PAPER

On beampattern design for beamspace music

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Abstract: The problem of beamforming and related beamspace high resolution direction-of-arrival (DOA) estimation is studied in this paper. All beamspace processing methods are based on the beam outputs and the beampattern design plays an important role in providing high quality beam output data for further processing. Three typical situations which are frequently encountered in practical sonar system working environment and the most widely studied MUSIC algorithm are considered herein. First, when isotropic noise is the dominant noise at sensors, conventional beamforming techniques provide the optimum performance in the sense that DOA estimate is the ML estimate. Good DOA estimates are obtainable by applying MUSIC to the beam outputs directly. Then, uncorrelated interferes with much higher strength than the wanted signals are assumed to be present in the sidelobe region, and low sidelobe Dolph-Chebyshev and adaptive MVDR beampatterns are designed to guarantee the performance of MUSIC. And finally, the robustness of conventional techniques is combined with the adaptivity of MVDR beamforming to deal with the situation when the interfere in the sidelobe region is strongly correlated with one of the wanted sources. Performance in all three situations is studied with numerical examples.

Keywords: Beamspace DOA estimation, MVDR beamformer, MUSIC algorithm, Interference rejection, High resolution

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1. INTRODUCTION

In modern sonar systems, estimation of directions-of-arrival (DOAs) of sources mainly depends on the outputs of preformed multi-beams, which are generally constructed via conventional techniques. The main reason of such popularity is that conventional beamformers are easy to implement in practice, robust to system errors, and insensitive to correlation among incident signals. On the other hand, it is difficult to resolve closely placed sources and estimate their directions with such systems. This is inherent limitation of the conventional multi-beam sonar systems and need new techniques to overcome.

The so-called high resolution DOA estimation algorithms have much stronger abilities to resolve closely spaced sources and seem to be very promising in solving DOA estimation problem after a large amount of theoretical and computer simulation work. However, there are difficulties prohibiting their applications to practical systems, including complexity in computation, robustness to system errors and model mismatch, higher resolution SNR threshold, and so on. Furthermore, outputs at sensors are not available in almost all sonar systems, and instead it is the data from the preformed beams that can be used in high resolution DOA estimation algorithms. Beamspace processing applies the methods and techniques, which are originally proposed for sensor data, to the outputs of the preformed beams. When conducted in the beam domain, the mentioned shortcomings of high resolution algorithms can be overcome to some extent: computational burden is reduced significantly since usually much less beams are involved in DOA estimation compared to the number of sensors, the processing is more robust due to the averaging operation in the beamforming procedure, and the array gain provided in beamforming would reduce the resolution threshold to a much lower level. Therefore, beamspace processing has received much attention during the past two decades or so.

To guarantee the performance of high resolution DOA estimation algorithms in beamspace processing, good beampatterns are required. Instead of the fixed properties of physical sensors, beampatterns can be designed to suit for different situations so as to improve the DOA estimation performance further. This paper is dedicated

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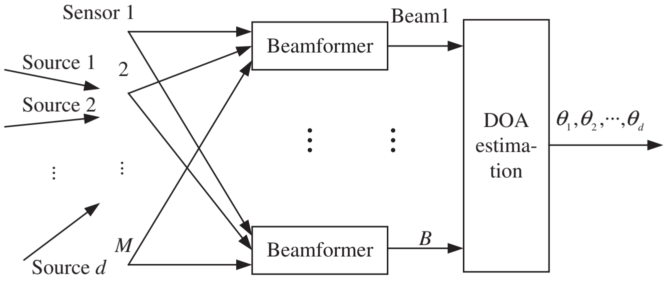


Fig. 1 Diagram of beamspace processing.

to narrowband beamforming techniques and related beamspace high resolution algorithms. After the presentation of the data model and a description of the algorithm, beamspace MUSIC is thoroughly studied by applied to outputs of the beams which are formed in different ways. Research work is presented with results from three numerical study cases, representing typical situations which are frequently encountered in practical sonar working environments.

2. DATA MODEL

The procedure of beamspace DOA estimation algorithms can be divided into two stages in implementation, as shown in Fig. 1. In the beamforming process, the output data at the array sensors from sources in different directions pass through the multi-beamformers and the beam output data are obtained. Beams can be formed in different ways, such as using conventional beamforming techniques or adaptive beamforming methods. And then in the DOA estimation process, based on the beam outputs, source directions are roughly determined by the peak positions of beam outputs and only the outputs from a small number of beams in the adjacent region of the peak are used for fine DOA estimation, either using conventional techniques or high resolution algorithms.

