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PERIODICITY IN IMPULSIVE PREDATOR-PREY SYSTEM WITH HOLLING III FUNCTIONAL RESPONSE

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Abstract

With the help of the continuation theorem in coincidence degree theory, we establish the existence of positive periodic solutions of impulsive predator-prey system with Holling III functional response.

1 Introduction

Recently, many authors devoted themselves to investigate the dynamics of nonautomous predator-prey system with periodic parameters and achieved many results, see for example [3–6, 9, 11, 17, 19]. As we all know, the predator-prey systems with Holling III functional response possess an important role in the predator-prey theory. Some authors have shown great interest in such systems with periodic parameters. For example, Jia [9] has investigated predator-prey model with Holling III functional response [8].

$$
\begin{aligned}
y_1'(t) &= y_1(t)\left(a_1(t) - a_{11}(t)y_1(t) - \frac{a_{12}(t)y_1(t)y_2(t)}{a_{13}^2(t) + y_1^2(t)}\right),\\
y_2'(t) &= y_2(t)\left(-a_2(t) - a_{21}(t)y_2(t) + \frac{a_{22}(t)y_1^2(t)}{a_{13}^2(t) + y_1^2(t)}\right),
\end{aligned}
\tag{1.1}
$$

and derived sufficient conditions for the persistence and the existence of periodic solutions.

In 2002, following the idea and the method of [6], Zhang [19] derives sufficient criteria for the existence of positive periodic solution of the discrete analogue of (1.1) governed by nonautomous difference equation with periodic parameters.

Key Words: Positive periodic solution, predator-prey system, Holling III functional response, coincidence degree, impulse.

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However, almost all the work on the predator-prey systems with Holling III functional response neglect the practical effects of some important impulsive factors existing in the real world. The birth of many populations is not continuous but happens at some regular time, for example, the birth of many wildlife is seasonal. The birth of population at those time can be viewed as impulse to the population. Another typical impulse is the harvest and stock of the population explored by human beings. If one incorporate such impulsive factors into the mathematical models modelling the population interactions, those models must be impulsive and governed by impulsive deferential equations [1, 2, 10, 12–16, 18].

The principal aim of this paper is to incorporate impulsive factors into system (1.1) and establish sufficient criteria for the existence of positive periodic solution of the following system

$$\begin{aligned}
y_1'(t) &= y_1(t)\left(-d_1(t) - a_{11}(t)y_1(t) - \frac{a_{12}(t)y_1(t)y_2(t)}{a_{13}^2(t) + y_1^2(t)}\right), \quad t \neq t_k,\ k = 1,2\ldots \\
y_2'(t) &= y_2(t)\left(-d_2(t) - a_{21}(t)y_2(t) + \frac{a_{22}(t)y_1^2(t)}{a_{13}^2(t) + y_1^2(t)}\right), \quad t \neq t_k,\ k = 1,2\ldots \\
\triangle y_i(t) &= y_i(t^+) - y_i(t^-) = (b_{ik} + h_{ik})y_i(t), \quad t = t_k,\ y_i(0) = y_{i0},\ i = 1,2,
\end{aligned} \tag{1.2}$$

where b_{ik}, h_{ik} stand for birth rate and harvesting (stocking) rate of y_i at time t_k, respectively; $d_1(t)$ is the death rate of the prey and $d_2(t)$ is that of the predator. Other parameters have the same meaning as that in (1.1). When $h_{ik} > 0$, it stands for stocking, while $h_{ik} < 0$ means harvesting. $y_i(t_k^+)$ and $y_i(t_k^-)$ represent the right and the left limit of y_i at t_k, respectively. In this paper, it is assumed that y_i is left-continuous at t_k.

