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TECHNICAL REPORT

Estimation of acoustic focal position in an ultrasonic field

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Abstract: We applied the ultrasonic diffraction method for recognizing an acoustic structure of muscle tissue and diagnosising a structural change of the part of disease disturbance. However, in this signal processing application, the recognition accuracy depends on the accuracy of location of the focal plane. It is also necessary to estimate the location of the focal plane within short time. On the other hand, The MUSIC algorithm is high-resolution algorithm for estimating arrival direction of a sound sources. In this paper, we propose a method to estimate the acoustic focal point using the MUSIC algorithm at two points, then calculate the location of the focus based on these two estimates of the focal point.

Keywords: Ultrasonic diffraction method, Concave Transducer, Autocorrelation Image, Microphone array, MUSIC Algorithm

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1. INTRODUCTION

Ultrasonic signal processing techniques are utilized in ultrasonic diagnosis, measurement and imaging, as well as acoustic holography and three-dimension object recognition. OKADA has developed a method for recognizing the acoustic structure of muscle tissue based on ultrasonic diffraction imaging at the focal plane [1,2]. In this signal processing application, the recognition accuracy of the structure of muscle tissue depends on the accuracy of location of the focal plane produced by a concave transducer. As the morphology and properties of muscle tissue change rapidly, it is necessary to estimate the location of the focal plane within a short time. The measurement of an acoustic field around using a miniature hydrophone is typically time consuming and may disturb the acoustic field. This paper describes a new estimation method for acoustic focal position without these faults.

2. FORMULATION OF THE PROBLEM

Figure 1 shows the system for obtaining the ultrasonic diffraction image. The system detects the root mean square value of sound pressure of the diffracted wave at the focal plane using a miniature hydrophone with a piezoelectric transducer element 0.14 [mm] diameter, 0.15 [mm] thick. The primary transducer is a circular concave transducer of 25 [mm] in diameter made of polyvinylidene fluoride. The emission is a 5.0 [MHz] pulse with a velocity of 1,466.0 [m/s] in water.

The distribution of sound pressure generated by the transducer is shown in Fig. 2, given as the root mean square of sound pressure measured at 55 points at interval of 2.0 [mm] on the v-axis and 241 points at an interval of 0.1 [mm] on the u-axis. The peak value of sound pressure corresponds to the location of the focus of the transducer, and defines the origin of the u-v coordinate system.

Figure 3 shows a schematic of the experimental sample. The sample consists of an array of nylon lines, each 0.3 [mm] in diameter and separated by 1.0 [mm]. The sample was placed at a position 15 [mm] ($= 50\lambda$, $\lambda = 0.3$ [mm]) away from the transducer. The distance between the focal plane and the transducer surface was 87.4 [mm] ($= 291.3\lambda$). Figure 4 shows an autocorrelation image of the sample, calculated from the data observed at the focal plane. Figure 5 shows an autocorrelation image calculated from the data observed 7.6 [mm] ($= 25.3\lambda$) along the v-axis in the positive direction away from the focal plane. The autocorrelation image calculated from data observed 17.4 [mm] ($= 58\lambda$) along the v-axis in the negative direction away from the focal plane is shown in Fig. 6

Clearly, it is difficult to recognize the structure of the sample at 7.6 [mm] or further away from the focal plane. In this system, the recognition precision depends on the

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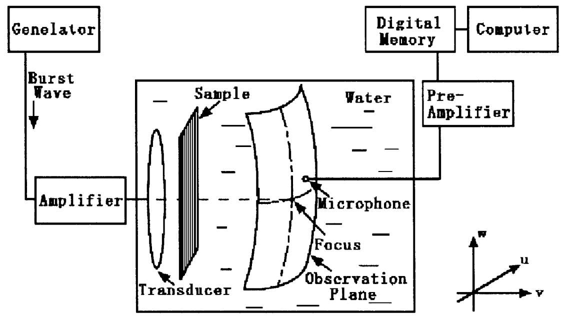


Fig. 1 Ultrasonic diffraction imaging system.

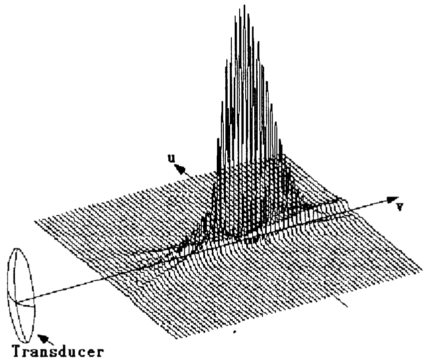


Fig. 2 Distribution of sound pressure in the tank.

