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Experimental investigation on large amplitude standing wave induced in closed tubes with varying cross section

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Abstract: When a gas-filled closed tube is driven near the resonant frequency of the gas-column, large amplitude standing wave is induced in the tube. The use of large amplitude standing wave in practical field is based on the exploitation of high amplitude pressure fluctuation together with the associated phenomena like acoustic streaming, thermoacoustic effect and mean pressure distribution. It is already known that shock wave appears in cylindrical tube and thereby the wave motion is limited in the peak pressure that can be generated. In shaped tubes, it is possible to avoid the problem of shock formation and as a result, large amplitude standing wave can be achieved. And also the mode of standing wave induced in shaped tubes depends on the tube geometry. For practical application of large amplitude standing wave, the selection of optimum tube shape must be required. In this report, experimental results are described for three types of tube geometry, such as conical, exponential and half cosine. HFC134a was selected as the working gas for the reason that the effects of tube geometry in the wide range of pressure amplitude can be investigated than air. The frequency responses and pressure fluctuation at closed end of each tube were monitored. The result revealed that the half cosine geometry is suitable in the case of design of the acoustic compressor.

Keywords: Finite amplitude standing wave, Area contraction ratio, Compression ratio, Frequency response

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1. INTRODUCTION

Large amplitude standing wave phenomena in closed tubes are classic but interesting research field for the applications in recent high technology. When the gas column in a closed tube is driven by a piston at one end at the resonant frequency, large amplitude standing wave is induced in the tube. The mode of the standing wave, the pressure waveform and its amplitude at the closed end strongly depend on the geometry of the tube shape. In the case of cylindrical tube, the amplitude of the waveform is restricted due to the nonlinear effect in propagating shock-wave generation by the steepening of wave front. Acoustic saturation comes quickly because the energy supplied to generate standing wave at exiting frequency mode is dissipated to higher harmonics instead of increasing the amplitude of the pressure waveform. On the other hand, in varying cross-sectional area tube, a large amplitude standing wave can be achieved and shock waves disappear with the wave motion. This large amplitude pressure fluctuation has a practical application in the development of acoustic compressor which can consist an oil-less compression system of gases. The large amplitude standing wave in closed tube also induces non zero mean pressure distribution along the tube axis, steady circulating streaming at every 1/4 wave length section, and thermoacoustic effect of cooling at the node of pressure fluctuation and heating at the loop [1–3]. The coupling of acoustic streaming with unstable convection not only transforms it into stable convection with cellular structure characterized by the wavelength of the sound wave but also increases the velocity of the convection current [4]. Thermoacoustic effect induced by the air column oscillation can be applied to design thermoacoustic engine and refrigerator [5].

Significant research work has been conducted to understand the characteristics of wave motion induced in

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tubes with cross-sectional area contraction towards the closed end [6]. The most earliest theoretical and experimental studies related to this field was limited to the large amplitude wave motion in tubes of constant cross-section with piston vibrating at one end and with the other end closed. Chester's analysis revealed that the discontinuity in the waveform corresponding to shock front appeared at the resonant frequency, and it disappeared just before and after the resonant frequency [7]. A second order theoretical analysis of sound wave was conducted by considering the effects of viscosity and wall friction, and it found to be in good agreement when compared experimentally [8,9]. Moreover experimental investigation of large amplitude standing wave in a tube with a modified midsection having an expansion or contraction represents that the geometry of the tube shape is the most important factor to obtain shock free waveform [10]. The study on wave motion in cone, horn cone and bulb shape tubes showed to be high amplitude shock free standing wave at compression ratio of required limit [11].

In this report, experimental result has been described to understand the effects of tube shape on large amplitude standing wave in closed acoustic tube. The performance of each tube shape has been described from the viewpoint of application to the compressor of gases called Acoustic Compressor or Sonic Compressor [11]. In order to investigate large amplitude wave phenomena in closed tubes under the practical restriction of experimental facility, the change of acoustic impedance of working gas can increase dynamic range of the experiments. In this experiments, HFC134a was selected as working gas considering the application to refrigerant compressor. The amplitude of the pressure fluctuation at the closed end and the frequency response for different tube geometry corresponding to piston acceleration have been presented. The tubes used here have conical, exponential and half cosine shaped cross-sectional area contraction towards the closed end. Finally, experimental results were compared with the numerical result and the performance of each tube geometries were evaluated.

