# PAPER

# Estimation of the area effect of sound absorbent surfaces by using a boundary integral equation

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**Abstract:** A method of predicting the area effect of an absorbent surface with finite dimensions by using a boundary integral equation is proposed and the reason why the area effect occurs is shown. In order to check the effectiveness of the method, some experiments are carried out in a reverberation room. These results are in good agreement with those obtained numerically from the proposed method.

**Keywords:** Sound absorbent surface, Area effect, Boundary integral equation, BEM

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## 1.   INTRODUCTION

It is known that the random incidence sound absorption coefficient increases when the dimensions of an absorbent surface become as small as the dimension of wavelength. This phenomenon is usually referred to as area effect, and it becomes evident when the absorption coefficient is large. Some researchers explain that the reason why this phenomenon occurs is due to edge effect, which includes diffraction or scattering at the edge of the absorbent surface [1,2]. It seems to be difficult, however, to say that this phenomenon is explainable by using edge length of the absorber. Though there exist theoretical analyses for an infinitely long strip of absorbent surface [3,4], they have a restricted application in practice. In this paper a method of estimating the area effect by using a boundary integral equation is proposed.

## 2.   THEORETICAL CONSIDERATION

Let us consider an infinitely large rigid flat, surface a part of which is an absorbent surface. Also, let semi-infinite space $\Omega_0$ bounded by the infinitely large semi sphere $\Sigma$, the rigid surface $S$ and the absorbent surface $F$ include a point source $P_s$ and a receiving point $P$, and $P_i$ be the image point of $P$ with respect to the surfaces $S$ and $F$ as shown in Fig. 1. In order to derive an integral formula, let us use

$$G(P, Q) = \frac{\exp(ikr)}{4\pi r} + \frac{\exp(ikr_i)}{4\pi r_i}, \quad (1)$$

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as a fundamental solution in which the image point $P_i$ is taken into account. Here, $r = \overline{PQ}$ and $r_i = \overline{P_iQ}$. The time factor $\exp(-i\omega t)$ is omitted throughout this paper. We apply here Green's theorem to the region $\Omega_0 - \sigma_s - \sigma$ where $\sigma_s$ and $\sigma$ are small spheres of centers $P_s$ and $P$ respectively, with radius $\epsilon$. Taking into consideration 1) the normal component of particle velocity vanishes on $S$, 2) $\partial G/\partial n = 0$ on $S$ and $F$, and 3) Sommerfeld's radiation condition [5], we can obtain

$$\Phi(P) = \Phi_D(P) + \Phi_D(P_i)$$
$$- \frac{1}{2\pi} \iint_F \frac{\partial \Phi(Q)}{\partial n} \frac{\exp(ikr)}{r} dS, \quad (P \in \Omega_0, S, F), \quad (2)$$

where $\Phi(P)$ denotes velocity potential at $P$ and $\Phi_D(P)$ the direct wave at $P$. Eq. (2) is valid when $P$ is located on $S$ or $F$ (i.e., $P = P_i$), since

$$\lim_{\epsilon \to 0} \iint_{\partial\sigma/2} dS = -\Phi(P) \quad (P \in S, F) \quad [6].$$

If the absorbent surface $F$ is assumed to be locally reactive, then we have

$$\frac{\partial \Phi}{\partial n} = -ikA\Phi, \qquad \text{on } F, \quad (3)$$

where $A$ denotes the specific admittance. Hence, Eq. (2) reduces to

$$\Phi(P) = \Phi_D(P) + \Phi_D(P_i)$$
$$+ \frac{ikA}{2\pi} \iint_F \Phi(Q) \frac{\exp(ikr)}{r} dS, \quad (P \in \Omega_0, S, F). \quad (4)$$
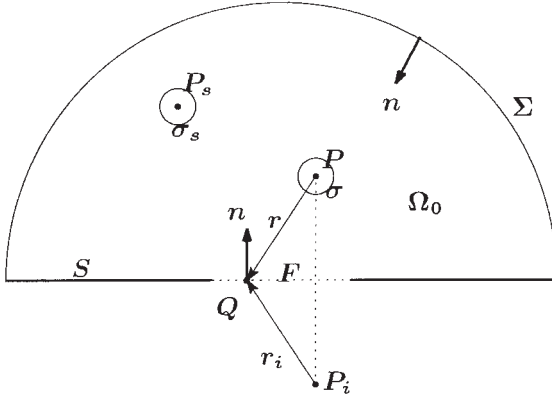
**Fig. 1** Definition of symbols: $\Omega_0$ semi-infinite space bounded by rigid surface $S$, absorbent surface $F$, and infinitely large semi sphere $\Sigma$; $n$ normal vector; $P_s$ point source; $P$ receiving point; $P_i$ the image of $P$ with respect to $S$ and $F$; $\sigma_s$ and $\sigma$ small spheres of centers $P_s$ and $P$ respectively.

