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Evaluation of quality of concrete by linear predictive coefficient method with multi-reflected elastic wave

Liang Tao[1], Yahiro Mori[2] and Seiichi Motooka[1,*]

[1]*Department of Electrical, Electronics and Computer Engineering, Faculty of Engineering, Chiba Institute of Technology, 2–17–1, Tsudanuma, Narashino, 275–0016 Japan*
[2]*Department of Architectural and Civil Engineering, Faculty of Engineering, Chiba Institute of Technology, 2–17–1, Tsudanuma, Narashino, 275–0016 Japan*

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1. Introduction

Concrete is a popular architecture material which is used in most of the modern buildings, tunnels, and some of roads, etc. Because the strength of the concrete will decrease extend a long term, the inspection and repair are extremely important for the safety of the construction. However, the inspection methods generally employed are "watching" the outlook and "listening" the hammer-strike response of the concrete up to now. Problems of instability and low accuracy can be pointed out. The technique of inspecting the quality of the concrete easily and accurately is expected.

In this paper, an approach of inspection method by the quality factor of the resonant peak of elastic wave propagating inside the concrete is proposed. An electromagnetic induced (EMI) sound source is employed to radiate impulsive elastic wave into the concrete. The elastic wave (direct wave and multi-reflected waves) propagating inside the concrete is received by a piezoelectric transducer (PZT) placed on the surface. However, if the concrete varies only the strength but has no markedly crack or unevenness inside, the time domain signals will show little difference. Here, considering that the attenuation properties of the elastic wave should be related with the strength of the propagating medium, a method of distinguishing the difference of the strength of the concrete by using the quality factor of the spectra of the elastic wave is contrived. In order to acquire stable quality factor, the linear prediction coefficient (LPC) and resonant analysis methods are employed for spectrum derivation and quality factor calculation, respectively.

Three kinds of concretes with same dimension but different strength are measured. The quality factors of received elastic wave, derived by LPC analysis, show a same tendency with the strengths of different concretes.

2. Method of inspection

2.1. Measuring system

Figure 1 shows the system for measurement. The EMI sound source consists of mainly a coil cemented and an aluminum diaphragm under it. Its structure and the principle of sound transmitting are similar as that of the sound source we used for the underground imaging [1]. The aluminum diaphragm is driven by the impulsive electromagnetic repulsion force brought forth by the coil while the condenser discharges suddenly, and vibrates as a sound source. A 0.1 μF condenser is employed for the main frequency of the elastic wave pulse irradiated from the EMI sound source can match the resonant frequency of PZT receiver, 100 kHz. And the charging energy is 1.0 J.

2.2. Signal processing

Because the attenuation property of the elastic wave should be related with the strength of the propagating medium, a method of inspecting the strength of the concrete by using the quality factor of the spectra of the elastic wave is contrived. Moreover, the linear prediction coefficient (LPC) analysis method is employed for derivation of stable quality factor.

The linear prediction can be described simply by the following equation. The current datum x_n is predicted by the linear synthesis of the former data $x_{n-i} (i = 1, 2, \dots, p)$

$$x'_n = x_n - e_n = -\sum_{i=1}^{p} a_i x_{n-i} \quad (n = p+1, p+2, \cdots, N), \quad (1)$$

where x'_n and x_n denote the prediction and the real value respectively, e_n is the prediction error, a_i is the LPC, p is the order of prediction, and N is the data length of the whole signal. Regarding a known signal x_m $(m = 1, 2, \dots, N)$, its LPC a_i can be derived by the least square condition of the prediction error e_n. The result is

$$\begin{vmatrix} R_0 & R_1 & \cdots & R_{p-1} \\ R_1 & R_0 & \cdots & R_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p-1} & R_{p-2} & \cdots & R_0 \end{vmatrix} \begin{vmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{vmatrix} = \begin{vmatrix} R_1 \\ R_2 \\ \vdots \\ R_p \end{vmatrix}, \quad (2)$$

where

$$R_i = \frac{1}{N-p} \sum_{n=p+1}^{N} x_n x_{n-i} \quad (i = 1, 2, \cdots, p) \quad (3)$$

is the self-correlation of the signal x_m. Then, the spectrum of the signal in Z-transform can be shown by its LPC as

