J. Acoust. Soc. Jpn. (*E*) **21**, 2 (2000)

PAPER

A new tuning method for glass harp based on a vibration analysis that uses a finite element method

Keiichi Oku, Atsushi Yarai, and Takuji Nakanishi

Faculty of Engineering, Osaka Sangyo University, 3-1-1, Nakagaito, Daito, Osaka, 574-8530 Japan

(*Received 17 May 1999*)

When using wine glasses as musical instruments (also referred to as the glass-harp), the pitch needs to be minutely adjusted. That is, it needs to be tuned. A wide adjustment range has been achieved by a new method that locally shaves the bottom of the cup of each vessel circumferencially. The pitch decreased in proportion to the quantity of glass shaved. This relationship between the quantity of glass shaved and the change in pitch was clarified both experimentally and analytically by Finite Element Method (FEM) analysis. The amount of pitch change accompanied with the shaving method is occasionally limited by the vessel shape. In such cases, pitch can be changed by filling wine glasses with specific quantities of water, a well-known conventional tuning method. This auxiliary method has been measured experimentally and analyzed by FEM to clarify the relationship between the water quantity in vessels and the amount of pitch change. A harmonics analysis was also performed. Using these procedures, prediction of vibration frequency could be done in advance, which means a desired pitch can be easily obtained.

Keywords: Glass harp, Vibration analysis, Finite element method, Tuning, Localized shaving

PACS number: 43. 75. Kk, 43. 75. Yy

1. INTRODUCTION

The glass harp has a long history as a musical instrument, because it can be simply composed from wine glasses. The harp is played by rubbing the rim of each glass with a moist finger. Despite the glass harp's simplicity, it has a very pleasing sound, which has been described as "a sound from heaven".

When using wine glasses as musical instruments, it is necessary to adjust the pitch minutely. The water-filling method, which has been widely performed, is one effective method for adjusting pitch. In spite of its effectiveness and the simplicity, however, it has some disadvantages. These include the need to adjust the water level just before the performance and the small extent that the pitch can be changed. We have achieved pitch change with a method that has many advantages over the conventional water-filling method. In this method, we shave the bottom part of each vessel locally and circumferentially. Our final goal for the glass harp as a musical instrument is to be able to use only commercially available glasses.

However, the number of pitches that can be obtained by using only commercially available glasses, even when the shaving method is fully adopted, is limited by the kinds of wine glasses available on the market (in other words, the finite variety of shapes and sizes) and thus the limits of pitch change that can be achieved by shaving. Therefore, the water-filling method must still be considered as a necessary auxiliary method for making minute pitch changes. Even when the water-filling method is used, the water volume required is much less with the shaving method than without it. This is important because better timber can be obtained with less water, as shown below.

This study performed a Finite Element Method (FEM) analysis, as well as an experimental investigation, to analyze the vibration (inherent fundamental and harmonic frequencies) of wine glasses. Previous research has been done on the wine glass vibration mechanism,[1-3] hitherto, the mode study[4,5] and on an analytical natural resonant frequency study of the simple wine glass model.[6] However, such research with regard to the tuning has not been conducted yet.

2. VIBRATION ANALYSIS USING THE FINITE ELEMENT METHOD

2.1 Conditions of the FEM Analysis

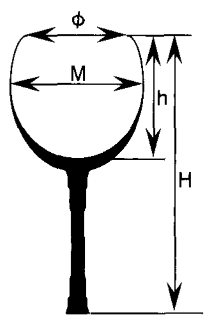
Three types (A,B,C) of commercially available crystal glasses (PdO 24%) of different sizes were prepared in order to experimentally confirm the FEM-analyzed results of the wine glass vibrations. Plural glasses were prepared for each type of glass. Table 1 lists the main dimensions measured, whose definitions are shown in Fig. 1. The thickness data of each glass which varies according to the height, were obtained through measurement. Though shapes of these three types of glasses resemble each other, they are not exactly the same.