Narrowband processing is considered herein. Assume that there are d narrowband signals arriving onto the array in directions $\{\theta_i, i = 1, \cdots, d\}$ and M sensors in the array have identical isotropic responses. In order to utilize different beamforming methods which might have constraints on the array geometry and also to simplify the problem under consideration, a uniform linear array is assumed with inter-element spacing being a half of wavelength corresponding to the central frequency of the narrowband signals. B beams are formed based on these M sensors to cover a spatial region $[\theta_\mathrm{a}, \theta_\mathrm{b}]$ and outputs from these beams are used to estimate the DOAs of sources incident onto the array within this region.

Put in vector-matrix notation, data at sensors can be described by a $M\times 1$ vector as

$$\boldsymbol{x}(t) = \boldsymbol{A}(\Theta)\boldsymbol{s}(t) + \boldsymbol{n}(t). \tag{1}$$

where $\boldsymbol{A}(\Theta) = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_d)]$ is the so-called array manifold containing information on DOAs of the incident signals, $\boldsymbol{s}(t)$ is the $d\times 1$ source signal vector, and $\boldsymbol{n}(t)$ the $M\times 1$ additive noise vector at sensors.

The outputs of these B beamformers can be written as

$$\boldsymbol{y}(t) = \boldsymbol{W}^\mathrm{H}\boldsymbol{x}(t). \tag{2}$$

where $\boldsymbol{W}$ is a $M \times B$ weighting matrix consisting of beamforming vectors for each beam and is referred to as the beamforming matrix. The transform described in Eq. (2) converts the element domain outputs into the beam domain outputs.

Under the narrowband assumption and for the case of multiple incident sources, we have the beamformer outputs as follows,

$$\boldsymbol{y}(t) = \boldsymbol{W}^\mathrm{H}\boldsymbol{A}(\Theta)\boldsymbol{s}(t) + \boldsymbol{n}_y(t) \tag{3}$$

where

$$\boldsymbol{n}_y(t) = \boldsymbol{W}^\mathrm{H}\boldsymbol{n}(t) \tag{4}$$

is the additive noise in the beam domain.

From Eq. (4), we have the direction response vectors after beamforming defined as

$$\boldsymbol{v}(\theta) = \boldsymbol{W}^\mathrm{H}\boldsymbol{a}(\theta) \tag{5}$$

which plays the same role in the beamspace processing as that of vector $\boldsymbol{a}(\theta)$ in the element-space processing.

In narrowband array signal processing, usually it is assumed that incident signals are uncorrelated and independent from the noise at sensors, which are in turn presumed to be uncorrelated from sensor to sensor. Furthermore, if we assume that noise at all sensors has identical power, then the array covariance matrix can be written as

$$\boldsymbol{R}_x = AE[\boldsymbol{s}(t)\boldsymbol{s}^\mathrm{H}(t)]\boldsymbol{A}^\mathrm{H} + E[\boldsymbol{n}(t)\boldsymbol{n}^\mathrm{H}(t)] = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^\mathrm{H} + \sigma^2\boldsymbol{I} \tag{6}$$

where $\boldsymbol{S} = E[\boldsymbol{s}(t)\boldsymbol{s}^\mathrm{H}(t)]$ is the source covariance matrix and σ^2 is the noise power at array sensors. And the argument vector Θ is dropped from $\boldsymbol{A}(\Theta)$ to simplify the expression.

From matrix theory, the array covariance matrix can be described by its eigenvalues and eigenvectors as

$$\begin{aligned}\boldsymbol{R}_x &= \boldsymbol{E}_s\Lambda_s\boldsymbol{E}_s{}^\mathrm{H} + \boldsymbol{E}_n\Lambda_n\boldsymbol{E}_n{}^\mathrm{H} \\ &= \sum_{j=1}^{d}\lambda_j\boldsymbol{e}_j\boldsymbol{e}_j{}^\mathrm{H} + \sigma^2\sum_{j=d+1}^{M}\boldsymbol{e}_j\boldsymbol{e}_j{}^\mathrm{H}\end{aligned} \tag{7}$$

where $\boldsymbol{E}_s = [\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_d]$ and $\boldsymbol{E}_n = [\boldsymbol{e}_{d+1}, \boldsymbol{e}_{d+2}, \cdots, \boldsymbol{e}_M]$ are the so-called signal and noise subspaces consisting of signal and noise eigenvectors, respectively, and $\Lambda = \mathrm{diag}[\lambda_1, \lambda_2, \cdots, \lambda_M]$ is a diagonal matrix containing the eigenvalues in descending order with d biggest ones being the signal eigenvalues and others the noise eigenvalues.

