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PAPER

A study on the focusing property of time reversal waves in shallow water

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Abstract: The focusing property of time reversal waves (phase conjugate waves) in shallow water or the deep sea has been discussed based on simulations of the normal mode method. In this study, the focusing property of time reversal waves was verified through tank experiments and it was concluded that time reversal waves enable acoustic waves to be converged to the focus. Further, the simulation results qualitatively concurred with the experimental results. This study also investigated how the configuration of the elements of the Time Reversal Mirror (TRM) array affects the focusing property, and the following matters were clarified: The signal received at the focus is not disrupted when the number of elements in the TRM array is reduced. Even if the TRM array is tilted or the heights of its elements are steadily shifted, the signal received at the focus is not disrupted. However, if the tilting or the height-shifting is not stationary, the time reversal waves lose their focusing property. Even a horizontal TRM array can generate time reversal waves if it is of adequate length.

Keywords: Underwater acoustics, Phase conjugate, Time reversal, Tank experiment

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1. INTRODUCTION

In recent times, extensive research has been conducted on time reversal waves (phase conjugate waves) in ocean acoustics [1–10]. In ocean acoustics, time reversal waves are used as follows: Acoustic waves transmitted from a source are received at the time reversal mirror (TRM) array; these signals are reversed in the time domain (which corresponds to phase conjugate signals in the frequency domain); these time-reversed signals are retransmitted from the TRM array; and they converge to the focus where the original source existed, even in unknown and inhomogeneous media.

Conventionally, the beam-forming method is used to focus acoustic waves to a specific point. This method, however, limits the direction of propagation of waves and the wave energy diverges at long distances. As time reversal waves converge to a "point" at any range, their focusing property is quite different from waves focused using the beam forming method. In other words, using time reversal waves is equivalent to collecting the reflected and refracted waves that are removed as unnecessary signals in the beam forming method.

In the oceanic environment, in which reflected and refracted waves take many paths, this property of time reversal waves can be conveniently used in acoustic communication, sonar, acoustic positioning, and other underwater acoustic technologies.

The focusing properties of time reversal waves in shallow water and the deep sea have been discussed based on simulations and only in the frequency domain [9,10]. In this study, tank experiments were conducted to study the properties of these waves in shallow water. These experiments confirmed that time reversal waves converge to the focus in shallow water, and the results obtained were compared with the simulation results in the time domain. In these experiments and simulations, the focusing property is discussed by treating a tone burst pulse, because it is regarded as a basic signal containing one frequency component and it is taken into account that time reversal waves in future are applied for underwater acoustic communication with frequency shift keying (FSK) or phase shift keying (PSK) modulations in which the signal is composed of continuous tone burst pulses.

Moreover, how the configuration of the TRM array elements affects the focusing property of time reversal waves was also investigated.

2. SIMULATION

2.1. Propagation Model and Simulation Method

The Pekeris solution of the normal mode method was used as the simulation method to study acoustic propaga-

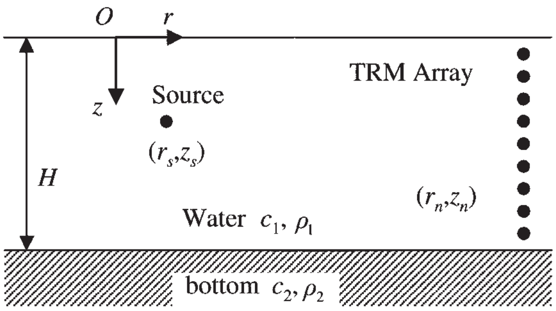

Fig. 1 Pekeris model and TRM array configuration.

tion in shallow water.

Shallow water was modeled in the Pekeris model as shown in Fig. 1. In this model, water depth H, sound velocity and density in water and at the sub-bottom, c_1, ρ_1, c_2, and ρ_2, respectively are all constant. The thickness of the sub-bottom is considered infinite and the transversal waves at the sub-bottom are ignored. The cylindrical coordinate is defined as shown in Fig. 1, in which the horizontal range is expressed as r and the depth is z.

Under these conditions, when acoustic waves are transmitted from a source at $\boldsymbol{r}_s = (r_s, z_s)$, the transfer function at an arbitrary receiving point $\boldsymbol{r} = (r, z)$, at an angular frequency ω is as follows [11]:

$$G_\omega(\boldsymbol{r}, \boldsymbol{r}_s) = \frac{2}{H}\sqrt{\frac{2}{\pi|r - r_s|}}\sum_{m=1}^{\infty}\Bigg[A(k_m) \times \sin(\beta_1 z_s)\sin(\beta_2 z)\exp j\left(-k_m(r - r_s) - \frac{\pi}{4}\right)\Bigg],$$

$$A(k_m) \equiv \frac{1}{\sqrt{k_m}} \times \frac{\beta_1 H}{\beta_1 H - \sin(\beta_1 H)\cos(\beta_1 H) - b^2\sin^2(\beta_1 H)\tan(\beta_1 H)},$$

$$\beta_1 = \sqrt{\frac{\omega^2}{{c_1}^2} - {k_m}^2}, \quad \beta_2 = -j\sqrt{{k_m}^2 - \frac{\omega^2}{{c_2}^2}}, \quad b = \frac{\rho_1}{\rho_2}. \quad (1)$$

Here, k_m is the horizontal wave number of the mth mode, and β_1 and β_2 are the vertical wave numbers in water and at the sub-bottom, respectively. Although propagation and attenuation modes are generated in this model, the latter are not considered in this paper because they do not propagate to long distances. The value of k_m is obtained as a solution of the following characteristic equation:

$$\beta_1\cos(\beta_1 H) + jb\beta_2\sin(\beta_1 H) = 0 \quad (2)$$

2.2. Acoustic Field by TRM Array

A TRM array is placed, as shown in Fig. 1, with the nth element positioned at $\boldsymbol{r}_n = (r_n, z_n)$. The acoustic field of time reversal waves is then generated as follows:

1) Acoustic signals are transmitted from a source at $\boldsymbol{r}_s = (r_s, z_s)$.
2) These acoustic signals are received by the elements of the TRM array.
3) The received signals are reversed in the time axis.
4) These signals are retransmitted from each element of the TRM array.