Assume that

(A_1) $b_{ik} \geq 0$, $b_{ik} + h_{ik} \geq 0$, and $d_i(t), a_{ij}(t)$ $(i = 1,2; j = 1,2,3.)$ are non-negative continuous ω-periodic functions

(A_2) there exists a positive integer q, such that $t_{k+q} = t_{k+\omega}$, $b_{i(k+q)} = b_{ik}$, $h_{i(k+q)} = h_{ik}$. Without loss of generality, we also assume that if $t_k \neq 0$ and $[0,\omega] \cap t_k = t_1, t_2 \cdots t_m$, then it follows that $q = m$.

It is trivial to show that the solutions of (1.2) with positive initial value remain positive too. So, we can make the change of variables

$$y_i(t) = e^{x_i(t)}, \quad i = 1,2,$$

and then (1.2) is reformulated as

$$\begin{aligned}
x_1'(t) &= -d_1(t) - a_{11}(t)e^{x_1(t)} - \frac{a_{12}(t)e^{x_2(t)+x_1(t)}}{a_{13}^2(t) + e^{2x_1(t)}}, \quad t \neq t_k,\ k = 1,2\ldots \\
x_2'(t) &= -d_2(t) - a_{21}(t)e^{x_2(t)} + \frac{a_{22}(t)e^{2x_1(t)}}{a_{13}^2(t) + e^{2x_1(t)}}, \quad t \neq t_k,\ k = 1,2\ldots \\
\triangle x_i(t) &= x_i(t^+) - x_i(t^-) = \ln(1 + b_{ik} + h_{ik}), \quad t = t_k,\ x_i(0) = \ln\{y_{i0}\} > 0,\ i = 1,2.
\end{aligned} \tag{1.3}$$

LEMMA 1.1. *If* $y(t) = (y_1(t), y_2(t))$ *is a positive* ω*-periodic solution of* (1.2), *then* $x_i(t) = \ln\{y_i(t)\}$ *is an* ω*-periodic solutions of* (1.3), *and vice versa.*

DEFINITION 1.1. The mapping $x : [0, \omega] \to R^2$ is called a solution of system (1.3) in $[0, \omega]$, if

(i) $x(t)$ is partly continuous, $t_k \cap [0, \omega]$ is discontinuous point of the first kind of $x(t)$ and left continuous.

(ii) $x(t)$ satisfies system (1.3) on $[0, \omega]$.

DEFINITION 1.2. The mapping $x : R \to R^2$ is called an ω-periodic solution of system (1.3), if

(i) $x(t)$ is a solution of (1.3).

(ii) $x(t)$ satisfies $x(t + \omega - 0) = x(t - 0)$, $t \in R$.

Obviously, if $x(t)$ is a solution of (1.3) satisfying $x(0) = x(\omega)$ in $[0, \omega]$, then from the periodicity of the vector field of (1.3), we know that the function

$$x^*(t) = \begin{cases} x(t - j\omega), & t \in [j\omega, (j+1)\omega] \\ x^*(t) \text{ is left continuous at } t_k. \end{cases}$$

is an ω-periodic solution of (1.3). So, in order to achieve the existence of periodic solution to (1.3), it is sufficient to find the solutions of (1.3) in $[0, \omega]$ satisfying $x(0) = x(\omega)$. That is to find solutions of the following system in $[0, \omega]$

$$\begin{aligned} x_1'(t) &= -d_1(t) - a_{11}(t)e^{x_1(t)} - \frac{a_{12}(t)e^{x_2(t)+x_1(t)}}{a_{13}^2(t) + e^{2x_1(t)}}, \quad t \neq t_k,\ k = 1, 2 \ldots \\ x_2'(t) &= -d_2(t) - a_{21}(t)e^{x_2(t)} + \frac{a_{22}(t)e^{2x_1(t)}}{a_{13}^2(t) + e^{2x_1(t)}}, \quad t \neq t_k,\ k = 1, 2 \ldots \qquad (1.4) \\ \triangle x_i(t) &= x_i(t^+) - x_i(t^-) = \ln(1 + b_{ik} + h_{ik}), \quad t = t_k,\ x_i(0) = x_i(\omega) > 0,\ i = 1, 2. \end{aligned}$$

For simplicity and convenience in the following discussion, throughout this paper, we will use the notation

$$\bar{f} := \frac{1}{\omega}\int_0^\omega f(t)\,dt,$$

where f is ω-periodic.