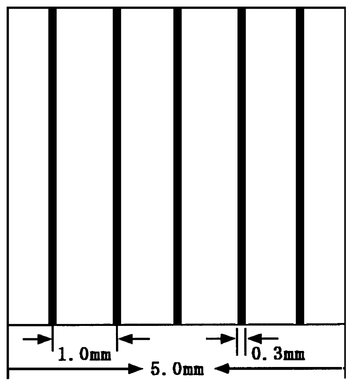


Fig. 3 Experimental sample.

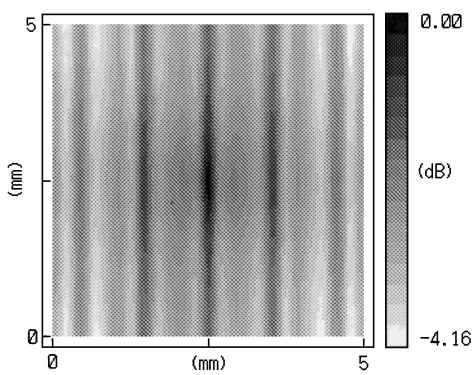


Fig. 4 Autocorrelation image of sample at focal plane.

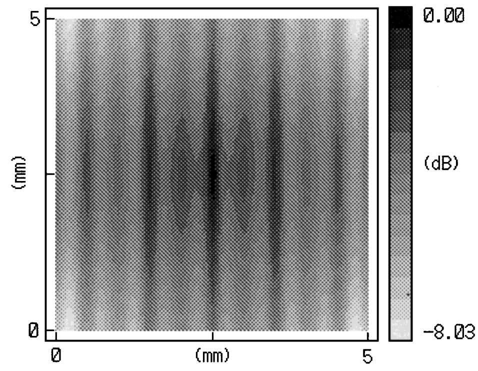


Fig. 5 Autocorrelation image of sample at 7.6 [mm] away from focal plane on v-axis.

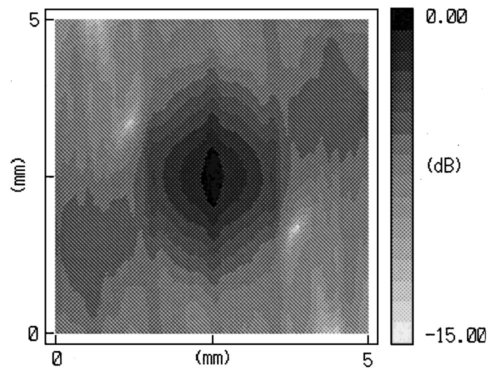


Fig. 6 Autocorrelation image of sample at −17.4 [mm] away from focal plane on v-axis.

accuracy of location of the focal plane. However, it is difficult to observe the distribution of sound pressure in a short time using a mobile hydrophone, and there is a possibility that the presence of the hydrophone may cause interference.

The estimation of arrival direction and location of a sound source in an audible acoustic field has been widely studied. The MUSIC algorithm is a high-resolution algorithm for estimating the arrival direction of a sound source [3,4]. In this paper, we propose a method of estimating the direction of the acoustic focal point using the MUSIC algorithm at two points. The location of the focus is calculated based on these two estimates of the focal point direction. In this study, we conduct experiments of focal direction estimation and examine the accuracy of the results.

3. ESTMATION OF ARRIVAL DIRECTION—MUSIC ALGORITHM

Figure 7 shows the block diagram for estimation of the acoustic focal direction. The microphone array consists of M microphone elements, and it is set in the far field.

The signal received by the array are linear combinations of the signal K and noise. Thus, the recevied signal

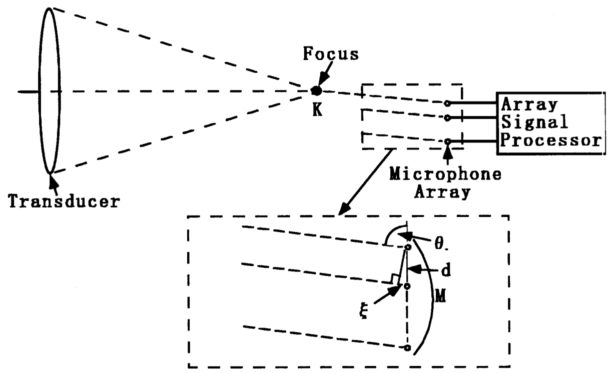


Fig. 7 Block diagram for estimation of a focus.

vector $\boldsymbol{x}(t)$ is expressed as follows [3]:

$$\boldsymbol{x}(t) = \boldsymbol{AS}(t) + \boldsymbol{n}(t) \tag{1}$$

where

$$\begin{aligned}\boldsymbol{x}(t) &= [x_1(t), x_2(t), \cdots, x_M(t)]^T\\ \boldsymbol{A} &= [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_K)]\\ \boldsymbol{S}(t) &= [S_1(t), S_2(t), \cdots, S_K(t)]^T\\ \boldsymbol{n}(t) &= [n_1(t), n_2(t), \cdots, n_M(t)]^T.\end{aligned}$$

