

# Calculation of transfer function of acoustic feedback path for in-the-ear hearing aids with a correction for specific acoustic impedance of a tubule

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## 1. Introduction

Hearing aids should be small and light since they are continuously worn by hearing-impaired people in their everyday life. Currently, an in-the-ear (ITE) hearing aid has a microphone, an amplifier circuit, and an earphone packed into a small housing unit so that it fits completely inside the external ear canal. For this type of hearing aid, an oscillation, also called howling, is due to the acoustic feedback through a small aperture between the external ear canal and the hearing aid, which is not an intentional vent. This aperture is due to the close proximity of the microphone and earphone in the small housing unit. Recently, digital hearing aids have been developed with various feedback reduction algorithms, such as simple notch and adaptive filterings [1–5]. However, these algorithms are often insufficient, particularly for severe-to-profound hearing-impaired people, who require an extremely high acoustic gain [6].

In addition to an ordinary microphone, an internal microphone might be useful for solving this problem. To reduce howling with this internal microphone, the transmission function of the feedback path must be properly estimated. Thus, in the present study, we examine the formulation of the feedback transmission function with an equivalent circuit model based on the acoustic feedback. The aperture between the external ear canal and the hearing aid, i.e., the “leakage element”, which causes the acoustic feedback, is assumed to be that of a cylindrical tube. Thus, the transfer function is calculated by assuming the acoustic system as a cascade of cylindrical tubes. Moreover, a correction term for the tubule in the cylindrical tube is introduced to improve the accuracy of the calculation.

## 2. Modeling

### 2.1. Hearing aid model

Figure 1 shows the hearing aid model used assumed in this study. Mic1, Ear, and Mic2 are the external microphone, earphone, and ITE microphone, respectively. The transfer function of the path from the earphone (EAR) to Mic2 in the ear cavity is denoted by  $g(s)$ , while the transfer function from the point near the earphone to the sound inlet of the external microphone, i.e., the leakage transmission function, is denoted

by  $h(s)$ . The three sections, ‘s,’ ‘v,’ and ‘e,’ shown in Fig. 1 represent the residual capacity in the ear with the hearing aid, the feedback path with a small aperture, and the radiation area, respectively. In this study, these three sections are assumed to be cylindrical tubes with rigid inner walls.

### 2.2. Transfer matrix of cylindrical tube cascade

The assumption that the three sections are rigid cylindrical tubes allows the application of the transfer-line-matrix (TLM) method to the analysis of the acoustic characteristics [7,8]. The basic matrix  $F$  and transmission matrix  $T$  of the  $n$ th tube are represented as,

$$F_n = \begin{pmatrix} \cosh(\gamma_n x) & j \cdot Z_{0n} \sinh(\gamma_n x) \\ j \cdot \frac{1}{Z_{0n}} \sinh(\gamma_n x) & \cosh(\gamma_n x) \end{pmatrix}, \quad (1)$$

$$T_n = \begin{pmatrix} e^{-\gamma_n x} & 0 \\ 0 & e^{\gamma_n x} \end{pmatrix}, \quad (2)$$

where  $Z_{0n}$  is the characteristic impedance,  $\gamma$  is a propagation constant [9,10], and  $x$  is the tube length. The reflection matrix  $D$  between sections ‘s’ and ‘v’ is given by,

$$D_{n,n+1} = \frac{1}{2\sqrt{Z_{0n} \cdot Z_{0n+1}}} \begin{pmatrix} Z_{0n} + Z_{0n+1} & Z_{0n} - Z_{0n+1} \\ Z_{0n} - Z_{0n+1} & Z_{0n} + Z_{0n+1} \end{pmatrix}, \quad (3)$$

$$n \rightarrow s, \quad n+1 \rightarrow v$$

where  $Z_{0s}$  and  $Z_{0v}$  are the characteristic impedances for sections ‘s’ and ‘v,’ respectively.