2 Existence of periodic solution

In order to obtain the existence of positive periodic solution of (1.2), for the readers' convenience, we shall present below a few of concepts and results from [7], which will be basic for this section.

Let X, Z be normed vector spaces, $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping, $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L < +\infty$ and $\operatorname{Im} L$

is closed in Z. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q)$, it follows that $L \,|\, \operatorname{Dom} L \cap \operatorname{Ker} P : (I - P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

Lemma 2.1 (Continuation Theorem). *Let L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$. Suppose*

(a) *For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;*
(b) *$QNx \neq 0$ for each $x \in \partial\Omega \cap \operatorname{Ker} L$ and*

$$\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0.$$

Then the operator equation $Lx = Nx$ has at least one solution lying in $\operatorname{Dom} L \cap \overline{\Omega}$.

Let

$$C[0,\omega; t_1, t_2, \ldots t_m] = \left\{ x : [0,\omega] \to R^2 \;\middle|\; \begin{array}{l} x(t) \text{ is continuous with respect to } t \neq t_1, \ldots, t_m; \\ x(t+0) \text{ and } x(t-0) \text{ exist at } t_1, \ldots, t_m; \\ x(t_k) = x(t_k - 0), \quad k = 1, 2, \ldots, m \end{array} \right\}$$

Define

$$\frac{\sum_{k=1}^{q} \ln(1 + b_{ik} + h_{ik})}{\omega} - \bar{d}_i := \triangle_i, \quad i = 1, 2.$$

Now we are ready to state and prove the main results of the present paper.

Theorem 2.1. *Assume that $(A_1)(A_2)$ hold. Moreover, if $\triangle_2 > 0$ and $\triangle_1 > (a_{12}/a_{13}^2)e^{H_1 + H_2}$, where*

$$H_1 = \ln\left\{\frac{\triangle_1}{\bar{a}_{11}}\right\} + 2\sum_{k=1}^{q} \ln(1 + b_{1k} + h_{1k}),$$

$$H_2 = \ln\left\{\frac{\bar{a}_{22} + \triangle_2}{\bar{a}_{21}}\right\} + 2\left(\sum_{k=1}^{q} \ln(1 + b_{2k} + h_{2k}) + \bar{a}_{22}\omega\right).$$

Then system (1.2) has at least one positive ω-periodic solution.

Proof. Let

$$X = \{x = (x_1, x_2)^T \in C[0, \omega; t_1, \dots t_m] \mid x(\omega) = x(0)\}, \quad Z = X \times R^{2q}.$$

Define

$$\|x\|_C = \sup_{t \in [0,\omega]} |x(t)|, \quad \|z\|_Z = \|x\|_C + \|y\|, \quad x \in X,\ y \in R^{2q},$$

where $|\cdot|$ and $\|\cdot\|$ are norms of R^2 and R^{2q}, respectively. Then it is trivial to show that X, Z are both Banach spaces when they are endowed with the above norm $\|\cdot\|_C$ and $\|\cdot\|_Z$, respectively.

Let

$$\operatorname{dom} L \subset X = \{x = (x_1, x_2)^T \in C[0, \omega; t_1, \dots t_m] \mid x(\omega) = x(0)\},$$

$$L : \operatorname{dom} L \to Z, \quad Lx = (x', \Delta x(t_1), \dots, \Delta x(t_m)),$$

$$N : X \to Z,$$

$$Nx = \left(\begin{pmatrix} -d_1(t) - a_{11}(t)e^{x_1(t)} - \dfrac{a_{12}(t)e^{x_2(t)+x_1(t)}}{a_{13}^2(t) + e^{2x_1(t)}} \\ -d_2(t) - a_{21}(t)e^{x_2(t)} + \dfrac{a_{22}(t)e^{2x_1(t)}}{a_{13}^2(t) + e^{2x_1(t)}} \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} \ln(1 + b_{11} + h_{11}) \\ \ln(1 + b_{21} + h_{21}) \end{pmatrix}, \dots \begin{pmatrix} \ln(1 + b_{1q} + h_{1q}) \\ \ln(1 + b_{2q} + h_{2q}) \end{pmatrix} \right)$$