When $P$ is located on $F$, Eq. (4) is the boundary integral equation and $\boldsymbol{\Phi}$ is an unknown surface function. Though Thomasson [7] solved the above equation to obtain the pressure on the absorbent surface by using the variational technique, the boundary element technique is more suitable for solving numerically.

The sound pressure $p$ and the particle velocity $v$ are connected with the velocity potential $\boldsymbol{\Phi}$ by

$$\begin{cases} p = -i\omega\rho\boldsymbol{\Phi} \\ v = -\dfrac{\partial\boldsymbol{\Phi}}{\partial n} \end{cases}, \qquad (5)$$

where $\rho$ is air density, and the incidence sound energy $I_i$ and absorbed sound energy $I_a$ can be obtained from the following equations

$$I_i = \iint_F \frac{1}{4}(p_i v_i^* + p_i^* v_i)dS, \qquad (6)$$

$$I_a = \iint_F \frac{1}{4}(p_a v_a^* + p_a^* v_a)dS, \qquad (7)$$

where $p_i$ and $v_i$ denote the incidence sound pressure and the particle velocity, and $p_a$ and $v_a$ the sound pressure and the particle velocity on the absorbent surface, respectively [8]. The symbols with $*$ represent complex conjugates.

As a numerical example of this method, sound intensity distribution near a locally reacting surface is calculated. The surface has dimensions of $2\,\mathrm{m} \times 2\,\mathrm{m}$, and normal incidence pressure reflection coefficient and phase shift are 0.2 and $\pi/12$ [rad] respectively. Figure 2 shows sound energy flow and pressure contour when a plane wave (63 Hz) is incident vertically on the absorbent surface.

In the vicinity of the absorber's edge sound energy flow from adjacent space to the absorber can be seen, the area effect thus being induced.
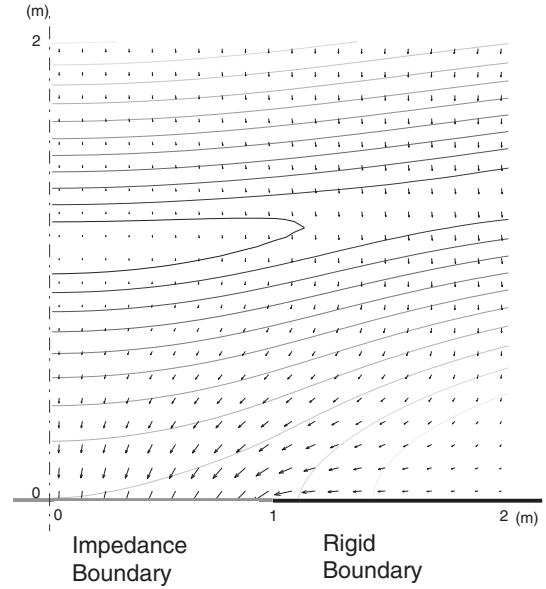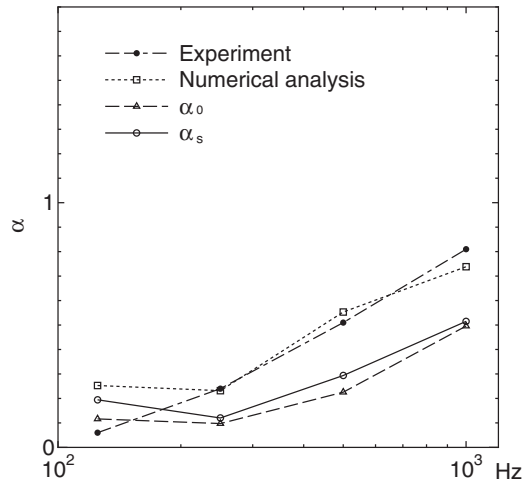


**Fig. 2** Sound intensity near a locally reacting surface ($2\,\mathrm{m} \times 2\,\mathrm{m}$) in case of normal incidence of a plane wave, 63 Hz.