The acoustic field at an arbitrary point $\boldsymbol{r} = (r, z)$ is expressed, using Eq. (1), as follows:

$$G_{c\omega}(\boldsymbol{r}, \boldsymbol{r}_s) = \sum_{n=1}^{N} G^*_\omega(\boldsymbol{r}_n, \boldsymbol{r}_s)G_\omega(\boldsymbol{r}, \boldsymbol{r}_n) \quad (3)$$

Here * indicates complex conjugate.

2.3. Simulation Results

Results of a simulation in which time reversal waves are generated using a vertical TRM array in shallow water are presented in this section. In these calculations, the depth of the water is 100 m, an original source is placed at $\boldsymbol{r}_s = (100\,\text{m}, 50\,\text{m})$, the distance between the source and the TRM array is 5,000 m, and c_1, ρ_1, c_2, and ρ_2 are 1,500 m/s, 1,000 kg/m^3, 1,600 m/s, and 1,500 kg/m^3, respectively. The same parameters are used in real-scale simulations described in this paper. The length of the TRM array is 100 m and the interval between its elements is 1.5 m, that is, the number of elements is 67.

Figure 2 shows the pressure field of the time reversal waves at 500 Hz, in which the waves converge to the focus point, which is equal to $\boldsymbol{r}_s = (100\,\text{m}, 50\,\text{m})$.

Figure 3 shows the responses in the frequency domain calculated at up to 1 kHz at the focus and at the typical neighboring receiving points. The coordinate of the receiving position (r, z) is indicated in each graph as (100, 40) (m). Regarding the focus point, its phase response is zero and its amplitude response is almost flat across the bandwidth. However, at the point (110, 50), 10 m ahead of the focus on the array side, its phase response resembles the teeth of a saw due to the time delay, and its amplitude is

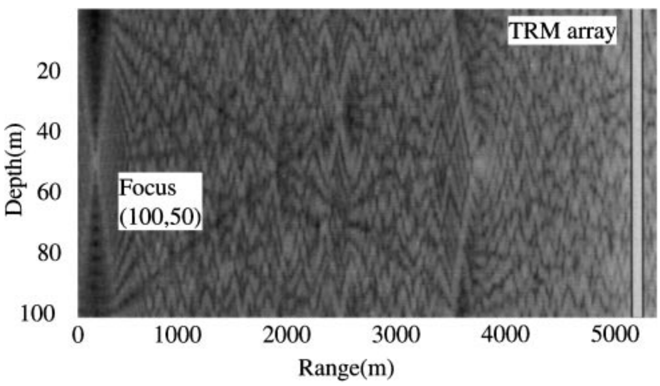

Fig. 2 Simulation of real ocean scale (Pressure field of time reversal waves at 500 Hz).