Then

$$\operatorname{Ker} L = \{x : x = A \in R^2, t \in [0, \omega]\},$$

$$\operatorname{Im} L = \left\{ z = (f, C_1 \cdots C_q) \in Z : \int_0^\omega f(s)\, ds + \sum_{k=1}^q C_k = 0 \right\}$$

and

$$\dim \operatorname{Ker} L = 2 = \operatorname{codim} \operatorname{Im} L.$$

Since $\operatorname{Im} L$ is closed in Z, L is a Fredholm mapping of index zero. Let

$$Px = \frac{1}{\omega} \int_0^\omega x(t)\, dt,$$

$$Qz = Q(f, C_1 \cdots C_q) = \left(\frac{1}{\omega} \left[\int_0^\omega f(s)\, ds + \sum_{k=1}^q C_k \right], 0 \cdots 0 \right).$$

It is easy to show that P, Q are continuous projectors such that

$$\operatorname{Im} P = \operatorname{Ker} L, \quad \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q).$$

Furthermore, the generalized inverse (to L) $K_P : \operatorname{Im} L \to \operatorname{Dom} L \cap \operatorname{Ker} P$ exists.

Now, we derive the explicit expression for K_p. Let $z = (f, C_1 \cdots C_q) \in \operatorname{Im} L$, then $x \in X$ satisfies

$$x'(t) = f(t), \quad t \neq t_k,\ k = 1, 2 \ldots$$
$$\triangle x(t)|_{t=t_k} = C_k.$$

Then

$$x(t) = \int_0^{\omega} f(s)\, ds + \sum_{t>t_k} C_k + x(0). \tag{2.1}$$

Note that $x(t) \in \operatorname{Ker} P$, i.e., $1/\omega \int_0^{\omega} x(s)\, ds = 0$. From (2.1), we get

$$\int_0^{\omega}\int_0^{t} f(s)\, dsdt + \int_0^{\omega} \sum_{t>t_k} C_k\, dt + \omega x(0) = 0,$$

and hence

$$x(t) = \int_0^{\omega} f(s)\, ds + \sum_{t>t_k} C_k - \frac{1}{\omega}\int_0^{\omega}\int_0^{t} f(s)\, dsdt - \sum_{t=1}^{q} C_k\, dt, \tag{2.2}$$

that is

$$K_P z = \int_0^{\omega} f(s)\, ds + \sum_{t>t_k} C_k - \frac{1}{\omega}\int_0^{\omega}\int_0^{t} f(s)\, dsdt - \sum_{t=1}^{q} C_k\, dt. \tag{2.3}$$

Thus

$$QNx = \left(\begin{pmatrix} \frac{1}{\omega}\left(\int_0^{\omega}\left[-d_1(s) - a_{11}(s)e^{x_1(s)} - \frac{a_{12}(s)e^{x_2(s)+x_1(s)}}{a_{13}^2(s) + e^{2x_1(s)}}\right] ds \right. \\ \left. \qquad + \sum_{k=1}^{q} \ln(1 + b_{1k} + h_{1k})\right) \\ \frac{1}{\omega}\left(\int_0^{\omega}\left[-d_2(s) - a_{21}(s)e^{x_2(s)} + \frac{a_{22}(s)e^{2x_1(s)}}{a_{13}^2(s) + e^{2x_1(s)}}\right] ds \right. \\ \left. \qquad + \sum_{k=1}^{q} \ln(1 + b_{2k} + h_{2k})\right) \end{pmatrix}, 0, \ldots 0\right),$$

