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L^2 HARMONIC 1-FORMS ON COMPLETE SUBMANIFOLDS IN EUCLIDEAN SPACE

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Abstract

Let M^n $(n \geq 3)$ be an n-dimensional complete noncompact oriented submanifold in an $(n+p)$-dimensional Euclidean space $\mathbf{R}^{n+p}$ with finite total mean curvature, i.e, $\int_M |H|^n < \infty$, where H is the mean curvature vector of M. Then we prove that each end of M must be non-parabolic. Denote by ϕ the traceless second fundamental form of M. We also prove that if $\int_M |\phi|^n < C(n)$, where $C(n)$ is an an explicit positive constant, then there are no nontrivial L^2 harmonic 1-forms on M and the first de Rham's cohomology group with compact support of M is trivial. As corollaries, such a submanifold has only one end. This implies that such a minimal submanifold is plane.

1. Introduction

Let us recall that the well-known Berstein's theorem asserts that an entire minimal graph $M^n \subset \mathbf{R}^{n+1}$ must be linear if $n \leq 7$. Moreover, the dimension restriction is necessary as indicated by the examples of Bombieri, De Giorgi and Giusti. Because of the stability of minimal entire graphs, one is naturally led to the generalization of the classical Bernstein theorem to the question of asking whether all stable minimal hypersurfaces in $\mathbf{R}^{n+1}$ are hyperplanes when $n \leq 7$. In the case when $n = 2$, this problem was solved independently in [6] and [7]. For higher dimension, this problem is still open. However, Cao-Shen-Zhu proved a topological obstruction for complete immersed stable minimal hypersurface M^n of $\mathbf{R}^{n+1}$ with $n \geq 3$ that M must have only one end [2]. Its strategy was to utilize a result of Schoen-Yau asserting that a complete stable minimal hypersurface of $\mathbf{R}^{n+1}$ can not admit a non-constant harmonic function with finite Dirichlet integral [15]. Assuming that M^n has more than one end, they constructed a non-constant harmonic function with finite Dirichlet integral in [2]. According to the work of Li-Tam [11], Li-Wang modified this proof to show that each end of a complete immersed minimal submanifold must be non-parabolic in [12]. Due to this connection with harmonic functions, this allows

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one to estimate the number of ends of the above hypersurface by estimating the dimension of the space of bounded harmonic functions with finite Dirichlet integral [11]. Since the exterior differential form of harmonic function is an L^2 harmonic 1-form, the theory of L^2 harmonic forms give one to study minimal submanifolds in $\mathbf{R}^{n+1}$ [12, 19].

Let M^n be an n-dimensional complete oriented submanifold isometrically immersed in an $(n+p)$-dimensional Euclidean space $\mathbf{R}^{n+p}$. Fix a point $x \in M$ and choose a local orthonormal frame $\{e_1, e_2, \dots, e_{n+p}\}$ such that $\{e_1, e_2, \dots, e_n\}$ are tangent fields. For each α, $n+1 \le \alpha \le n+p$, define a linear map $A_\alpha : T_xM \to T_xM$ by

$$\langle A_\alpha X, Y \rangle = \langle \tilde{\nabla}_X Y, e_\alpha \rangle,$$

where X, Y are tangent fields and $\tilde{\nabla}$ denotes the Euclidean connection on $\mathbf{R}^{n+p}$. We denote by H the mean curvature vector of M, i.e.,

$$H = \frac{1}{n} \sum_{\alpha=n+1}^{n+p} (Tr\ A_\alpha) e_\alpha.$$

For each α, $n+1 \le \alpha \le n+p$, define a linear map $\phi_\alpha : T_xM \to T_xM$ by

$$\langle \phi_\alpha X, Y \rangle = \langle X, Y \rangle \langle H, e_\alpha \rangle - \langle A_\alpha X, Y \rangle,$$

and a bilinear map $\phi : T_xM \times T_xM \to T_xM^\perp$ by

$$\phi(X, Y) = \sum_{\alpha=n+1}^{n+p} \langle \phi_\alpha X, Y \rangle e_\alpha.$$

It is easy to see that the tensor ϕ is traceless. Denote by A the second fundamental form of M. We have

$$|A|^2 = |\phi|^2 + n|H|^2.$$

By Hoffman and Spruck's Sobolev inequality, there exists an explicit positive constant $S(n)$ depending only on the dimension n such that

$$(1.1) \qquad \left(\int_M |f|^{n/(n-1)} \right)^{(n-1)/n} \le S(n) \int_M (|\nabla f| + n|H|\,|f|), \quad \forall f \in C_0^1(M).$$

$\int_M |\phi|^n * 1$ and $\int_M |H|^n * 1$ are called the total curvature of M and the total mean curvature of M respectively. Let $H^1(L^2(M))$ denote the space of L^2 harmonic 1-forms on M, $H_0^1(M)$ denote the first de Rham's cohomology group with compact support of M and Δ denote the Laplacian on M.

