

PAPER

Active control of a sound field with a state feedback electro-acoustic transducer

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Abstract: This study presents an alternative control system in which the acoustic impedance of the diaphragm of an electro-acoustic transducer can be manipulated by modifying the design parameters of the control system. This system involves a state-space description of an electro-acoustic transducer that is derived from its electrical equivalent circuit using modern control theory. The optimal quadratic regulator was used in the control system design, and the quadratic performance index was formulated to relate to the square of the sound pressure near the diaphragm of the control system. Computer simulations were performed to test the proposed control system and indicated that significant reductions in the acoustic impedance density could be achieved near the assumed vibration frequency that was used in the formulation of the quadratic performance index. A computer model of the proposed control system was used to illustrate effective active noise control in a duct and indicated that the control system brings about an effect similar to that of a resonator type muffler.

Keywords: Electro-acoustic transducer, State equation, Feedback control, Active noise control

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1. INTRODUCTION

Acoustic physical restriction or architectural design sometimes makes it difficult to control a sound field by passive methods such as installing sound absorbers or changing room shape. In such cases, it is possible to use electro-acoustic transducers to actively control the sound field. This technology, active sound field control, has been described in several studies.

To date, most research involving active sound field control employed methods based on the inverse filtering of the transfer functions from sound sources to receiving points in a sound field and simulating the desired transfer functions as strictly as possible. However, it appears likely that an effective method of active sound field control would not change the transfer functions completely, but would change the transmission characteristics.

The transmission characteristics between a sound source and a receiving point in a sound field are affected substantially by the poles of the transfer function between them; these poles determine the global acoustic characteristics of the sound field. The poles of a transfer function can be controlled by introducing a feedback loop to the system.

Olson and May's "electronic sound absorber" [1, 2]

is one of the most efficient methods of applying feedback control to active sound field control. In their system, the sound pressure at a microphone collocated with a loudspeaker is fed back through an electro-acoustic transducer. By choosing an appropriate feedback gain, the local sound pressure near the microphone can be reduced.

Nishimura *et al.* proposed an "active acoustic treatment" [3] that actively controls the acoustic impedance on the surface of this acoustic treatment using a method similar to that of Olson and May. In an effort to control the mechanical impedance of the diaphragm of a loudspeaker as seen from a sound field, Yagi *et al.* developed an active sound absorption control system that feeds back not only the sound pressure near the loudspeaker but also the vibrating velocity of the diaphragm [4]. Okda *et al.* presented an active noise control system that uses a motion feedback loudspeaker and can control the internal impedance of an electro-acoustic transducer without microphones [5].

These prior studies used classical control theory based on transfer function analysis. In the present study, we reformulated the control system that Olson and May proposed by involving modern control theory based on the state-space description of a control object, and developed an alternative control system that could control the

acoustic impedance of its diaphragm. Furthermore, we expect that the effectiveness of this control can be optimized.

2. THE PROPOSED CONTROL SYSTEM

2.1. State-Space Description of an Electro-Acoustic Transducer

Figure 1 shows the electrical equivalent circuit of an electro-acoustic transducer of the electro-dynamic type. Its equation of motion is expressed as follows:

$$\begin{cases} E_0 - Z_{e0}I = Z_{ed}I - Av \\ F = F_0 - Z_{m0}v = Z_{md}v + Z_a v + AI \end{cases}, \quad (1)$$

where E_0 is the electrical source voltage, I is the current, A is the force factor, Z_{e0} is the internal impedance of the power source, Z_{ed} is the electrical impedance of the voice coil; F_0 is the vibromotive force, v is the vibrating velocity of the diaphragm, Z_{m0} is the radiation impedance, Z_{md} is the mechanical impedance of the diaphragm, and Z_a is the acoustic impedance of the cabinet. By using the relations such that

$$\begin{aligned} Z_{e0} &= R_{e0} + j\omega L_0 + \frac{1}{j\omega C_0}, & Z_{ed} &= R_{ed} + j\omega L_d + \frac{1}{j\omega C_d}, \\ Z_{md} &= R_{md} + j\omega M_d + \frac{K_d}{j\omega}, \end{aligned}$$

where ω is the angular frequency, and assuming that Z_a consists only of the stiffness K_a , Eq. (1) can be transformed to a state-space description expressed as follows:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}_m F + \mathbf{b}_e E_0 \\ \mathbf{y} = \mathbf{c}\mathbf{x} \end{cases}, \quad (2)$$

where,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_d + K_a}{M_d} & -\frac{R_{md}}{M_d} & 0 & -\frac{A}{M_d} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{A}{L_0 + L_d} & -\frac{1}{C_0} + \frac{1}{C_d} & -\frac{R_{e0} + R_{ed}}{L_0 + L_d} \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} \int v dt \\ v \\ \int I dt \\ I \end{bmatrix}, \quad \mathbf{b}_m = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{L_0 + L_d} \end{bmatrix},$$