2. EXPERIMENTAL SETUP

The schematic of experimental setup is shown in Fig. 1. The gas-filled tubes were closed at one end, and the gas-column in the tube was excited by a piston at the other end. The closed end was formed by heavy metal plug through which small hole was provided for the insertion of the pressure transducer. Experiments were conducted in three different tube geometries: one with linearly reduced diameter towards the closed end, one with exponentially reduced diameter towards closed end, and one with half cosine shaped with reduced diameter towards closed end. The tubes are made of aluminum. The length of each

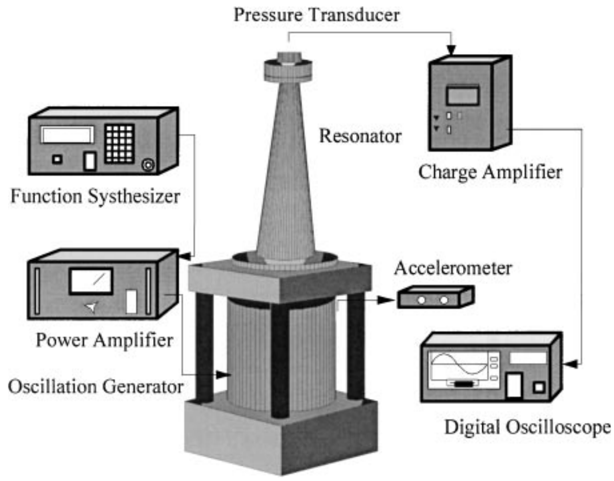

Fig. 1 Schematic diagram of the experimental setup.

axisymmetric tube is 0.5 m with closed end diameter of 0.01 m and the diameter at piston end is 0.1 m. The inside of the tube was filled with HFC134a at normal atmospheric pressure and temperature. The standing waves in the tubes were induced by sinusoidal vibration of piston. The piston has flat surface and is made by aluminum. O-ring seals the gap between the inner tube wall and piston. The piston was driven by a vibrator of electromotive force type with frequency range of 5 Hz $\sim$ 10 kHz, vibration force of 500 N, limit acceleration of 653 m/s^2, and maximum displacement in peak-to-peak of 8 mm. Piston acceleration was measured by using accelerometer with the range of $0 \sim 550$ m/s^2. Pressure fluctuation at the closed end of tube was measured by using a pressure transducer (Kistler 5011B). The waveform was monitored in digital oscilloscope and the data stored in computer from it. Experiments were executed at fundamental resonant frequency condition.

3. TUBE GEOMETRY

The fundamental equation in dimensional form for one-dimensional analysis of the wave motions in the tubes with varying cross-section is as follows.

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + F\frac{1}{A}\frac{dA}{dx} = H \tag{1}$$

where

$$Q = \begin{bmatrix} \rho \\ \rho u \\ E \end{bmatrix}, \quad F = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ (E+p)u \end{bmatrix},$$

$$H = \begin{bmatrix} 0 \\ p\dfrac{1}{A}\dfrac{dA}{dx} - \dfrac{4}{3}\mu\dfrac{\partial^2 u}{\partial x^2} + f \\ -\dfrac{4}{3}\mu u\dfrac{\partial^2 u}{\partial x^2} \end{bmatrix}$$

and ρ is the density in kg/m^3, u is the velocity in m/s, p is the absolute pressure in Pa, E is the total energy which is the sum of potential energy and kinetic energy in J, μ is the coefficient of viscosity in Pa·s, f is the wall friction in Pa/m and A is cross-sectional area in m^2. In the above equation, Q denotes the acoustic variables, F is the flux term and H is the source term.