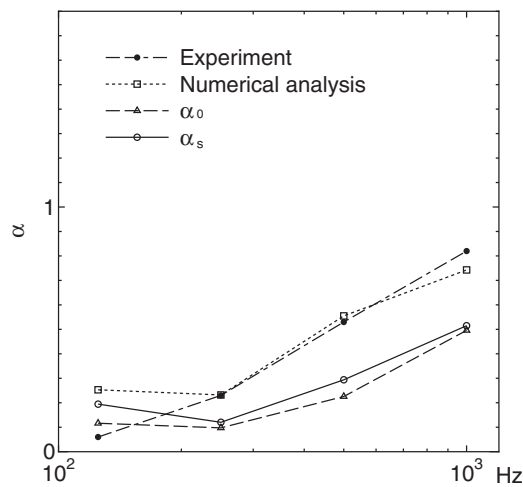## 3. MEASUREMENT OF REVERBERATION ABSORPTION COEFFICIENT

In order to verify the effectiveness of the method introduced in the previous section, reverberation absorption coefficients of glass wool patches (25 mm thickness, 32 kg/m$^3$, no air space) are measured in the reverberation room in the General Building Research Corporation (GBRC). The reverberation room used for the measurements has a volume of 317.4 m$^3$. Six curved polyvinyl chloride panels with dimensions of $915 \times 1,830\,\mathrm{mm}^2$ are hung in the room in order to diffuse sound.

Figures 3(a) and (b) show the results of the measurements for 25 mm thick glass wool with dimensions of $3.5\,\mathrm{m} \times 3.5\,\mathrm{m}$ and $3.0\,\mathrm{m} \times 4.0\,\mathrm{m}$ respectively. In these figures dashed dotted lines denote experimental results for 1/3 octave band noise, dotted lines the values obtained from the numerical calculations of the boundary integral equation, dashed lines normal incidence absorption coefficients $\alpha_0$ and solid lines statistical random incidence absorption coefficients $\alpha_s$ for infinitely large absorbent surface. Normal impedances used for obtaining the above calculated absorption coefficients were measured by using the tube method (B&K 4206 Two-microphone impedance measurement tube) (see Table 1).

To simulate random incidence noise, the numerical calculations were carried out for $m$ plane waves that are incident on the absorbent surface from all directions at regular solid angle ($m = 213$ in this calculation). The energy both of the incident wave and of the absorbed one are calculated for each condition and summed up separately. The random incidence absorption coefficient $\alpha$ can
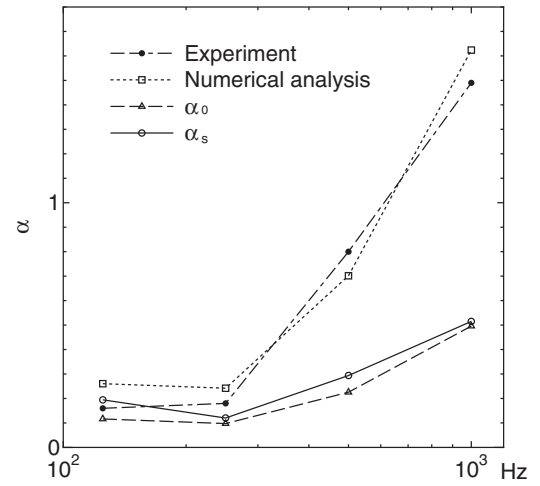
(a)



(b)

