

PAPER

A new percussion instrument “hokyo” made of Sanukite

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Abstract: In this paper, we introduce a new percussion instrument, “hokyo,” made of a particular stone, “Sanukite,” and study its vibrational properties. The hokyo has a unique and somewhat complicated structure. Vibrational modes of the hokyo were analyzed by the finite element method, and their existence was verified by fast Fourier analysis of its tone and experimental modal analysis. The vibrational modes of the hokyo are principally determined by the rather simple behavior of the centered inner rod with a quasi-fixed end and a free end. The term “quasi-fixed end” means that the inner rod is not fixed exactly but only approximately at the base. The important modes of the inner rod are the fundamental bending mode, the torsional mode, and the longitudinal mode. The out of phase motion between the inner rod and the outer frame of a hokyo, coupled to each other by the base, produces a quasi-fixed boundary condition at the base. The quasi-fixed end gives a practical advantage to the hokyo in that it can shorten the length of the instrument very much compared to the instrument with free ends.

Keywords: Sanukite, Percussion instrument, Vibrational analysis, Finite element method, Modal analysis

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1. INTRODUCTION

In the past, percussion instruments made of stone were quite common and they are still used in some countries. However, they are less familiar in the musical world than percussion instruments such as the glockenspiel [1] (“tek-kin” in Japanese) made of metal, and the xylophone (“mok-kin” in Japanese) made of wood.

Chime stones in China originated long before written history, and were developed to a complete set of tuned chime stones, Pien-ch’ing [2] (“Hen-Kei” in Japanese). Pien-ch’ing were used mostly in court music and ritual ceremonies in ancient China, and they were replaced by chimes made of bronze around 750 A.D.

Stone chimes are also known as “lithophones,” in England, Iceland, and Indochina, for example [2, 3]. Unfortunately, we cannot describe them further because we have little information about them at this time. Further information will soon appear in a book titled “Science of Percussion Instruments [4].”

One of the authors, H. Maeda, has devised and produced various kinds of percussion instruments made of “Sanukite.” Sanukite (Sanuki rock) [5] is a special lava rock used for stone arrowheads and stone axes in the

Stone Age and named scientifically by a German, Dr. E. Weinshenk in 1891. Tuned percussion instruments made of Sanukite were first produced in 1981 as “sek-kin,” an instrument with two and a half octaves and similar to the glockenspiel. Since that time, about a thousand Sanukite percussion instruments have been produced, and some of them were presented to Buddhist temples in Japan, the National Center (Confucius) Hall in Taiwan, the University of Munich, Washington State University, and others. Many well-known percussionists and composers, such as Stomu Yamash’ta, Mutsuko Fujii, Lionel Hampton [6], and Maki Ishii, admire these instruments and have used them in their plays and compositions. Furthermore, CD’s (compact discs) containing works played with Sanukite percussion instruments by S. Yamash’ta and M. Fujii are currently on the market.

Sanukite percussion instruments can be divided into four major classes, “sek-kin,” “sou,” “rou,” and “kei” [7]. The sou can be further subdivided into “hokyo” with a rectangular outside cross section, and “hensho” with a circular outside cross section [8]. The structure of the sou was discovered by chance while constructing a chime-like instrument with a thick and short cylindrical tube. The sou with a cylindrical deep groove (“sou” means a deep

groove in Chinese) has a unique and somewhat complicated structure, so we may say that the sou is a new percussion instrument.

In this paper, we analyze the vibrational modes of a piece of “hokyo” by using the finite element method and verify them by fast Fourier transform analysis of its tone and modal analysis. These analyses, revealed that the hokyo has various vibrational modes, two kinds of bending modes, two kinds of torsional modes, and a longitudinal mode. Furthermore, the frequencies of a few vibrational modes, including those of the fundamental modes (the pitch) of a hokyo, can be predicted not only by the finite element method (FEM) for a whole hokyo but also by the simple one dimensional theory for a cantilever, and by the rather simple finite element method applied to a centered cylindrical rod with a fixed end.

2. SHAPE AND DIMENSIONS OF HOKYO

The shape and dimensions of a standard hokyo analyzed here are shown in Fig. 1. The coordinate axes are given as in Fig. 1. The thickness in the y -direction is 55.6 mm and is a little smaller in the x -direction at 51.7 mm. Thus, the hokyo has two planes of symmetry parallel to the vertical (z) axis. The hokyo can be considered to be constituted of three parts, an inner cylindrical rod with an outside diameter of 30.8 mm, an outer frame with a cylindrical hole with an inside diameter of 39.8 mm, and a rectangular solid base that couples the outer frame with the inner rod. In the testing stage, the hokyo did not have an inner cylindrical rod as described in section 1, however, the chime-like structure could not produce enough sound when struck with an ordinary blow.