Equation (1) is linearized as

$$\frac{\partial^2 u}{\partial t^2} = a^2\left[\frac{\partial^2 u}{\partial x^2} + \frac{1}{A}\frac{dA}{dx}\frac{\partial u}{\partial x} + u\frac{d}{dx}\left(\frac{1}{A}\frac{dA}{dx}\right)\right]. \quad (2)$$

where a is the speed of sound (m/s) determined by $a^2 = (\partial p/\partial \rho)_{ad}$.

As shown in Eqs. (1) and (2), the effect of tube geometry on the wave motion characterized by the term of $1/A \cdot dA/dx$. Detailed phenomena of standing waves in the tubes are analyzed by numerical calculation of Eq. (1). However, fundamental characteristics of the modes of standing wave and resonant frequencies can be obtained by analysis of Eq. (2). The estimation of resonant frequencies by linear theory with considering of tube geometry in the function of $1/A \cdot dA/dx$ was very close to experimental results [12]. Here the effect of tube geometry has been investigated experimentally and discussed on the basis of the function $1/A \cdot dA/dx$. Among different tube geometries three cases were mainly taken into the consideration, which cover all possible simple tube geometries. Experiments were conducted in exponential, conical and half cosine shaped area contraction closed tubes. The geometry of each tube is shown in Fig. 2. All those tubes are axisymmetric, and their geometries are described by the following equations.

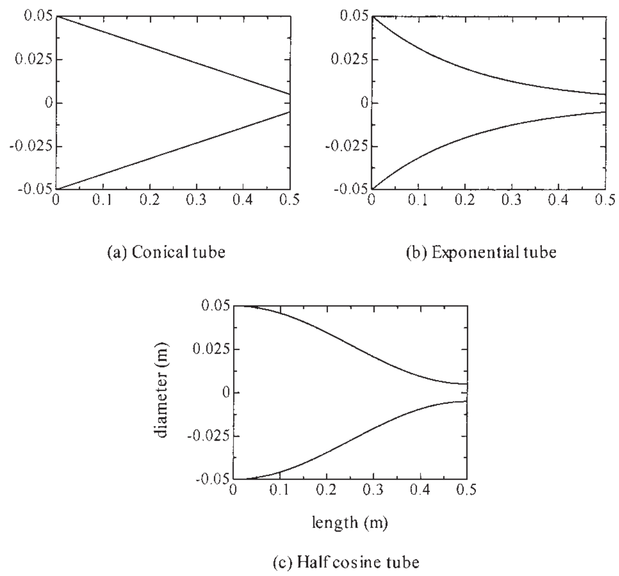

Fig. 2 Geometry of the tube shape. (a) Conical tube, (b) Exponential tube, (c) Half cosine tube.

For the exponential tube, the radius is

$$r(x) = r_p \exp(mx) \quad \text{for} \quad 0 \le x \le L \quad (3)$$

where, $m = \ln(r_0/r_p)/L$

Here r_p, r_0 and L are the radius at piston end, the radius at closed end and the length of the tube in m, respectively. For the conical tube, the radius is

$$r(x) = r_p + mx \quad \text{for} \quad 0 \le x \le L \quad (4)$$

where, $m = (r_0 - r_p)/L$

The geometry of the half cosine tube is expressed by the following equation

$$r(x) = \frac{r_p - r_0}{2}\cos\left(\frac{\pi}{L}x\right) + \frac{r_p + r_0}{2} \quad \text{for} \quad 0 \le x \le L \quad (5)$$

The radius at piston end r_p and the radius at closed end r_0 for each tube are 0.05 m, 0.005 m, respectively. This indicates that the cross-sectional area contraction ratio of each tube is kept same which are 100. The length of each tube is 0.5 m. The reason why these geometries have been selected is that the functions $1/A \cdot dA/dx$ are different typically from one to the others as shown in Fig. 3. The rate of change of cross-sectional area contraction ratio is constant in the exponential tube. It is zero at both ends of the half cosine tube while it increases towards the closed end in the conical tube. It is necessary to mention that though the length and diameter at both ends of the tube is kept same, however, the volume in the tube has not taken into the consideration.