**Fig. 3** Sound absorption coefficients of glass wool patches (25 mm thickness, 32 kg/m³, no air space) with dimensions of (a) 3.5 m × 3.5 m and (b) 3.0 m × 4.0 m.
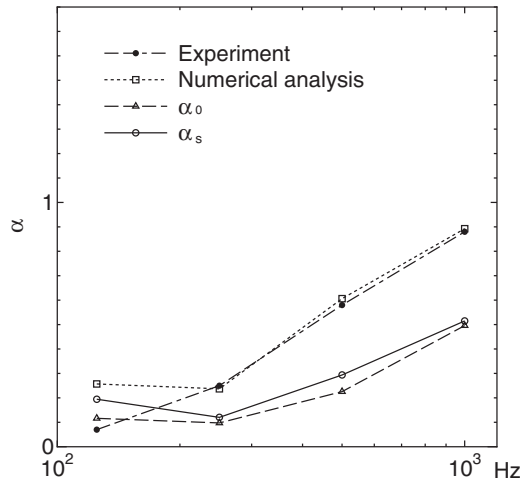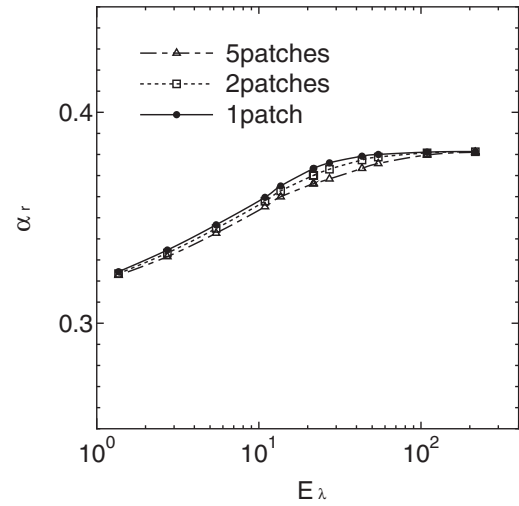


(a)



(b)

**Fig. 4** Sound absorption coefficient of glass wool patches (25 mm thickness, 32 kg/m³, no air space) with dimensions of (a) 0.5 m × 0.5 m and (b) 1.0 m × 1.0 m.

**Table 1** Measured values of absorption coefficient $\alpha_0$, sound pressure reflection coefficient $r_p$, phase shift $\delta$ and impedance ratio $z$ at normal incidence for glass wool with density of 32 kg/m³, 25 mm thickness and no air space.

| Freq. | $\alpha_0$ | $r_p$ | $\delta$ (deg) | $\Re(z)$ | $\Im(z)$ |
|---|---|---|---|---|---|
| 125 Hz | 0.1164 | 0.94 | 7.393 | 6.55742 | 12.6577 |
| 250 Hz | 0.0975 | 0.95 | 15.0864 | 1.33294 | 7.28891 |
| 500 Hz | 0.2256 | 0.88 | 31.2328 | 0.78677 | 3.41015 |
| 1,000 Hz | 0.4959 | 0.71 | 57.6460 | 0.66382 | 1.61481 |

be obtained by

$$\alpha = \sum_{j=1}^{m} (I_a)_j \Bigg/ \sum_{j=1}^{m} (I_i)_j. \qquad (8)$$

Since the averaged absorption coefficients within 1/3 octave band have approximately same values as those for the center frequencies of the bands, the latter values are used in Fig. 3. Figure 3 shows that the theoretical values are in good agreement with the experimental values, the effectiveness is thus being verified.
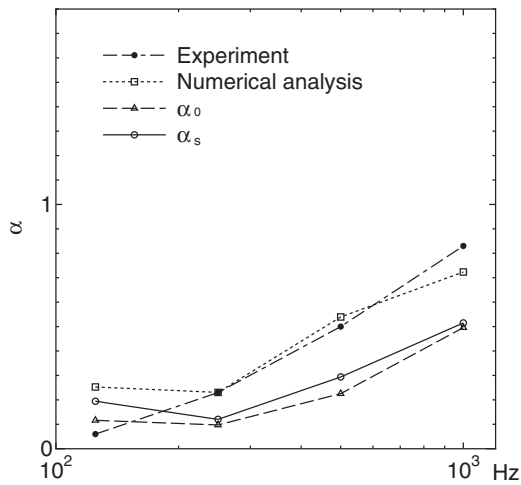
Figures 4 and 5 show the results for four specimens with dimensions of 0.5 m × 0.5 m, 1.0 m × 1.0 m, 2.0 m × 2.0 m and 4.0 m × 4.0 m. We can see that the area effect becomes evident when the absorption coefficient is large and the dimensions of the specimens are small, and also that the theoretical values are in good agreement with the experimental values. Since the specimen with dimensions of 0.5 m × 0.5 m has little absorption, measurement error becomes large. Therefore, four specimens were set sufficiently apart from each other in the reverberation room in the measurement.
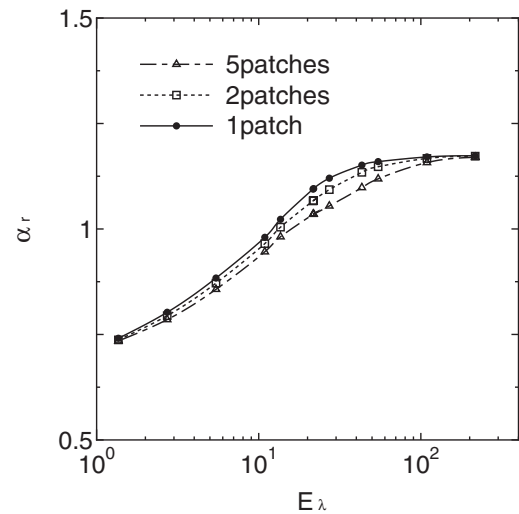
(a)