Similarly, in the beam domain, the beam covariance matrix can be written as

$$\boldsymbol{R}_y = E[\boldsymbol{y}(t)\boldsymbol{y}^{\mathrm{H}}(t)] = \boldsymbol{W}^{\mathrm{H}}\boldsymbol{R}_x\boldsymbol{W} = \boldsymbol{W}^{\mathrm{H}}\boldsymbol{ASA}^{\mathrm{H}}\boldsymbol{W} + \sigma^2\boldsymbol{W}^{\mathrm{H}}\boldsymbol{W} \tag{8}$$

If the beamforming matrix $\boldsymbol{W}$ is designed to satisfy the orthogonal condition, i.e.

$$\boldsymbol{W}^{\mathrm{H}}\boldsymbol{W} = \boldsymbol{I} \tag{9}$$

then Eq. (8) can be simplified as

$$\boldsymbol{R}_y = \boldsymbol{W}^{\mathrm{H}}\boldsymbol{ASA}^{\mathrm{H}}\boldsymbol{W} + \sigma^2\boldsymbol{I} \tag{10}$$

which also can be written in terms of eigenvalues and corresponding eigenvectors as

$$\boldsymbol{R}_y = \boldsymbol{W}^{\mathrm{H}}\boldsymbol{ASA}^{\mathrm{H}}\boldsymbol{W} + \sigma^2\boldsymbol{I} = \boldsymbol{E}_{\mathrm{Bs}}\Sigma_{\mathrm{Bs}}\boldsymbol{E}_{\mathrm{Bs}}{}^{\mathrm{H}} + \sigma^2\boldsymbol{E}_{\mathrm{Bn}}\boldsymbol{E}_{\mathrm{Bn}}{}^{\mathrm{H}} \tag{11}$$

where $\boldsymbol{E}_{\mathrm{Bs}}$ and $\boldsymbol{E}_{\mathrm{Bn}}$ represent the beam domain signal and noise eigenvectors, respectively, and Σ_{Bs} is the beam domain eigenvalues matrix.

3. BEAMSPACE MUSIC

The element-space MUSIC algorithm is well known and can be described as

$$P_{\mathrm{E\text{-}MUSIC}} = \frac{\boldsymbol{a}(\theta)^{\mathrm{H}}\boldsymbol{a}(\theta)}{\boldsymbol{a}(\theta)^{\mathrm{H}}\hat{\boldsymbol{E}}_n\hat{\boldsymbol{E}}_n{}^{\mathrm{H}}\boldsymbol{a}(\theta)} \tag{12}$$

where the estimated noise subspace $\hat{\boldsymbol{E}}_n$ from finite data record at sensors is used. When this algorithm is applied to the outputs of the beamformers, beamspace MUSIC yields [1,2]. Using the beam covariance matrix estimated from N snapshots to replace $\boldsymbol{R}_y$ defined in Eq. (8), the beamspace MUSIC spectrum is given by

$$\begin{aligned} P_{\mathrm{B\text{-}MUSIC}} &= \frac{[\boldsymbol{W}^{\mathrm{H}}\boldsymbol{a}(\theta)]^{\mathrm{H}}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{a}(\theta)}{[\boldsymbol{W}^{\mathrm{H}}\boldsymbol{a}(\theta)]^{\mathrm{H}}\hat{\boldsymbol{E}}_{\mathrm{Bn}}\hat{\boldsymbol{E}}_{\mathrm{Bn}}{}^{\mathrm{H}}\boldsymbol{W}^{\mathrm{H}}a(\theta)} \\ &= \frac{\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{W}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{a}(\theta)}{\boldsymbol{a}^{\mathrm{H}}(\theta)[\boldsymbol{W}\hat{\boldsymbol{E}}_{\mathrm{Bn}}][\boldsymbol{W}\hat{\boldsymbol{E}}_{\mathrm{Bn}}]^{\mathrm{H}}\boldsymbol{a}(\theta)} \end{aligned} \tag{13}$$

4. COMPUTER SIMULATIONS

In all computer simulations, a uniform linear array of 16 sensors with half-wavelength inter-element spacing is assumed, and 16 beams are formed pointing to directions $\pm\arcsin((2i-1)/16)$ $(i = 1,2,\cdots,8)$, which are ±3.6°, ±10.8°, ±18.2°, ±25.9°, ±34.2°, ±43.4°, ±54.3°, and ±69.6°, respectively.