Figure 2(a) shows a cross-sectional view of a glass, which is divided into finite elements. As is shown in the figure, each element has a quadrilateral shape that has four nodal points. This division into finite elements required a finer treatment, *i.e.*, smaller elements, for the area around the shaved part than for the other parts of the glass. The shaving method will be described later. This cross section was rotated 360 degrees and the solid glass was divided equally into 36 circular arcs around the circumference, resulting in cubic finite elements that were each composed of eight nodal points, as shown in Fig. 2(b). The boundary conditions required the bottom of the cup of the glass to be fixed. The material constants of the glasses are shown in Table 2.[7] For these glasses, we first analyzed the vibration frequencies using FEM analysis (MARC4 soft-

Table 1 Main dimensions of the glasses.

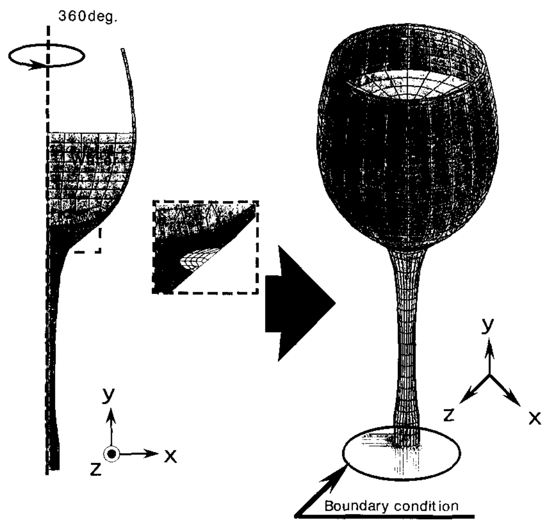
	Glass type		
	A	B	C
H	190	170	140
h	91	76	62
ϕ	66	60	50
M	79	70	58

Unit : mm



H : Total height
h : Vessel's height
ϕ : Diameter of the top of the glass
M : Maximum diameter

Fig. 1 Definitions of wine glass dimensions.



(a) two dimensional cross-sectional view of a glass

(b) three dimensional view

Fig. 2 A wine glass divided into its finite elements.

Table 2 Material constants.

	Wine glass
Young's modulus	6.15×10^{10}N/m^3
Poisson's ratio	0.25
Mass density	3.0×10^3 kg/m^3

ware was used). The eigenvalue analysis was adopted for the glasses without water, and the fluid/solid-coupled analysis was adopted for the glasses with water.

Figure 3 shows the analytical results of the relationship between the number of circumferencial divisions and the inherent frequency of the multiple vibration modes for the A-type glass without the vessel bottom shaved and without water. Here, the definition of each mode corresponds to the previous research.[4] A typical example of these results is shown in Fig. 4. (The vibration, however, is exaggerated here.) Figure 3 indicates that each frequency becomes almost constant when the division number is between 30 and 40. From these results, 36 was adopted as the division number.

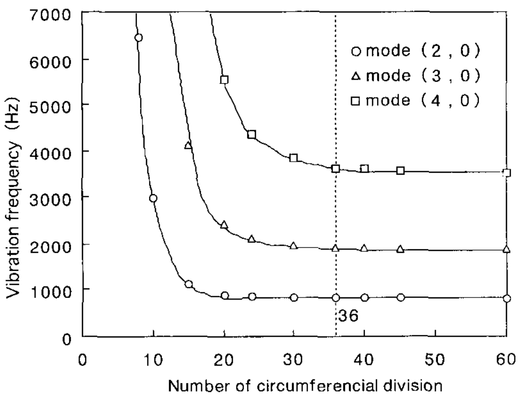
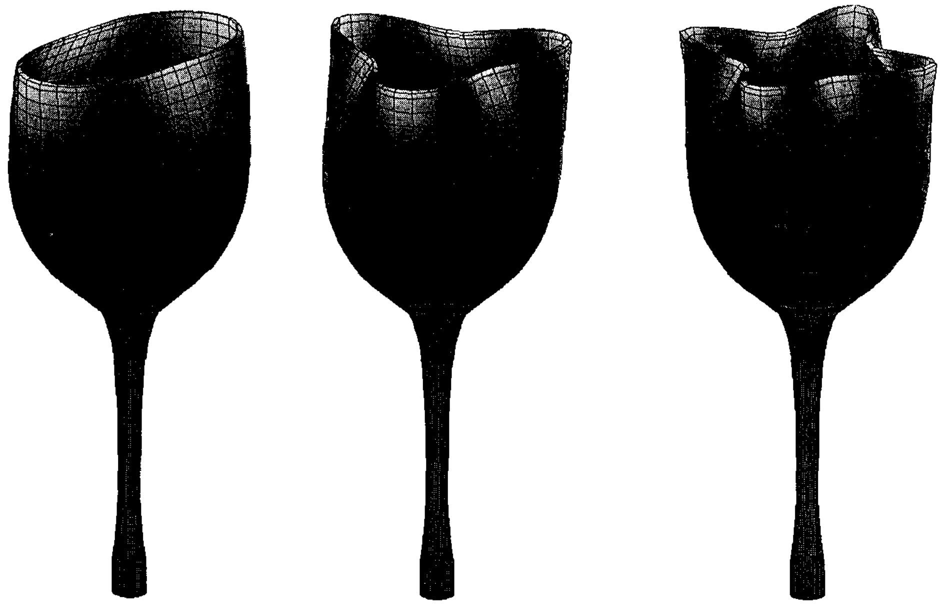