From Eqs. (2) and (3), the transmission matrix  $H$  can be shown as

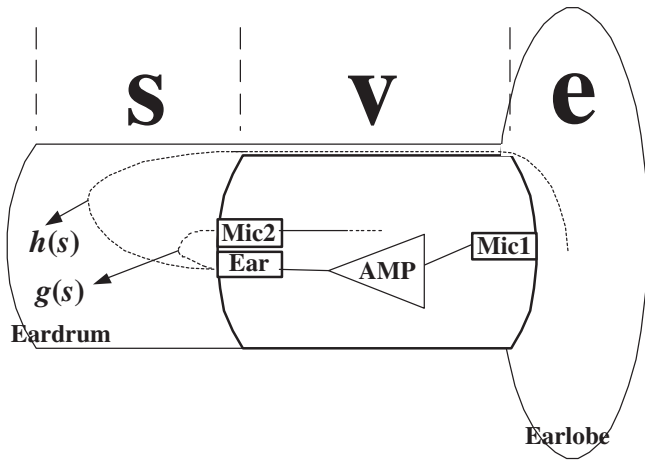
$$H = \left( \prod_{i=0}^{n-1} (D_{i,i+1} \cdot T_i) \right) \cdot D_{n,n+1}. \quad (4)$$

From Eq. (4), the transfer function is obtained from the matrix element  $H_{11}$ . Figure 2 shows the transmission matrix model that corresponds to the hearing aid model depicted in Fig. 1.

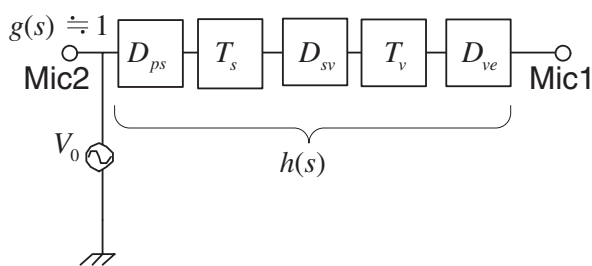
### 2.3. Sound characteristics of tubule

The sound equivalent circuit of a cylindrical tube can be represented as a distributed constant circuit, which includes the acoustic characteristic impedance  $Z_0$  and propagation constant  $\gamma$  of the transmitted sound wave. However, the loss resistance of a thin tubule is higher than that of a tube due to

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**Fig. 1** Proposed hearing aid model.



**Fig. 2** Transmission matrix model.

the decreasing diameter. Hence, in this study, a correction for the specific acoustics impedance  $\Omega(\beta)$  of a tubule, which is given by Hayakawa and Kinsler in the following equation, is introduced to account for the effect [11,12],

$$\Omega(\beta) = \frac{8 \cdot \mu}{r^2} \cdot \frac{1}{\frac{8}{\beta^2} - \frac{16}{\beta^3} \cdot \frac{I_1(\beta)}{I_0(\beta)}}, \quad (5)$$

$I_0(\beta)$  and  $I_1(\beta)$  are Bessel functions.

where  $r$ ,  $\mu$ ,  $\rho$ , and  $f$  are the radius of the tube, viscous coefficient, air density and frequency, respectively, and  $\beta$  is the index of the mass effect [11] defined by

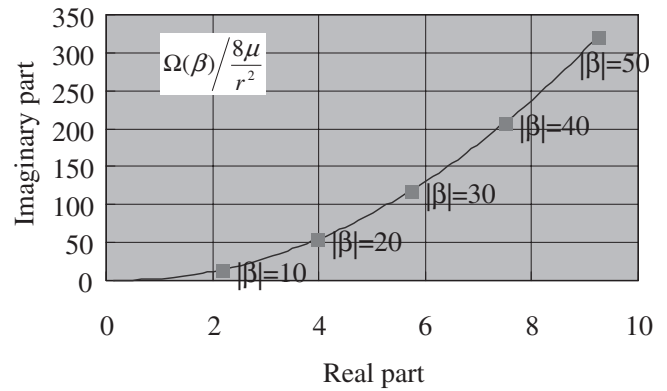
$$\beta = r \cdot \sqrt{\frac{\rho \cdot \lambda}{\mu}} \quad \lambda = j \cdot \omega = j \cdot 2\pi f, \quad (6)$$

where  $\omega$  is the angular frequency.

Depending on  $\beta$ , the corrected viscosity  $\rho'$ , velocity  $c'$ , wave number  $k'$ , and attenuation constant  $\alpha'$  for a tubule are given by [11,12]

$$\rho' = \frac{4}{3}\rho, \quad c' = \frac{c}{2} \left( \frac{r}{\sqrt{\frac{a \cdot \mu}{\rho \cdot \omega}}} \right), \quad k' = \alpha' = \frac{\omega}{c'} \quad \text{for } |\beta^2| < 100, \quad (7a)$$

where  $a = 1.408$  is an experimentally measured constant [12].