In [19], Yun proved that if M^n $(n \ge 3)$ is a complete oriented minimal hypersurface in $\mathbf{R}^{n+1}$ and if

$$\int_M |A|^n < \left(\frac{n-2}{2(n-1)S(n)} \right)^n,$$

then there are no L^2 harmonic 1-forms on M, and M has only one end. When M^n $(n \geq 3)$ is a complete oriented immersed minimal submanifold in $\mathbf{R}^{n+p}$, Ni showed that if

$$\int_M |A|^n < \left(\sqrt{\frac{n}{n-1}}\,\frac{n-2}{2(n-1)S(n)}\right)^n,$$

then M has only one end in [14]. Seo improved the upper bound of total scalar curvature and proved if M has the same upper bound of total scalar curvature, there is no nontrivial L^2 harmonic 1-form on M in [16]. In [8], the first author and Xu studied complete noncompact oriented submanifolds with bounded total curvature and bounded total mean curvature in $\mathbf{R}^{n+p}$. In this paper, following the work due to Li-Wang and Yun, we study complete noncompact oriented submanifolds with bounded total curvature and finite total mean curvature in $\mathbf{R}^{n+p}$. Throughout this article, we always assume that M is a complete, noncompact, connected Riemannian manifold without boundary. In this case, we will simply say that M is a complete manifold.

Our main result in this paper is stated as follows:

THEOREM 1.1. *Let M^n $(n \geq 3)$ be an oriented complete submanifold with finite total mean curvature vector in $\mathbf{R}^{n+p}$. Then there exists an explicit positive constant $C(n)$ such that if*

$$\int_M |\phi|^n < C(n),$$

then $H^1(L^2(M)) = 0$, and M must have only one end. Moreover, $H_0^1(M) = 0$.

2. Preliminary

Let M^n $(n \geq 3)$ be an oriented complete immersed submanifold with mean curvature vector H in $\mathbf{R}^{n+p}$.

If $\int_M |H|^n < \infty$, then there exists a compact subset $D \subset M$ such that

$$\left(\int_{M\setminus D} |H|^n\right)^{1/n} \leq \frac{1}{\alpha n S(n)}, \quad \alpha > 1.$$

Thus

$$\begin{aligned} nS(n)\int_{M\setminus D} |H|\,|f| &\leq nS(n)\left(\int_{M\setminus D} |H|^n\right)^{1/n}\left(\int_{M\setminus D} |f|^{n/(n-1)}\right)^{(n-1)/n} \\ &\leq \frac{1}{\alpha}\left(\int_{M\setminus D} |f|^{n/(n-1)}\right)^{(n-1)/n}. \end{aligned}$$

Substituting the above inequality into (1.1), we have

$$\left(\int_{M\setminus D}|f|^{n/(n-1)}\right)^{(n-1)/n} \le \frac{\alpha}{\alpha-1}S(n)\int_{M\setminus D}|\nabla f|, \quad \forall f \in C_0^1(M\setminus D). \tag{2.1}$$

Putting $f = g^{2(n-1)/(n-2)}$ with $g \in C_0^1(M\setminus D)$ in (2.1), we obtain

$$\left(\int_{M\setminus D}|g|^{2n/(n-2)}\right)^{(n-2)/n} \le 4\frac{\alpha^2(n-1)^2}{(\alpha-1)^2(n-2)^2}S^2(n)\int_{M\setminus D}|\nabla g|^2. \tag{2.2}$$

According to [4], from (2.2) we have

$$\left(\int_{M}|g|^{2n/(n-2)}\right)^{(n-2)/n} \le 4\frac{\alpha^2(n-1)^2}{(\alpha-1)^2(n-2)^2}S^2(n)\int_{M}|\nabla g|^2, \quad \forall g \in C_0^\infty(M). \tag{2.3}$$

If $H = 0$, H. Muto [13] improved on Hoffman and Spruck's Sobolev inequality as follows:

$$\left(\int_{M}|f|^{2n/(n-2)}\right)^{(n-2)/n} \le \frac{96}{\pi^2}\int_{M}|\nabla f|^2, \quad \forall f \in C_0^1(M). \tag{2.4}$$

In this paper, we will discuss the number of ends of submanifolds. Now we state some definitions and well-known theorems.