3. FINITE ELEMENT ANALYSIS OF A HOKYO

In order to analyze the vibrations of a hokyo by the finite element method (FEM analysis), not only is data

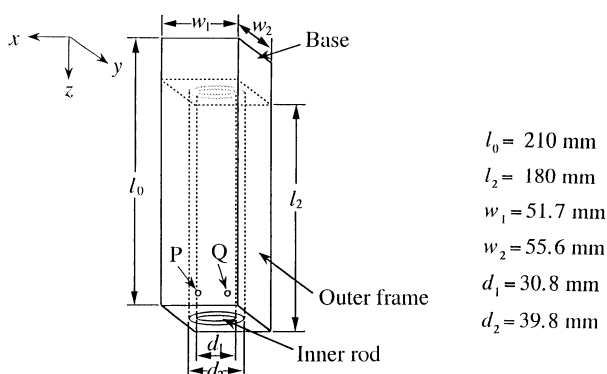


Fig. 1 Structure of a standard hokyo.

required to define the geometry but information regarding the material constants, density, Young's modulus, and Poisson's ratio is also required, at least on the assumption that the material is isotropic. This assumption seems to be reasonable if we see the microscopic pictures of Sanukite [9]. Unfortunately, our research on the basis of vibrational testing of the thin Sanukite strips indicates that the Young's moduli can vary by more than 10% among the samples. The differences seem to originate from the location of the samples and/or their anisotropy. We cannot identify which of these has a greater effect on the variation of the Young's moduli of the Sanukite strips at this time, but we have confirmed that the mode frequency is exactly proportional to the square root of the Young's modulus by FEM analysis. In contrast, the densities of Sanukite samples remain almost constant. Poisson's ratios cannot be obtained here, but afterward we will see this has little effect on the mode shapes and frequencies determined by FEM analysis. We thus adopted a typical value for glasses, 0.25. The density used in this paper is $2.60 \times 10^3 \text{ kg/m}^3$, and the Young's modulus, $8.56 \times 10^{10} \text{ N/m}^2$ [10]. The standard model of a hokyo is subdivided into 120 parabolic solid elements [11] as shown in Fig. 2 (also see Fig. 10). Sixty-four six-sided solid elements with 20 nodes and 56 five-sided solid elements with 15 nodes are used here. There is a total of 711 grid points or node points for the hokyo shown in Fig. 2. The approximation to the cylindrical surface configuration is good because 20-noded and 15-noded parabolic isoparametric elements are used. The general-purpose finite element code “NASTRAN” [12] is used in this study.

The lowest nine modes of a hokyo obtained by FEM analysis are shown in Fig. 3 and their mode frequencies are given in Table 1. In Fig. 3, mode shapes are shown as cross-section displacements in two planes of symmetry, except for the fifth and eighth modes in which the horizontal displacements (bottom view) of every element are overdrawn. Note that the z -axis in the length direction, shown vertically in Figs. 1 and 2, is drawn horizontally

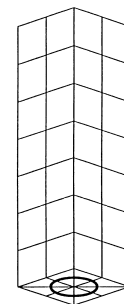


Fig. 2 Subdivision of a hokyo into 120 elements. Also see Fig. 10 for the subdivision of the inner rod.

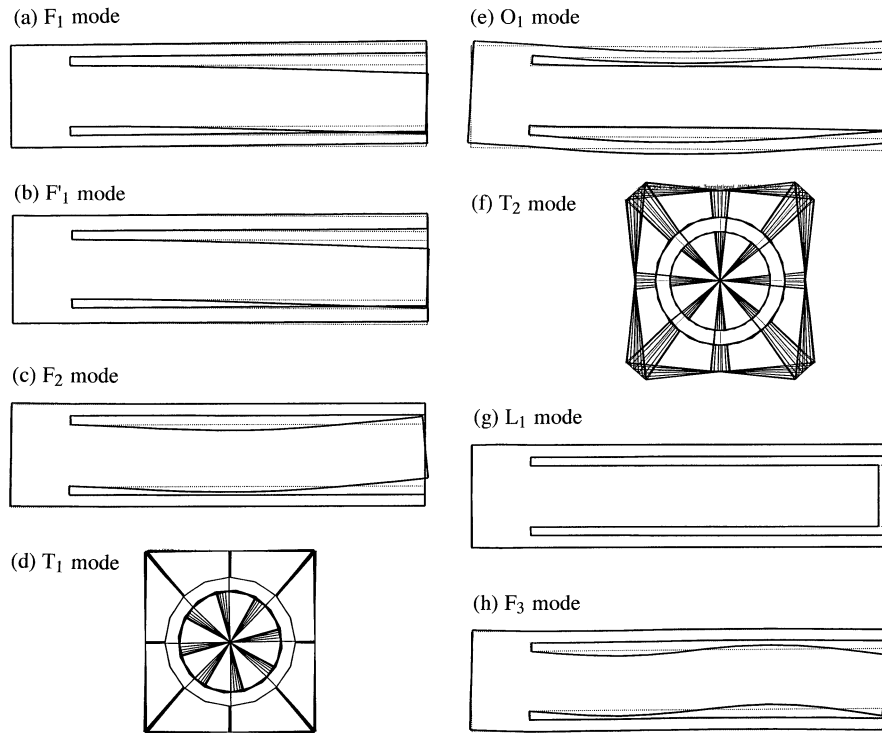


Fig. 3 Mode shapes of the lowest 11 modes of a hokyo obtained by the finite element analysis. Three of 11 modes are omitted because of similarity. (a) F_1 mode; (b) F'_1 mode; (c) F_2 mode; (d) T_1 mode; (e) O_1 mode; (f) T_2 mode; (g) L_1 mode; (h) F_3 mode.

in Fig. 3 to save the space. Thus, the top and bottom surfaces of hokyo in Fig. 1 correspond to the left and right side in Fig. 3. Displacements in Fig. 3 are exaggerated in order to make the features of the mode shapes more visible. Therefore, the inner cylindrical rod seems to indent the outer frame for the lowest two modes [Fig. 3(a), (b)]. The thin dotted lines indicate the rest position of the cross section of hokyo.

