

Optical visualization of ultrasonic waves propagating in a fluid waveguide

Ken Yamamoto*

Kobayasi Institute of Physical Research,
3-20-41, Higashi-Motomachi, Kokubunji, 185-0002 Japan

(Received 26 January 2005, Accepted for publication 1 March 2005)

Keywords: Visualization, Waveguide, Ultrasonic wave, Fresnel diffraction, Group velocity
PACS number: 43.35.Bf [DOI: 10.1250/ast.26.378]

1. Introduction

Ultrasonic waves traveling in a medium with boundaries, that is, a “waveguide,” have a large number of propagation modes and show dispersion characteristics [1]. The propagation mode is a resonance phenomenon caused by the structure of the waveguide and the frequency of the ultrasonic wave. Each mode propagates with a different phase velocity according to the dimensions of the waveguide and the frequency. A typical wave that propagates along a solid plate is a Lamb wave, and waves propagating along a solid with boundaries are generally named “guided waves.” Alternatively, an ultrasonic wave in a fluid with rigid boundaries, that is, a “fluid waveguide,” has the propagation modes of a guided wave and shows dispersion characteristics. The aim of this study is to understand the dispersion characteristics of ultrasonic waves traveling in the fluid waveguide. In this study, an optical visualization technique was used to determine the phase velocity of each traveling mode, and the measurement of the group velocity in the waveguide was also carried out.

2. Analysis of the fluid plate with rigid boundaries

We consider a waveguide formed by a plate with fluid thickness d in the z -direction, that is infinitely long in the x - and y -directions, as illustrated in Fig. 1. We deal with the propagation of the guided wave in this system using the wave reflections at the interface and describe the direct analysis for finding solutions to the wave equation that satisfies the boundary conditions. Here, coupling of the wave propagating in the fluid, the “fluid mode,” with that of the solid plate mode, the “Lamb mode,” can be neglected when the incident angle θ is greater than Rayleigh angle θ_R . Experimentally, it is possible to make this waveguide by sandwiching a fluid between two large metal plates in which the velocity normal to the boundaries is very nearly zero at the boundaries. We introduce a potential function ϕ and solve the wave equation

$$\nabla^2 \phi = \frac{1}{c_1^2} \frac{\partial^2 \phi}{\partial t^2}, \quad (1)$$

with boundary conditions given by

$$\left[\frac{\partial^2 \phi}{\partial z \partial t} \right]_{z=\pm d/2} = 0. \quad (2)$$

Here, c_1 is the sound velocity of the longitudinal wave in the fluid. We have the following two solutions using the amplitude constant A and angular frequency ω . One corresponds to the symmetrical mode and describes the cosine part of the solution for ϕ with subscript of S, which corresponds to both the potential ϕ and the sound pressure p being symmetrical about the axis.

$$\begin{aligned} \phi_S &= A \cos(k_z z) \exp(-ik_x x) \exp(i\omega t) \\ (k_z d/2)_S &= m\pi/2, \quad m = 0, 2, 4, \text{etc.} \end{aligned} \quad (3)$$

The other has sine terms that give the asymmetrical modes with A.

$$\begin{aligned} \phi_A &= A \sin(k_z z) \exp(-ik_x x) \exp(i\omega t) \\ (k_z d/2)_A &= m\pi/2, \quad m = 1, 3, 5, \text{etc.} \end{aligned} \quad (4)$$

Here, the wave vector in the x direction is $k_x (= \omega/c_1 \sin \theta)$, and that along the z axis is $k_z (= \omega/c_1 \cos \theta)$. Even values of m give modes with symmetrical pressure or potential distributions, while odd values of m correspond to asymmetrical modes. There are m “half-cycles” on the z -axis. Each mode has a characteristic potential distribution and a characteristic phase velocity c_p . The phase velocity is given by

$$c_p = \frac{\omega}{k_x} = \frac{c_1}{\sqrt{1 - k_z^2 \left(\frac{c_1}{\omega} \right)^2}}. \quad (5)$$

The energy of the guided wave propagates at the group velocity c_g :

$$c_g = \frac{\partial \omega}{\partial k_x} = c_1 \sqrt{1 - k_z^2 \left(\frac{c_1}{\omega} \right)^2}. \quad (6)$$