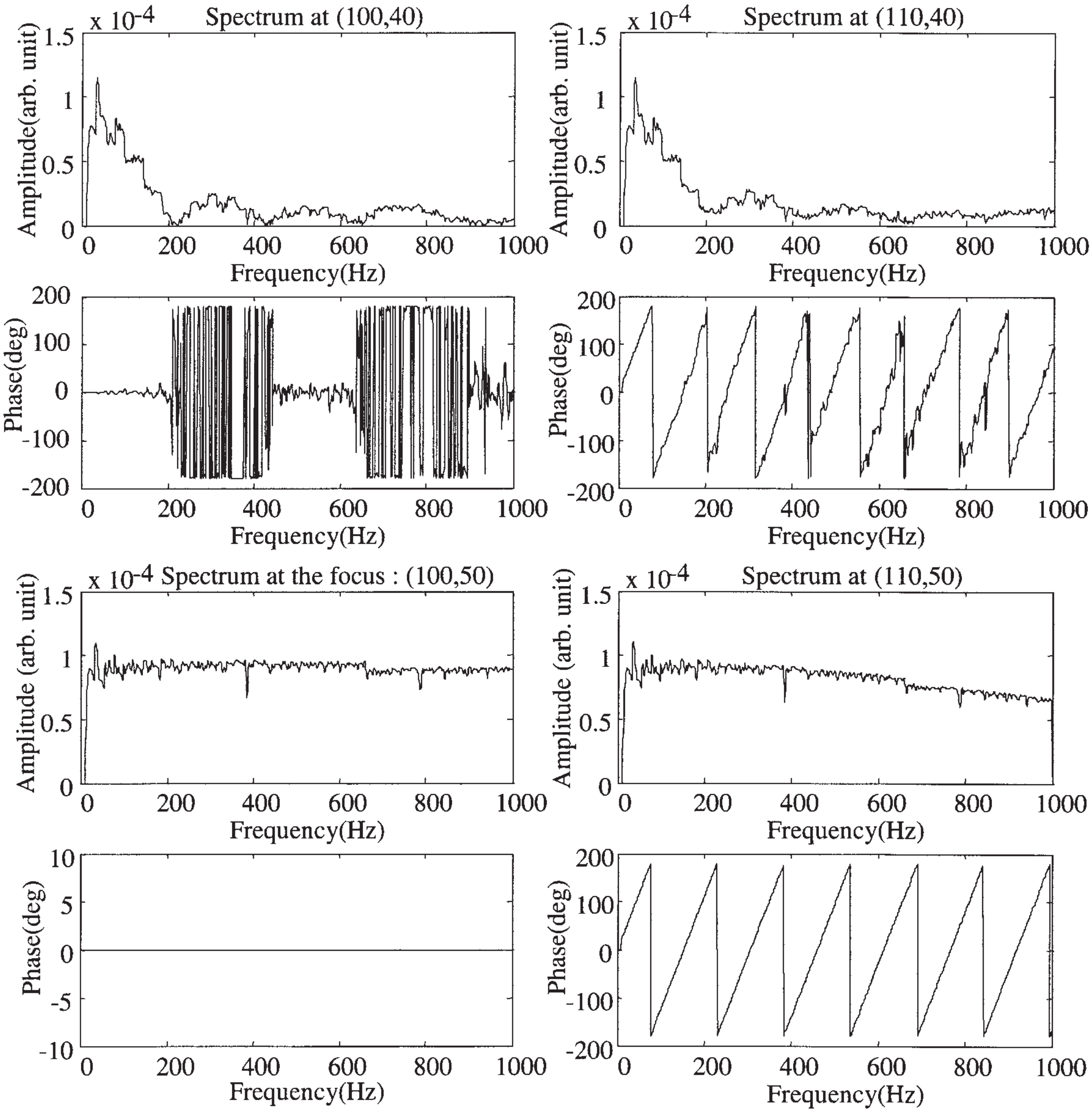

Fig. 3 Frequency responses of the focus and neighboring points (Simulation, Range: 5,000 m).

slightly lowered in the high-frequency band. At the point (100, 40), 10 m above the focus point, both amplitude and phase are very disturbed.

The received time reversal wave signals are shown in Fig. 4. In this case, the original signal transmitted from the source point is a burst pulse, 10 waves of a 500 Hz sinusoidal wave. As shown in Fig. 4, a signal with almost the same shape as the original signal (i.e., the burst pulse) can be received at the focus. At the point (110, 50), the burst pulse can be also received, though its amplitude is a little low. At the point (100, 40), the pulse does not maintain its shape.

3. TANK EXPERIMENTS

3.1. Outline of the Experiments

As shown in Fig. 5, the tank experiments were performed in the shallow part (Length 6 × Width 9 × Depth 0.6(m)) of the anechoic tank at JAMSTEC to simulate conditions of shallow water. As shown in the figure, the transducer, used as a source, is placed at a depth of 0.3 m and the five transducers of the TRM array are placed at intervals of 0.1 m.

The transmitted wave from the source to the TRM array is a burst pulse, 10 waves of a 100 kHz sinusoidal wave. The acoustic signals are sampled at 1 MHz in both AD and DA converting.

All the transducers are ITC 1042, which is omnidirectional, in other words, whose transmitting and receiving responses are not dependent on the vertical or horizontal directivity. In Fig. 6, the transmit frequency response of ITC1042 is shown. When the burst pulse is transmitted by this transducer, the wave directly received is not so broken, as shown later in Fig. 9, because its bandwidth is relatively broad.

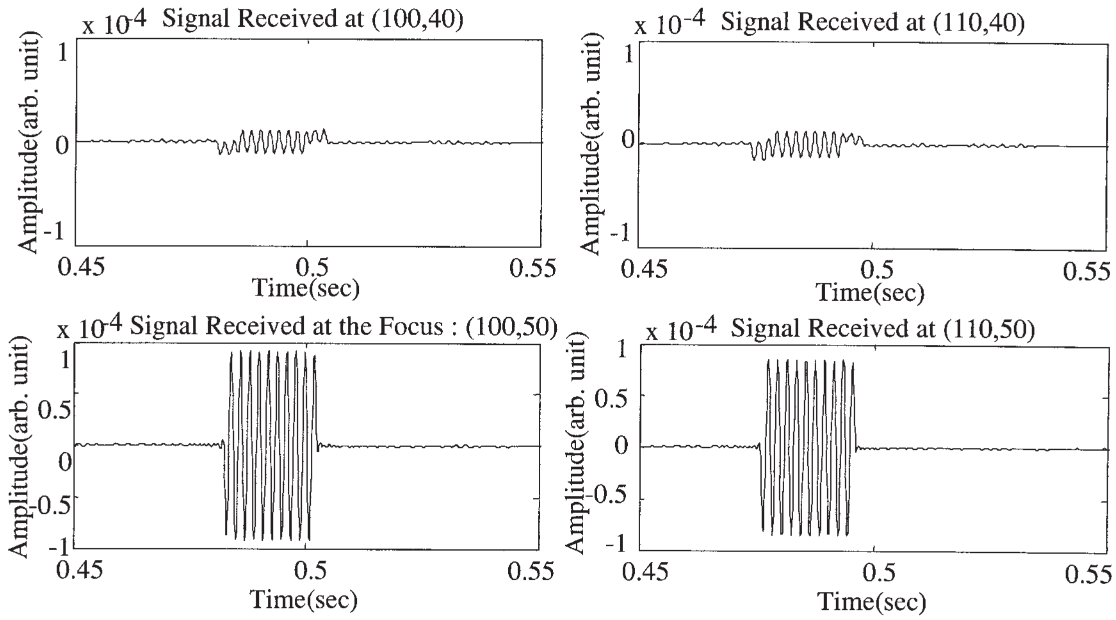

Fig. 4 Received signals at the focus and neighboring points (Simulation, Range: 5,000 m).

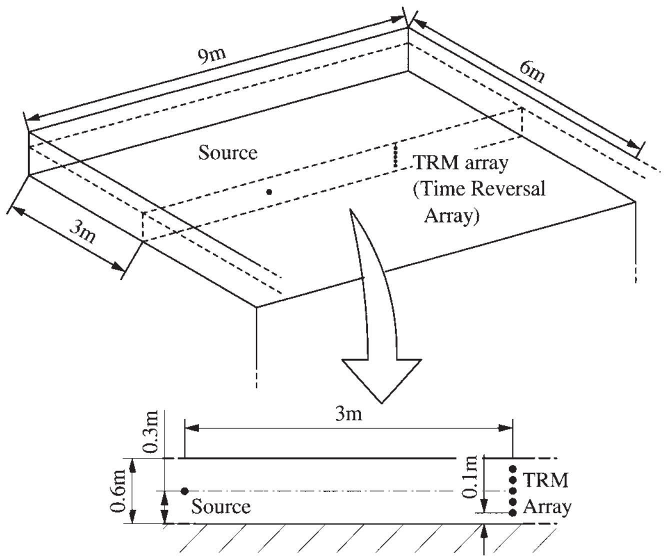

Fig. 5 Arrangement of tank experiments.