In this paper, we assume a number of foci K in direction θ. The steering vector $\boldsymbol{a}(\theta_k)$ $(1 \leq k \leq K)$ is given by

$$\boldsymbol{a}(\theta_k) = [1, \epsilon^{-j\omega\tau_k}, \epsilon^{-j\omega 2\tau_k}, \cdots, \epsilon^{-j\omega(M-1)\tau_k}] \tag{2}$$

$$\tau_k = \frac{d\cos\theta}{c} \tag{3}$$

The expected covariance matrix $\boldsymbol{R}$ of $\boldsymbol{x}(t)$ is given by

$$\boldsymbol{R} = E[\boldsymbol{x}(t)\boldsymbol{x}^*(t)] = \boldsymbol{ASA}^* + \sigma^2\boldsymbol{I} \tag{4}$$

where the asterisk represents the conjugation, σ^2 denotes the power of noise, and $\boldsymbol{I}$ is the identity matrix. Let $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_M$ denote the eigenvalue of $\boldsymbol{R}$. Since rank $(\boldsymbol{ASA}^*) = K$, it follows that

$$\begin{aligned}&\lambda_m > \sigma^2 \text{ for } m = 1, \cdots, K; \qquad \text{and}\\ &\lambda_m = \sigma^2 \text{ for } m = K+1, \cdots, M.\end{aligned} \tag{5}$$

The MUSIC spectrum is defined as follows, and is a function of θ corresponding to the focal direction of the transducer [3,5,6].

$$P_{\text{MUSIC}}(\theta) = \frac{1}{\boldsymbol{a}^H(\theta)\boldsymbol{R}_n{\boldsymbol{R}_n}^H\boldsymbol{a}(\theta)} \tag{6}$$

Because the signal subspace is orthogonal to noise subspace, we can find K peaks of P_{MUSIC} at θ corresponding to the focal direction.

In order to apply the MUSIC algorithm to estimating the focal direction, it is necessary to know the focus number K. There are numerous methods for determining K, such as a information criterion (AIC) [7] and the minimum description length (MDL) principle [8,9]. Here, we applied the AIC method to estimate K as follows.

$$\text{AIC}(K) = (M-K)\ln\left(\frac{\dfrac{1}{M-K}\displaystyle\sum_{i=K+1}^{M}\lambda_i}{\displaystyle\prod_{i=K+1}^{M}\lambda_i^{-(M-K)}}\right) + K(2M-K) \tag{7}$$

where λ_i is the eigenvector corresponding to covariance matrix $\boldsymbol{R}$.

4. EXPERMENTAL CONDITIONS

The experimental system for estimating the focal direction is shown in Fig. 8.

In this system, the origin is the assumed location of the focus, and the other experimental parameters are as follows.

- Tank size (width×length×height): 200 [mm]×148 [mm]×50 [mm]
- Frequency: $f = 5.0$ [MHz]
- Sound velocity: $c = 1,482.7$ [m/s].
- Temperature: 20 [°C]
- Number of microphones: $M = 11$
- Space between microphones: $d = 0.1$ [mm]
- Coordinates of measuring points correspond to the coordinate of microphone No. 1 in the microphone array
  - (a) v-axis = 23.5 [mm](78.3λ)
    u-axis = −5.0 [mm](16.7λ)
    w-axis = 0.0 [mm]
    Focal direction angle θ from microphone No. 1 = 80.3 [deg]
  - (b) v-axis = 23.5 [mm](78.3λ)
    u-axis = 4.0 [mm](13.3λ)
    w-axis = 0.0 [mm]
    Focal direction angle θ from microphone No. 1 = 102.0 [deg]

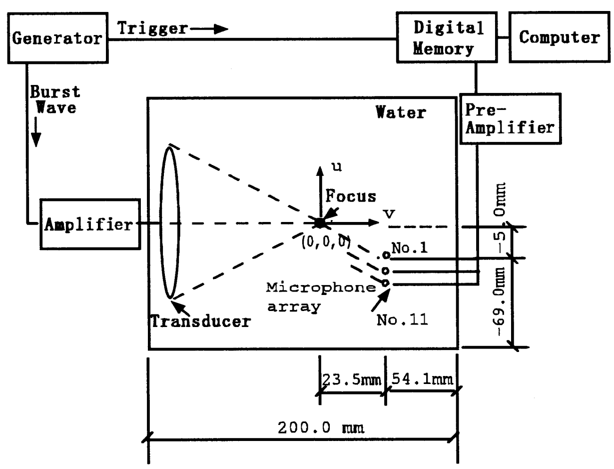
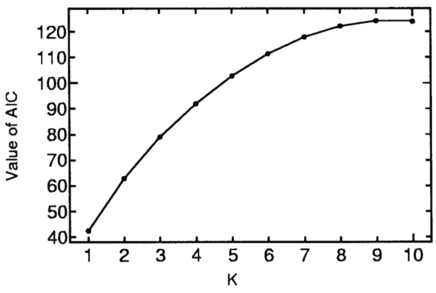
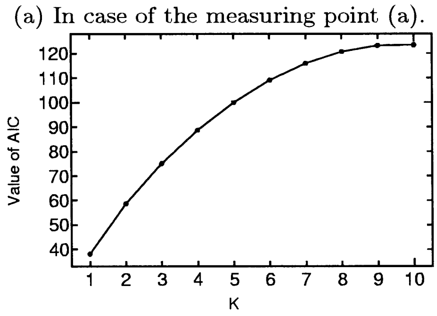


