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Numerical analysis of sound absorption characteristics of the sound absorbing wedge

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1. Introduction

Generally, the sound absorption coefficient of sound absorbing material with unevenness in the surface like a sound absorbing wedge is higher than that of even surface. Because, a sound wave reflects repeatedly to a sound-absorbing material. An acoustic field was analyzed using a finite element method, and it was found that the sound absorption characteristics could be improved by using a unevenness in the surface. Moreover, numerical results by the finite difference method and the finite element method were compared.

2. Analysis of a 1-dimensional acoustic field using the finite element method

Based on the principle of minimum potential energy, we approximately solve the sound field by the finite element method. Regarding the sound pressure $p(x,y,z)$ in the solid element e, we give the kinetic energy, the potential energy, the work field made by the displacement of the particle u applied from the outside as follows [1].

$$T_e = \frac{1}{2}\frac{1}{\rho_0\omega^2}\iiint_e (\text{grad } p)^2 dv \tag{1}$$

$$V_e = \frac{1}{2}\frac{1}{k_0}\iiint_e p^2 dv \tag{2}$$

$$W_e = \int_A pudS \tag{3}$$

ρ_0: Density of air, ω: Angular frequency, k_0: Bulk modulus of air ($= \rho_0 c^2$), A: The border plane about one element specified by the displacement of the particle, u: volume displacement. Therefore, the functional about the whole system is given as follows

$$L = \sum_e (V_e - T_e - W_e) \tag{4}$$

We assume that the sound pressure at the arbitrary point between x_i and x_j in Fig. 1 can be expressed with the following linear function shown in Eq. (5).

$$p(x) = \alpha_0 + \alpha_1 x \tag{5}$$

We can write this equation in matrix form.

$$\begin{bmatrix} p_i \\ p_j \end{bmatrix} = \begin{bmatrix} 1 & x_i \\ 1 & x_j \end{bmatrix}\begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} \tag{6}$$

Equations (1) and (2) can be expressed with sound pressure vector, at each node taking the 1-dimensional problem into consideration.

$$T_e = \frac{1}{2}\frac{1}{\rho_0\omega^2 l_e}\boldsymbol{P}_e^{\mathrm{T}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\boldsymbol{P}_e \tag{7}$$

$$V_e = \frac{1}{2}\frac{l_e}{6k_0}\boldsymbol{P}_e^{\mathrm{T}}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\boldsymbol{P}_e \tag{8}$$

Variation of the functional is set to $dL = 0$ and the following equation for element e is obtained.

$$\frac{1}{\omega^2\rho_0 l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\boldsymbol{P}_e - \frac{l_e}{6k_0}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\boldsymbol{P}_e = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_e \tag{9}$$

The right side of the equation expresses volume displacement which passes a node. Multiplying $i\omega$ by Eq. (9), we obtain the following Eq. (10).

$$\frac{1}{i\omega\rho_0 l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\boldsymbol{P}_e + \frac{i\omega l_e}{6k_0}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\boldsymbol{P}_e = -\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_e \tag{10}$$

In the sound-absorbing material, there are following relations shown in Eqs. (11) and (12) is substituted in Eq. (10).

$$\lambda^2 = -\omega^2/c^2 \tag{11}$$

$$\rho_0 = Z\lambda/i\omega \tag{12}$$

λ: Propagation constant c: Acoustic velocity Z: Acoustic impedance. The results by the finite element method and the finite difference method and the experimental value are shown in Fig. 2. Acoustic impedance and propagation constant were measured by 2-microphone method. From Fig. 2, although there are some differences between the finite element method and experiment in a low frequency region, we found that overall it is almost same value.

3. Analysis of a 2-dimensional acoustic field using the finite element method

We use the following linear function for the triangular element shown in Fig. 3.

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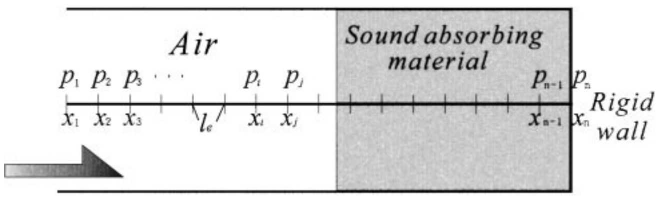

Fig. 1 Line element.

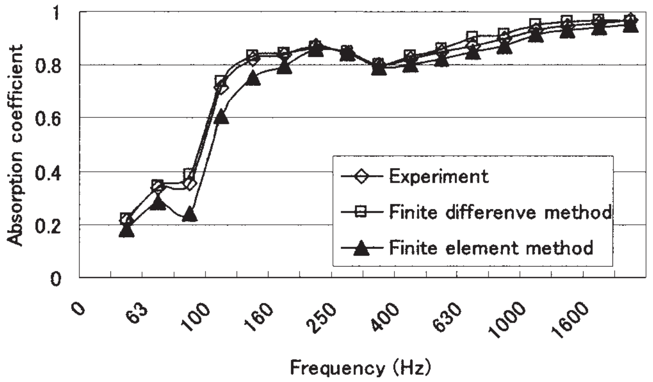

Fig. 2 Comparison of the sound absorption coefficient glass wool 32 kg/m^3 thickness 200 mm.