*e-mail: motooka@es.it-chiba.ac.jp

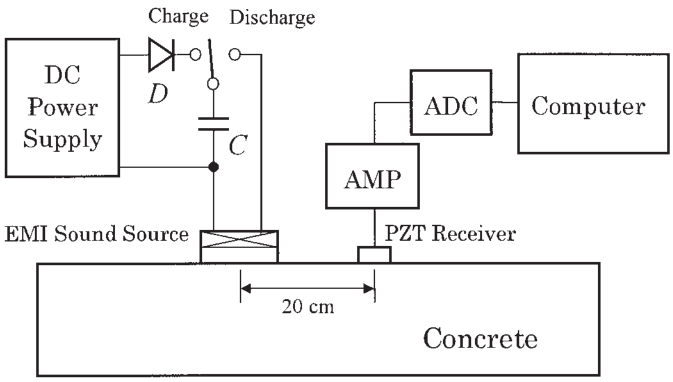

Fig. 1 Measuring system.

$$X(z) = \frac{1}{1 + \sum_{k=1}^{p} a_k \cdot z^{-k}}, \tag{4}$$

where $z = e^{j\omega\tau}$ and τ is the sampling interval.

Equation (4) shows an all-poles model, where $X(z)$ should be a pole while z is a root of the denominator. Assume b_n and $b_n{}^*$ $(n = 1, 2, \ldots, p/2)$ are the $p/2$ (p is even) pairs of complex conjugate roots, Eq. (4) can be rewritten as

$$X(z) = \frac{1}{(1 - b_1 Z^{-1})(1 - b_1^* Z^{-1}) \cdot \cdot (1 - b_n Z^{-1})(1 - b_n^* Z^{-1}) \cdot \cdot (1 - b_{\frac{p}{2}} Z^{-1})(1 - b_{\frac{p}{2}}^* Z^{-1})}. \tag{5}$$

The n-th factor of the denominator $Y_n(z)$ can be represents as following by substituting $b_n = c_n e^{j\theta_n}$ (c_n and θ_n are the modulus and the argument of b_n, respectively) and $z = e^{j\omega\tau}$

$$\begin{aligned} Y_n(z) &= 1 - b_n Z^{-1} = 1 - c_n e^{j\theta_n} e^{-j\omega\tau} \\ &= 1 - c_n e^{j(\theta_n - \omega\tau)} \quad (c_n \approx 1). \end{aligned} \tag{6}$$

Then the n-th resonant frequency ω_{0n} and its band width $\Delta\omega_n$ can be drawn from the following conditions, respectively.

$$\arg\{Y(z)\}|_{\omega=\omega_0} = 0; \quad \arg\{Y(z)\}|_{\omega=\omega_0+\Delta\omega} = \frac{\pi}{4}. \tag{7}$$

That gives

$$\omega_{0n} = \frac{\theta_n}{\tau}; \quad \Delta\omega_n = \frac{1 - c_n}{\tau}. \tag{8}$$

Finally, the quality factor of the n-th pole is derived as

$$Q_n = \frac{\omega_{0n}}{\Delta\omega_n} = \frac{\theta_n}{1 - c_n}. \tag{9}$$

In order to obtain a spectrum with characteristically resonant peaks of concrete, the order of LPC employed in this study is 30 [2], i.e., $p/2 = 15$ poles with 15 quality factors can be calculated theoretically. Here, only that of the pole at the resonant frequency of PZT receiver is employed to evaluate the strength of the concrete.

3. Measurement results

Three kinds of concrete specimen, with same shape and dimension (10 cm × 10 cm × 40 cm), manufactured from different water contents are employed. The velocity of longitudinal elastic wave in the concrete specimen is 4000 m/s. Because the strength of a concrete block is proportional to the cement water ratio (CWR) while manufacturing [3], in this paper, these three concrete specimen are employed to study the relationship of the strength and the quality factor. The water contents and the CWRs of three specimens are concluded in Table 1.

Table 1 Cement-water ratio (CWR) of concrete specimens.

Specimen	No. 1	No. 2	No. 3
Water content	50%	60%	70%
CWR	2.00	1.67	1.43

Figure 2 shows example of the signals received by the PZT placed with 20 cm interval from the EMI sound source. The signals are recorded from the start of the discharging (driving) process. Though a short electromagnetic induction wave is coupled into the receiving signal at the beginning, it is shown that the elastic wave start from 50 μs are received separately from it. The time is calculated by the propagating distance of sound source with PZT and the sound velocity in concrete. Similarly, it can easily be calculated that the first reflect wave should start from 71 μs. Because the pulse elastic wave propagates in the concrete via multi-paths and all the direct wave and multi-reflected waves are received with inference between each other, the signals show complicated waveforms, and the rule in time domain signals corresponding to different concrete specimen can hardly be drawn, as shown in the figure.

