Acoust. Sci. & Tech. **22**, 6 (2001)

ACOUSTICAL LETTER

Feedback active noise control system based on $\mathcal{H}_\infty$ control

Shuichi Adachi[1], Michiya Ogawa[1], Akira Takahashi[2] and Hisashi Sano[2]

[1]*Faculty of Engineering, Utsunomiya University,*
7–1–2 Yoto, Utsunomiya, 321–8585 Japan
[2]*Honda R&D Co., Ltd.,*
4730 Haga-machi, Haga-gun, Tochigi, 321–3393 Japan

(*Received 4 June 2001, Accepted for publication 27 July 2001*)

Keywords: Active noise control, Feedback control, Acoustic field, $\mathcal{H}_\infty$ control
PACS number: 43.50.Ki

1. Introduction

Feedback active noise control (FB-ANC) [1] can eliminate low-frequency noise below 300 Hz which substantially constitutes the road noise inside an automobile. An FB-ANC system is developed by first constructing a lower-order infinite impulse response (IIR) model for the acoustic transmission system, this model incorporates a loudspeaker, an acoustic field and a microphone. A feedback controller is then developed by $\mathcal{H}_\infty$ control based on the model. Digital control is implemented by a digital signal processor (DSP) and the effectiveness of the proposed FB-ANC system is verified through experiments.

2. Modeling the acoustic transmission system

The FB-ANC system shown in Fig. 1 is considered, where P is the transfer function from the secondary source (loudspeaker) to the error sensor (microphone) and C represents the transfer function of the FB-controller which attenuates the noise level around the error sensor.

The system identification technique has been widely used as a modeling method for feed-forward (FF) ANC systems [1] where a finite impulse response (FIR) model with a number of parameters is identified by the least mean square (LMS) algorithm. However, in order to apply $\mathcal{H}_\infty$ control as an FB-ANC design, it is necessary to describe the nominal model by a lower-order finite-dimensional rational transfer function, that is, not an FIR but an IIR model. Consequently, an IIR model was adopted for the plant model.

The plant is composed of three parts: a loudspeaker, an acoustic field and a microphone. The physical model made up from these elements is described by

$$P_p(s) = \frac{2.03 \times 10^{-2} s^2}{9.03 \times 10^{-3} s^2 + 12.43 s + 3512.6} \cdot \exp(-2.94 \times 10^{-4} s). \tag{1}$$

A Panasonic WS-A10-K is used as the loudspeaker and the distance from the loudspeaker to the microphone is 0.1 m [2]. The representation of the plant in Eq. (1) is a 2nd-order lumped-parameter system and a distributed-parameter system with a time delay. Latter involves an infinite-dimension, so the following Pade approximation is applied to give a finite-dimension system.

$$P_d(s) = \exp(-2.94 \times 10^{-4} s) \approx \frac{-s + 80}{s + 80} \tag{2}$$

The dynamics of the DSP need to be considered since it is being used for control. This can be approximated by

$$P_f(s) \approx \frac{-0.0008 s + 1}{0.0008 s + 1} \tag{3}$$

for the Active-5 DSP (Redec Co.) that we use.

The plant can thus be described by the 4th-order IIR model represented by

$$\begin{aligned} P(s) &= P_p(s) P_d(s) P_f(s) \\ &= \frac{0.0203 s^2}{0.00903 s^2 + 12.43 s + 3512.6} \\ &\quad \cdot \frac{-s + 80}{s + 80} \frac{-0.0008 s + 1}{0.0008 s + 1}. \end{aligned} \tag{4}$$

A Bode plot of the model is shown in Fig. 2, in which the dashed line shows the system identification based on the FIR model with 64 parameters. It is clear that the lower-order IIR model has a good accordance with the conventional FIR model over frequency range from 50 to 300 Hz.

3. Feedback controller design incorporating $\mathcal{H}_\infty$ control

Feedback controller $C(s)$ is designed based on the model Eq. (4). The control specifications of the FB-ANC system are as follows:

1. Stabilization of the closed-loop system
2. Attenuation of the sensitivity function in the low-frequency range, especially around 150 Hz

The sensitivity function, which is the transfer function from the noise d to the error e is defined by

$$S(s) = \frac{1}{1 + P(s) C(s)}.$$

$\mathcal{H}_\infty$ control has been adopted because it can assign the appropriate attenuation performance according to the sensitivity function. The nominal performance can be described by

$$\|W_S(s) S(s)\|_\infty < 1 \tag{5}$$

where $W_S(s)$ is a frequency weighting function.

To meet the control specifications, the frequency weighing function

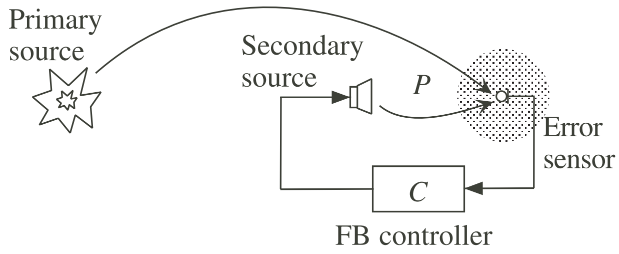

Fig. 1 Configuration of FB-ANC system.