In implementing beamspace MUSIC, only the outputs of a small number of beams with the sources to be estimated incident onto the main lobes are used for precise DOA estimation. Throughout this paper, we use outputs from 3 adjacent beams, the one with biggest response and two neighboring as the input data to the beamspace MUSIC algorithm. Three situations are investigated to illustrate the cases where isotropic noise is present at sensors, interferes at the sidelobe region of the beampattern are much stronger that the wanted sources, and the case when an interfere in the sidelobe region is strongly correlated with one of the incident signals.

4.1. Suppression of Isotropic Noise at Sensors

When isotropic noise is the dominant noise at sensors, conventional beamformers provide the optimum performance in determination DOAs and suppression of noise. In this case, the beams are formed in the conventional manner, as shown in Fig. 2. Five uncorrelated sources are assumed to be present in the far-field and arrive at the array in directions −42°, −40°, −5°, 30° and 33° with equal SNR = 5 dB at sensors.

Outputs from these beams based on 1,000 snapshots are given in Fig. 3, where it can be seen that outputs of beams pointing to −43.4°, 3.6° and 34.2° depict maximum

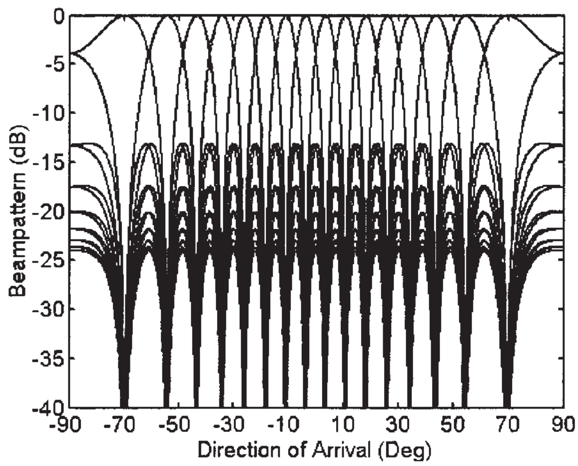


Fig. 2 Sixteen conventional beams from a uniform linear array of 16 isotropic sensors with half-wavelength spacing.

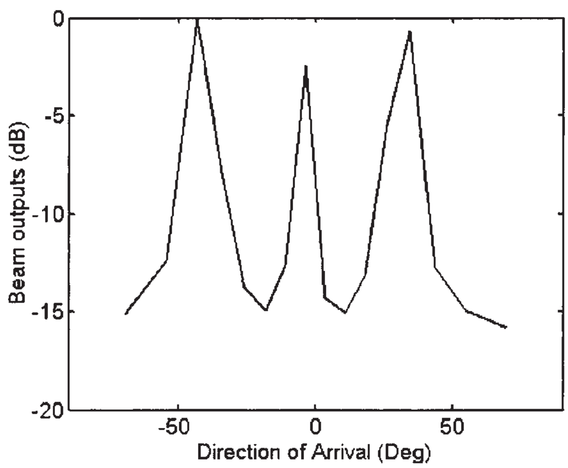


Fig. 3 Outputs from beams in Fig. 2.