Definition 2.1. Let $D \subset M$ be a compact subset of M. An end E of M with respect to D is a connected unbounded component of $M\setminus D$. When we say that E is an end, it is implicitly assumed that E is an end with respect to some compact subset $D \subset M$.

Definition 2.2. A manifold is said to be parabolic if it does not admit a positive Green's function. Conversely, a nonparabolic manifold is one which admits a positive Green's function. An end E of a manifold is said to be nonparabolic if it admits a positive Green's function with Neumann boundary condition on ∂E. Otherwise, it is said to be parabolic.

Theorem 2.3 ([11]). *Let M be a complete manifold. Let $\mathscr{H}_D^0(M)$ denote the space of bounded harmonic functions with finite Dirichlet integral. Then the number of non-parabolic ends of M is at most the dimension of $\mathscr{H}_D^0(M)$.*

Theorem 2.4 ([12]). *Let E be an end of a complete manifold. Suppose for some $v \ge 1$, E satisfies a Sobolev type inequality of the form*

$$\left(\int_E |f|^{2v}\right)^{1/v} \le C\int_E |\nabla f|^2, \quad \forall f \in C_0^1(E).$$

then E must either have finite volume or be non-parabolic.

Proposition 2.5. *Let M^n $(n \geq 3)$ be a complete manifold. If there exists a constant $C(n) > 0$ depending only on n such that*

$$\left(\int_M |f|^{2n/(n-2)}\right)^{(n-2)/n} \leq C(n) \int_M |\nabla f|^2, \quad \forall f \in C_0^\infty(M).$$

then the volume of each end E of M is infinite, and each end of M is non-parabolic.

Proof. By the definition of end, every end E of M is contained in $M \backslash D$, then

$$\left(\int_E |f|^{2n/(n-2)}\right)^{(n-2)/n} \leq C(n) \int_E |\nabla f|^2, \quad \forall f \in C_0^\infty(E). \tag{2.5}$$

According to proposition 2.4 in [3], we know the sobolev inequality (2.5) implies a uniform lower bound on the volume of geodesic ball:

$$\forall x \in M,\ \forall r \geq 0 : vol\ B_r(x) \geq C'(n) r^n$$

for some explicit positive constant $C'(n)$ depending only on n.

Let $R > 0$ large enough so that $D \subset B_R(p)$. For $k \in \mathbf{N}$ we choose $x \in \partial B_{R+(2k+1)\varepsilon}(p) \cap E$ and consider $\gamma : [0, R + (2k+1)\varepsilon] \to M$ a minimizing geodesic from p to x. Necessary for all $t \in (R, R + (2k+1)\varepsilon]$ we have $\gamma(t) \in E$; moreover for $l = 0, 1, \ldots, k$, the open geodesic ball $B_\varepsilon(R + (2l+1)\varepsilon)$ are in E and disjiont. Hence

$$vol\ E \geq \sum_{l=0}^{k} vol\ B_\varepsilon(R + (2l+1)\varepsilon) \geq (k+1) C'(n) \varepsilon^n.$$

Therefore the volume of every end E of M is infinite. By Theorem 2.4 and (2.5), every end of M is non-parabolic. □

By (2.3) and Proposition 2.5, we get

Corollary 2.6. *Let M^n $(n \geq 3)$ be an oriented complete submanifold with finite total mean curvature in $\mathbf{R}^{n+p}$. Then each end of M must be non-parabolic.*

Remark 2.7. It is showed that each end of the oriented complete minimal submanifold M^n $(n \geq 3)$ in $\mathbf{R}^{n+p}$ must be non-parabolic in [2, 10].

3. Proof of the theorems

For each $\omega \in H^1(L^2(M))$, we have the following well-known Bochner formula.

$$\Delta |\omega|^2 = 2(|\nabla \omega|^2 + Ric(\omega, \omega)). \tag{3.1}$$

On the other hand, we have

$$\Delta|\omega|^2 = 2(|\omega|\Delta|\omega| + |\nabla|\omega||^2). \tag{3.2}$$

From (3.1), (3.2) and the generalized Kato's inequality $\frac{n}{n-1}|\nabla|\omega||^2 \leq |\nabla\omega|^2$, we obtain

$$|\omega|\Delta|\omega| \geq Ric(\omega,\omega) + \frac{1}{n-1}|\nabla|\omega||^2. \tag{3.3}$$

In [18], Shiohama and Xu proved that the following estimate holds for Ricci curvature of a submanifold M in the simply connected space form $N^{n+p}(c)$ with constant sectional curvature c.

$$Ric \geq \frac{n-1}{n}\left(nc + 2n|H|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}|H|\sqrt{|A|^2 - n|H|^2} - |A|^2\right).$$