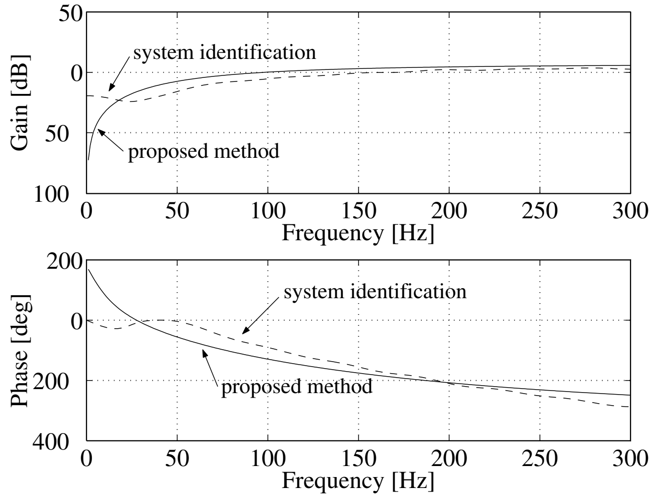

Fig. 2 Bode plot of the proposed model (solid line) and system identification model (dashed line).

$$W_S(s) = \frac{Ks^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{6}$$

was applied, where $K = 0.6$, $\zeta = 1/14$ and $\omega_n = 2\pi \times 150$. The $\mathcal{H}_\infty$ controller was designed by the Glover and Doyle's method [3]. The resulting sensitivity function is shown in Fig. 3, indicating a noise attenuation by the feedback controller of about 15 dB at a frequency of 150 Hz.

A bilinear transformation was applied to obtain discrete-time controller $C(z)$, where the sampling rate was 5 kHz. The values for these coefficients are not specified due to space limitation.

4. Experiment

The performance of the proposed feedback controller was tested by the experimental apparatus shown in Fig. 4. Pseudo road noise was used to simulate the noise inside an automobile.

The plot of the sound pressure level shown in Fig. 5 indicates that the FB-ANC system attenuated the noise level at 150 Hz about 15 dB, this being the predicted attenuation level based on the sensitivity function.

5. Conclusion

The FB-ANC system described here was designed systematically by $\mathcal{H}_\infty$ control based on the lower-order IIR model which was built by use of the physical insight of the acoustic transmission system. The effectiveness of this system was confirmed by experiments applying simulated road noise.

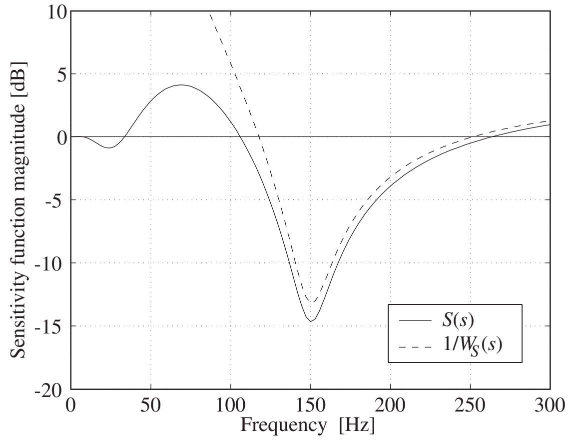

Fig. 3 Sensitivity function.

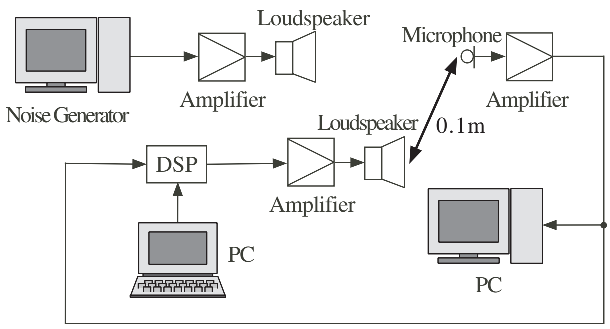

Fig. 4 Configuration of experimental apparatus.

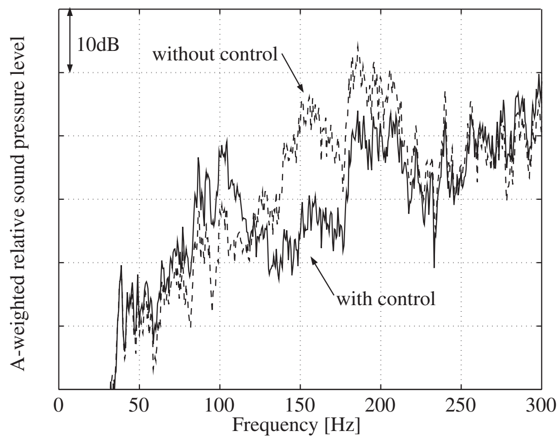

Fig. 5 Sound pressure level.

References

[1] P. A. Nelson and S. J. Elliot, *Active Control of Sound* (Academic Press, London, 1992).

[2] S. Adachi and H. Sano, "Modeling of acoustic field for feedback active noise control", *MOVIC '98*, 683, Zurich (1998).

[3] K. Glover and J. C. Doyle, "State-space formulae for all stabilizing controller that satisfy an $\mathcal{H}_\infty$-norm bound and relations to risk sensitivity", *Syst. Control Lett.*, **11**, 167 (1988).