Fig. 8 Experimental system for estimating focal location.

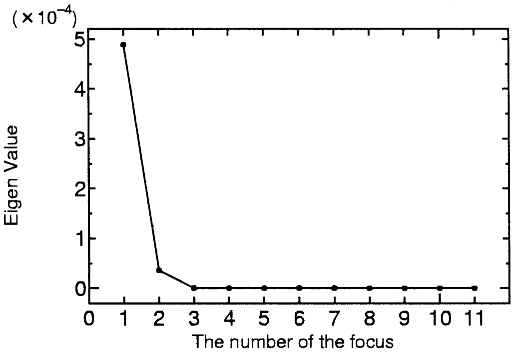


(a) In case of the measuring point (a).

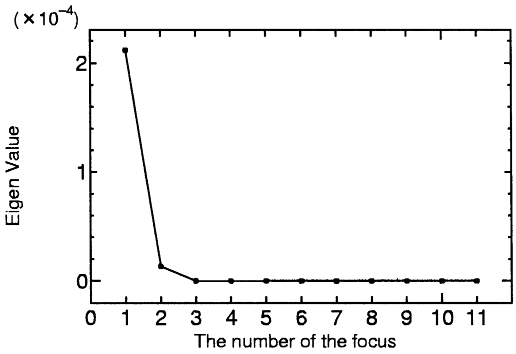


(b) In case of the measuring point (b).

Fig. 9 Determination of *K* by AIC.



(a) Measurement point (a)



(b) Measurement point (b)

Fig. 10 Determining of *K* from distribution of eigenvalues.

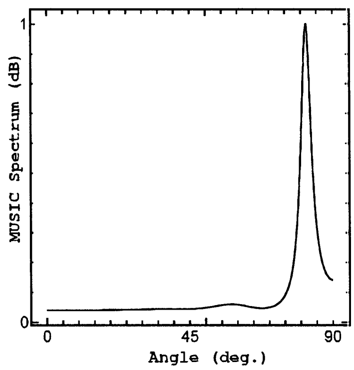


Fig. 11 Estimation by MUSIC algorithm at measuring point.

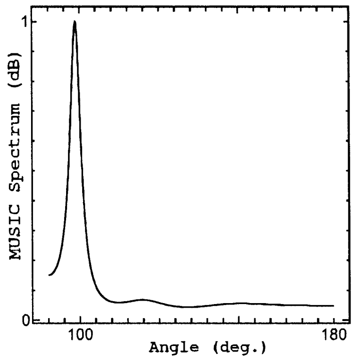


Fig. 12 Estimation by MUSIC algorithm at measuring point.

5. RESULTS AND DISCUSSION

The AIC values calculated from Eq. (7) at measuring points (a) and (b) are shown in Fig. 9. The minimum value of K in AIC(K) of corresponds to the actual number of foci, in this experiment there is 1 focus ($K = 1$). In order to validate this result, the distribution of eigenvalues λ is examined, as shown in Fig. 10. The peak value in this distribution represents the number of foci, in this case 1.

Figures 11 and 12 show estimated results of the focal direction θ using the MUSIC algorithm at two measurement points (a) and (b). The estimated focal direction θ at measuring point (a) is 81.1 [deg], and the error between the estimated and actual angle (80.3 [deg]) is 0.8 [deg]. The estimated focal direction θ at measuring point (b) is 98.6 [deg], which differs from the actual (102.0 [deg]) by 3.4 [deg].

The true position of the focal point can then be calculated geometrically from these two estimates. In this case, the calculated focal location is 0.4 [mm] on the u-axis and 5.7 [mm] on the v-axis from the true location (0,0). As mentioned above, it is possible to obtain the structure of muscle tissue if the position of the focal plane is estimated to within 7.6 [mm]. Therefore, the present results indicate that the MUSIC algorithm is potentially useful for estimating the focal direction and location with sufficient accuracy for muscle tissue imaging.

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