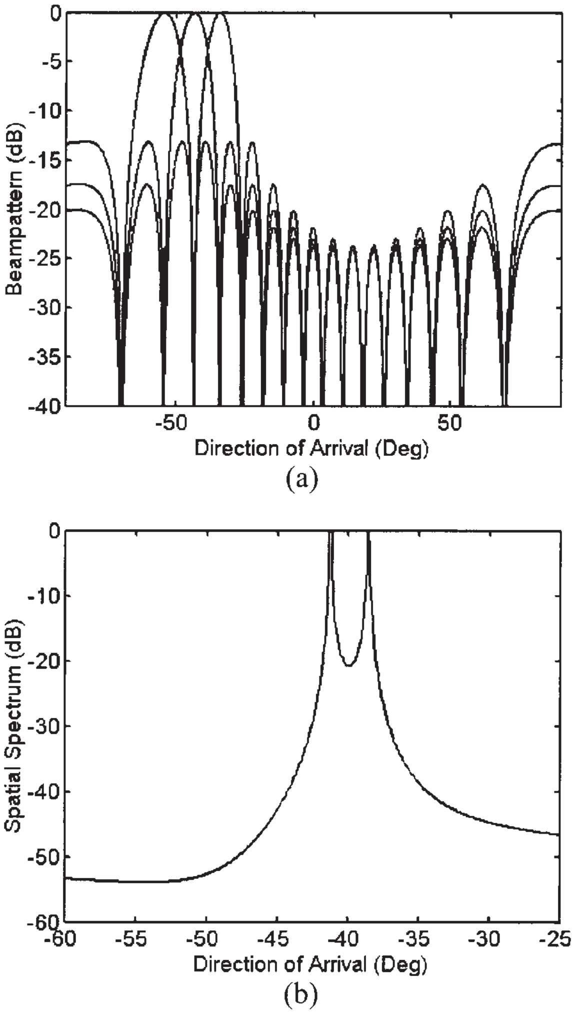


Fig. 4 (a) Beams pointing to −54.3°, −43.4° and −34.2°, and (b) fine estimates of two close sources.

energies and sources are regarded to be present around these directions. For fine estimates of DOAs, 3 beams centered at each of these directions are used in the beamspace MUSIC algorithm. For example, for the peak at −43.4°, outputs from three beams pointing to −54.3°, −43.4° and −34.2°, as shown in Fig. 4(a) are used in beamspace MUSIC and the resulting spatial spectrum is shown in Fig. 4(b), with two peaks in directions of −41.5° and −38.5°, which are good estimates of −42° and −40°. In the similar way, DOA estimates of other 3 sources can be obtained, which are −5°, 29.4° and 32.6°, respectively. By applying beamspace MUSIC, closely spaced sources are resolvable and DOAs are estimated with high accuracy.

4.2. Rejection of Strong Interferes outside Beam Region

The essence of beamforming is to let the signals incident onto the array within the mainlobe of the beampattern pass through and meanwhile reject all other signals in the sidelobe region. Since contributions from all sources in the sidelobe region are suppressed to some extent by the low sidelobes, better resolutions are expected in the beam domain processing, which has been verified by the example in previous subsection. However, unwanted sources might exist in the sidelobe region with strength much higher than that of the wanted sources, such as man-made jammers, then the performance of DOA estimation in beamspace will degrade seriously if conventional beamforming is applied to the sensor outputs directly. We will use another numerical example to illustrate the situation and how to overcome the problem.

In this case, only two uncorrelated sources were assumed in directions 30° and 33° with SNR = 5 dB at sensors. In addition, 7 uncorrelated interferes were assumed evenly distributed within [−3°,0°] with INR = 15 dB when arriving at the array. Interferes were assumed to be uncorrelated with signals and the noise.

Corresponding outputs of the 16 beams with 1,000 snapshots are given in Fig. 5, where 2 peaks show up in directions −3.6° and 34.2°. Obviously, the amplitude in direction 34.2° is much lower than that in −3.6° since SNR is lower than INR. If we use the 3 beams pointing in 25.9°, 34.2° and 43.4° in Fig. 4(a) for finer DOA estimates, the resulting beamspace MUSIC spectrum is given in Fig. 6, with a peak in direction 31.6°. Due to strong interference in the sidelobe region, the beamspace MUSIC algorithm failed to resolve two sources.

4.2.1. Low sidelobe beampattern design

To reduce the effect from interference in the sidelobe region, one way at hand is low sidelobe design. For the situation considered, Dolph-Chebyshev weighting with −50 dB desired sidelobe level was applied [3]. Figure 7(a) is the 3 beams therefore obtained pointing to 25.9°, 34.2° and 43.4°, and Fig. 7(b) is the spectrum based on these 3 beams, with 2 peaks in directions 34.2° and 30.5°. It can be seen that the two closely placed sources were resolved even though strong interferes exist in the sidelobe region.

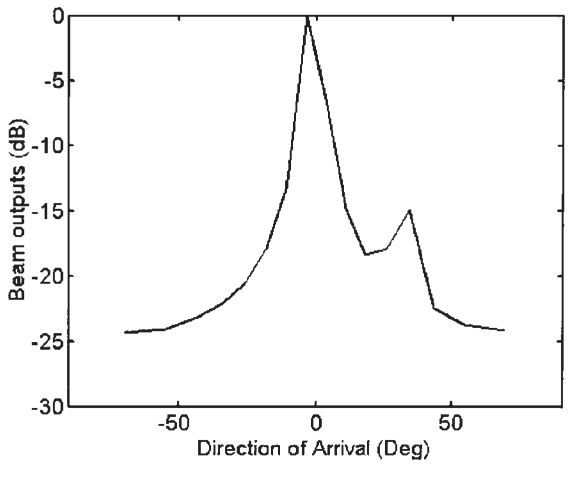


Fig. 5 Outputs of beams.