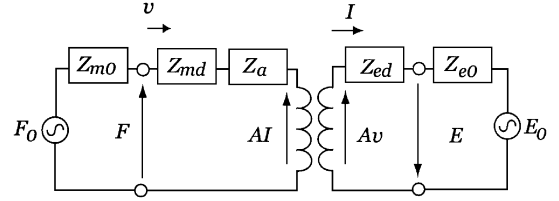


Fig. 1 Electrical equivalent circuit of an electro-acoustic transducer of the electro-dynamic type.

and y is an output of the system. The formulation of the output equation $y = \mathbf{c}\mathbf{x}$ depends on which state variable of the components of \mathbf{x} is measured.

Now, if we carry out state feedback such as $E_0 = -\mathbf{f}\mathbf{x}$, where \mathbf{f} is a state feedback gain vector, Eq. (2) can be changed into the following state-space description for a closed loop system:

$$\begin{cases} \dot{\mathbf{x}} = (\mathbf{A} - \mathbf{b}_e \mathbf{f})\mathbf{x} + \mathbf{b}_m F \\ \mathbf{y} = \mathbf{c}\mathbf{x} \end{cases}. \quad (3)$$

If the system is controllable, the root of the closed loop system can be determined arbitrarily by choosing an appropriate state feedback gain vector. Thus, the characteristics of the electro-acoustic transducer can be controlled.

2.2. Configuration of the Control System

Here, a practical method to realize the feedback controller is described that uses a state estimator of a linear dynamic system, and an example of a configuration of the control system is shown.

When we use the Kalman filter to estimate the state variables \mathbf{x} from the output y of the system, the state estimate \mathbf{x}_e is given by

$$\dot{\mathbf{x}}_e = (\mathbf{A} - \mathbf{b}_e \mathbf{f} - \mathbf{k}\mathbf{c})\mathbf{x}_e + \mathbf{k}y, \quad (4)$$

where

$$\mathbf{k} = \mathbf{P}\mathbf{c}^T \mathbf{r}^{-1} \quad (5)$$

and \mathbf{P} is the solution of the Riccati equation:

$$\mathbf{0} = (\mathbf{A} - \mathbf{b}_e \mathbf{f})\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{b}_e \mathbf{f})^T + \mathbf{Q} - \mathbf{P}\mathbf{c}^T \mathbf{r}^{-1} \mathbf{c}\mathbf{P}, \quad (6)$$

where \mathbf{Q} is the covariance matrix of the process noise vector \mathbf{w}_1 contaminating the input of the system and r is the variance of the observation noise w_2 contaminating the output of the system [6]. By carrying out $E_0 = -\mathbf{f}\mathbf{x}_e$ instead of $E_0 = -\mathbf{f}\mathbf{x}$, the transfer function $H(s)$ from y to the control command E_0 can be expressed as follows:

$$\begin{aligned} E_0 &= \mathbf{f}(s\mathbf{I} - \mathbf{A} + \mathbf{b}_e \mathbf{f} + \mathbf{k}\mathbf{c})^{-1} \mathbf{k}Y(s) \\ &= H(s)Y(s), \end{aligned} \quad (7)$$

where the Laplace transforms of y and E_0 are $Y(s)$ and $E_0(s)$, respectively. Thus, the feedback controller can be realized as a single-input, single-output filter. Figure 2

shows the block diagram of the electro-acoustic transducer to which the feedback controller described above is introduced.

If an extra coil is used to pick up the voltage proportional to v , i.e., c of the output equation is given by

$$c = [0 \ A' \ 0 \ 0], \quad (8)$$

where A' is the force factor of the extra coil, then the control system can be arranged as shown in Fig. 3.

2.3. Control System Design

For control system design, the optimal quadratic regulator is employed. A quadratic performance index of this method is a time integral expressed as follows:

$$J = \int_0^\infty (x^T R_1 x + E_0 r_2 E_0) dt, \quad (9)$$

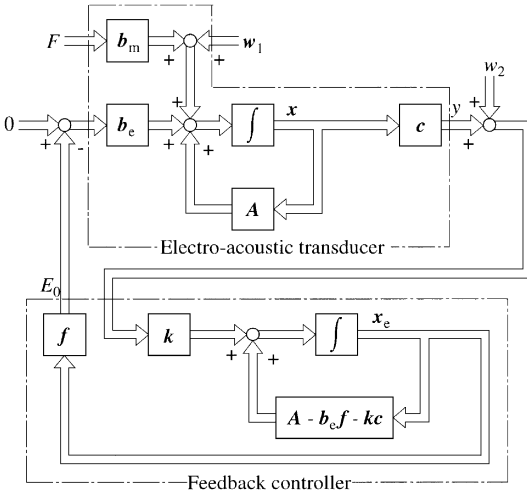


Fig. 2 Block diagram of the electro-acoustic transducer to which the feedback controller is introduced.