Fig. 3 Relationship between the number of circumferencial divisions and vibration frequency.

2.2 The Relationship between the Glass Shape and Vibration Frequency

Figure 5 shows the relationship between the magnification power n ($0 < n \leq 3$) and the fundamental vibration frequencies of a B-type glass. (In this case, the magnification power n relates to every portion of the vessel, including the height, diameter and thickness.) This was obtained through FEM analysis. The relationship reveals an inverse proportion, whose numerical equation is put inside the figure. The reason for this inverse proportion can be explained as follows. The eigenvalue analysis for the vibration is governed by the following equation of motion:

$$Mx'' + Kx = 0 \qquad (1)$$



(2, 0) Mode (3, 0) Mode (4, 0) Mode

Fig. 4 Vibration modes of a glass.

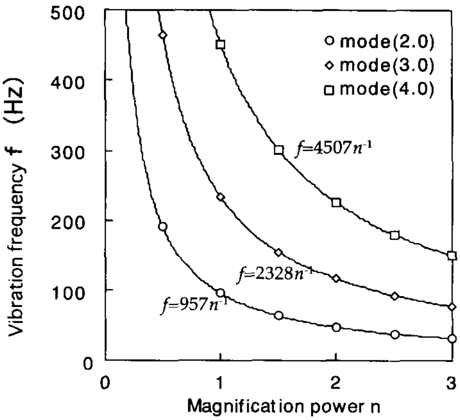


Fig. 5 Relationship between the magnification power n and the vibration frequency (Type B).

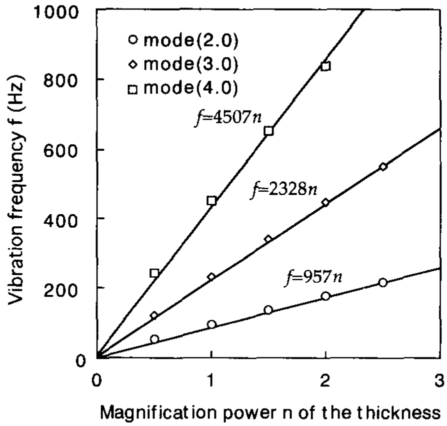


Fig. 6 Relationship between the magnification power n of the thickness and the vibration frequency (Type B).

Here, M, x and K are the mass matrix, displacement vector and stiffness matrix, respectively. When the periodic solution for x is set as

$$x = X\exp(j\omega t), \tag{2}$$

Equation (1) can be described as follows:

$$[-\omega^2 M + K]X = 0 \tag{3}$$

From this equation, the angular vibration frequency can be obtained. When glass shape varies similarly (the magnification power is n), the mass and stiffness are proportioned as n^3 and n, respectively. Consequently, M and K in Equation (3) should be set as n^3M and nK. Substituting these into Equation (3), and setting the angular frequency to be ω', the following equation can be obtained:

$$[-(n\omega')^2 M + K]X = 0 \tag{4}$$

Accordingly, the relationship $\omega' = \omega/n$ can be obtained. This shows that the vibration frequency is proportional to $1/n$ even for a wine glass with a very complicated shape.