**Fig. 3**  $\Omega(\beta) / \frac{8 \cdot \mu}{r^2}$  as function of  $|\beta|$ .

$$\begin{aligned}\rho' &= \rho \left( 1 + \frac{1}{r} \sqrt{\frac{2 \cdot \mu}{\rho \cdot \omega}} \right), & c' &= c \left( 1 - \frac{1}{2 \cdot r} \sqrt{\frac{2 \cdot \mu}{\rho \cdot \omega}} \right), \\ \alpha' &= \frac{1}{r \cdot c} \sqrt{\frac{\omega \cdot \mu}{2 \cdot \rho}}, & k' &= \frac{\omega}{c'} \\ &\text{for } |\beta^2| > 100.\end{aligned}\tag{7b}$$

Figure 3 shows the relationship between the index of the mass effect  $|\beta|$  and  $\Omega(\beta)/\frac{8-\mu}{r^2}$  for the  $|\beta|$  range from 1 to 50 [11].

Consequently, the characteristic impedance  $Z_0$  and propagation constant  $\gamma$  of a tubule are given by,

$$Z_0 = \frac{c' \rho'}{\pi \cdot r^2}, \quad (8)$$

$$\gamma = \alpha' + j \cdot k'. \quad (9)$$

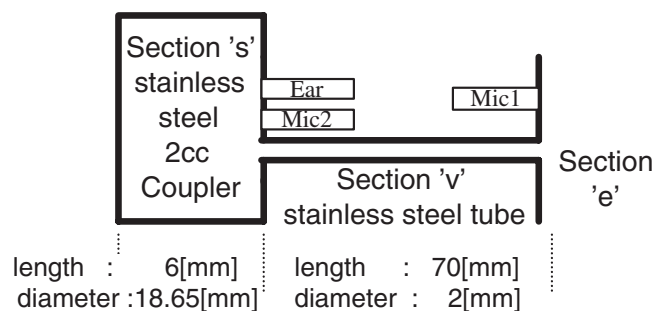
### 3. Experiment

### 3.1. Method

The transfer function of the hearing aid model shown in Fig. 1 was measured by comparing the calculation results for cascade cylinders with and without a correction for the specific acoustics impedance of a tubule and the experimental results. Figure 4 shows the arrangement of the acoustic measurement. The material of the cylindrical tube in section 'v' was stainless steel. For section 's,' a 2cc coupler specified by IEC60126 was used.

### 3.2. Parameters for calculations

In the calculations, the viscous coefficient of the tube,



**Fig. 4** Experimental constitution conditions.

speed of sound, and air density were assumed to be  $\mu = 0.186 \times 10^{-5}$  [kg·s/m<sup>2</sup>],  $c = 341$  [m/s] and  $\rho = 1.205$  [kg/m<sup>3</sup>], respectively. The symbols  $T_s$ ,  $T_v$ , and  $T_e$  represented the transmission parameters in sections 's,' 'v' and 'e,' respectively, where  $D_{sv}$  and  $D_{ve}$  were the reflection parameters at the borders of for consistency sections 's' and 'v,' and sections 'v' and 'e,' respectively. To obtain the transmission matrices, the propagation constants  $\gamma$  at sections 's' and 'v' derived from Eq. (7b) were substituted into Eq. (2). Similarly, the reflection matrices were calculated by substituting the characteristic impedance  $Z_0$  of Eq. (8) into Eq. (3). To calculate the reflection matrix  $D_{ps}$ , the earphone tube with the radius  $r = 0.8$  [mm] was assumed Eq. (8) and the characteristic impedance in sections 's.' The characteristic impedance of section 'e' was  $\rho \cdot c$ .

In approximating a distributed parameter acoustic system by sectioning, the section length  $\Delta X$  must be shorter than  $1/4$  of the wavelength. Moreover, to keep errors by the phase difference and the reflection wave of the traveling wave in an acoustics system enough small, it is desirable to make the section length  $\Delta X$  as short as  $1/10$  of the wavelength. Therefore, in this study, section length  $\Delta X$  [mm] was set to 3.4 mm, i.e.  $1/10$  wavelength at 10 kHz, because the upper limit frequency for the hearing aids is regarded as some 10 kHz.