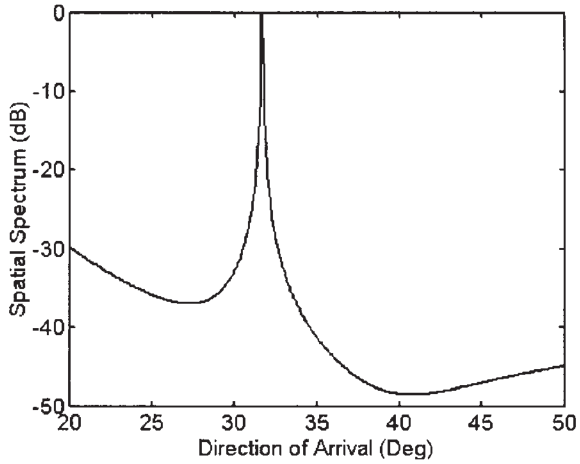


Fig. 6 Spectrum of beamspace MUSIC.

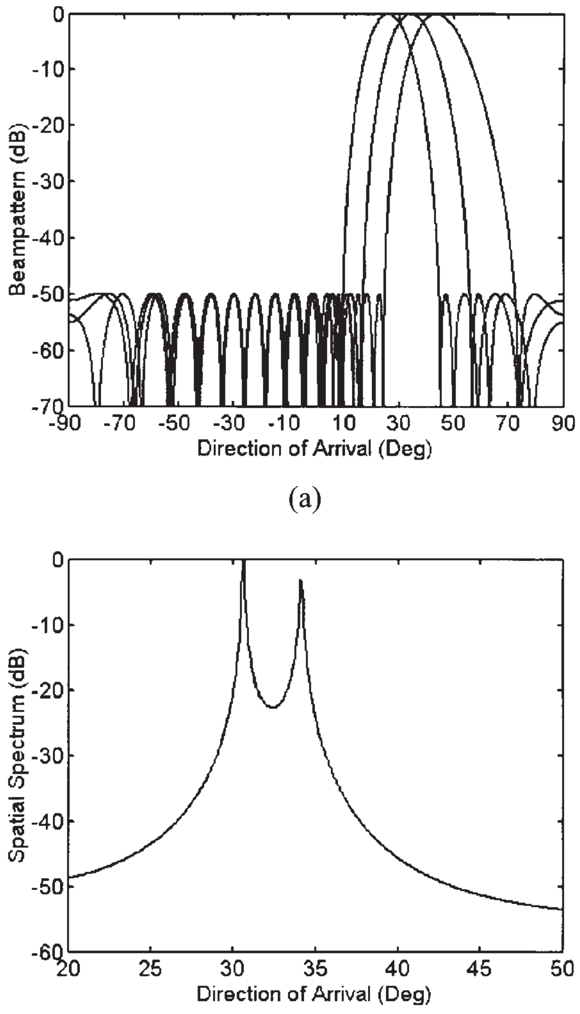


Fig. 7 (a) Beams with DC weighting, and (b) corresponding spectrum of beamspace MUSIC.

4.2.2. Adaptive beampattern design

With low sidelobe beampattern design, the sidelobe level is reduced at the cost of wider mainlobes, and at the same time array gain will also drop [4]. In fact, it is not necessary to retain low levels in all sidelobe regions. In stead, a deep null in the corresponding direction would be enough for interfere suppression. MVDR adaptive beamforming is a good choice for this circumstance [5]. For the parameters in theis case, results by directly applying the outputs from MVDR beamformers for DOA estimation are given in Fig. 8(a). Deep nulls were formed as desired but distortion in the beampatterns led to deficiency in DOA estimation, as shown in Fig. 8(b). Utilize diagonal-loading technique in MVDR beamforming to reduce the beampattern distortion which might exist due to ill-conditioned covariance matrix, results for the same situation are depicted in Fig. 9. From the plots we can see that, although high sidelobes exist in the beampattern compared to those in Fig. 7(a), good DOA estimates were obtained since the beampattern nulls out strong interference and retain information of wanted sources in the mainlobes.