The elastic wave (direct wave and multi-reflected waves) from 50 µs is employed for the LPC analysis described in section 2. As the interval between the start times of direct wave and multi-reflected waves is rather small, it is apparently that the signals used for analysis consists of mainly the multi-reflected waves. The quality factors of the resonant peak at about 100 kHz corresponding to each concrete specimen are derived, and the result is shown in Fig. 3. Though it is hard to discriminate the difference with time domain signals shown in Fig. 2, the result of quality factors show clear difference, and the tendency agree well with that of the strength of the concrete specimen. Moreover, similar results are acquired for measurements with varies driving frequencies of EMI sound source and varies alignment positions of sound source and PZT receiver.

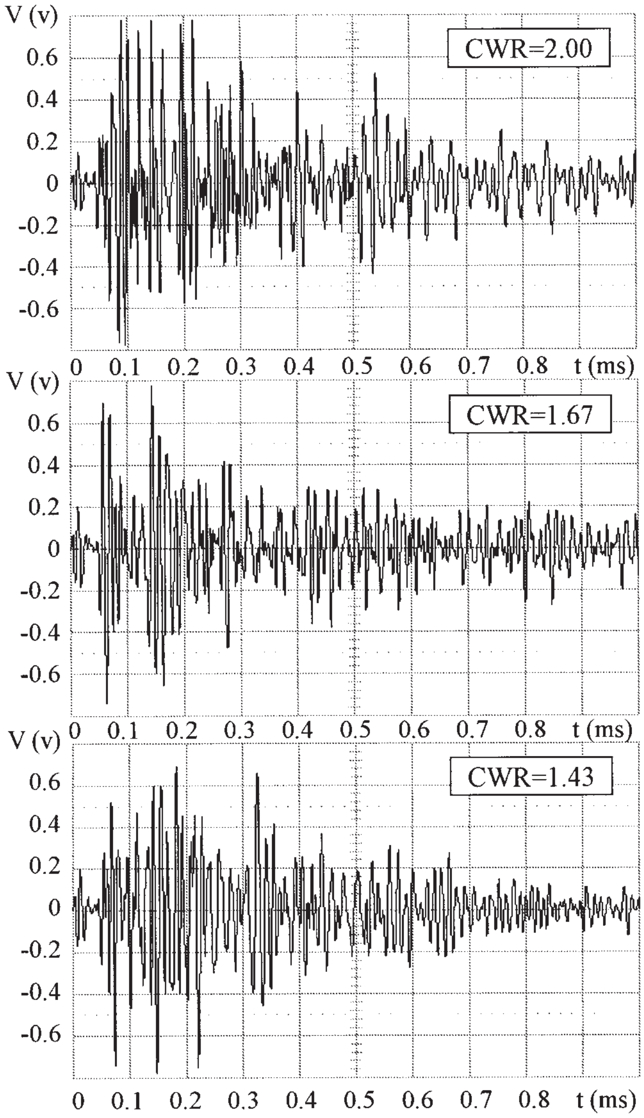

Fig. 2 Examples of received signals from different concretes.

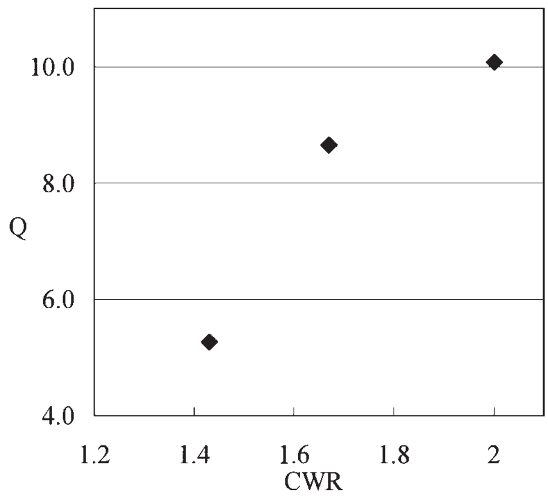

Fig. 3 Results of the Quality factor vs. CWR.

4. Conclusion

Method of inspecting the strength of the concrete by the quality factor of the resonant peak of elastic wave is proposed. Three kinds of concrete specimen with different strengths are measured, and the quality factors derived by linear prediction coefficient analysis show a same tendency with the strengths. Further study on more concrete specimen and discussion on the optimal order of LPC are expected.

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