The lowest two modes in which the inner rod essentially vibrates in flexural or bending modes have almost the same frequencies, so they may produce a beat. For this reason we labeled these modes as F_1 and F'_1 . The mode shapes of the F_1 and F'_1 modes closely resemble each other as shown in Figs. 3(a) and (b) and also resemble the fundamental bending mode of a cantilever. In these modes, the thick inner rod vibrates out of phase with the outer frame and has an amplitude of 2.6 times that of the outer frame.

The mode shape of the third mode labeled F_2 is shown in Fig. 3(c). The mode shapes and frequencies of the third and fourth (F_2 and F'_2) modes are similar to each other and resemble the second bending mode of a cantilever. The shape of the F'_2 mode is omitted from Fig. 3, as it is nearly the same as the F_2 mode.

The fifth mode, labeled T_1 in Fig. 3(d), resembles the fundamental torsional mode of a cylindrical rod with a clamped end. In this mode, the inner rod also vibrates

more than, and out of phase with, the outer frame. The eight thin solid lines between the inner rod and the outer frame indicate that the amplitude of the rectangular solid base is low.

The sixth and seventh modes, labeled O_1 and O'_1 , look like the bending mode of the outer frame with free ends, and the vibrational amplitude of the inner rod is very small compared with that of the outer frame. Only the O_1 mode is shown in Fig. 3(e). The frequencies of the O_1 and O'_1 modes (see Table 1) are approximately the same as in the case where the inner rod is removed from a hokyo (5,901 Hz and 6,175 Hz, respectively). As we have mentioned in section 1, these modes correspond to the fundamental modes of a chime-like hokyo from which the inner rod is removed and thus cannot produce a strong tone.

The eighth mode, labeled T_2 [Fig. 3(f)], is a torsional mode of the hokyo as a whole and resembles that of a thick rod with a square cross section and with free ends. The base of the hokyo also vibrates considerably in this mode.

The ninth mode, labeled L_1 in Fig. 3(g), is the longitudinal mode of the inner rod vibrating out of phase with the outer frame, and so the base behaves as a quasi-fixed wall for this mode.

The tenth mode labeled F_3 and the eleventh modes F'_3 are the third bending modes of the inner rods, and the vibrational amplitude of the base and outer frame is very

Table 1 Mode frequencies of the lowest nine modes of a hokyo. Frequency deviations in percent of other Poisson's ratios from those with the Poisson's ratio, $\nu = 0.25$, are also listed.

| <i>n</i> | Mode | Mode frequencies (Hz) | | | Deviation (%) |
|----------|-----------------|-----------------------|--------------|--------------|---------------|
| | | $\nu = 0.20$ | $\nu = 0.25$ | $\nu = 0.30$ | |
| 1 | F ₁ | 805.5 | 807.3 | 809.3 | 0.53 |
| 2 | F' ₁ | 809.1 | 811.0 | 831.5 | 0.54 |
| 3 | F ₂ | 4,082 | 4,082 | 4,084 | 0.06 |
| 4 | F' ₂ | 4,098 | 4,098 | 4,101 | 0.08 |
| 5 | T ₁ | 4,500 | 4,409 | 4,324 | 3.99 |
| 6 | O ₁ | 6,024 | 6,010 | 5,996 | 0.46 |
| 7 | O' ₁ | 6,268 | 6,250 | 6,232 | 0.59 |
| 8 | T ₂ | 7,516 | 7,365 | 7,223 | 3.98 |
| 9 | L ₁ | 7,495 | 7,505 | 7,519 | 0.31 |

small compared with that of the inner rod. Only the F₃ mode is shown in Fig. 3(h).

Mode frequencies calculated by FEM analysis for three Poisson's ratios are given in Table 1. The percentage of frequency deviations from the case where the Poisson's ratio is 0.25 is listed in the sixth column in Table 1. From Table 1, the deviations are smaller than 0.6% with the exception of the torsional modes. Thus, it is reasonable to proceed with FEM analysis of the hokyo on the assumption that the Poisson's ratio is 0.25.