2.3 The Relationship between Wine Glass Thickness and Vibration Frequency

Figure 6 shows the relationship between the magnification power n ($0 < n \leqq 3$) of the thickness for a B-type glass, and the fundamental vibration frequencies. (In this case, the magnification power n relates to only the thickness of the vessel.) The relationship reveals a direct proportion, whose numerical equation is put inside the figure. The reason for this proportion can be explained with a procedure similar to the previous section. When glass thickness varies with the magnification power n, the mass is proportional to n and the stiffness is approximately proportional to n^3. Then, the following Equation (5) based on the Equation (3) can be obtained:

$$[-(\omega''/n)^2 M + K]X = 0 \tag{5}$$

Accordingly, the relationship $\omega'' = n\omega$ can also be obtained. This shows that the vibration frequency is directly proportional to n even for a wine glass with a very complicated shape.

3. MINOR CHANGES IN PITCH DUE TO LOCALIZED SHAVING OF THE CUP BASE

In order to achieve minor changes in the pitch of a glass, we tried a method involving localized shaving of the bottom of the vessel of each glass both analytically and experimentally.[8] The shaving was performed circumferencially at the base of the cup, just above the stem of the glass, as shown in Fig. 7. This method was chosen to make the mechanical shaving itself and the analysis easier. The experimental shaving was performed using the following method. First, a glass was set on a manually rotating plate and then ground using a grinder, set at 30,000 rpm. The processing time needed for shaving several grams of glass was roughly several minutes. The drill diameter of the

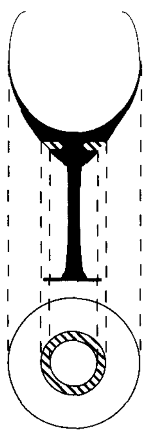


Fig. 7 Method for shaving a glass circumferencially.

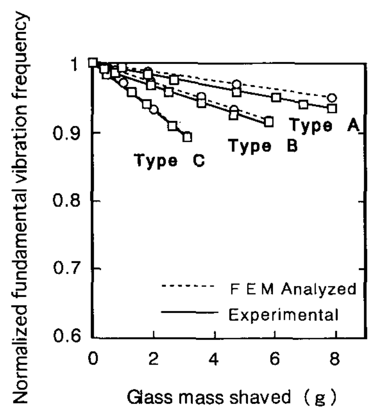


Fig. 9 Relationship between the glass mass shaved and the normalized fundamental vibration frequency (Types A, B, C).

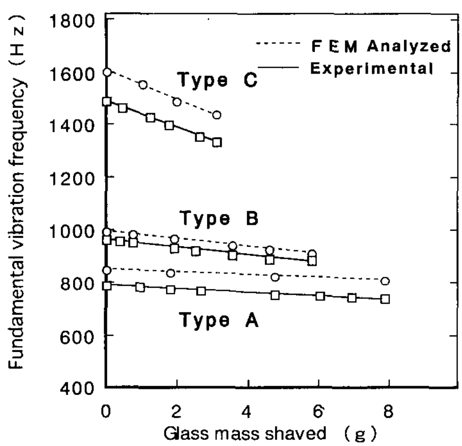


Fig. 8 Relationship between the glass mass shaved and the fundamental inherent frequencies (Types A, B, C).

grinder was 5 mm.

Figure 8 shows the analytical and experimental relationship between the glass mass shaved and the fundamental inherent vibration frequencies for the A-, B-, and C-type glasses of Table 1. A sound and vibration signal analyzer (SA-74, Rion Corp.) was used for the experiment. Figure 8 shows that every vibration frequency decreases linearly with the quantity of mass shaved, keeping both experimental and analytical gradients almost the same. The quantitative differences between the analytical and the experimental results are likely due to the differences between the cross-sectional shapes of the real glasses and those analyzed. The harmonic frequencies also decreased linearly, keeping the ratios of these frequencies to those of the respective fundamental frequencies being constant. The maximum decreases in the vibration frequencies by the shaving were 51 Hz, 76 Hz and 158 Hz for the A, B and C glasses, respectively. These values correspond to about 0.6, 0.8 and 1.0 times a semitone, respectively. Figure 9 shows similar results when these frequencies are normalized against glasses that have not been shaved. From the figure, it can be concluded that the smaller the glass, the larger the decrease in the frequencies.

The basic mechanism for the characteristic linear decrease tendency is as follows. When a glass is shaved, the K value should be set smaller than for glasses that are not shaved. However, the M value in Equation (3) can be regarded as constant because the shaved mass quantity is small compared with the total mass. This leads to a lower vibration frequency than for non-shaving glasses.