$K_P(I - Q)Nx$

$$= \begin{pmatrix} \int_0^{t}\left[-d_1(s) - a_{11}(s)e^{x_1(s)} - \frac{a_{12}(s)e^{x_2(s)+x_1(s)}}{a_{13}^2(s) + e^{2x_1(s)}}\right] ds + \sum_{t>t_k} \ln(1 + b_{1k} + h_{1k}) \\ \int_0^{t}\left[-d_2(s) - a_{21}(s)e^{x_2(s)} + \frac{a_{22}(s)e^{2x_1(s)}}{a_{13}^2(s) + e^{2x_1(s)}}\right] ds + \sum_{t>t_k}^{q} \ln(1 + b_{2k} + h_{2k}) \end{pmatrix}$$

$$-\begin{pmatrix} \frac{1}{\omega}\int_0^\omega\int_0^t\left[-d_1(s)-a_{11}(s)e^{x_1(s)}-\frac{a_{12}(s)e^{x_2(s)+x_1(s)}}{a_{13}^2(s)+e^{2x_1(s)}}\right]dsdt+\sum_{k=1}^q\ln(1+b_{1k}+h_{1k}) \\ \frac{1}{\omega}\int_0^\omega\int_0^t\left[-d_2(s)-a_{21}(s)e^{x_2(s)}+\frac{a_{22}(s)e^{2x_1(s)}}{a_{13}^2(s)+e^{2x_1(s)}}\right]dsdt+\sum_{k=1}^q\ln(1+b_{2k}+h_{2k}) \end{pmatrix}$$

$$-\begin{pmatrix} \left(\frac{t}{\omega}-\frac{1}{2}\right)\left(\int_0^t\left[-d_1(s)-a_{11}(s)e^{x_1(s)}-\frac{a_{12}(s)e^{x_2(s)+x_1(s)}}{a_{13}^2(s)+e^{2x_1(s)}}\right]ds \right. \\ \left. \qquad +\sum_{k=1}^q\ln(1+b_{1k}+h_{1k})\right) \\ \left(\frac{t}{\omega}-\frac{1}{2}\right)\left(\int_0^\omega\left[-d_2(s)-a_{21}(s)e^{x_2(s)}+\frac{a_{22}(s)e^{2x_1(s)}}{a_{13}^2(s)+e^{2x_1(s)}}\right]ds \right. \\ \left. \qquad +\sum_{k=1}^q\ln(1+b_{2k}+h_{2k})\right) \end{pmatrix}.$$

Obviously, QN and $K_P(I-Q)N$ are continuous. It is not difficult to show that $K_P(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega\subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega\subset X$.

Now we reach the position to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $Lx=\lambda Nx$, $\lambda\in(0,1)$, we have

$$\begin{aligned} x_1'(t)&=\lambda\left[-d_1(t)-a_{11}(t)e^{x_1(t)}-\frac{a_{12}(t)e^{x_2(t)+x_1(t)}}{a_{13}^2(t)+e^{2x_1(t)}}\right], \quad t\neq t_k,\ k=1,2\dots \\ x_2'(t)&=\lambda\left[-d_2(t)-a_{21}(t)e^{x_2(t)}+\frac{a_{22}(t)e^{2x_1(t)}}{a_{13}^2(t)+e^{2x_1(t)}}\right], \quad t\neq t_k,\ k=1,2\dots \qquad (2.4) \\ \triangle x_i(t)&=x_i(t^+)-x_i(t^-)=\lambda\ln(1+b_{ik}+h_{ik}), \quad t=t_k,\ x_i(0)=x_i(\omega),\ i=1,2. \end{aligned}$$