3. Visualization of mode patterns in a fluid waveguide

The experimental system is illustrated in Fig. 2. The stroboscopic light source delivers collimated light pulses at a repetition rate of about 100 flashes per second to the water tank consisting of optical glass. A small PZT ceramic transducer, which repeatedly generates tone-burst ultrasonic waves of 1.15 MHz with about a 20 μ s duration, is immersed in water and excites the symmetric or asymmetric modes in the waveguide. To excite the target mode, the thickness of the waveguide and the angle of ultrasonic incidence must satisfy Eqs. (3) or (4). The stroboscopic light is triggered synchronously with ultrasonic wave excitation after a certain delay

*e-mail: ken-yamamoto@kobayasi-riken.or.jp

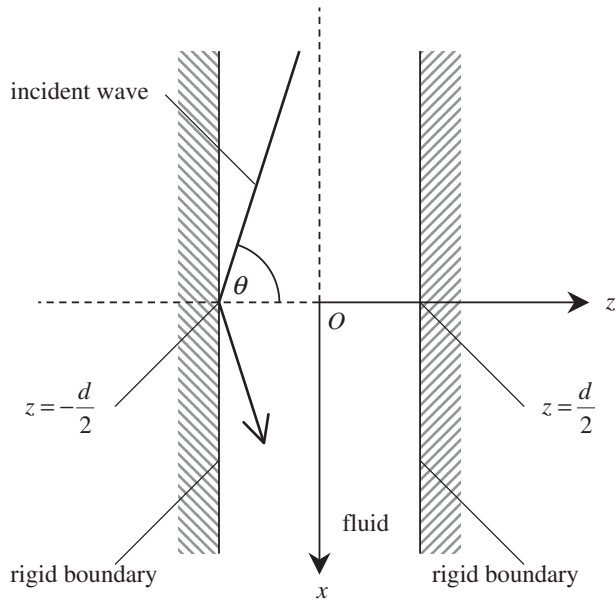


Fig. 1 Coordinate system for a fluid plate of thickness d with rigid boundaries.

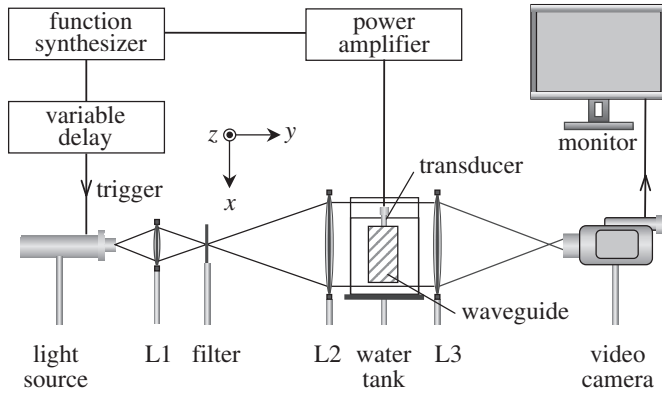


Fig. 2 Experimental setup for the visualization system of the stroboscopic Fresnel diffraction technique.

time. In this experiment, slight defocusing of the camera lens makes it possible to visualize ultrasonic waves in water through Fresnel refraction [2]. Figures 3(a) and (b) show the visualized ultrasonic waves traveling through the waveguide from top to bottom when frequency $f = 1.15$ MHz and the thickness of the fluid plate $d = 3.3$ mm. Figure 3(a) was taken $15.0\mu\text{s}$ after ultrasonic wave excitation and shows the mode of $m = 2$ propagating in the upper portion of the waveguide. Both the $m = 2$ and $m = 5$ modes travel through the waveguide at the same time, as shown in Fig. 3(b) taken $39.0\mu\text{s}$ after Fig. 3(a). We can see that the $m = 2$ mode has bright spots symmetrically arranged with respect to the median plane of the fluid plate, with a separation of one wavelength between bright spots. On the other hand, the $m = 5$ mode has bright spots located asymmetrically in the upper portion of the waveguide, as shown in Fig. 3(b). In this experiment, the thickness of the waveguide and the angle of ultrasonic incidence was adjusted to excite the $m = 2$ mode, however, the higher $m = 5$ mode was simultaneously generated at the