4. RESULTS AND DISCUSSIONS

4.1. Pressure Waveform

Experiments were conducted to analyze the characteristics of large amplitude standing wave in three different shaped tubes and to come out with the best tube shape to design the compressor. The tubes were filled with

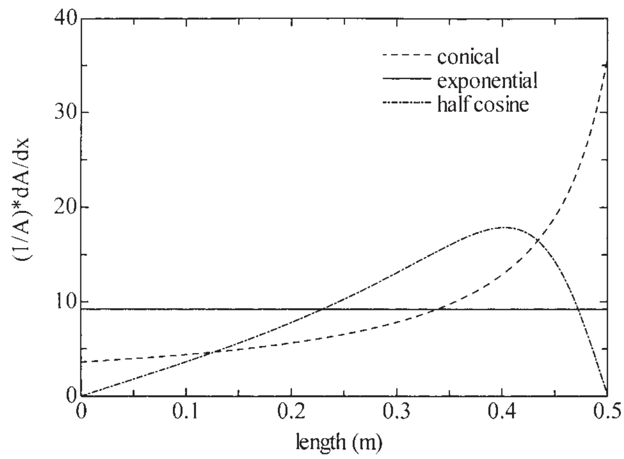

Fig. 3 Rate of change of cross-sectional area contraction ratio in three shaped tubes.

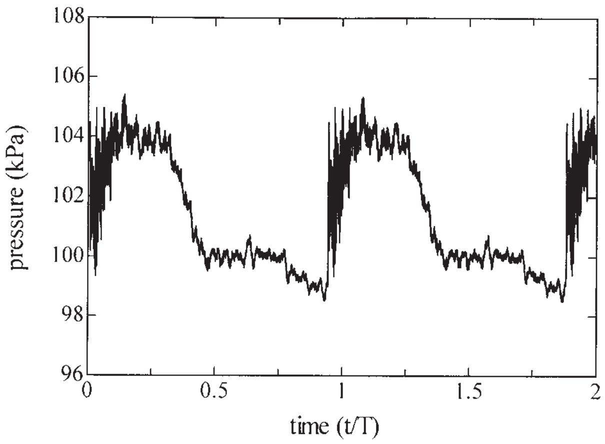

Fig. 4 Pressure fluctuation obtained at closed end of cylindrical tube for piston acceleration amplitude of $100\,\mathrm{m/s^2}$.

HFC134a, which is a kind of refrigerant gases, at atmospheric state. The driving frequency was kept at the first mode resonant frequency corresponding to half wave-length resonance.

First, the pressure fluctuation generated at the closed end of reference cylindrical tube filled with HFC134a at piston acceleration of $100\,\mathrm{m/s^2}$ is shown in Fig. 4. It is observed that the waveform distorted and formed into shock wave even for small piston acceleration. Several researchers have explained the reason why shock wave appears in cylindrical tube as follows. The higher mode frequencies in cylindrical tube are coincided with the frequencies of higher harmonics of pressure waveform. Therefore, the energy supplied to the fundamental mode is transferred to the higher harmonics by fluid dynamic nonlinear processes. It is brought about the suppression of increasing rate of amplitude which called acoustic saturation.

Figure 5 shows the pressure fluctuations at closed end of the conical, the exponential and the half cosine tube for three different piston acceleration. In contrast to the cylindrical tube, strong shock wave does not appear in spite of large amplitude of the acoustic pressure, but the waveform in each tube is different. The difference is caused by the difference between the higher modal frequencies appeared in each tube and the frequencies of higher harmonics of fundamental oscillation as shown in Fig. 6. The solid lines in Fig. 6 are obtained by linear theory. It shows that the experimental results agree well with the theoretical ones. The fact that the waveform obtained in the half cosine tube is almost sinusoidal except appearing of shock let as shown in Fig. 5 is explained by that the difference described above is largest in the case of the half cosine tube.