Suppose that $x\in X$ is a solution of system (2.4) for a certain $\lambda\in(0,1)$. Integrating on both sides of (2.4) from 0 to ω, we obtain

$$\begin{aligned} \int_0^\omega\left[a_{11}(t)e^{x_1(t)}+\frac{a_{12}(t)e^{x_2(t)+x_1(t)}}{a_{13}^2(t)+e^{2x_1(t)}}\right]dt&=\triangle_1\omega, \\ \int_0^\omega\left[a_{21}(t)e^{x_2(t)}-\frac{a_{22}(t)e^{2x_1(t)}}{a_{13}^2(t)+e^{2x_1(t)}}\right]dt&=\triangle_2\omega. \qquad (2.5) \end{aligned}$$

It follows from (2.4) and (2.5) that

$$\int_0^\omega |x_1'(t)|\,dt \le \int_0^\omega \left[d_1(t) + a_{11}(t)e^{x_1(t)} + \frac{a_{12}(t)e^{x_2(t)+x_1(t)}}{a_{13}^2(t) + e^{2x_1(t)}}\right] dt + \sum_{k=1}^q \ln(1 + b_{1k} + h_{1k})$$
$$= 2\sum_{k=1}^q \ln(1 + b_{1k} + h_{1k}),$$
$$\int_0^\omega |x_2'(t)|\,dt \le \int_0^\omega \left[d_2(t) + a_{21}(t)e^{x_2(t)} + \frac{a_{22}(t)e^{2x_1(t)}}{a_{13}^2(t) + e^{2x_1(t)}}\right] dt + \sum_{k=1}^q \ln(1 + b_{2k} + h_{2k})$$
$$= 2\left(\sum_{k=1}^q \ln(1 + b_{2k} + h_{2k}) + \bar{a}_{22}\omega\right). \tag{2.6}$$

Since $x \in X$, there exist $\xi_i \in [0, \omega]$ such that

$$x_i(\xi_i) = \min_{t\in[0,\omega]} x_i(t), \quad i = 1, 2. \tag{2.7}$$

On the other hand, note that $\sup_{t\in[0,\omega]} x_i(t)$ exist and there exist $\eta_i \in [0, \omega]$ such that

$$x_i(\eta_i^+) = \sup_{t\in[0,\omega]} x_i(t), \quad i = 1, 2. \tag{2.8}$$

If $\eta_i \ne t_k$, then $x_i(\eta_i^+) = x_i(\eta_i)$ while if $\eta_i = t_k$, we have $x_i(\eta_i^+) = x_i(t_k^+)$.
From (2.5) and (2.7), we obtain

$$\triangle_1\omega \ge \int_0^\omega a_{11}(t)e^{x_1(\xi_1)}\,dt = \bar{a}_{11}\omega e^{x_1(\xi_1)},$$
$$\triangle_2\omega \ge \int_0^\omega \left[a_{21}(t)e^{x_2(\xi_2)} - a_{22}(t)\right] dt = \bar{a}_{21}e^{x_2(\xi_2)} - \bar{a}_{22},$$

and hence,

$$x_1(\xi_1) \le \ln\left\{\frac{\triangle_1}{\bar{a}_{11}}\right\}, \quad x_2(\xi_2) \le \ln\left\{\frac{\triangle_2 + \bar{a}_{22}}{\bar{a}_{21}}\right\}. \tag{2.9}$$

From (2.6) and (2.9), we obtain

$$x_1(t) \le x_1(\xi_1) + \int_0^\omega |x_1'(t)|\,dt < \ln\left\{\frac{\triangle_1}{\bar{a}_{11}}\right\} + 2\sum_{k=1}^q \ln(1 + b_{1k} + h_{1k})) := H_1,$$
$$x_2(t) \le x_2(\xi_2) + \int_0^\omega |x_2'(t)|\,dt < \ln\left\{\frac{\triangle_2 + \bar{a}_{22}}{\bar{a}_{21}}\right\}$$
$$+ 2\left(\sum_{k=1}^q \ln(1 + b_{2k} + h_{2k})) + \bar{a}_{22}\omega\right) := H_2. \tag{2.10}$$