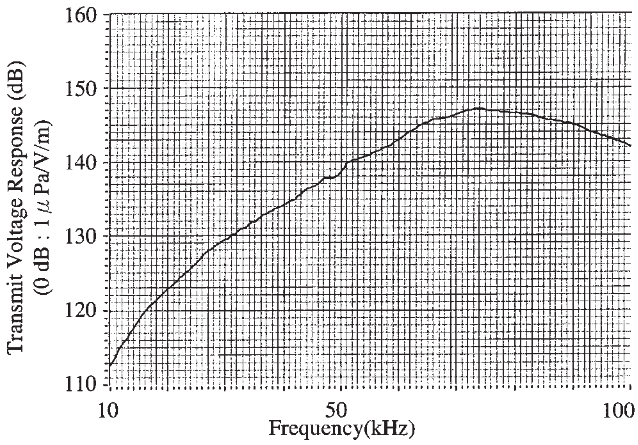

Fig. 6 Transmit Frequency Response of ITC1042.

3.2. Experimental Results

Figure 7 shows the signals received at the elements of the TRM array when tone burst signals are transmitted from the source. The burst pulse cannot be recognized in these signals, because many reflected waves at the water surface and the tank bottom overlap with the direct waves. These received signals also include the waves reflected from side-walls.

The signal received at the focus, when the time reversal waves are transmitted from the TRM array, is shown in Fig. 8. It can be seen from the figure that the burst pulse is received clearly. Figure 9 shows the pulse directly received, using the same transducers, at a range of 1 m. Comparing Fig. 8 and Fig. 9, it is seen that these two signals are similar in shape. This implies that it is not the time reversal process, but the frequency response of the transducers, that breaks the edges of the pulse. In other words, it is possible to focus an arbitrary, desired waveform signal, using time reversal waves, within the constraints of the hardware.

A comparison of the signals received at the focus and at neighboring points is presented in Fig. 10. The coordinates on these graphs indicate the positions of the receiving points. The focus is at $\boldsymbol{r} = (0, 0.3)$ (m). A comparison of these signals reveals that at the focus, the burst pulse is received at a very high signal-to-noise ratio (SNR), while at the neighboring points, the pulse does not appear. Thus, it is revealed that, spatially, the time reversal waves converge very sharply.

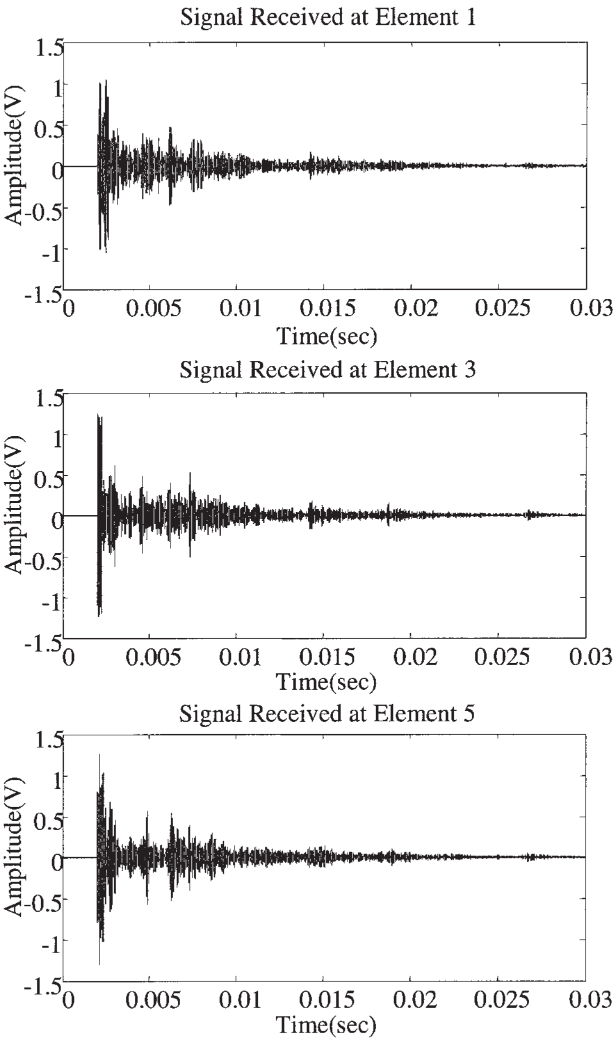

Fig. 7 Received signal at TRM array (experiment).

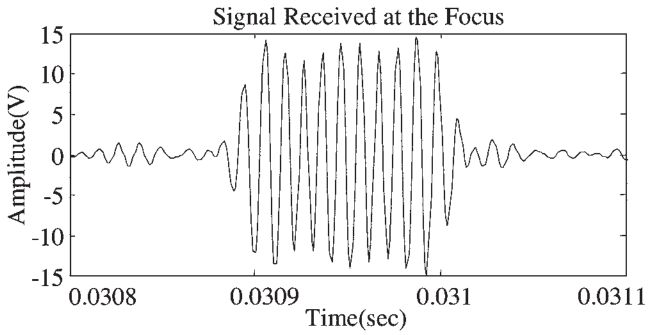

Fig. 8 Signal of time reversal waves received at the focus (experiment).