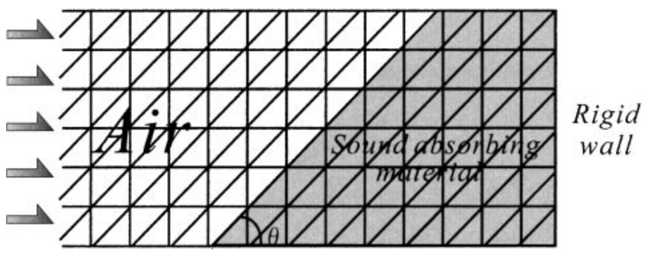

Fig. 3 Triangular element.

$$p = \alpha_1 + \alpha_2 x + \alpha_3 y = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \tag{13}$$

Kinetic energy and potential energy can be expressed with sound pressure vector $\boldsymbol{P}_e$.

$$T_e = \frac{1}{2}\frac{1}{4\rho_0\omega^2\Delta_e}\boldsymbol{P}_e{}^{\mathrm{T}} \begin{bmatrix} \Theta_2+\Theta_3 & -\Theta_3 & -\Theta_2 \\ -\Theta_3 & \Theta_1+\Theta_3 & -\Theta_1 \\ -\Theta_2 & -\Theta_1 & \Theta_1+\Theta_2 \end{bmatrix} \boldsymbol{P}_e \tag{14}$$

$$V_e = \frac{1}{2}\frac{\Delta_e}{12k_0}\boldsymbol{P}_e{}^{\mathrm{T}} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \boldsymbol{P}_e \tag{15}$$

Δ_e: Area of an element Θ_i: $\cot\theta_i$ Variation of the functional is set to $\partial L = 0$, then the following linear equation for element e is obtained. The calculated results of the finite element method are shown in Fig. 4. When the degree of angle of inclination is 90 degrees, the result of one dimension and the result of two dimensions became the same. Although sound absorption coefficient in the case of $\theta = 60°$ and $75°$ is high at the low frequency region, from 50 to 80 Hz sound absorption coefficient becomes higher as θ becomes smaller on the whole.

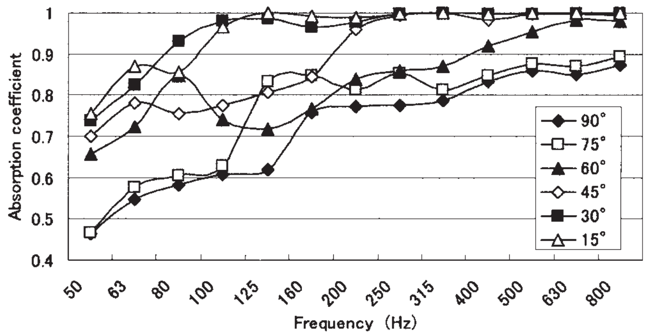

Fig. 4 Comparison of the sound absorption coefficient glass wool 32 kg/m^3 thickness 200 mm.

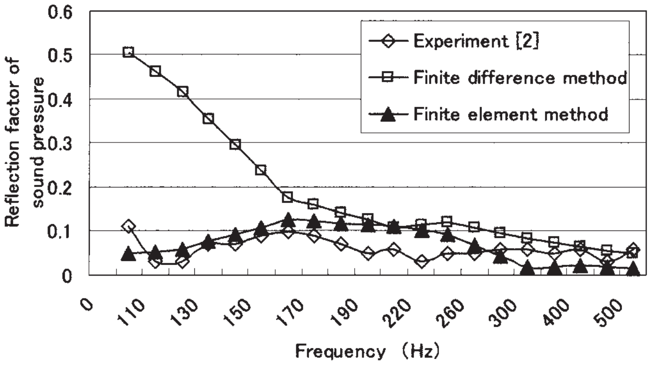

Fig. 5 Comparison of the sound pressure reflection coefficient.

Then the calculated results by the finite element method and the finite difference method and experimental value [2] for sound absorbing wedge are shown in Fig. 5. The inclination angle of the sound absorbing wedge is $\theta = 9.46°$. A sound pressure reflection coefficient γ is used. Because, the absorption coefficient is nearly equal to 1, it is hard to know change of the value.

$$\gamma = \sqrt{1-\alpha} \tag{16}$$

α: sound absorption coefficient

In the low frequency region, there is a difference between the calculated value by the finite difference method and the experimental value. On the other hand, the finite element method and the experimental value agree well in all frequency regions.

4. Conclusions

The acoustic field was analyzed using a finite element method, and the numerical result agreed well with the experimental value. We found that the sound absorption characteristics could be improved by changing the inclination angle of the sound absorbing wedge.

References

[1] Y. Kagawa, *Shindou·Onkyoukougaku/Kiso to Ouyou* (Baifukan Co., Tokyo, 1990), p. 160.
[2] Data by Kobayasi Institute of Physical Research.