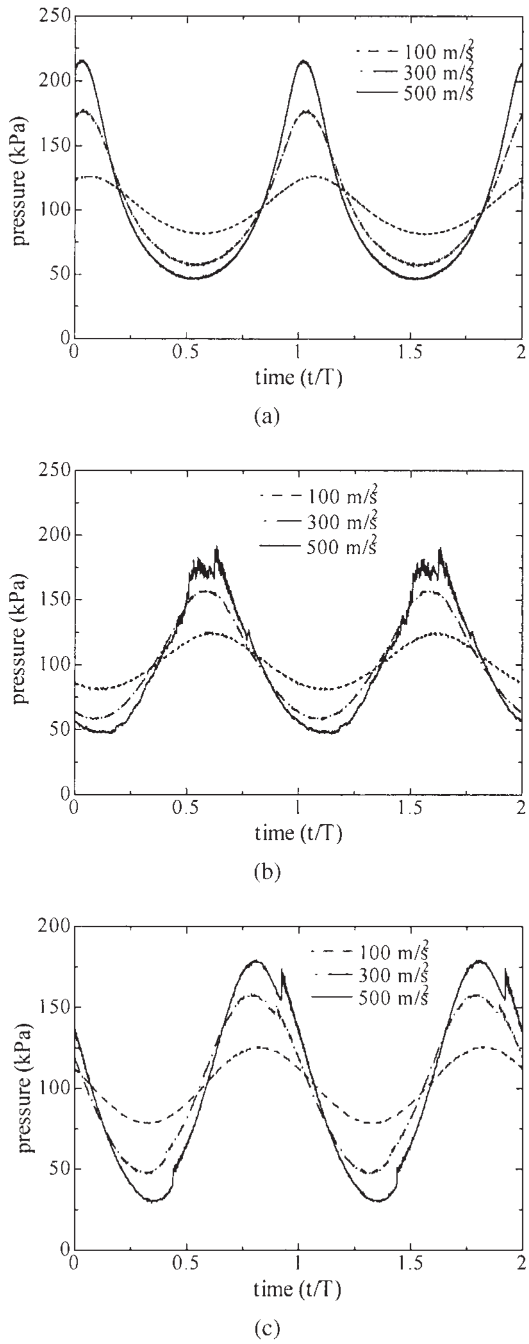

Fig. 5 Pressure fluctuations obtained at closed end of area change tubes for three different piston accelerations. (a) Conical tube, (b) Exponential tube, and (c) Half cosine tube.

4.2. Frequency Response Curve

The effect of tube geometry is also appeared in the resonant condition. Figure 7 shows frequency response curve obtained in each tube for three different piston acceleration amplitudes. The peak-to-peak pressure amplitude at closed end of the tube was monitored during the frequency sweeps in the vicinity of resonance. The curves are plotted for upward and downward frequency sweeps. As shown in the figure, in the case of conical tube, it is seen that for high piston acceleration amplitudes, the resonance curve obtained at upward frequency sweep leans towards the upper frequency. The resonance curve obtained at

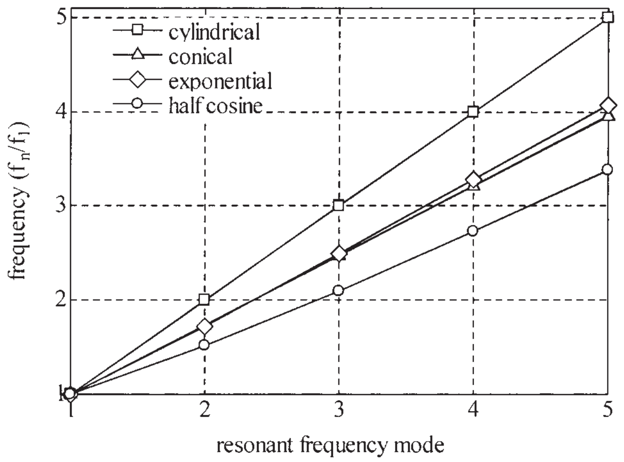

Fig. 6 Relation between resonant frequency modes with harmonics in different tube geometries.