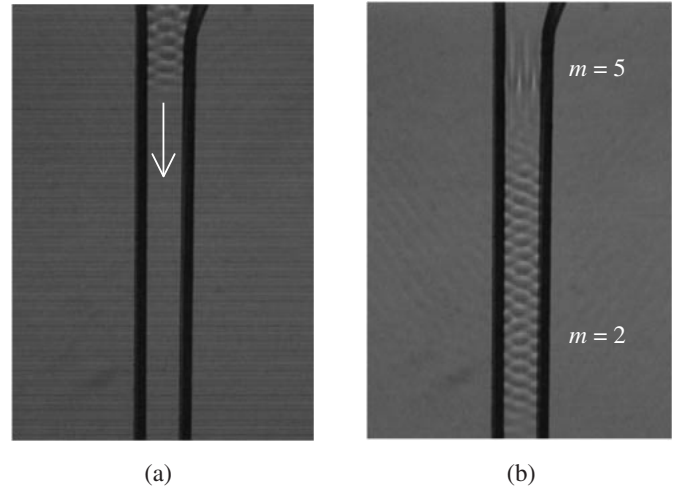


Fig. 3 Visualized patterns of the symmetrical $m = 2$ mode and the asymmetrical $m = 5$ mode traveling through the fluid waveguide with metal boundaries. (a) The image taken $15.0\mu\text{s}$ after ultrasonic excitation. (b) The image taken $39.0\mu\text{s}$ after (a).

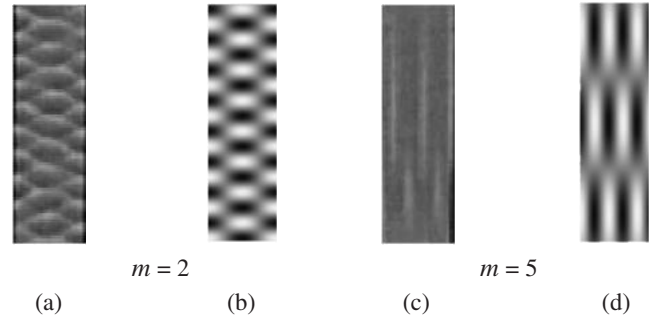


Fig. 4 Enlarged visualized patterns (a, c) and theoretical patterns (b, d) for the symmetric $m = 2$ and asymmetric $m = 5$ modes when the product of frequency and the plate thickness fd is $3.795 [10^3 \text{ Hz}\cdot\text{m}]$.

approach of the waveguide where metal plates are not parallel to each other. Furthermore, each mode travels at a different speed through the waveguide, as shown in Figs. 3(a) and (b). This means that the lower mode has greater group velocity than the higher mode when fd [Hz·m] is the same. Figure 4 shows the enlarged views of each mode (a, c) and their numerical calculations (b, d). Here, experimental fields do not have the perfect sinusoidal pattern because the brightness of the visualized image is not completely proportional to the sound pressure in this technique. We can easily obtain wavelength λ and the phase velocity $c_p (= f \times \lambda)$ from the visualized pattern. Figures 5 to 7 show typical visualized patterns (a, c) and its numerical calculations (b, d) in the waveguide for various fd . Figure 5 shows visualized patterns of the same mode as shown in Fig. 4, but the thickness of the fluid is slightly wider, $d = 3.7$ mm. Upon comparing Figs. 4(c) and 5(c) or Figs. 4(d) and 5(d), we can see clearly that the phase velocity becomes slower when fd becomes larger. Visualized higher order symmetric and asymmetric mode patterns of guided waves and their calculation patterns

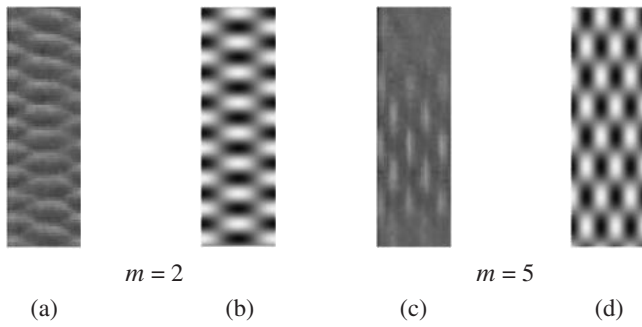


Fig. 5 Visualized patterns (a, c) and theoretical patterns (b, d) for the symmetric $m = 2$ and asymmetric $m = 5$ modes when fd is $4.255 [10^3 \text{ Hz}\cdot\text{m}]$.

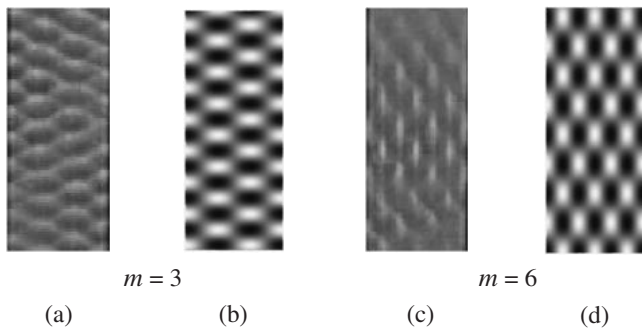


Fig. 6 Visualized patterns (a, c) and theoretical patterns (b, d) for the asymmetric $m = 3$ and symmetric $m = 6$ modes when fd is $5.175 [10^3 \text{ Hz}\cdot\text{m}]$.