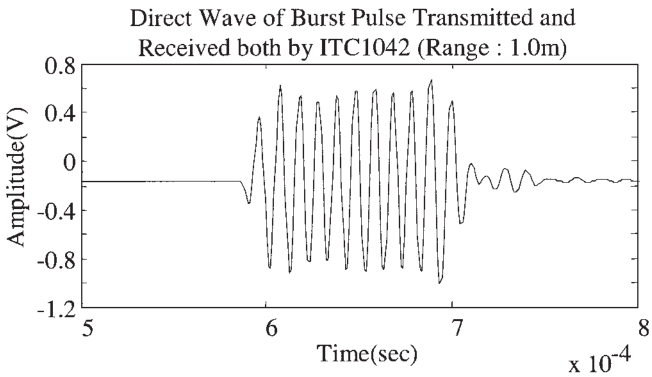

Fig. 9 Signal received directly using same transducers.

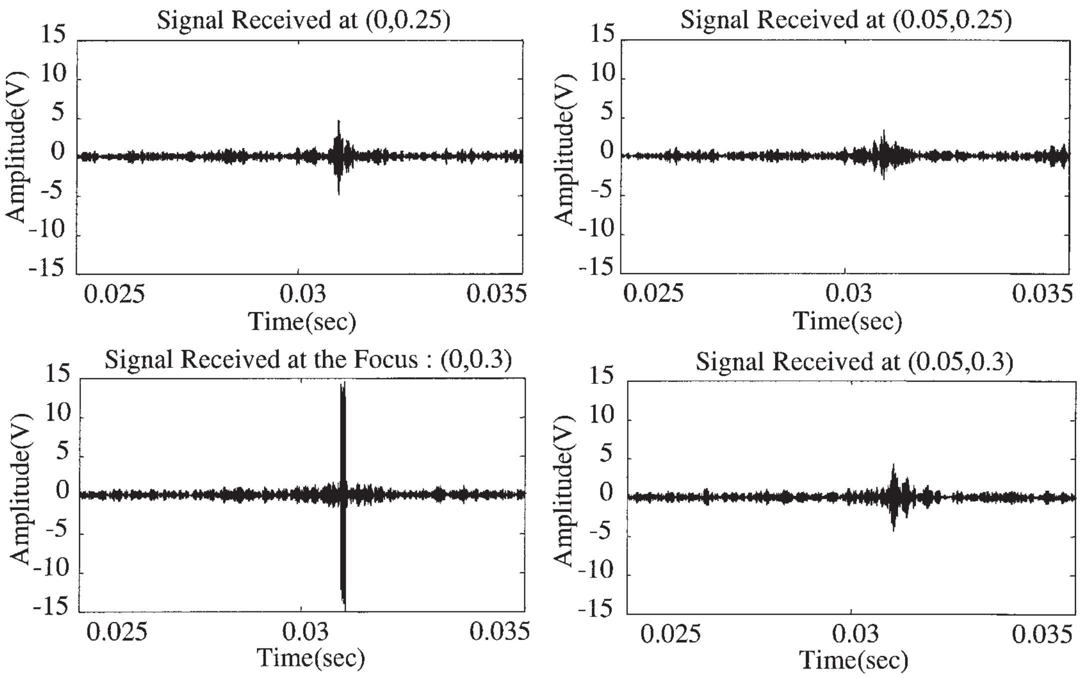

Fig. 10 Signal received at the focus and neighbor points (experiments).

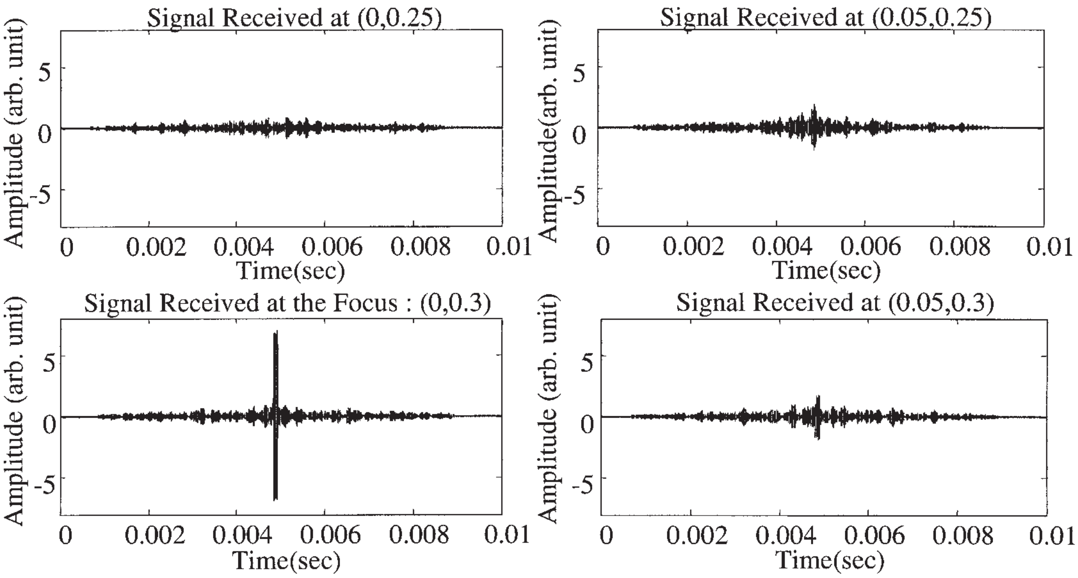

Fig. 11 Simulation results under same conditions as tank experiments.

Calculation results of simulations of tank experiments are shown in Fig. 11. In the experimental results, the time reversal waves include waves reflected from the side-walls. Since the normal mode calculation does not simulate such reflected waves, it is meaningless to quantitatively compare Fig. 10 and Fig. 11. However, qualitatively, the simulation results resemble those of the tank experiments, because the pulse is received clearly. Further, around the focus, the pulse disappears in both results.

4. EFFECT OF THE TRM ARRAY CONFIGURATION

4.1. Effect of the Interval between Array Elements

In the real ocean-scale simulations mentioned in Chap. 2, the interval between the TRM array elements was 1.5 m, which is half the wavelength of 500 Hz. When the depth of the water was 100 m, the number of elements was 67; this is not a realistic scenario. Therefore, the acoustic fields of the time reversal waves are investigated in order to decrease the number of elements and extend the intervals.