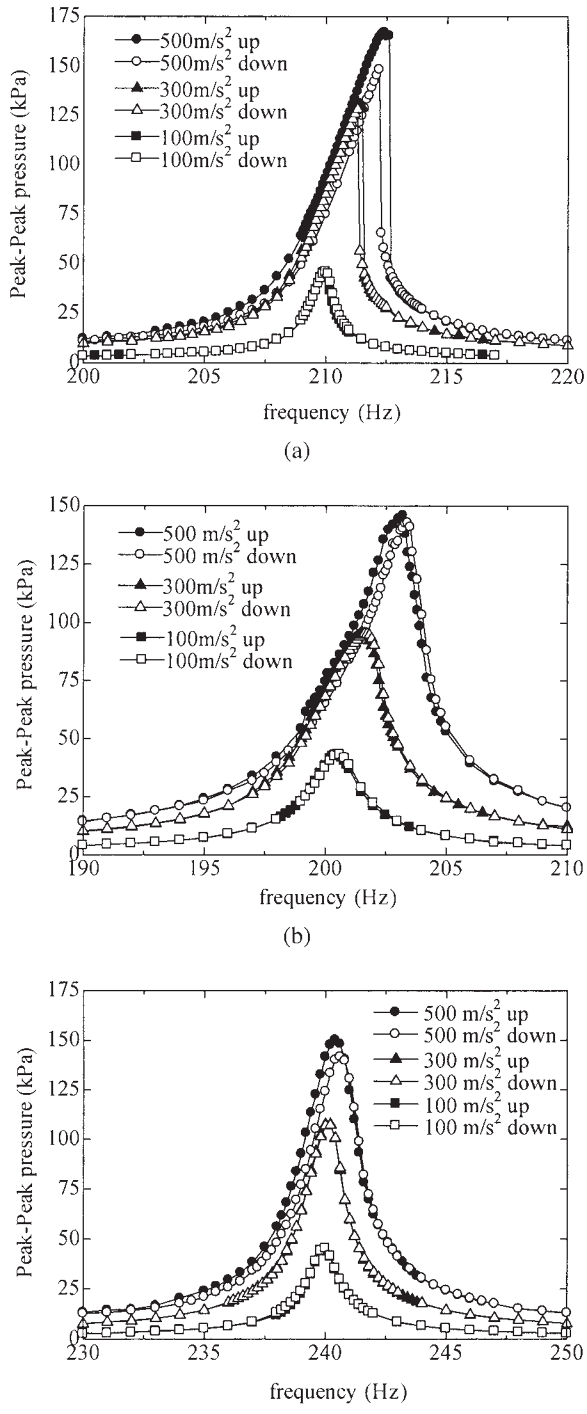

Fig. 7 Frequency response curves obtained in three tube geometries. (a) Conical tube, (b) Exponential tube, and (c) Half cosine tube.

downward frequency sweep does not follow the resonance curve obtained at upward frequency sweep. Furthermore, for piston acceleration amplitude of $300\,m/s^2$ and $500\,m/s^2$, the frequency responses indicate hysteresis.

The frequency response of the exponential tube shows the similar tendency with the conical tube, but the hysteresis does not appear. The response curves in the case of half cosine are showing completely different phenomena as seen in exponential and conical tube. In contrast to the conical and exponential tube, the resonance frequencies for different driving conditions have almost the same value, and the frequency response curves are symmetrical for the resonance frequencies.

In each case, the effect of mean temperature rising of the gas with increase of pressure amplitude on the resonant frequency was negligible because of that the maximum temperature rising at the closed end was less than 3°C which was measured by cupper-constantan thermo-couple.