4. TONES OF A HOKYO

The tones of a hokyo were picked up with a low-noise free-field 1/2 inch condenser microphone (B&K, type 4190) about one meter from the hokyo in an anechoic room and recorded on a DAT (Digital Tape Recorder, SONY, type DTC-ZA5ES). An example of the envelopes of the tones when the outer frame of the hokyo is struck with a wooden mallet at the center near the bottom (designated as point P in Fig. 1), is shown in Fig. 4. An example of the sound spectra obtained by the FFT (fast Fourier transform) analysis of the 0.2 s after being struck is also

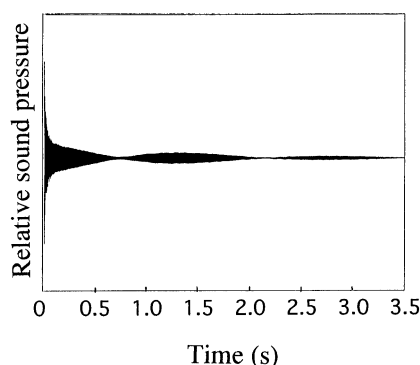


Fig. 4 Example of waveform envelopes of hokyo tones.

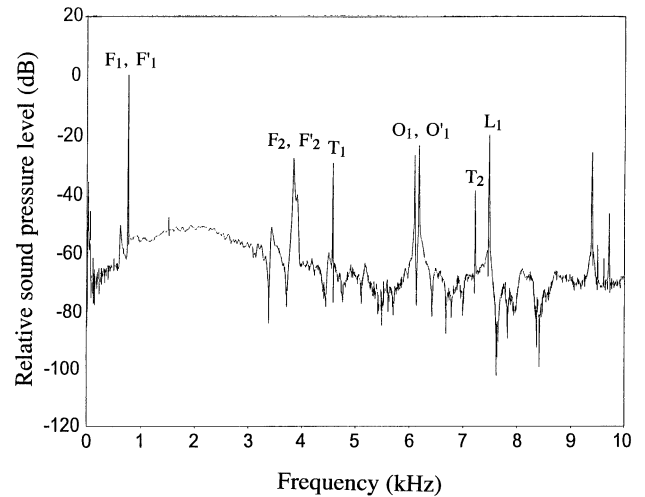


Fig. 5 Example of the spectra of hokyo tones.

shown in Fig. 5. The sampling frequency was 48 kHz and the size of the FFT was 16,384 points, thus the frequency resolution is 5.86 Hz. We can see a beat with a periodicity of 1.28 s from Fig. 4 and spectrum peaks near the predicted frequencies of respective modes from Fig. 5. The F₁ and F'₁ modes have the highest level and so determine the pitch of the hokyo. The torsional mode, T₁, and longitudinal mode, L₁, are excited strongly when the outer frame of the hokyo is struck off center (designated as point Q in Fig. 1), and the relative sound pressure levels of the T₁ and L₁ modes can be increased by 20 dB at most. Two frequencies of the mode pairs F₂ and F'₂ and O₁ and O'₁ are observed as adjacent peaks in Fig. 5. The frequencies of the F₁ and F'₁ mode pair cannot be observed separately in Fig. 5 because the FFT analysis does not have sufficient frequency resolution. Nonetheless, the beat observed in Fig. 4 gives evidence of the existence of the two modes with very close frequencies. The difference of the frequencies can be calculated to be about 0.78 Hz from the beat.

5. MODAL ANALYSIS OF A HOKYO

We performed modal analysis in order to confirm the predicted mode shapes of a hokyo. In this experimental method, we used a subminiature accelerometer (B&K, type 4374) with 0.65 g weight, an impact hammer (B&K, type 8203), two charge amplifiers (B&K, type A/S-2690), a dual channel signal analyzer (B&K, type 2034), and a microcomputer. Nodal points on grids or impact points, and a point A where an accelerometer is mounted, are shown in Fig. 6. As the impact points are connected merely by lines in the software, the shapes of the bottom surfaces of the inner rod and the inside of the outer frame seem to be rather octagons than circles. The cylindrical surfaces of the inner rod and the inside of the outer frame

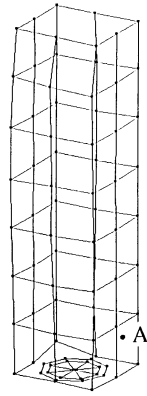


Fig. 6 Set of grid points for modal analysis of an actual hokyo.

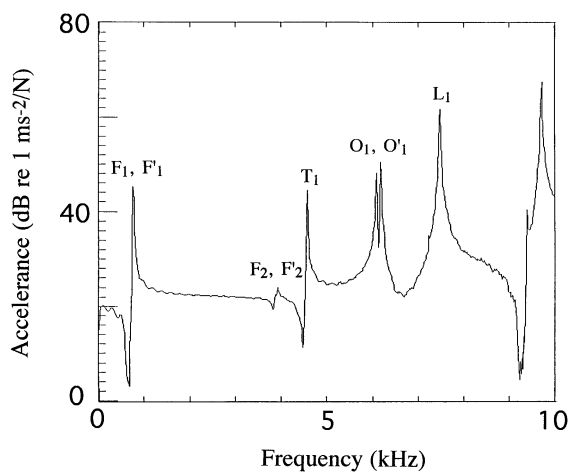


Fig. 7 Example of the frequency response functions of a hokyo.