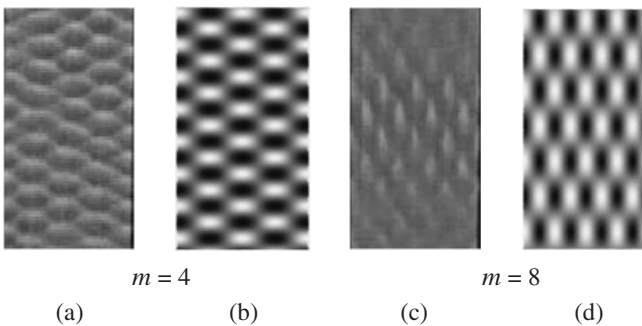


Fig. 7 Visualized patterns (a, d) and theoretical patterns (b, d) for the symmetric $m = 4$ and $m = 8$ modes when fd is $6.555 [10^3 \text{ Hz}\cdot\text{m}]$.

are shown for $fd = 5.175 [10^3 \text{ Hz}\cdot\text{m}]$ in Fig. 6 and $fd = 6.555 [10^3 \text{ Hz}\cdot\text{m}]$ in Fig. 7.

4. Phase velocity and group velocity of guided wave

Visualization of propagation modes provides a means of measuring the wavelength and phase velocity of several guided waves. The experimental results of the phase velocity c_p versus the value of fd are shown in Fig. 8. The solid curves indicate the theoretical curves calculated from Eq. (5). The experimental results (open circles) are in good agreement with

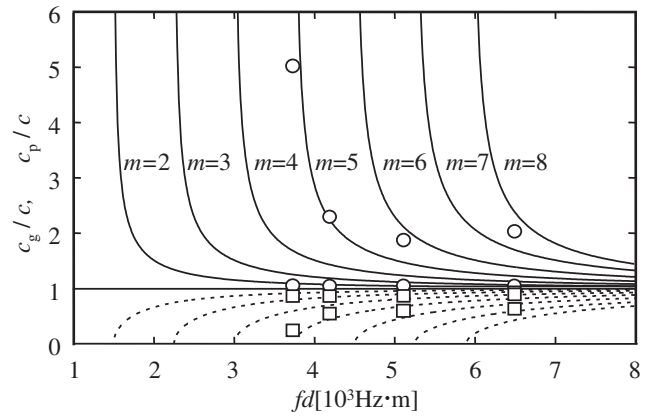


Fig. 8 Phase velocities and group velocities of guided modes in the fluid plate with metal boundaries as a function of fd . Solid lines: theoretically calculated phase velocity, open circles: experimental data of phase velocity, dashed lines: theoretically calculated group velocity, open squares: experimental data of group velocity.

the theoretical curves. In addition, we can estimate the group velocities $c_g (= \Delta l / \Delta t)$ for each mode in order to compare two still images observed with different delay times. As indicated above, the stroboscopic light is triggered by the ultrasonic wave excitation after a certain delay time, and the fixed delay time allows to obtain a still image of the ultrasonic wave. The Δt is equivalent to the time difference between the two images, and Δl is easily estimated by measuring the pattern positions in each image. The group velocity of the $m = 2$ mode for $fd = 3.795 [10^3 \text{ Hz}\cdot\text{m}]$ was estimated to enable the comparison between the images in Figs. 3(a) and (b). The dashed lines show the theoretical lines calculated using Eq. (6) and open squares indicate experimental results. The observed group velocities were consistent with the predicted values within the range of experimental error.

5. Conclusions

Using the stroboscopic Fresnel technique, several symmetric and asymmetric mode patterns of guided waves in a fluid waveguide with solid boundaries were visualized. Observed results qualitatively coincided with those of the calculated patterns. It was also shown that the phase velocity and the group velocity can be obtained from the visualized mode patterns.

The visualization technique is not only a tutorial method but also a powerful tool for clarifying acoustic fields such as the mode pattern in the waveguide and for obtaining the dispersion relationship and the group velocity of guided waves.

References

- [1] M. R. Redwood, *Mechanical Waveguides* (Pergamon Press, Oxford, 1960), pp. 57–99.
- [2] K. Patorski, "Optical testing of ultrasonic phase gratings using Fresnel diffraction method," *Ultrasonics*, **19**, 169–172 (1981).