(b)

**Fig. 5** Sound absorption coefficient of glass wool patches (25 mm thickness, 32 kg/m$^3$, no air space) with dimensions of (a) 2.0 m × 2.0 m and (b) 4.0 m × 4.0 m.
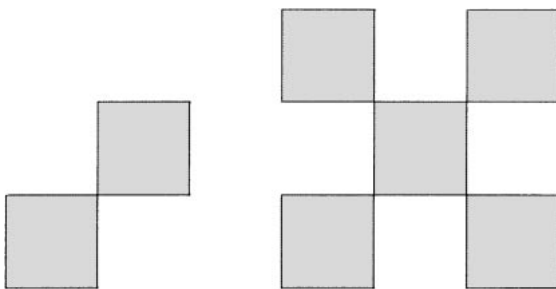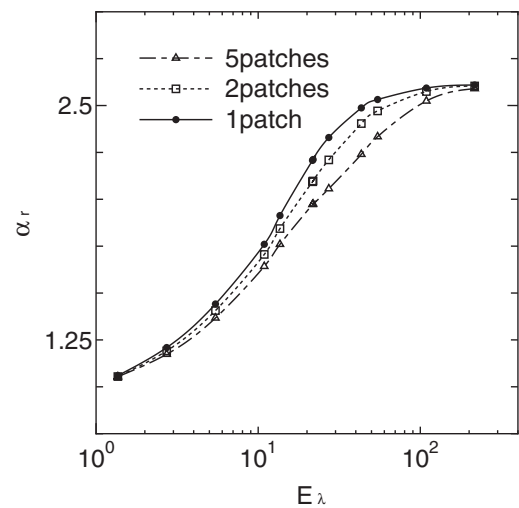


**Fig. 6** Checkered pattern of absorbent patches.

## 4.  INTERACTION BETWEEN ABSORBENT SURFACES

Since, as mentioned above, the area effect occurs due to energy inflow from neighboring area of the absorbent surface, it is considered that the effect decreases when



(a)



(b)



(c)

**Fig. 7** Sound absorption coefficient of square absorbent patches with checkered pattern, (a) $\alpha_0 = 0.2$, (b) $\alpha_0 = 0.5$, (c) $\alpha_0 = 0.8$.

some absorbent patches are collocated closely. In this section, the area effect of the absorbent patches collocated checkered pattern is examined.

The author and his coworker have already shown that the area effect of a rectangular absorbent patch does not depend on its aspect ratio ($a : b$), but it depends on $\alpha_0$ and the following parameter $E_\lambda$

$$E_\lambda = 2\left(\frac{\lambda}{a} + \frac{\lambda}{b}\right), \qquad (9)$$

where $\lambda$ denotes wavelength.

Figures 7(a), (b), (c) show that random incidence absorption coefficients calculated numerically for a square patch, two patches and five patches collocated as shown in Fig. 6. We can see that the area effect of absorbent patches collocated like these (checkered pattern) does not much different from that of simple one.

## 5. CONCLUSIONS

The area effect of a sound absorbent surface with finite dimensions was analyzed by using a boundary integral equation, and the validity of the method was examined through experiments. These numerical results seem to show that the method is appropriate for effective estimation of the area effect. Through the numerical calculations it is shown that the area effect of rectangular absorbent patches collocated checkered pattern can be approximately estimated by a simple absorbent patch.

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