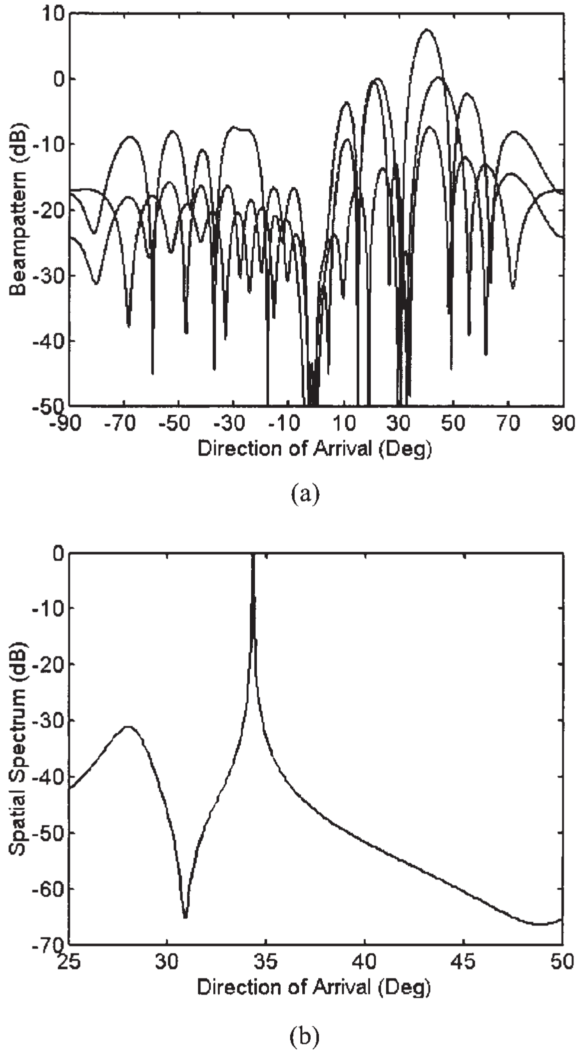


Fig. 8 Beampatterns and beamspace MUSIC spectrum based on ordinary adaptive MVDR beamformers.

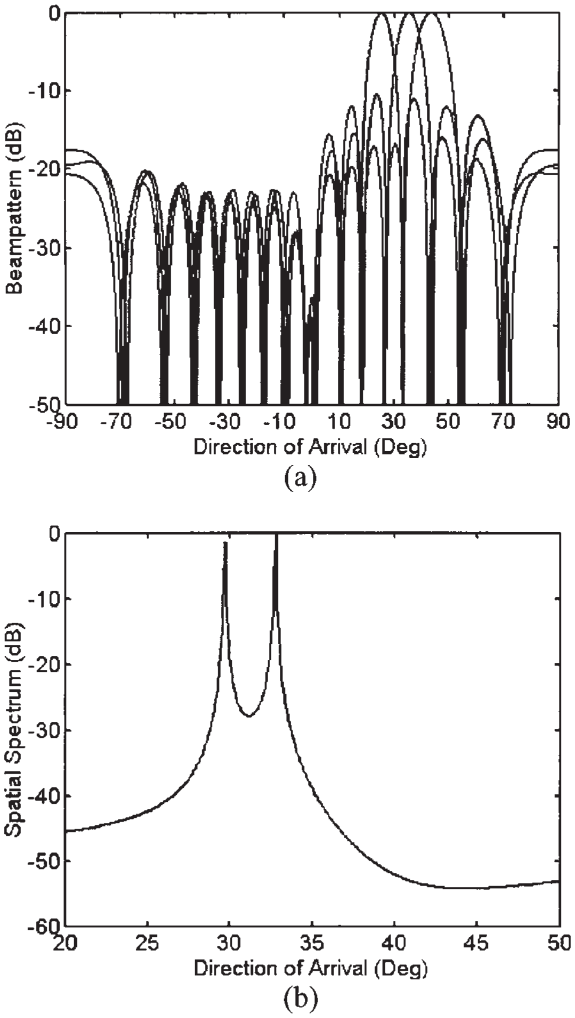


Fig. 9 Beampatterns and beamspace MUSIC spectrum based on diagonal-loading adaptive MVDR beamformers.

4.3. Rejection of Strong Correlated Interfere Outside Beam Region

In practical applications, it is quite possible that the man-made interfere is correlated with one of the wanted sources. Under this circumstance, the MVDR beamforming based on the inverse of the sample covariance matrix will fail to work. Especially when the interfere with much higher strength is fully correlated with wanted signals, the wanted signals will also be suppressed if no other remedy is applied.

For this situation, the robustness of conventional techniques is combined with the adaptivity of the MVDR beamforming method, with the beamforming vectors being generated by virtual interferes in the unwanted source region rather than from the sample covariance matrix.