do not appear in Fig. 6 because they cannot be struck by an impact hammer. The set of 83 grid points in three dimensions is struck five times at a right angle with each surface to increase accuracy, and we used the software LMS CADA-PC (type 5010) to determine the natural frequencies, mode shapes, and damping parameters of an actual hokyo. Although the shape of the actual hokyo shown in Fig. 6 differs a little from that of the analyzed hokyo in Fig. 1, we can expect that the difference between the frequencies of the two hokyos will be little. An example from 83 transfer frequency response functions calculated from impulse responses is shown in Fig. 7. We can see from Fig. 7 that there are several peaks at frequencies corresponding to those in Fig. 5 obtained from the FFT analysis of the tone of the same hokyo. Note that their levels in the high frequency range are emphasized in Fig. 7 because the acceleration [13] was adopted as a frequency response function. Next, the mode shapes obtained by the modal analysis are shown in Fig. 8. Note that every mode shape except for the T_1 mode [Fig. 8(c)] is shown horizontally to save the space, as in Fig. 3.

The mode shape of the F_1 mode [Fig. 1(a)] shows that the outer frame vibrates like a thick bar with a fixed end. Although we cannot access the cylindrical surface of an inner rod, it is possible to compare the amplitudes and phases of the tips or bottom surfaces of the inner rod and the outer frame. In order to measure the amplitude ratio between the bottom surfaces, two accelerometers (B&K, type 4500), sensitive to in-plane motion of the surfaces,

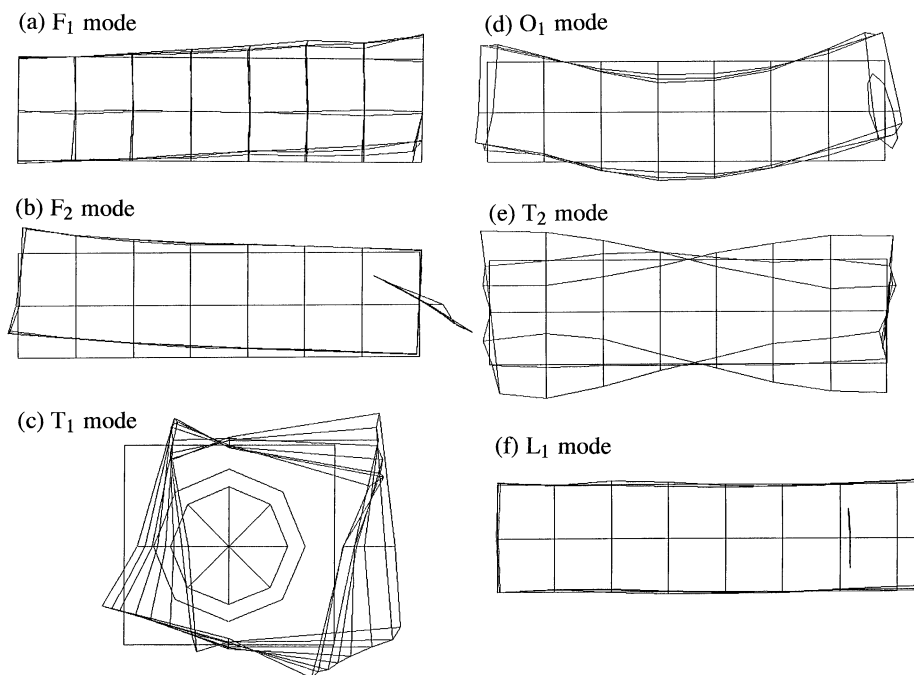


Fig. 8 Mode shapes of the lowest nine modes of a hokyo obtained by the modal analysis. Three of 9 modes are omitted because of similarity. (a) F_1 mode; (b) F_2 mode; (c) T_1 mode; (d) O_1 mode; (e) T_2 mode; (f) L_1 mode.

Table 2 Mode frequencies of the lowest nine modes determined by modal analysis and FEM for a hokyo. The FEM analysis (I) was done for a standard model (Fig. 1) and FEM analysis (II) was done for the particular case where the outer frame has a square outside cross section. The asterisk * shows frequencies of the mode pairs that degenerate.

| <i>n</i> | Mode | Mode frequencies (Hz) | | | Damping ratio for modal analysis (%) |
|----------|-----------------|-----------------------|---------------------|----------------------|--|
| | | Modal analyses | FEM analyses (I) | FEM analyses (II) | |
| 1 | F ₁ | 756.7 | 807.3 [6.67] | 820.2* | 0.05 |
| 2 | F' ₁ | 757.5 | 811.0 [7.07] | 820.2* | 0.05 |
| 3 | F ₂ | 3,801 | 4,082 [7.39] | 4,080* | 0.81 |
| 4 | F' ₂ | 3,884 | 4,098 [5.51] | 4,080* | 0.18 |
| 5 | T ₁ | 4,572 | 4,409 [−3.57] | 4,408 | 0.02 |
| 6 | O ₁ | 6,094 | 6,010 [−1.38] | 6,022* | 0.18 |
| 7 | O' ₁ | 6,177 | 6,250 [1.18] | 6,022* | 0.15 |
| 8 | T ₂ | 7,224 | 7,365 [1.95] | 7,364 | 0.11 |
| 9 | L ₁ | 7,485 | 7,505 [0.28] | 7,521 | 0.03 |

were attached to the bottom surfaces. The accelerometers used here weighed 3.5 g and these may affect the vibration of the hokyo somewhat. Thus, we observed that the surface of the inner rod vibrates out of phase with the outer frame and has an amplitude of 2.4 times that of the outer frame. This value is approximately the same as that obtained by the FEM (2.6).