### 3.3. Results and discussion

Figure 5 shows the measured and calculated frequency response curves. The resonance between 200 and 300 Hz is the Helmholtz resonance (HHR) of sections 'v' and 's.' As shown in Fig. 5, the resonance frequencies from the calculations with and without the correction are similar identical; however, they are slightly higher than the experimental results.

On the other hand, the resonance frequencies above 2 kHz are the open-tube resonance (OTR) frequencies of section 'v.' The measured OTR frequencies agree well with the calculated OTR frequencies. Moreover, it should be noted that the calculation with the correction enables the estimation of the sharpness of the OTR resonances much better than that

without the correction. However, the difference between the measured and estimated data increases with the frequency.

The above results indicate that the proposed calculation method with the correction is useful for analyzing the feedback frequencies and their peak levels in ITE hearing aids since they usually have severe feedback problems from 1 kHz to 4 kHz.

## 4. Conclusion

The acoustic feedback transmission function of ITE hearing aids was modeled using a transmission matrix. The proposed calculation model with a correction for the specific acoustics impedance of a tubule, which is a path through a small aperture between the external ear canal and the hearing aid, can accurately simulate the acoustic characteristics of the feedback path between the ear cavity and the emission area. Therefore, the proposed method is useful for analyzing the feedback frequencies and their peak levels in ITE hearing aids, which are important to effectively control howling. In the future, the validity of the proposed method for estimating the "leakage" of real human ears will be examined by comparing the calculation results and measurement results using human ears.

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## References

- [1] H. Josen, F. Asano, Y. Suzuki and T. Sone, "A consideration on adaptive howling suppression for hearing aids," *IEICE*, EA91-40 (1991).
- [2] H. Josen, F. Asano, Y. Suzuki and T. Sone, "Adaptive howling suppression with frequency compression," *IEICE*, EA92-36 (1992).
- [3] H. Josen, F. Asano, Y. Suzuki and T. Sone, "Use of adaptive notch filters to suppress howling in hearing aids," *IEICE*, EA93-31 (1993).
- [4] B. Friedlander and J. O. Smith, "Analysis and performance evaluation of an adaptive notch filter," *IEEE Trans. Inf. Theory*, **IT-30**, 283–295 (1984).
- [5] M. R. Schroeder, "Improvement of acoustic-feedback stability by frequency shifting," *J. Acous. Soc. Am.*, **36**, 1718–1724 (1964).
- [6] J. Agnew, "Application of a notch filter to reduce acoustic feedback," *Hear J.*, **46**(3), 37–43 (1993).
- [7] R. Sato, *Transfer Circuit* (Corona, Tokyo, 1963), pp. 66–68.
- [8] T. Ozeki, *Optical Transfer Circuit* (Corona, Tokyo, 2000), pp. 88–90.
- [9] S. Yoshikawa, *Principles of Acoustics* (Kodansha, Tokyo, 2002), pp. 141–156.
- [10] T. D. Rossing, *Principles of Vibration and Sound* (Springer, Berlin, 1995), pp. 175–183.
- [11] T. Hayakawa, *Sound Vibration Theory* (Maruzen, Tokyo, 1943), pp. 727–735.
- [12] L. E. Kinsler, *Fundamentals of Acoustics* (John Wiley & Sons Inc., New York, 1950), pp. 231–243.

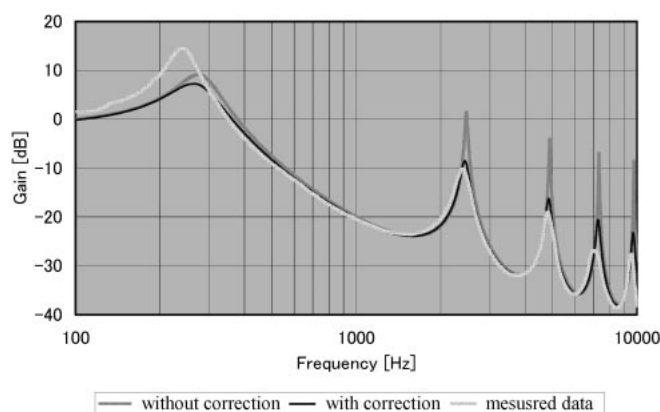


Fig. 5 Measured and simulated frequency response curves.