On the other hand from (2.5) and (2.8)

$$\int_0^\omega \left[a_{11}(t)e^{x_1(t)} + \frac{a_{12}(t)e^{x_2(t)+x_1(t)}}{a_{13}^2(t)} \right] dt \geq \triangle_1 \omega,$$

that is

$$\bar{a}_{11} e^{x_1(\eta_1^+)} + \overline{\left(\frac{a_{12}}{a_{13}^2}\right)} e^{H_1+H_2} \geq \triangle_1,$$

then

$$x_1(\eta_1^+) \geq \ln \left\{ \frac{\triangle_1 - \overline{\left(\frac{a_{12}}{a_{13}^2}\right)} e^{H_1+H_2}}{\bar{a}_{11}} \right\},$$

and hence

$$x_1(t) \geq x_1(\eta_1^+) - \int_0^\omega |x_1'(t)|\, dt \geq \ln \left\{ \frac{\triangle_1 - \overline{\left(\frac{a_{12}}{a_{13}^2}\right)} e^{H_1+H_2}}{\bar{a}_{11}} \right\} - 2\sum_{k=1}^q \ln(1 + b_{1k} + h_{1k}))$$
$$:= H_3. \tag{2.11}$$

Similarly, one can derive that

$$\int_0^\omega \left[a_{21}(t)e^{x_2(\eta_2^+)} - \frac{a_{22}(t)e^{2H_3}}{a_{13}^2(t) + e^{2H_1}} \right] dt \geq \triangle_2 \omega.$$

$$x_2(\eta_2^+) \geq \ln \left\{ \frac{\overline{\left(\frac{a_{22}e^{2H_3}}{a_{13}^2 + e^{2H_1}}\right)} + \triangle_2}{\bar{a}_{21}} \right\},$$

then

$$x_2(t) \geq x_2(\eta_2^+) - \int_0^\omega |x_1'(t)|\, dt \geq \ln \left\{ \frac{\overline{\left(\frac{a_{22}e^{2H_3}}{a_{13}^2 + e^{2H_1}}\right)} + \triangle_2}{\bar{a}_{21}} \right\}$$
$$- 2\left(\sum_{k=1}^q \ln(1 + b_{2k} + h_{2k}) + \bar{a}_{22}\omega \right) := H_4, \tag{2.12}$$

which, together with (2.10) and (2.11), implies

$$\sup_{t \in [0,\omega]} |x_1(t)| < \max\{|H_1|, |H_3|\} := M_1,$$
$$\sup_{t \in [0,\omega]} |x_2(t)| < \max\{|H_2|, |H_4|\} := M_2. \tag{2.13}$$
$$\|x\|_C \le (M_1^2 + M_2^2)^{1/2} := M$$

Clearly, M_1 and M_2 are independent of λ.

Now consider the following system of algebraic equations

$$\begin{aligned} \triangle_1 - \bar{a}_{11} e^{x_1} - \mu \overline{\left(\frac{a_{12}(t) e^{x_1+x_2}}{a_{13}^2(t) + e^{2x_1}} \right)} &= 0 \\ \triangle_2 - \bar{a}_{21} e^{x_2} + \overline{\left(\frac{a_{22}(t) e^{2x_1}}{a_{13}^2(t) + e^{2x_1}} \right)} &= 0 \end{aligned} \tag{2.14}$$

where $x = (x_1, x_2)^T \in R^2$ and $\mu \in [0,1]$ is a parameter. Using the similar technique and the previous estimates, we get $\bar{H}_3 < x_1 < \bar{H}_1$, $\bar{H}_4 < x_2 < \bar{H}_2$, where

$$\bar{H}_1 = \ln\left\{\frac{\triangle_1}{\bar{a}_{11}}\right\}, \quad \bar{H}_2 = \ln\left\{\frac{\triangle_2 + \bar{a}_{22}}{\bar{a}_{21}}\right\},$$