Figure 8(b) shows the F₂ mode, and it appears that the base of the hokyo vibrates greater than its bottom surface (or tip). However, if we look carefully, the bottom surface motion of the inner rod in the *z* direction inclines very much compared with that of the top surface of the base. Thus, we may conclude that this mode corresponds to the F₂ modes obtained from the FEM [Fig. 3(c)]. If we look at Fig. 3(a), we can also see that the displacement of the top surface is greater than that of the bottom surface of the outer frame (this is not readily apparent in Fig. 3(c) due to the small presentation).

Apparently, the other four modes shown in Fig. 8(c), (d), (e), and (f) correspond to those shown in Fig. 3(d), (e), (f), and (g).

The mode frequencies of these modes are given in Table 2 with the mode frequencies obtained already by FEM (specified by “FEM analysis (I)”). The percentage deviations of the mode frequencies obtained by FEM from those obtained by the modal analysis are also given in the brackets of the same column.

As shown in Table 2, in general the lower the mode frequency, the larger the deviations. As this tendency contradicts with our experiences [14, 15], it seems better to consider that the Young's modulus estimated in this study was a little larger than the true value. As it is, the largest deviation is 7.39% (slightly more than a semitone), and so we proceeded with the FEM analysis without changing the geometry shown in Fig. 1 and the values of the material constants.

The frequency difference between the F₁ and F'₁ modes in the modal analysis is 0.8 Hz and approximately equal to the 0.78 Hz obtained from the beat (see Fig. 4). However, the frequency difference obtained by FEM is 3.7 Hz and far larger than the actual value. This discrepancy may originate from a little structural difference between Fig. 1 and Fig. 6 and/or the anisotropic property of a hokyo in the horizontal (*xy*) plane.

To confirm if the mode pairs F₁ and F'₁, F₂ and F'₂, and O₁ and O'₁ degenerate in a particular case where the thickness in the *y*-direction is made the same dimension as that in the *x*-direction, 51.7 mm, another FEM analysis (specified by “FEM analysis (II)”) was executed for this case. The fifth column of Table 2 shows that all the frequencies of the three mode pairs exactly coincide with each other.

The damping ratios in percent obtained by the modal analysis were added in the sixth column of Table 2. The damping ratio is equal to the quantity $1/2Q$ (where, *Q* is a quality factor). Thus, the smaller the damping ratio is, the longer the vibration continues. From the Table 2, we can see that the damping ratios are particularly small for the fundamental modes of each three mode families, that is, F₁, T₁, and L₁ modes.

6. DEPENDENCE OF THE MODE FREQUENCIES ON THE LENGTH OF A HOKYO

We have discussed that the F₁, T₁, and L₁ modes of the hokyo seem to vibrate like a simple thick cylindrical rod with a fixed end at the base. This quasi-fixed condition is caused by an out of phase vibration of the outer frame. Therefore, their mode frequencies may be approximated by the following respective equations deduced from one dimensional theory [16] within the appropriate range of the length of the inner rod of the hokyo,

l_2 :

$$f_{F1} = \frac{\pi a}{16l_2^2} \sqrt{\frac{E}{\rho}} \times 1.194^2, \quad (1)$$

$$f_{T1} = \frac{1}{4l_2} \sqrt{\frac{E}{2\rho(1+\nu)}}, \quad (2)$$

and

$$f_{L1} = \frac{1}{4l_2} \sqrt{\frac{E}{\rho}}. \quad (3)$$

where, ρ is the density, E is Young's modulus, ν is Poisson's ratio, a is the radius of the inner rod, and l_2 is its length.

From Eqs. (1) to (3) it is clear that the length, l_2 affects the frequency of the F_1 mode more than those of the T_1 and L_1 modes. The frequencies of the F_1 , T_1 , and L_1 modes are obtained by the FEM analysis by changing the length, l_2 of the inner rod and the outer frame simultaneously and keeping the dimensions of the base constant. The frequencies of the F_1 mode are shown in Fig. 9. Where, l is the total length of the hokyo and $l_0 = 210$ mm is that of a standard hokyo. The mode frequencies of the same rod as the inner rod obtained under the boundary condition, that its end at the base of hokyo is perfectly fixed to the rigid wall as shown in Fig. 10, are obtained by the FEM analysis and drawn by a dotted curve. This approach may be called the second approximation. As the second approximation reduces the total node points from 711 for the hokyo to 229 for the inner rod only, the calculation time for the latter (the second approximation) is

reduced to be less than 0.104 [= (227/711)²] times that for the former. Values calculated by two FEM analyses are obtained at every incremental value of 0.1 in normalized length and are shown in solid and broken lines. From Fig. 9, we can change the F_1 mode frequency or the pitch of the hokyo from 335 Hz to 6,586 Hz in the range of $0.4 \leq l/l_0 \leq 1.5$. Therefore, more than four octaves can be obtained by just changing the total length. However, a short hokyo will not produce a sufficiently strong and long tone because of its high mechanical impedance and increased damping at its higher fundamental frequency. The F_1 mode frequencies calculated from Eq. (1) are also shown in Fig. 9 with a smooth solid curve.