Simulations were executed under the same conditions as mentioned in Sect. 2.3, except for the intervals between elements. Figure 12 shows the frequency response at the focus when the interval was 10 times half the wavelength, 15 m, that is, the number of elements is 7. As shown in the figure, the phase response is zero, and the amplitude is uniformly flat across the bandwidth though it is minutely disturbed.

Figure 13 shows the signals received at the focus when the original signal was a 500 Hz burst pulse. As shown in the figure, the pulse is received clearly, although its level is lowered due to the smaller number of elements. These results show that time reversal waves maintain their

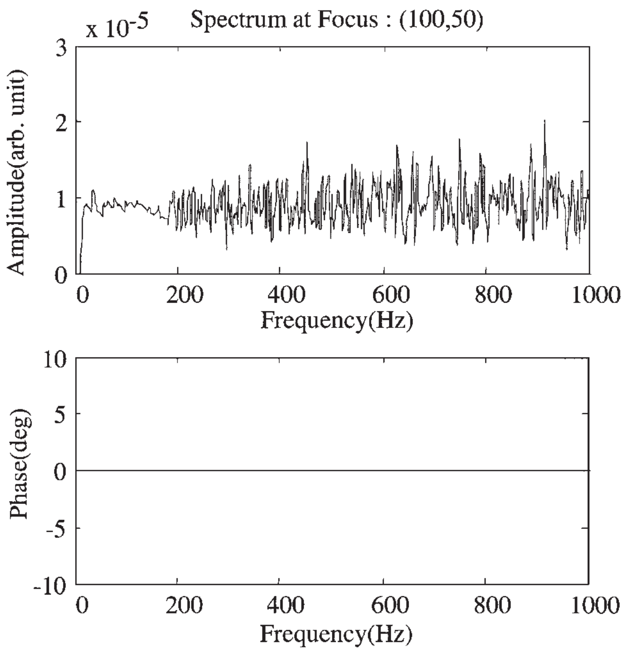

Fig. 12 Frequency response at the focus (simulation) (The interval of TRM elements is 15 m).

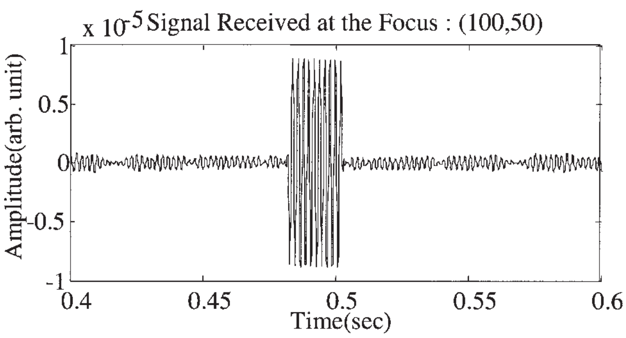

Fig. 13 Signal received at the focus (simulation) (The interval of TRM elements is 15 m).

focusing property even if the number of elements is reduced as long as the array length is kept. Of course, the property is not maintained in the environment where is no reflection or refraction.

4.2. Effects of Tilting the TRM Array

It is supposed that the vertical TRM array is installed by tensioned mooring in the ocean to use time reversal waves. In practice, however, it is expected that the moored array is inclined due to currents or ocean waves. Therefore, how the tilting of the TRM array affects the focusing property of time reversal waves is studied.

4.2.1. Stationary tilting of TRM array

Tank experiments and simulations were conducted in which the TRM array was steadily inclined. Figure 14 shows the results of one of the experiments, in which the TRM array was inclined at 5.0 degrees to the side of the focus. Figure 15 shows the results of real-scale simulations

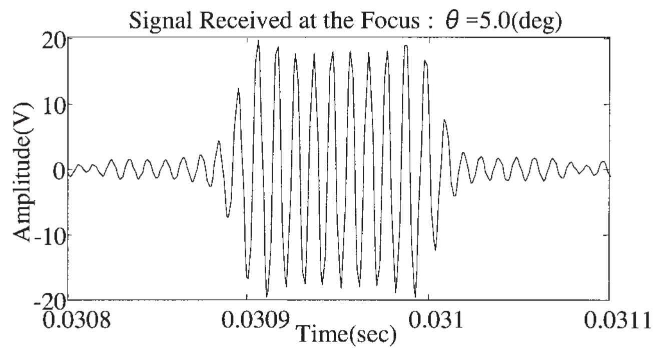

Fig. 14 Experiment result with stationary tilting TRM array.

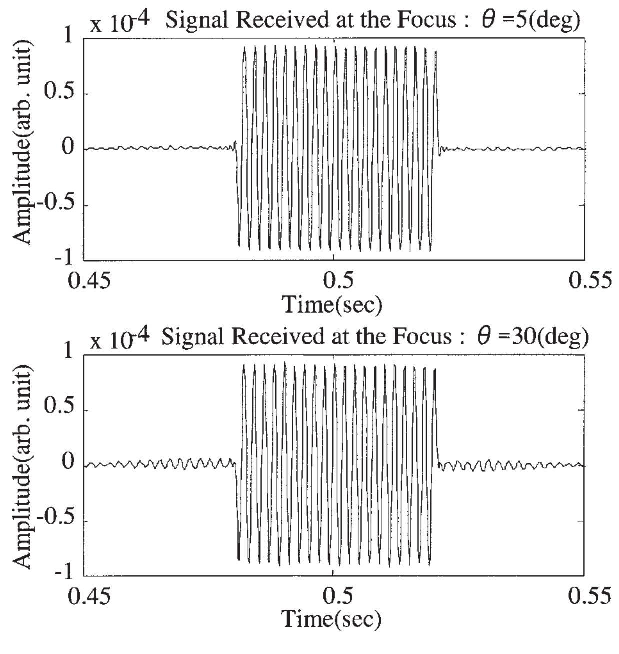

Fig. 15 Simulation result with stationary tilting TRM array (real scaled).