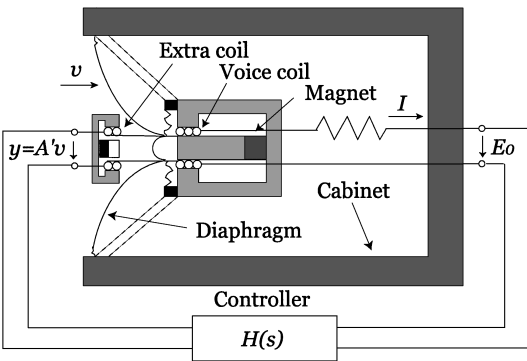


Fig. 3 Schematic diagram of the proposed control system.

where R_1 is the weighting matrix and r_2 is the weighting factor. The state feedback gain vector f that minimizes this performance index is formulated as follows:

$$f = r_2^{-1} b_e^T P, \quad (10)$$

where P is the solution of the Riccati equation.

$$0 = PA + A^T P + R_1 - P b_e r_2^{-1} b_e^T P \quad (11)$$

In the control system proposed by Olson and May, the sound pressure detected by the microphone, which is related to F in Fig. 1, is fed back into the vibromotive force AI via the controller. The amplitude and phase characteristics of the controller are designed to make the sound pressure at the microphone as small as possible. To obtain f suitable for the same objective as Olson and May used, R_1 was formulated as follows:

The driving force F is given by

$$\begin{aligned} F &= (Z_{md} + Z_a)v + AI \\ &= (K_d + K_a) \int v dt + R_{md}v + M_d \dot{v} + AI \\ &= [(-\omega_a^2 M_d + K_d + K_a) \quad R_{md} \quad 0 \quad A] x, \end{aligned} \quad (12)$$

where harmonic time dependence of the angular frequency ω_a is assumed because the acceleration of the diaphragm is not involved in x . Thus, if R_1 is expressed as

$$\begin{aligned} R_1 &= [(-\omega_a^2 M_d + K_d + K_a) \quad R_{md} \quad 0 \quad A]^T \\ &\quad \times [(-\omega_a^2 M_d + K_d + K_a) \quad R_{md} \quad 0 \quad A], \end{aligned} \quad (13)$$

then the first term $x^T R_1 x$ in the integral of Eq. (9) is equal to squared F . By adopting this state feedback control which minimizes the performance index expressed as Eqs. (9) and (13), the sound pressure near the diaphragm of the control system can be expected to decrease as fast as possible when the diaphragm of the control system is excited by a disturbance. In addition, since this control system is designed in the time domain, there is no lack of causality about realization of the feedback controller.

3. NUMERICAL STUDIES ON THE PROPOSED CONTROL SYSTEM

To assess the effectiveness of the proposed control system, a computer simulation was performed on the theoretical basis presented in the previous section.

In general, when the optimal quadratic regulator theory is applied to practical control problems, it is difficult to find suitable values of the weighting matrix R_1 and the weighting factor r_2 without actually calculating the response of the control system; this difficulty is one disadvantage of the optimal quadratic regulator theory. In the

previous section, R_1 was formulated so that the performance index was related to the sound pressure on the diaphragm of the control system. Thus, the design parameters in the computer simulation are r_2 , and the assumed vibration frequency $f_a (= \omega_a/(2\pi))$ of the diaphragm used in Eq. (12).

Table 1 Physical constants of the electro-acoustic transducer.