Next, the frequency deviations from the F_1 mode frequencies of a hokyo were calculated for the first (solid curve) and the second (dotted curve) approximations and are shown in Fig. 11. The first approximation is good, that is, the deviation is less than 6% (corresponding to a semitone), in the range of $1.0 \leq l/l_0 \leq 1.5$, whereas the second approximation is good for a rather short hokyo in the range of $0.5 \leq l/l_0 \leq 0.9$.

The T_1 and L_1 modes are not as important as the F_1 mode, but they can be strongly excited when the hokyo

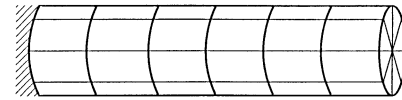


Fig. 10 Subdivision of an inner rod rigidly clamped at the base into 48 elements.

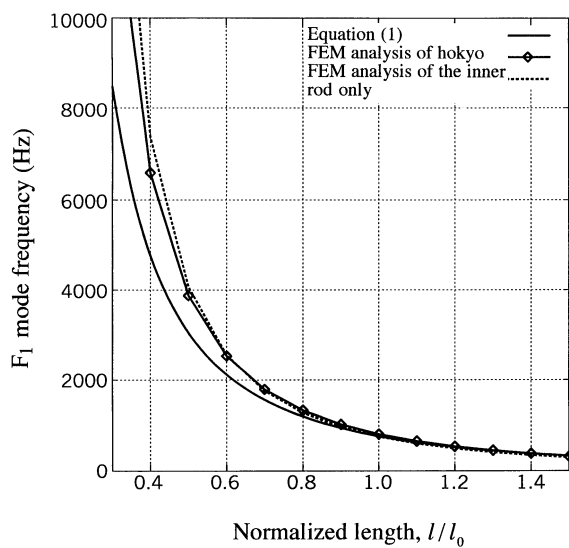


Fig. 9 Dependence of F_1 mode frequency of the fundamental bending mode on normalized total length of a hokyo. Dotted curve; FEM analysis for the inner rod shown in Fig. 10. Solid curve; calculated from Eq. (1). Plots; FEM analysis for a hokyo.

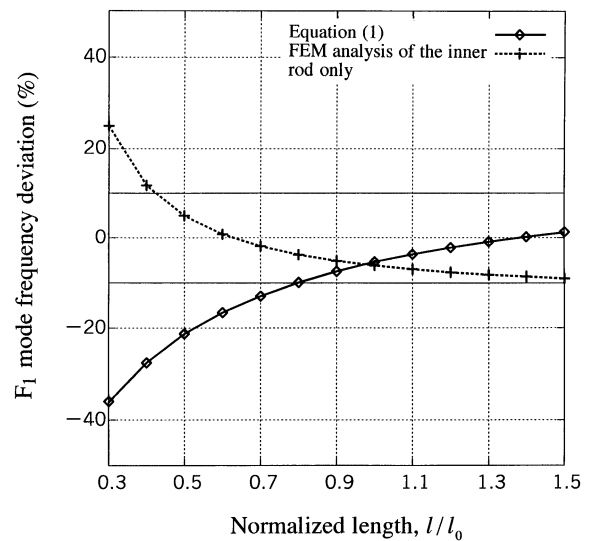


Fig. 11 Frequency deviations from the F_1 mode frequencies of a hokyo. Solid curve and dotted curve are obtained from Eq. (1) and FEM analysis for a thick rod shown in Fig. 10.

is struck off-center (at point Q shown in Fig. 1). So their mode frequencies were also calculated. The results are shown in Figs. 12 and 13. For the T_1 mode shown in Fig. 12, the curves for the second approximation and for the hokyo never intersect each other, and the former is always a little larger. However, their deviation is less than 6% in the wide range of $0.6 \leq l/l_0 \leq 1.5$. In contrast, the curves calculated from Eq. (2) and the hokyo intersect

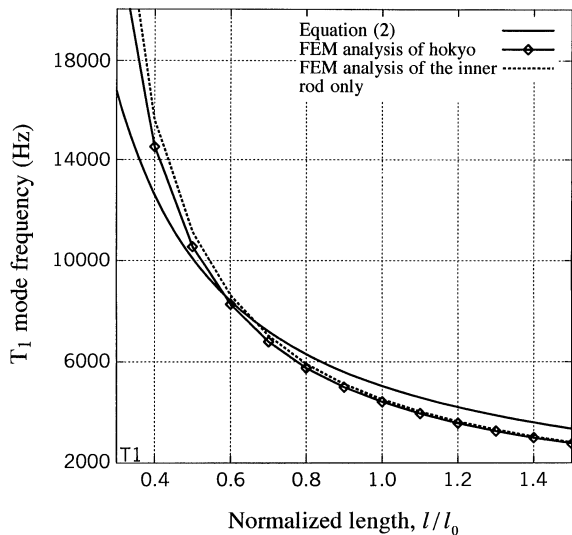


Fig. 12 Dependence of T_1 mode frequency of the fundamental bending mode on normalized total length of a hokyo. Dotted curve; FEM analysis for the inner rod shown in Fig. 10. Solid curve; calculated from Eq. (2). Plots; FEM analysis for a hokyo.