As will be described later in more detail, our results have proven that the amount of shaving required can be analytically predicted beforehand if the inherent vibration frequency before shaving has been measured.

4. MINOR CHANGES IN PITCH DUE TO THE WATER-FILLING METHOD

As described above, even if the shaving method is fully adopted, the pitch range obtained using only commercially available glasses is limited. This is due the limited variety of wine glasses available on the market (in other words, the finite variety of their shapes and sizes) and thus a limit in the pitch change that can be achieved by shaving. The water-filling method, therefore, must sometimes be considered as a necessary auxiliary method for making minute pitch changes.[9] Even when the water-filling method is adopted, however, the water volume required is much less with the shaving method than without it. Furthermore, using the shaving method combined with the water-filling method also provides better timber because better timber can be obtained with less water, as shown later.

An FEM that adopts the fluid/solid coupled analysis as well as an experimental investigation was also performed to analyze the vibration (inherent fundamental and harmonic frequencies) of wine glasses. Other procedures conducted were similar to those in the cup base shaving experiment. The additional material constant necessary for the analysis is a mass density of water of 1.0×10^3 kg/m^3.

Figure 10 shows the FEM-analyzed and experimental results of the relationship between the water volume ratio (%) and the inherent fundamental (2,0) mode frequencies for the three types of glasses in Table 1. Here, the water volume ratio is expressed as the ratio of the water in each vessel to that of full capacity. As shown in the figure, the vibration frequency gradually decreases non-linearly with water volume. The tendencies of each curve for both the experimental and the FEM-analyzed results coincide well with each other. The reason for the

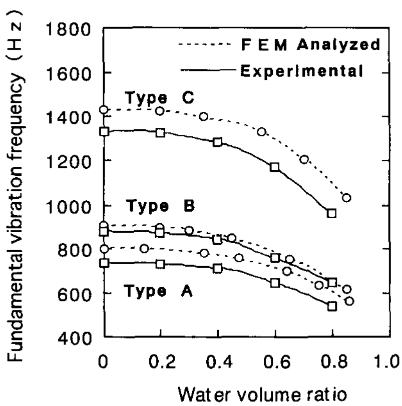


Fig. 10 Relationship between the water volume ratio and the fundamental vibration frequency.

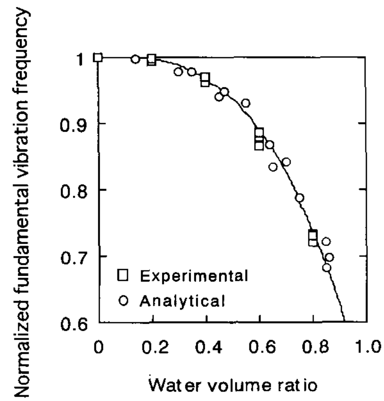


Fig. 11 Relationship between the water volume ratio and the normalized fundamental vibration frequency.

quantitative differences in the absolute values is the same as in the case of Fig. 8. The harmonic frequencies also decreased non-linearly, keeping the ratios of these frequencies to those of the respective fundamental frequencies being constant.

Figure 11 shows similar results when the inherent fundamental vibration frequencies are normalized by those without water. As shown in the figure, the normalized frequencies decrease similarly regardless of glass size. This curve fits the following experimental Equation (6):

$$f_{\mathrm{norm.}} = 1 - 0.5\,p^3 \qquad (6)$$

Here, $f_{\mathrm{norm.}}$ is a normalized frequency, and p is the ratio of the water volume in each vessel to the volume at full capacity. The correlation coefficient between the equation and the experimental data was calculated to be 0.99.

The reason for the frequency decrease can be considered as follows. In the fluid/solid-coupled analysis of FEM, water is set to be an ideal liquid, that is, a non-compressible liquid. The equation of motion can be rewritten based on Equation (3):

$$[-\omega'''^2(M_g + M_f) + K]X = 0 \qquad (7)$$

Here, M_g and M_f are the glass mass and water mass, respectively. The K values in Eq.(7) and Eq.(3) are equal. Comparing Eq.(7) with Eq.(3), ω''' should decrease corresponding to the increase in mass from M_g (without water) to $M_g + M_f$ (with water).

The timber, however, gradually worsens as the water volume increases, becoming clearly unpleas-

Table 3 The maximum adaptable decreases in the vibration frequency.