Force factor	$A = 12.8 \times 10^{-4}$ Web/m
Inductance of voice coil	$L_d = 1 \times 10^{-3}$ Henry
Resistance of voice coil	$R_{ed} = 6.7 \Omega$
Capacity of voice coil	$C_d = 1 \mu F$
Mass of voice coil, cone, etc.	$M_d = 2.8 \times 10^{-2}$ kg
Mechanical damping coefficient	$R_{md} = 4.0$
Mechanical stiffness of suspension	$K_d = 1,400$ N/m
Effective area of diaphragm	$S_s = 3.14 \times 10^{-2}$ m ²
Volume of cabinet	$V = 3.5 \times 10^{-3}$ m ³

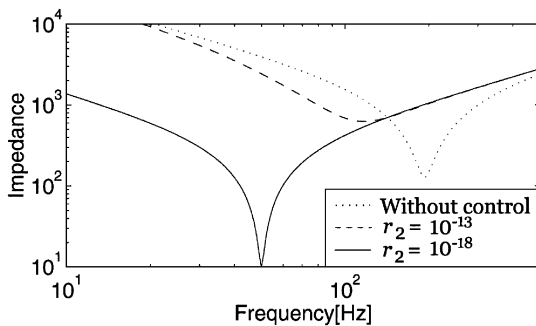


Fig. 4 Acoustic impedance density when the assumed vibration frequency f_a is set to 50 Hz and the weighting factor r_2 is changed.

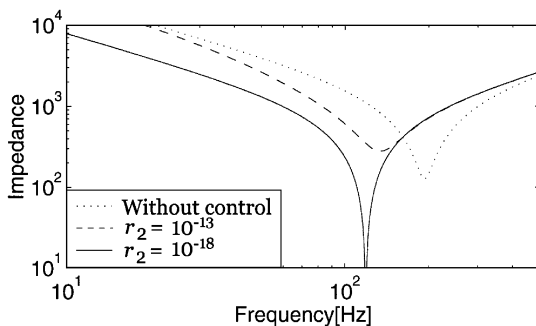


Fig. 5 Acoustic impedance density when the assumed vibration frequency f_a is set to 119 Hz and the weighting factor r_2 is changed.

3.1. Conditions for Calculation

Table 1 gives the physical constants of the electro-acoustic transducer modeled in the computer simulation. Most of these constants were measured directly or estimated with sufficient accuracy [7]. On the assumption that all state variables were directly observable, the frequency responses of the vibrating velocity v of the diaphragm to the vibromotive force F were calculated from the state equation of the control system presented in Eq. (3). The effectiveness of the state feedback control was evaluated as the acoustic impedance density of the diaphragm seen from a sound field, *i.e.*, $F/(vS_s)$.

3.2. Results

Figure 4 shows the acoustic impedance density when the assumed vibration frequency f_a is set to 50 Hz and the weighting factor r_2 is changed. Without the control, *i.e.*, opening the electric terminal of the system, the mechanical resonance of the electro-acoustic transducer results in a dip. As r_2 decreases, the dip frequency moves to 50 Hz. In addition, the dip value when $r_2 = 10^{-18}$ becomes smaller than that without the control.

Figure 5 shows the acoustic impedance density when f_a is set to 119 Hz. In this case, as r_2 decreases, the dip frequency moves to 119 Hz.

These results indicate that the resonance frequency of the electro-acoustic transducer changes to f_a Hz and its mechanical resistance decreases. As such, the sound pressure near the diaphragm of the system decreases around the dip frequency.

In both the above cases, the acoustic impedance density of the diaphragm can be reduced in the frequency range lower than f_a by introducing this control. This is an advantageous feature for suppressing acoustic noise because conventional passive methods such as sound absorbers and intervening barriers do not work well at low frequencies.

4. APPLICATION TO ACTIVE NOISE CONTROL IN A DUCT

Application of the proposed control system was demonstrated by a computer simulation of active noise control in a duct. The active control of one-dimensional acoustic waves propagating in a duct at low frequencies has become one of the classic problems in active noise control. However, there are some problems in the practical implementation of conventional active noise control systems using microphones and loudspeakers. This type of system has disadvantages under severe conditions such as extreme dustiness, high temperatures, or high humidity. In addition, it is more effective in the higher frequency range than the resonance frequency range of the loud-

speaker. On the other hand, the only part of the proposed control system seen from a sound field is the diaphragm, and it acts solely as an impedance surface. Thus, it is expected that installation of the proposed control system to a duct should prove simple and that the control system should run reliably.

4.1. Conditions for Calculation

The proposed control system is mounted on the duct wall as shown in Fig. 6. Assuming that the sound field in the duct is one-dimensional and that the effective diameter of the diaphragm of the control system is small compared with the wavelength, the duct can be treated as a transmission line as shown in Fig. 7. If the radiation resistance of the outlet of the duct $Z_2 \ll \rho c$, the insertion loss IL due to the proposed control system can be given by

$$IL = 20 \log \left| 1 + j \frac{S_s \rho c}{S Z_s} \frac{\sin k l_1 \sin k l_2}{\sin k(l_1 + l_2)} \right|, \quad (14)$$

where ρ is the density of the medium, c is the speed of sound, Z_s is the acoustic impedance density of the diaphragm of the control system seen from a sound field; S is the section area of the duct, S_s is the effective area of the diaphragm of the control system, l_1 and l_2 are the equivalent lengths between the noise source and the control system and between the control system and the outlet of the duct, respectively [8].