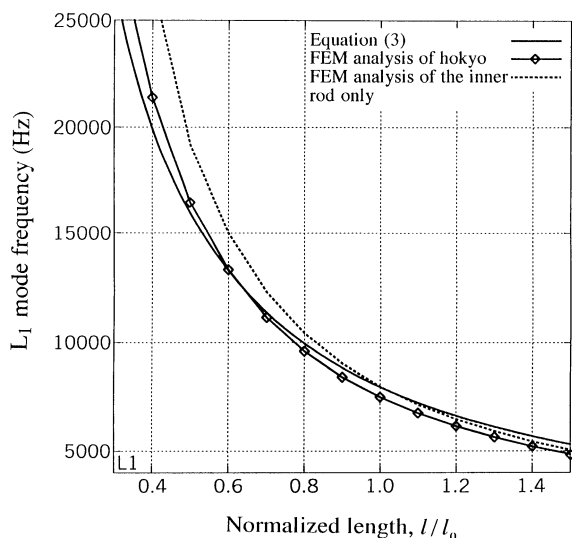


Fig. 13 Dependence of L_1 mode frequency of the fundamental bending mode on normalized total length of a hokyo. Dotted curve; FEM analysis for the inner rod shown in Fig. 10. Solid curve; calculated from Eq. (3). Plots; FEM analysis for a hokyo.

each other at about $l/l_0 = 0.6$, but their deviation is less than 6% within only the narrow range of $0.5 \leq l/l_0 \leq 0.7$. For the L_1 mode shown in Fig. 13, the curves for the second approximation and for the hokyo again never intersect each other, and the former is always larger than the latter. Their deviation is less than 6% within the rather narrow range of $1.2 \leq l/l_0 \leq 1.5$. However, the curves calculated from Eq. (3) and the hokyo intersect each other at about $l/l_0 = 0.6$, and their deviation is less than 6% in the wide range of $0.5 \leq l/l_0 \leq 0.9$.

As can be seen in Figs. 9, 12, and 13, the mode frequencies obtained from the second approximation do not coincide with, and are a little greater than those of the hokyo, except the range of $0.6 \leq l/l_0 \leq 1.5$ for the F_1 mode. As a result, the inner rod cannot be considered to have a perfectly fixed end, and so we should say that it has a quasi-fixed end. Of course, the length of the hokyo affects all mode frequencies and thus the timbre. We will publish a more precise paper on the timbre of a hokyo sometime in the future.

7. CONCLUSIONS

This paper introduced percussion instruments made of Sanukite stones. One of them, the hokyo with a unique and somewhat complicated structural shape, was analyzed by the finite element method in order to study its modes of vibration. The shapes and frequencies of the lowest nine modes were then verified by FFT analysis of the recorded tones and by experimental modal analysis. The principal results obtained in this study are as follows:

1. Almost all modes of the hokyo resemble those of an inner cylindrical rod with a free end and a quasi-fixed end. Thus, the lowest two modes (F_1 , F'_1) are the fundamental bending modes and their frequencies determine the pitch of the hokyo.
2. Additionally, there are the second bending (F_2 , F'_2), torsional (T_1), and longitudinal (L_1) modes, which also resemble those of an inner cylindrical rod with a free end and a quasi-fixed end.
3. The boundary condition, such as a quasi-fixed end, is established by the out of phase motions between the inner rod and the outer frame of the hokyo.
4. The inner rod cannot radiate tones directly, but it controls tones radiated from the outer frame, although the vibrational amplitude of the outer frame is less than that of the inner rod.
5. When a hokyo is struck off-center (at point Q shown in Fig. 1), the T_1 and L_1 modes can be excited sufficiently and partial tones can be produced.
6. There are two bending modes (O_1 , O'_1) and a torsional mode (T_2) that resemble those of a very thick outer frame with free ends, and their mode frequen-

cies are much higher than the respective modes expected for the outer frame with a fixed end.

Finally, we can conclude that the structure of the hokyo has the practical advantage of obtaining a quasi-fixed end and thus of considerably reducing the total length of the hokyo [17]. As we can easily predict, the mode shapes and mode frequencies of a “hensho” belonging to the same “sou” family may be approximately the same as those investigated here, except that the tones due to torsional modes radiate less. The research on the vibration of a hensho, accompanied with a more detailed study of the material constants of Sanukite, will be published at the first opportunity.

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