$$\bar{H}_3 = \ln\left\{\frac{\triangle_1 - \overline{\left(\frac{a_{12}}{a_{13}^2}\right)} e^{\bar{H}_1 + \bar{H}_2}}{\bar{a}_{11}}\right\}, \quad \bar{H}_4 = \ln\left\{\frac{\overline{\left(\frac{a_{22} e^{2\bar{H}_3}}{a_{13}^2 + e^{2\bar{H}_1}}\right)} + \triangle_2}{\bar{a}_{21}}\right\}.$$

Take $\bar{M}_1 = \max\{|\bar{H}_1|, |\bar{H}_3|\}$, $\bar{M}_2 = \max\{|\bar{H}_2|, |\bar{H}_4|\}$, it is easy to know that $\bar{M}_i$, $i = 1, 2$ is independent of μ such that

$$\|x\| \le (\bar{M}_1^2 + \bar{M}_2^2)^{1/2} := \bar{M}. \tag{2.15}$$

so $\bar{M}$ is independent of μ. It is proved that $\|x\|_C \le \bar{M}$ for every solution $x = (x_1, x_2)$ of (2.14). Now, we take $M^* > \max\{M, \bar{M}\}$ such that $\|x(t_k + 0)\| < M^*$. Let $\Omega := \{x = (x_1, x_2) \in X \mid \|x\|_C < M^*\}$. It is clear that $Lx \ne \lambda Nx$ for $x \in \partial\Omega$ and $\lambda \in (0,1)$, thus Ω verifies the requirement (a) in Lemma 2.1. When $x \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap R^2$, x is a constant vector in R^2 with $\|x\| = M$. Then from (2.15)

$$QNx = \left(\begin{pmatrix} \triangle_1 - \bar{a}_{11} e^{x_1} - \overline{\left(\frac{a_{12}(t) e^{x_1+x_2}}{a_{13}^2(t) + e^{2x_1}} \right)} \\ \triangle_2 - \bar{a}_{21} e^{x_2} + \overline{\left(\frac{a_{22}(t) e^{2x_1}}{a_{13}^2(t) + e^{2x_1}} \right)} \end{pmatrix}, (0 \cdots 0)_{2\times 1} \right) \ne 0. \tag{2.16}$$

Now let us consider homotopy $h_\mu(x) = \mu QNx + (1-\mu)Gx$, $\mu \in [0,1]$, where

$$Gx = \left(\begin{pmatrix} \triangle_1 - \bar{a}_{11}e^{x_1} \\ \overline{\triangle_2 - \bar{a}_{21}e^{x_2} + \left(\dfrac{a_{22}(t)e^{2x_1}}{a_{13}^2(t) + e^{2x_1}} \right)} \end{pmatrix}, (0 \cdots 0)_{2\times 1} \right). \tag{2.17}$$

We know that $0 \notin h_\mu(\partial\Omega \cap \operatorname{Ker} L)$ for $\mu \in (0,1)$ and noting that $J = I$, hence

$$\deg(JQN, \Omega \cap \operatorname{Ker} L, 0) = \deg(G, \Omega \cap \operatorname{Ker} L, 0) \neq 0$$

by the property of homotopy invariance of topological degree. By now we have proved that Ω verifies all requirements of Lemma 2.1, then $Lx = Nx$ has at least one solution in $\operatorname{Dom} L \cap \overline{\Omega}$, i.e., (1.4) has at least one ω-periodic solution in $\operatorname{Dom} L \cap \overline{\Omega}$, say $x = (x_1^*(t), x_2^*(t))^T$. Set $y^* = (y_1^*(t), y_2^*(t))^T = (e^{x_1^*(t)}, e^{x_2^*(t)})^T$, then $y^* = (y_1^*(t), y_2^*(t))^T$ is one positive ω-periodic solution of system (1.2). The proof is complete. $\square$

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