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Acoustical transfer function of a flat plate with bending wave and its stochastic response — A unification of deterministic and stochastic analysis —

Mitsuo Ohta* and Kiminobu Nishimura**

**The Emeritus Professor of Hiroshima University,*
E-mail: ohta-3322@mdd.spacetown.ne.jp
***Faculty of Engineering, Kinki University,*
E-mail: nisimura@hiro.kindai.ac.jp

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1. Introduction

As is well-known, a number of evaluation methods for several type sound insulation systems are developed from theoretical and/or experimental viewpoints, *e.g.*, as shown in the literature.[1-3] Many of these methods however were based on the frequency analysis only from a deterministic viewpoint. So, it seems difficult to directly apply these methods to evaluate stochastically the output noise fluctuation especially in a time domain. On the other hand, for stochastically evaluating a whole output fluctuation form of insulation systems inside and/or outside room as minutely as possible, it seems necessary to unify not only an equivalent noise level, L_{eq} but also conventional percentiles L_x of arbitrary level x.

In this study, we first propose some new trial of unified evaluation method to combine the deterministic and stochastic methods, based on first the physical mechanism of the system and then the stochastic evaluation of output response fluctuation. Concretely, the acoustic transfer function of a plate applicable to arbitrary input fluctuation form is derived after finding an equivalent circuit model (especially connected with the bending motion) and its specific acoustic impedance. The probabilistic evaluation of transmitted noise fluctuation can be evaluated by positively employing the previous method on probabilistic evaluation with system eigenvalues,[4,5] by considering explicitly the built-in rms circuit of sound level meter, insulation system and stochastic input.

In the experimental consideration, first as a principle confirmation, the proposed method has been applied to the transmitted noise evaluation of a single-wall.

2. Acoustic transfer function of a flat plate with bending wave

Assume a plane sound wave incidents to an elastic plate with incident angle θ, as well-known, the differential equation governing the vibration of its motion is given by

$$m\frac{\partial^2 y}{\partial t^2}+B\frac{\partial^4 y}{\partial x^4}=(P_i-P_t) \tag{1}$$

with the displacement y, the flexural rigidity B and the surface mass m. Here, P_i and P_t denote the input and output sound pressures. By solving Eq. (1) in a frequency domain under excitation of angular frequency ω, an impedance of the plate Z_ω is given by

$$Z_\omega \triangleq \frac{P_i-P_t}{dy/dt}=j\omega m+\frac{1}{j\omega}\frac{1}{C_B}$$
$$=j\omega m\left[1-\left(\frac{f}{f_c}\right)^2\sin^4\theta\right], \tag{2}$$

where

$$C_B=\frac{1}{Bk^4},\quad k=\frac{\omega\sin\theta}{c}\ \text{and}\ f_c=\frac{c^2}{2\pi}\sqrt{\frac{m}{B}}.$$

Here, f_c and c denote the minimum frequency of coincidence and velocity of sound. Furthermore, by additionally considering the internal loss of a plate, $2r/\cos\theta$, and impedance of air at the front and back side of plate, we can derive an equivalent circuit (a so-called force-voltage method) as shown in Fig. 1.

By considering Fig. 1, we directly have

$$P_t=\rho c\cdot I=\frac{2P_i\cdot\rho c_\perp}{Z\omega+2\rho c_\perp} \tag{3}$$

with

$$c_\perp=c/\cos\theta.$$

The sound pressure transfer function from P_i to P_t can be derived from Eq. (3) by substituting $C_B=c^4/EI(\sin^4\theta)s^4$, as follows:

$$G(s)=\frac{2\rho c_\perp}{\frac{EI\sin^4\theta}{c^4}s^3+ms+2\left(\frac{r}{\cos\theta}+\rho c_\perp\right)}. \tag{4}$$

It is nothing new to say that this transfer function is universally applicable to arbitrary input waveform, differing from usual frequency expression only based on a sinusoidal wave.