In this computer simulation, the values of the acoustic impedance density shown in Figs. 4 and 5 were used as Z_s , and the duct sizes were set as follows:

$$\begin{aligned} l_1 &= 1.645 \text{ m}, \\ l_2 &= 0.185 \text{ m}, \text{ and} \\ S &= 0.29^2 \text{ m}^2. \end{aligned}$$

4.2. Results

Figure 8 shows the insertion loss IL when the assumed vibration frequency f_a is set to 50 Hz and the weighting factor r_2 is changed. A positive value of IL indicates that the sound power radiating from the outlet of the duct is reduced.

Without the control, *i.e.*, opening the electric terminal of the system, the sharp peaks due to the acoustic resonance of the duct such as $\sin k(l_1 + l_2) = 0$ are observed at equal intervals. The mechanical resonance of the electro-acoustic transducer also results in a sharp peak since Z_s becomes the minimum value. The resonance frequency of the electro-acoustic transducer in this computer simulation is 182 Hz. Thus, a peak at about 188 Hz is due to the mechanical resonance of the electro-acoustic transducer as well as the acoustic resonance of the duct.

When $r_2 = 10^{-18}$, the proposed control system is effective in the frequency range from approximately 40 Hz

to 90 Hz, and the obtained IL has a sharp peak at about 50 Hz, *i.e.*, f_a Hz. These results stem from the fact that the control system acts as a resonator type muffler.

5. CONCLUSIONS

A novel control system was presented that can control the acoustic impedance of its diaphragm by reformulating the control system that Olson and May proposed by involving modern control theory. The control system described herein can be realized without microphones; thus, it can be easily installed to a sound field, and it is a reliable and stable system even under severe conditions. The optimal quadratic regulator was used in the control system design, and the quadratic performance index was formulated to relate to the square of the sound pressure near the diaphragm of the control system.

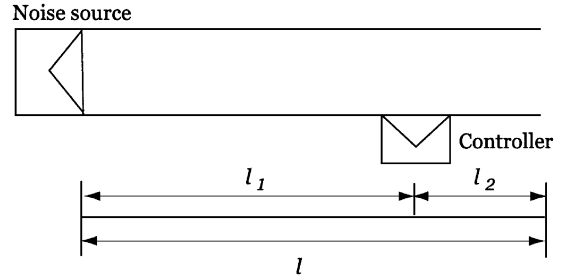


Fig. 6 Schematic diagram of a duct equipped with the proposed control system for computer simulations.

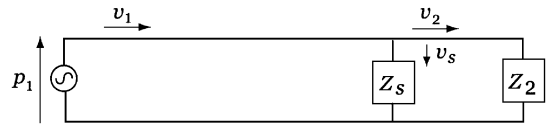


Fig. 7 Electrical equivalent circuit of a duct equipped with the proposed control system.

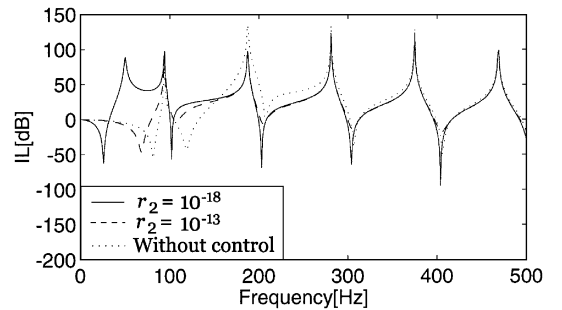


Fig. 8 Insertion loss IL when the assumed vibration frequency f_a is set to 50 Hz and the weighting factor r_2 is changed.

Computer simulations, used to test the effectiveness of the proposed control system, indicated that significant reductions in the acoustic impedance density could be achieved near a single frequency. However, it was not possible to get the desired effectiveness in a wide frequency range. This is likely to be the case because the assumed vibration frequency appears in the quadratic performance index formulated for this control system. Thus, formulation without the assumed vibration frequency is required for a more effective control system.

The proposed control system serves as an acoustic impedance surface which can be changed arbitrarily by modifying the design parameters of the control system. Thus, it provides a method of tuning the characteristics of a sound field. A computer model was used to illustrate effective active noise control in a duct and indicated that the proposed control system brings about an effect similar to that of a resonator type muffler.

In future studies we will modify the control system for a broadband effect and experimentally confirm the re-

sults obtained in this preliminary investigation.

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