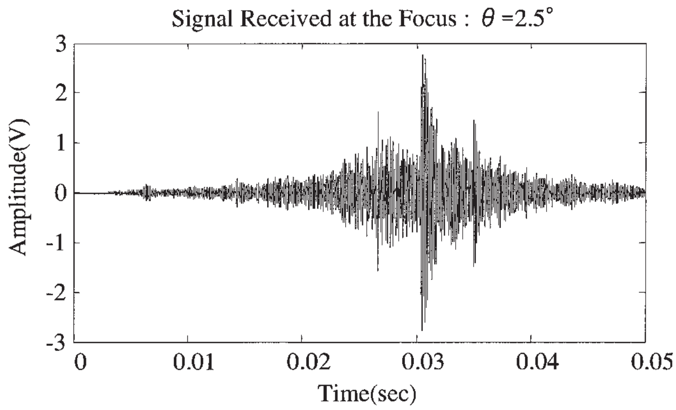

Fig. 16 Experiment result with "non-stationary" tilting TRM array.

in which the TRM array was inclined at 5.0 and 30.0 degrees. These figures demonstrate that time reversal waves can converge when the TRM array is steadily inclined.

4.2.2. Non-stationary tilting of TRM array

Tank experiments and simulations in which the TRM array was dynamically, rather than steadily, tilted were conducted as follows: When the signals from the source were received at the array to generate the time reversal waves ("receiving process"), it was tilted to the side of the focus. The array was then tilted to the opposite side when the time reversal waves were transmitted from the array ("retransmitting process"). Tank experiments were conducted at every 2.5 degrees of the tilting angle, which is limited by the instruments to control the array position. One of the experiment results, in which the tilting angle was ± 2.5 degrees, are shown in Fig. 16. Real-scaled simulations were also executed at every 0.2 degrees in order to search the boundary over which the focusing property is broken. The results of simulations, in which the tilting angles were ± 0.6 and ± 0.8 degrees, are shown in Fig. 17. These figures show that the tilting of the array between the "receiving" and "retransmitting" processes has a great influence on the focusing property of time reversal waves.

4.3. Effects of Vertical Deviation of TRM Array Elements

It is expected that each element of the TRM array is vertically shifted from the desired height because they may not be precisely installed at the desired height, or the mooring ropes may be extended or shrunk. Therefore, the effects of the vertical fluctuation of each TRM array element are discussed in this section.

4.3.1. Stationary deviation

Simulations in which the height of each TRM array element was fixedly shifted were performed. Figure 18 shows the results of each element being alternately shifted

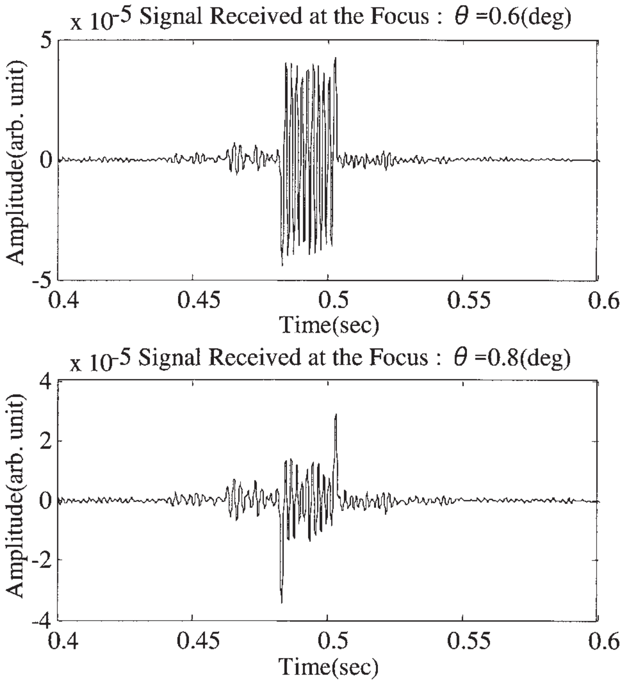

Fig. 17 Simulation result with "non-stationary" tilting TRM array (real scaled).

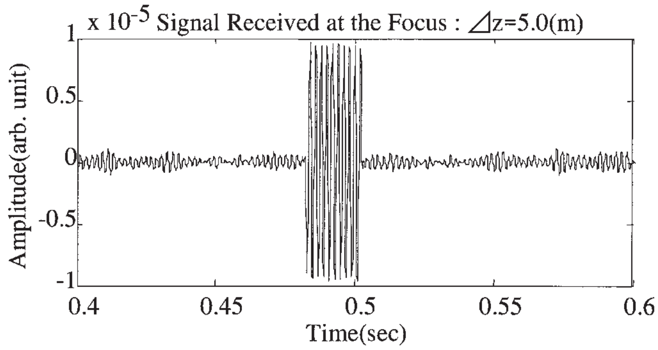

Fig. 18 Real-scaled simulation result of TRM array with the height deviation of each element stationary (The interval of element is 15 m and the height deviation Δz is 5 m).

upward and downward individually from the desired height. "Stationary deviation" of each element was found to have little effect on the focusing property of time reversal waves.

4.3.2. Non-stationary deviation

Assuming either one of the "receiving process" or the "retransmitting process" is actually performed in the ocean, and the other is executed by computer simulation, the deviation of the height of each TRM array element (i.e., the difference between the actual height in the ocean and the virtual height assumed in the simulation) is supposed to affect the focusing property of time reversal waves.