Applying the above inequality to the traceless second fundamental form $|\phi|$ and using the identity $|A|^2 = |\phi|^2 + n|H|^2$, we get

$$Ric \geq (n-1)c + (n-1)|H|^2 - \frac{(n-2)\sqrt{n(n-1)}|\phi|\,|H|}{n} - \frac{(n-1)|\phi|^2}{n}. \tag{3.4}$$

Substituting (3.4) into (3.3), we obtain

$$\begin{aligned} |\omega|\Delta|\omega| \geq{}& \frac{1}{n-1}|\nabla|\omega||^2 + (n-1)c|\omega|^2 \\ &- \left[\frac{(n-2)\sqrt{n(n-1)}|\phi|\,|H|}{n} + \frac{(n-1)|\phi|^2}{n} - (n-1)|H|^2\right]|\omega|^2. \end{aligned} \tag{3.5}$$

THEOREM 3.1. *Let M^n $(n \geq 3)$ be an oriented complete submanifold with finite total mean curvature in $\mathbf{R}^{n+p}$. If*

$$\int_M |\phi|^n < \left(\frac{(n-2)}{(n-1)\sqrt{n-1}S(n)}\right)^n,$$

then $H^1(L^2(M)) = 0$, moreover $H_0^1(M) = 0$.

Proof. Let $\omega \in H^1(L^2(M))$. Fixing a point $p \in M$ and for $r > 0$, we choose a C^1 cut-off function η satisfying $0 \leq \eta \leq 1$, $\eta \equiv 1$ on $B_r(p) \subset M$, $\eta \equiv 0$ on $M \backslash B_{2r}(p)$, and $|\nabla\eta| \leq \frac{1}{r}$ on $B_{2r}(p)\backslash B_r(p) \subset M$. Multiplying (3.5) by η^2 and integrating by parts over M, we get

$$
\begin{aligned}
(3.6)\qquad 0 \le & \int_M \left(\eta^2 |\omega| \Delta |\omega| - \frac{1}{n-1} \eta^2 |\nabla |\omega| \, |^2 \right) \\
& + \int_M \eta^2 \left(\frac{(n-2)\sqrt{n(n-1)} |\phi| \, |H|}{n} + \frac{(n-1)|\phi|^2}{n} - (n-1)|H|^2 \right) |\omega|^2 \\
= & -2 \int_M \eta \langle \nabla \eta, \nabla |\omega| \rangle |\omega| - \frac{n}{n-1} \int_M \eta^2 |\nabla |\omega| \, |^2 + \frac{n-1}{n} \int_M \eta^2 |\phi|^2 |\omega|^2 \\
& + \frac{(n-2)\sqrt{n(n-1)}}{n} \int_M |\phi| \, |H| \eta^2 |\omega|^2 - (n-1) \int_M |H|^2 \eta^2 |\omega|^2 \\
\le & -2 \int_M \eta \langle \nabla \eta, \nabla |\omega| \rangle |\omega| - \frac{n}{n-1} \int_M \eta^2 |\nabla |\omega| \, |^2 + \frac{n}{4} \int_M \eta^2 |\phi|^2 |\omega|^2 .
\end{aligned}
$$

On the other hand, it follows from (2.3) and Hölder inequality that

$$
\begin{aligned}
(3.7)\qquad \frac{n}{4} \int_M \eta^2 |\phi|^2 |\omega|^2 \le & \frac{n}{4} \left(\int_M |\phi|^n \right)^{2/n} \left(\int_M (\eta |\omega|)^{2n/(n-2)} \right)^{(n-2)/n} \\
\le & \frac{n\alpha^2 (n-1)^2 \phi_0}{(\alpha-1)^2 (n-2)^2} S^2(n) \int_M |\nabla (\eta |\omega|)|^2 \\
= & \frac{n\alpha^2 (n-1)^2 \phi_0}{(\alpha-1)^2 (n-2)^2} S^2(n) \int_M (|\omega|^2 |\nabla \eta|^2 + \eta^2 |\nabla |\omega| \, |^2) \\
& + \frac{n\alpha^2 (n-1)^2 \phi_0}{(\alpha-1)^2 (n-2)^2} S^2(n) \int_M 2\eta \langle \nabla \eta, \nabla |\omega| \rangle |\omega|,
\end{aligned}
$$

where $\phi_0 = (\int_M |\phi|^n)^{2/n}$. Substituting (3.7) into (3.6), we have

$$
\begin{aligned}
(3.8)\qquad 0 \le & \, 2 \left(\frac{n\alpha^2 (n-1)^2 \phi_0}{(\alpha-1)^2 (n-2)^2} S^2(n) - 1 \right) \int_M \eta \langle \nabla \eta, \nabla |\omega| \rangle |\omega| \\
& + \left(\frac{n\alpha^2 (n-1)^2 \phi_0}{(\alpha-1)^2 (n-2)^2} S^2(n) - \frac{n}{n-1} \right) \int_M \eta^2 |\nabla |\omega| \, |^2 \\
& + \frac{n\alpha^2 (n-1)^2 \phi_0}{(\alpha-1)^2 (n-2)^2} S^2(n) \int_M |\omega|^2 |\nabla \eta|^2
\end{aligned}
$$