Glass type	Shaving	Water filling	Total	M.P*
A	51 Hz	23	74	~0.9
B	76	35	111	~1.2
C	158	50	208	~1.4

*Magnifying power to semitone.

ant when the water volume ratio exceeds roughly 40%. Based on these considerations, we set the maximum adaptable water volume ratio to be 40%. Therefore, it can be concluded that the maximum decreases in the vibration frequency for each glass is 23 Hz, 35 Hz and 50 Hz for A, B and C glasses, respectively. These values correspond to about 0.3, 0.4 and 0.4 times one semitone, respectively.

Table 3 summarizes the maximum adaptable decreases in the vibration frequency (Hz) and the corresponding magnifying power in the semitones for A, B and C glasses, when both the cup shaving and the water-filling methods are adopted.

5. TIMBER ESTIMATION

The influence of the vessel-base shaving and the water volume ratio on timber was studied through a harmonics estimation. As mentioned earlier, timber gradually worsens as the water volume exceeds a certain percent (~40%) of the vessel's volume. However, we found that when the base of each vessel was shaved, the timber did not worsen at all.

Figure 12 shows the relationship between relative amplitudes of the second and third harmonics (dBr: the ratios of the second and third harmonic amplitudes to those of the respective fundamental frequencies for three types of glasses) for both the shaved-mass and the water-volume ratios. Water was poured into each vessel after the base was shaved as much as possible. The experimental results showed that 1) the relative amplitudes of every harmonic were almost independent of the mass shaved, and 2) all relative amplitudes for every harmonic, except that of the second harmonic, showed little relation to water volume, but the relative amplitude of the second harmonic increased as water volume increased. At around the 40% water-volume ratio, the relative amplitude of the second harmonic was less than about 20 dB. These results clarified that the timber aggravation that accompanies an increase in water volume is mainly due to the increase in the relative amplitude of the second harmonic.

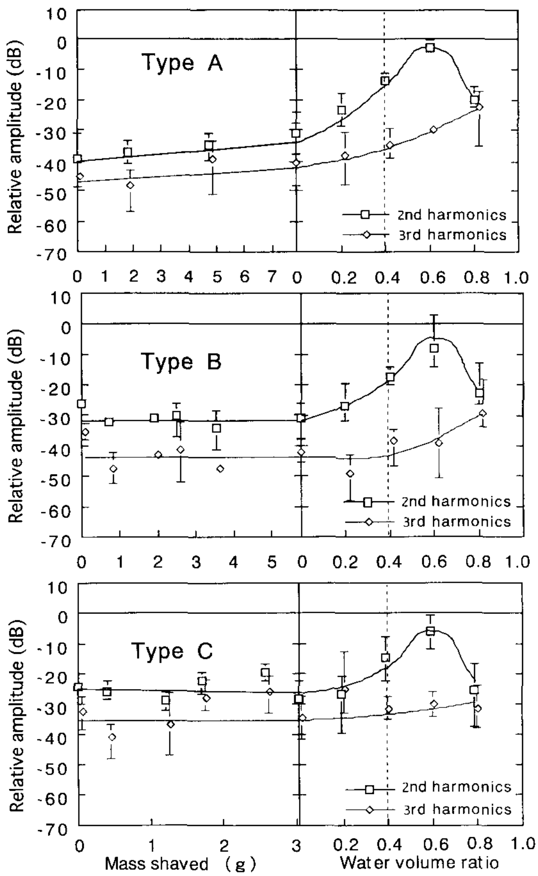


Fig. 12 The relationship between the ratios of the second and the third relative amplitudes and both the mass shaved and the water volume ratio.

6. PREDICTION OF THE PITCH

Based on the reasons above, the FEM analysis was proven to be effective in predicting the pitch achievable from glasses of various shapes that have different amount of mass shaved and water volume. The concrete method for estimating pitch in advance is performed according to the following procedures.

1) When a certain pitch is required, the procedures in Section 2.2 and 2.3 can be used to find an approximate size and shape for a glass.

2) When a glass is obtained that has a shape close to that found in procedure 1, we can accurately establish both its analyzed and measured inherent

fundamental vibration frequencies, f_{0c} and f_{0m}, before the cup base is shaved.

3) The gradient value b for the relationship between the mass shaved from the vessel and the frequency can also be calculated using the FEM analysis of Chapter 3.