Therefore, experiments and simulations were conducted, in which the height of each element in the "receiving process" and the "retransmitting process" was different. Figures 19 and 20 show the results of experiments and simulations, in which the height of each element was alternately shifted upward and downward individually

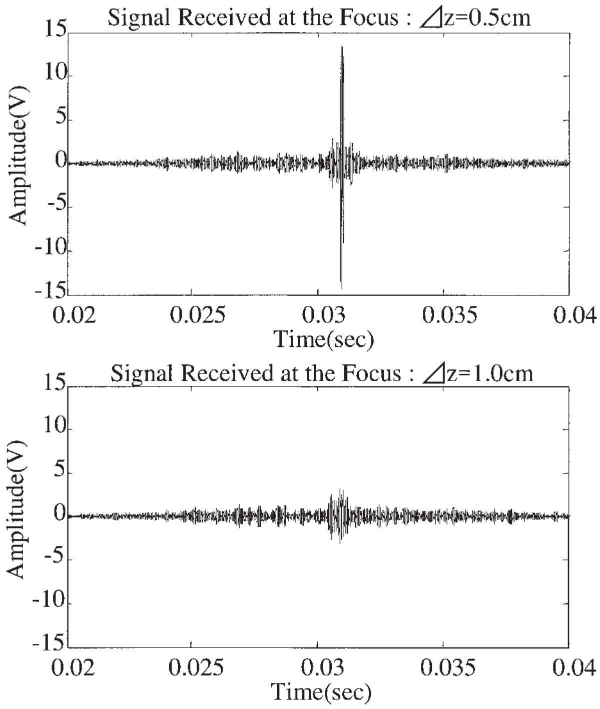

Fig. 19 Experiment result of TRM array with the height deviation of each element "non-stationary" (Δz is the height deviation and the intervals of elements is 10 cm).

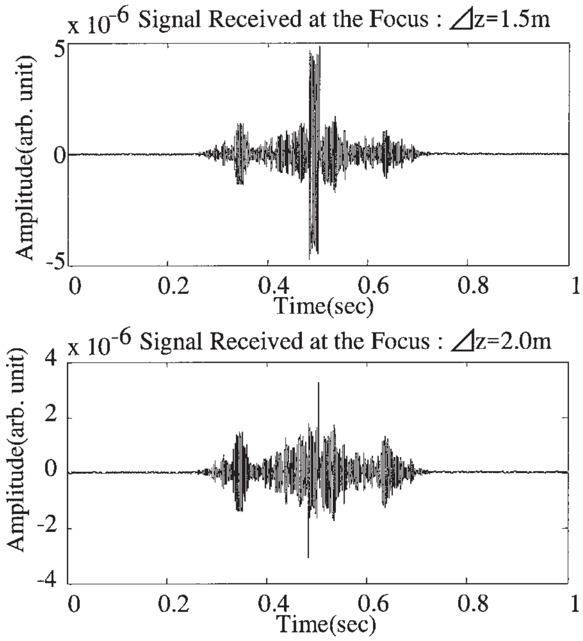

Fig. 20 Real-scaled simulation results of TRM array with the height deviation of its elements "non-stationary" (Δz is the height deviation and the intervals of elements is 15 m).

during the "receiving process" just like as mentioned in 4.3.1 and each element was shifted to the opposite direction during the "retransmitting process." In short, each deviation is non-stationary. These results show that the focusing property of time reversal waves is destroyed if the non-stationary fluctuation height is more than half the wavelength.

4.4. Acoustic Field of Horizontal TRM Array

JAMSTEC has several horizontal arrays such as the Multi-Channel Seismic Profiler or the Ocean Bed Seismometers. If these existent horizontal arrays are utilized by adopting the time reversal wave process, it would be possible to develop a new observation method. Hence, the possibility of generating focused acoustic fields of time reversal waves, using a horizontal array installed on the seabed, was examined.

Figure 21 shows the results of a real ocean-scale simulation, in which the time reversal wave signal, transmitted from a 200 m-long horizontal array, is received at the focus. It is obvious that the focused acoustic field of time reversal waves can be sufficiently generated even with a horizontal array whose length is twice that of the water depth.

5. CONCLUSION

This study confirmed that acoustic waves can be converged to the focus using time reversal waves. This conclusion was reached through tank experiments whose results qualitatively concur with simulation results.

The effects of the TRM array configuration were also discussed and the following matters were clarified: 1) Even though the number of elements of the TRM array is decreased, the desired signal can be received at the focus, as long as the array length is kept. 2) The focused acoustic field can be generated even with "stationary tilting" of the TRM array or even with a TRM array whose elements are steadily, vertically shifted. 3) If the TRM array is tilted or vertical fluctuation of its elements occurs between the "receiving" and "retransmitting" processes, the time reversal waves lose their focusing property. 4) It is possible to converge acoustic waves to the focus even with a horizontal TRM array.

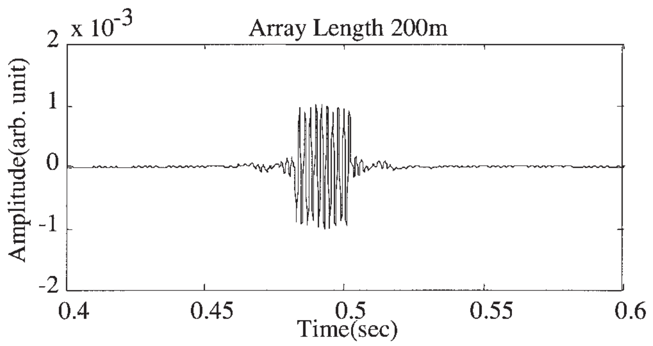

Fig. 21 Signal received at the focus by horizontal TRM array (real scaled simulation).

In this study, the property of time reversal waves is discussed only in the vertical plane including the source and the TRM array, and only in shallow water in which the sound velocity and the water depth are constant. It will be necessary to investigate the focusing property of time reversal waves in three dimensions or in the deep sea in which the sound velocity or the water depth is not constant.

We plan to promote such studies of time reversal waves and to conduct research on the application of time reversal waves in underwater acoustic technologies such as acoustic communication, sonar, and acoustic positioning. Especially, we have a plan to apply time reversal waves to horizontal long-range acoustic communication with autonomous underwater vehicle (AUV) which is expected to cruise under the ice in the Arctic ocean.

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