Using Schwarz inequality, we get

$$
(3.9)\qquad 2 \left| \int_M \eta \langle \nabla \eta, \nabla |\omega| \rangle |\omega| \right| \le \varepsilon \int_M \eta^2 |\nabla |\omega| \, |^2 + \frac{1}{\varepsilon} \int_M |\omega|^2 |\nabla \eta|^2 .
$$

From (3.8) and (3.9), we obtain

$$\left(\left(\frac{n}{n-1}-\frac{n\alpha^2(n-1)^2\phi_0}{(\alpha-1)^2(n-2)^2}S^2(n)\right)-\left|1-\frac{n\alpha^2(n-1)^2\phi_0}{(\alpha-1)^2(n-2)^2}S^2(n)\right|\varepsilon\right)\int_M \eta^2|\nabla|\omega||^2$$
$$\leq\left(\frac{1}{\varepsilon}\left|1-\frac{n\alpha^2(n-1)^2\phi_0}{(\alpha-1)^2(n-2)^2}S^2(n)\right|+\frac{n\alpha^2(n-1)^2\phi_0}{(\alpha-1)^2(n-2)^2}S^2(n)\right)\int_M |\omega|^2|\nabla\eta|^2$$
$$\leq\left(\frac{1}{\varepsilon}\left|1-\frac{n\alpha^2(n-1)^2\phi_0}{(\alpha-1)^2(n-2)^2}S^2(n)\right|+\frac{n\alpha^2(n-1)^2\phi_0}{(\alpha-1)^2(n-2)^2}S^2(n)\right)\frac{1}{r^2}\int_{B_{2r}(p)} |\omega|^2.$$

Since $\int_M |\phi|^n < \left(\frac{(n-2)}{(n-1)\sqrt{n-1}S(n)}\right)^n$, choosing $\varepsilon > 0$ sufficiently small and letting $r\to\infty$, we get $\nabla|\omega| = 0$ on M, i.e., $|\omega|$ is constant. Since $\int_M |\omega|^2 < \infty$, and the volume of M is infinite by Proposition 2.5, we have $\omega = 0$, thus get $H^1(L^2(M)) = 0$. Hence we obtain $H_0^1(M) = 0$ by Proposition 2.11 in [4]. □

Observe that if f is a harmonic function with finite Dirichlet integral then its exterior df is an L^2 harmonic 1-form. Moreover, $df = 0$ if and only if f is identically constant. By Theorems 2.3 and 3.1 and Corollary 2.6, one has the following result.

Corollary 3.2. *Let M^n $(n \geq 3)$ be an oriented complete submanifold with finite total mean curvature in $\mathbf{R}^{n+p}$. If*

$$\int_M |\phi|^n < \left(\frac{(n-2)}{(n-1)\sqrt{n-1}S(n)}\right)^n,$$

then there are no non-constant harmonic functions on M with finite Dirichlet integral, and M has only one end.

Corollary 3.3. *Let M^n $(n \geq 3)$ be an oriented complete submanifold with parallel mean curvature in $\mathbf{R}^{n+p}$. If*

$$\int_M |A|^n < \left(\frac{n\pi}{4\sqrt{6}(n-1)}\right)^n,$$

then $H^1(L^2(M)) = 0$, and M has only one end. Moreover M is a plane.

Proof. From the main theorem in [17], we see that if M^n $(n \geq 3)$ is an oriented complete submanifold with parallel mean curvature and finite total curvature in R^{n+p}, then M must be minimal. Applying the same argument as in the proof of Theorem 3.1 and (2.4), we conclude that under assumption of Corollary 3.3, there are no nontrivial L^2 harmonic 1-forms on M and M has only one end. A theorem due to Anderson [1] says that the minimal submanifold with only one end and finite total curvature in $\mathbf{R}^{n+p}$ is an affine plane. Hence M must be a plane. □

Remark 3.4. Theorem 1.1 can be considered as a generalization of Ni and Yun's results [14, 19].

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