circular coil of wire rotating at constant speed about a vertical axis, with a horizontal magnetised galvanometer needle at the centre. The deflection of this needle from the magnetic meridian, due to the current generated in the coil by its motion in the earth's magnetic field, is independent of the magnitude of the field, and depends only on the dimensions and resistance of the coil and the speed of rotation. The deflection of the needle was measured by reflecting a light beam from an attached mirror. Although a team working under Maxwell had obtained a value for the standard ohm, there was some controversy about its accuracy.

The equipment used in the original British Association experiment had been transported to Cambridge when Maxwell became Cavendish Professor. Rayleigh found the apparatus "a good deal out of repair," and carried out various modifications and improvements; eventually a set of results were obtained which achieved international acceptance [10]. While he was working on the equipment, he and his assistant Horace Darwin noticed that the orientation of the small mirror attached to the needle was surprisingly sensitive to even a gentle finger tap on the case. Rayleigh concluded that the mirror was being acted on by the oscillations in the surrounding air. He was aware that a suspended disc tends to orient itself in the plane perpendicular to the direction of a steady flow of air, since at any other angle the differing pressure distributions on either side of the disc result in a net torque about the axis of suspension. Since this effect does not depend on the direction of the flow, Rayleigh reasoned that it should also apply to oscillating flows.

He devised a simple experiment to validate this hypothesis.

'A small disk of paper, about the size of a sixpence, was hung by a fine silk fibre across the mouth of a

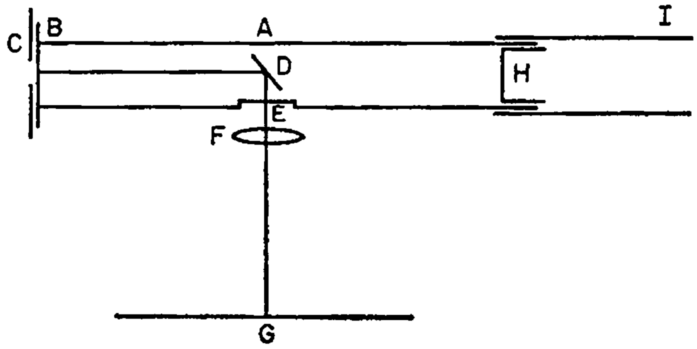

Fig. 1 The Rayleigh disc. A: brass tube; B, E: glass windows; C: lamp and slit; D: suspended mirror; F: lens; G: scale; H: paper diaphragm.

resonator of pitch 128. When a sound of this pitch is excited in the neighbourhood, there is a powerful rush of air in and out of the resonator, and the disk sets itself promptly across the passage.' [10].

From this simple observation Rayleigh developed an absolute method of measuring the acoustic particle velocity (and hence the intensity) in a sound wave. The Rayleigh disc, as it came to be known, was first described in a paper in the Philosophical Magazine in 1882 [11], from which Fig. 1 is reproduced. The galvanometer mirror is suspended in a brass tube, at a distance from the closed end equal to a quarter wavelength of the sound to be measured. The mirror is then at a velocity antinode, giving maximum sensitivity. In *The Theory of Sound* Rayleigh describes how an arrangement of double resonators can be used to further enhance the sensitivity.

Despite its simplicity, the Rayleigh disc remained in use by the National Physical Laboratory as the standard method for absolute calibration of microphones until 1971, when it was superseded by a method based on reciprocal transducers.

5. RAYLEIGH'S PRINCIPLE

Rayleigh's energy-based approach to the study of vibrating systems led him to an important variational theorem which has come to be known as "Rayleigh's Principle." In *The Theory of Sound* he gave the following statement of the principle [12]:

'The period of a conservative system vibrating in a constrained type about a position of stable equilibrium is stationary in value when the type is normal.'