In computer simulations, two uncorrelated sources were assumed in directions 30° and 33° with SNR = 5 dB at sensors. In addition, one interfere was assumed in −3° and fully correlated with the source in 30°. The ratio of the interfere strength to that of the source was 25 dB.

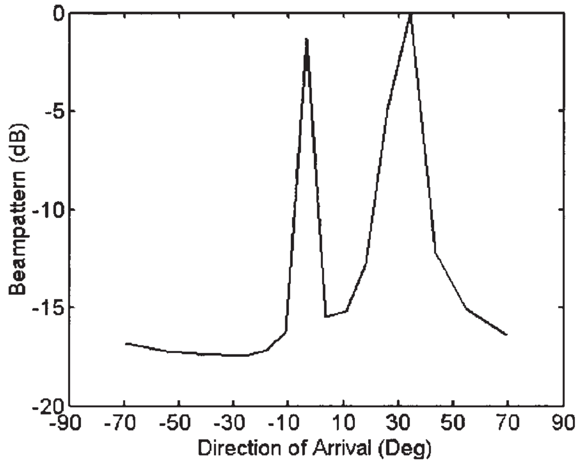


Fig. 10 Beam outputs.

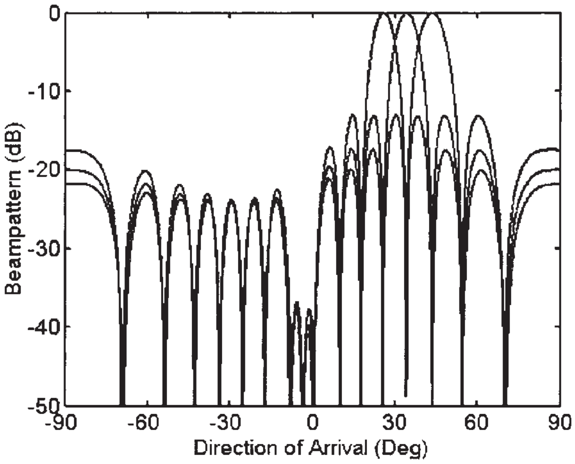
From the outputs of conventional beamformers, it can be seen that there were sources around −3.6° and 34.2° (Fig. 10). To resolve sources around 34.2°, we could set interferes around −3.6°, and vise versa. For the former case, virtual interferes were assumed around−3.6° and the resulting non-adaptive beampatterns pointing in 25.9°, 34.2° and 43.4° are shown in Fig. 11(a). If the sensor outputs passed through these 3 beams and then beamspace MUSIC was applied, the spatial spectrum in Fig. 11(b) yielded, showing two distinct peaks at 30° and 33°.

4.4. Performance of Beamspace MUSIC

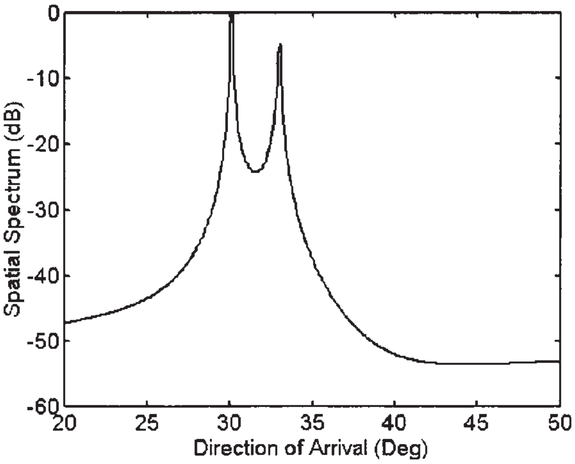
The make this paper self-contained, we briefly mention the performance of beamspace MUSIC herein, in comparison with that of its element-space counterpart. It has been verified that 1) beamspace MUSIC reduces the SNR threshold for resolution [6]; 2) variances in DOA estimates are increased no matter how the weighting matrix $\boldsymbol{W}$ is designed [1,7,8]; and 3) robustness to sensor error is improved [9].

5. SUMMARY

The beamspace MUSIC algorithm is studied with three numerical examples to illustrate the importance of beam design so as to reach the expected performance of beamspace methods. From the results it can be concluded that, with properly designed beams, beamspace MUSIC offers better resolution abilities and, for both weakly and strongly correlated interferes, beamforming based on the loading-diagonal MVDR technique offers superior performance.



(a)



(b)

Fig. 11 Beampatterns and beamspace MUSIC spectrum based on MVDR beamformer outputs with virtual interferes.

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