3. Probability expression for output response

The environmental input sound pressure wave which contribute to the system output is actually some random signal cut up into strips with every time length T

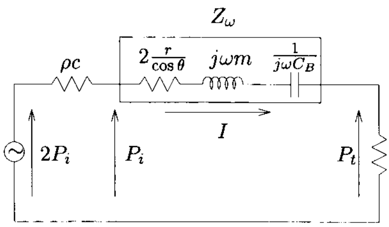

Fig. 1 Equivalent circuit of sound insulation system.

because of rms circuit in a sound level meter. The sample process from a sound level meter can be first expressed in a form of Fourier expansion series (a so-called Rice's Fourier model[6,7]). Furthermore, the output power fluctuation of an rms circuit can be expressed in terms of Fourier coefficients according to the Parseval's complete relation. Then, the sound intensity observed at the output of an insulation system can be evaluated by the power frequency component ε_n of incident noise and Eq. (4) as follows:

$$E_y=\sum_{n=1}^{N}\alpha_n\cdot\varepsilon_n \tag{5}$$

with $\alpha_n \triangleq |G(j\omega)|^2_{\omega=2n\pi/T}$, where T and N denote the time constant of a sound level meter and the number of sample points. For a Gaussian input of sound wave, the probability density function $P(E_y)$ can be explicitly derived as follows[5]:

$$P(E_y)=\sum_{K=1}^{M}\sum_{n_1+\cdots+n_M=r_k-1}P_r(Ey;\lambda_k,n_k+1)\times\prod_{m\neq k}g(n_m,r_m,-\lambda_m/\lambda_k), \tag{6}$$

where

$$g\left(n_m,r_m,\frac{-\lambda_m}{\lambda_k}\right)\triangleq\left(-\frac{\lambda_m}{\lambda_k}\right)^{n_m}\times\binom{r_m-1+n_m}{n_m}\left(1-\frac{\lambda_m}{\lambda_k}\right)^{-r_m-n_m} \tag{7}$$

and

$$P_r(E_y;\lambda_k,n_k+1)=\frac{E_y^{n_k}e^{-\frac{E_y}{\lambda_k}}}{\lambda_k^{n_k+1}\gamma(n_k+1)}. \tag{8}$$

Here, the eigenvalue λ_k denotes the intensity of output due to the kth frequency group and n_k denotes the band number of each bandwidth.

4. Experimental consideration

A single-wall system of 12 mm thick plywood panel with $m=8.45$ kg/m^2 and $B=900$ Nm[3] has been fixed between two reverberant rooms.

The cumulative probability distribution of transmitted noise fluctuation are calculated by Eq. 6. The parameters ε_k, α_k and r_k are shown in Table 1. Figure 2 shows some comparison between theoretically calculated curves and experimental points of cumulative distribution on transmitted noise. Though the noise levels in the reception room are originally different between two observation points, the theoretical curve with correction of absorption can illustrate well the both experimental results nearly within 1 dB, in spite of Gaussian input type approximation.

Table 1 Parameters used in Eq. (6).

Center freq.[Hz]	400	500	630	
k	1	2	3	
r_k	12	14	19	
ε_k	4.3	37.2	5.26	$[\times10^{-5}]$
α_k	4.97	3.30	2.18	$[\times10^{-3}]$

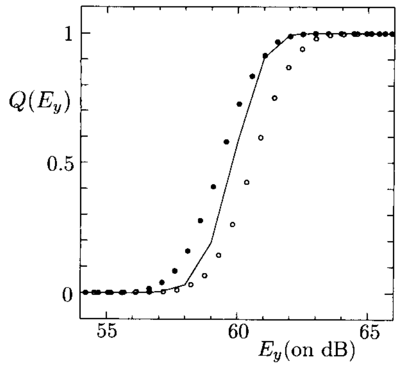

Fig. 2 A comparison between experimental points and theoretical results on the cumulative distribution on the output sound level of a single-wall. ● and ○ denote the experimental points observed at different two points in a reverberation room. A curve is drawn by Eq. (6) with correction on absorption of room.

5. Conclusion

A new stochastic evaluation method based on a physical model with a bending wave is theoretically proposed by first finding a sound pressure transfer function for a single-wall sound insulation under the consideration of coincidence effect. The output response probability distribution of a single-wall under Gaussian type sound pressure incidence is theoretically evaluated by employing several eigenvalues of the frequency transfer character of the plate and the dynamics of a sound level meter. Finally, since this is at an early stage of quantitative research on unification of deterministic and stochastic evaluations for an insulation wall, the proposed method has been applied to the actual noise evaluation problems of transmitted noise of single-wall sound insulation system, as one of principle experiments. The theoretical estimation result has illustrated well the experimental result of transmitted noise level distribution nearly within 1 dB, in spite of Gaussian input type approximation.

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