The characteristics of frequency responses for different tube geometries have been shown clearly, and it can be considered that the primary factor on the difference of the frequency responses is the difference of the function $1/A \cdot dA/dx$ for each tube, however, the mechanism inducing the difference has not been explained precisely for the time being. It has been clarified that, in practical applications of large amplitude standing wave realized at the resonant condition, the control for keeping the resonant condition induced in the half cosine is easier than other tubes.

4.3. Performance Evaluation

A comparison of peak-to-peak pressure amplitudes with regards to piston acceleration in exponential, conical and half cosine tube are shown in Fig. 8. It is observed that the pressure amplitude increases with the increase in piston acceleration amplitude in tubes with area change, on the other hand, the increase of amplitude in the cylindrical tube quickly saturates. In comparison for three tube geometries driven with the same piston acceleration, the maximum pressure amplitude is obtained in conical tube.

In consideration of design of resonant chamber for acoustic compressor, the compression ratio is one of the practical factors to understand the performance of the acoustic compressor. The compression ratio is defined by the ratio of maximum and minimum absolute pressure

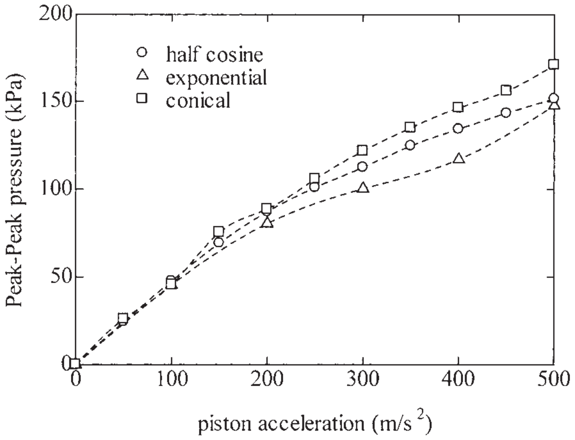

Fig. 8 Peak-Peak pressure amplitude with piston acceleration in three tubes.

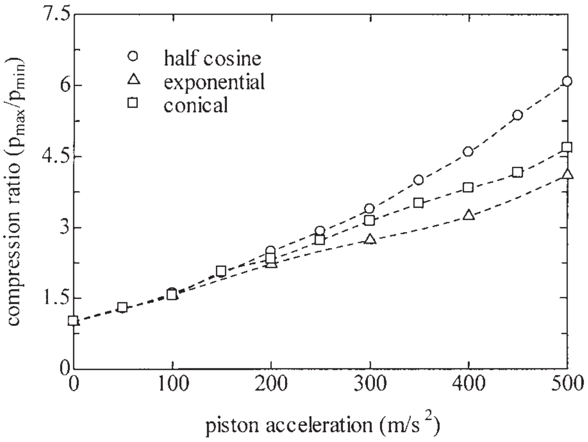

Fig. 9 Compression ratio with piston acceleration in conical, exponential and half cosine shaped tubes.

amplitude of the waveform. Figure 9 shows change of compression ratio with regards to piston acceleration in three different shaped tubes. The compression ratio in each tube geometry increases with the increase in piston acceleration amplitude. It is seen that for same piston acceleration amplitude, the compression ratio is maximum in half cosine tube. The compression ratio in half cosine tube reaches to 6 for piston acceleration amplitude of $500\,m/s^2$.

5. CONCLUSIONS

Large amplitude standing waves induced in three shaped tubes filled with HFC134a at normal atmospheric pressure and temperature was studied experimentally. The tubes were closed at one end by a solid metal plug and at the other end by a piston oscillating sinusoidally.

Experimental results obtained are summarized as follows. The characteristics of large amplitude standing wave is strongly influenced by the variation of the cross-sectional area contraction ratio along the tube axis. The rate of cross-sectional area contraction among the three tube geometries is quite different. The frequency responses and pressure fluctuation vary according to the variation of the tube shape. The stable resonant frequency is found in half cosine tube, which is very important to design practical devices. In consideration of stability of the resonant frequency and higher compression ratio in half cosine tube, the half cosine tube must be an appropriate candidate to design the resonance cavity of acoustic compressor.

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