He considers a system with a number of normal modes of free vibration, described by normal co-ordinates ϕ_1, ϕ_2, ϕ_3, etc. In order to calculate the modal frequencies of the system, a modification is introduced in which the system is constrained to have a single degree of freedom θ in which it vibrates sinusoidally with angular frequency p. This constrained motion is a superposition of the normal modes of the free system, such that

$$\phi_1 = A_1\theta, \quad \phi_2 = A_2\theta, \quad \text{etc.}$$

The kinetic energy T and potential energy V of the constrained motion are then given by

$$T = \left[\frac{1}{2}a_1A_1{}^2 + \frac{1}{2}a_2A_2{}^2 + \dots\right]\dot{\theta}^2 \tag{1}$$

$$V = \left[\frac{1}{2}c_1A_1{}^2 + \frac{1}{2}c_2A_2{}^2 + \dots\right]\theta^2 \tag{2}$$

where the a_i and c_i represent modal masses and stiffnesses respectively. Since in simple harmonic motion the maximum values of T and V are equal, substitution of $\theta = \theta_0 \cos pt$ in Eqs. (1) and (2) yields an expression for the square of the angular frequency of the vibration:

$$p^2 = \frac{c_1A_1{}^2 + c_2A_2{}^2 + \dots + c_mA_m{}^2}{a_1A_1{}^2 + a_2A_2{}^2 + \dots + a_mA_m{}^2}. \tag{3}$$

This expression is stationary when all but one of the coefficients A_i are zero; in this situation the vibration pattern coincides with one of the normal modes of the free system, and no constraints are necessary.

Rayleigh's Principle has proved to be of great practical utility in fields as diverse as seismology and quantum mechanics, because it can yield a good approximation to the mode frequencies (eigenvalues) of a system without the necessity for a derivation of the mode shapes (eigenfunctions). In particular, it yields an upper bound for the lowest mode frequency. Rayleigh makes the important point that first order deviations in the mode shape give rise to second order changes in the mode frequency:

'By means of the principle that the value of the free periods is stationary, we may easily calculate corrections due to any deviation in the system from theoretical simplicity. If we take as a hypothetical type of vibration that proper to the simple system, the period so found will differ from the truth by quantities depending on the square of the irregularities.' [12].

He gives the example of a uniform stretched string. If the analytically known sinusoidal mode shape is approximated by a parabolic function, the first mode frequency predicted by the use of Rayleigh's Principle is less than 0.7% above the true value; if the mode shape is even more crudely approximated by a simple triangle (as if the whole mass of the string were concentrated at the centre), the approximation to the frequency is still less that 10% higher than the true value.

6. CONCLUSION

Rayleigh made important contributions to so many areas of acoustics that it is impossible even to summarise them all in a short article. His remarkable physical intuition repeatedly led him to initiate or develop studies which would later become foundations for whole fields of research. An example is his pioneering work on non-linear sound propagation [13,14], in which he applies his approach based on considerations of energy flow and energy conservation to the discussion of discontinuous sound waves (now known as shock waves). Related to this is his detailed theoretical treatment of the circulating patterns generated by high amplitude standing waves in closed tubes, now described as Rayleigh streaming [15].

Another example which must be mentioned is the seminal paper of 1885 [16], in which Rayleigh presented the first treatment of waves propagating on the surfaces of elastic solids. Rayleigh waves, as they are now described, involve an elliptical motion of the particles of the solid, whose amplitude decreases rapidly with increasing depth. However, the amplitude of propagation along the surface from a point source decreases only with the square root of the distance from the source. As a consequence, Rayleigh waves generated on the earth's surface by an earthquake are largely responsible for the transmission of the energy of the disturbance over very long distances.

Rayleigh clearly foresaw the potential significance of his discovery in seismology: his paper of 1885 concludes:

'It is not improbable that the surface waves here investigated play an important part in earthquakes, and in the collision of elastic solids. Diverging in two dimensions only, they must acquire at a great distance from the source a continually increasing preponderance.'

What might have surprised even such a far-sighted scientist is the recent dramatic growth in the production of Surface Acoustic Wave (SAW) devices, based on the propagation of Rayleigh waves on the surface of a piezoelectric crystal. Miniaturised SAW filters are now very widely used in mobile phone and wireless technology [17], providing yet another illustration of the profound and lasting impact of the work of this dedicated and inspired classical physicist and acoustician.

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