4) Combining the values f_{0m} and b, we can calculate the exact amount of shaved mass required to obtain the objective frequency, and the objective pitch can be attempted.

5) When the pitch accomplished in the above procedures is still out of the target one, the water volume needed for the precise pitch adjustment should subsequently be calculated using experimental Equation (6).

Through these procedures, the objective pitch can be obtained.

7. CONCLUSION

Due to the limited variety of commercially available glasses (in other words, a small variety of shapes and sizes) for the glass harp, a glass shaving method has been developed for making minute pitch changes. The wine glass vibration was analyzed using Finite Element Method (FEM) analysis and an experimental investigation. The vibration mode was clarified through the FEM analysis. It was found that the experimental and analyzed frequency changes for the shaved mass and water volume nearly coincided with each other. This shows that FEM analysis can be effective for predicting the pitch of various shapes of glasses. It was also found that the timber aggravation corresponding to an increase in water volume was caused mainly by an increase in the second harmonic amplitude.

REFERENCES

1) T. D. Rossing, "Wine glasses, bell modes, and Lord Rayleigh," Phys. Teach. **28**, 582–585 (1990).
2) R. E. Apfel, "Whispering waves in a wineglass," Am. J. Phys. **53**, 1070–1073 (1985).
3) T. D. Rossing, *The Science of Sound*, 2nd ed. (Addison-Wesley Publishing Company, U. S., 1990).
4) T. D. Rossing, "Acoustics of some less-common percussion instruments," Proc. Int. Symp. Musical Acoustics, PA-1, 1–8 (1992).
5) U. J. Hansen, T. D. Rossing, and G. J. Jelatis, "Holographic studies of vibration modes in noncircular cylinders," Proc. Int. Symp. Musical Acoustics, 4A-3, 223–8226(1992).
6) A. P. French, "In Vino Verritas: A study of wine-glass acoustics," Am. J. Phys. **51**, 683–694 (1983).
7) Technical data obtained by the private communications with Sasaki Glass Co. Ltd., which is a manufacturer of the glasses.
8) T. Nakanishi, I. Nakai, and A. Yarai, "Vibration analysis of a glass-harp by finite element method —Minor pitch change due to localized shaving of the bottom part—," Proc. 3rd Jt. Meet. Acoust. Soc. Am. and Acoust. Soc. Jpn., 397–400 (1996).
9) T. Nakanishi, K. Oku, and A. Yarai, "Vibration analysis of a glass-harp by finite element method —Minor pitch change due to water filling in the vessel—," Proc. Japan-China Jt. Meet. Musical Acoustics, 15–20 (1997).

Keiichi Oku was born in Kyoto, Japan on November 26, 1974. He received the B. E. and M. E. degrees in Faculty of Engineering from Osaka Sangyo University, Osaka, Japan, in 1997, 1999 , respectively. He joined the Electrical Communication Laboratories, Nippon Advanced Technology Corporation in 1999. He is a member of the Acoustical Society of Japan.

Atsushi Yarai was born in Kyoto, Japan in 1966. He received the B. E. and M. E. degrees in electronics from Osaka Sangyo University, Japan, in 1989 and 1991, respectively. He also received the Ph.D. degree in Applied Physics and Chemistry from University of Electro-Communications in 1996. He joined the Department of Electrical Engineering and Electronics, Faculty of Engineering, Osaka Sangyo University in 1991 as a Research Associate, and has been Assistant Professor since 1998. Dr. Yarai is a member of the Japan Society of Applied Physics, Institute of Electronics, Information and Communication Engineers of Japan, Society of Instrument and Control Engineers of Japan, IEEE, and Optical Society of America.

Takuji Nakanishi was born in Kyoto, Japan on November 11, 1943. He received the B. E., M. E. and Ph.D. degrees in communication engineering from Osaka University, Osaka, Japan, in 1967, 1969 and 1984, respectively. He joined the Electrical Communication Laboratories, Nippon Telegraph and Telephone Corporation (NTT) in 1969. In 1991, he joined the Faculty of Engineering, Osaka Sangyo University, as a professor. Dr. Nakanishi is a member of the Acoustical Society of Japan, the Japan Society of Applied Physics, the Institute of Electronics, Information and Communication Engineers and Information Processing Society of Japan.