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DAC TESTING USING IMPULSE SIGNALS

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Abstract

The Multi-Tone (MT) signal with uniform amplitudes can be used for DAC testing. This paper shows an easier way to generate a MT signal using several impulse signals. The article also analyzes qualities of methods for testing the dynamic parameters of Digital to Analog Converters using an impulse signal. The MT, Damped Sine Wave (DSW) and Sinx/x (SINC) signals will be used as the source for these tests. The Effective Number of Bits (*ENOB*) and Signal to noise and distortion (*SINAD*) are evaluated in the frequency domain and they are modified using the *Crest Factor* (*CF*) correction and compared with the standard results of the Sine Wave FFT test. The first advantage of the test using an impulse signal is that you need fewer input parameters to create the band signal for testing the DAC. The second one is to reduce the testing time using a band signal in comparison with multiple tests using a single sine wave.

Keywords: DAC, ENOB, SINAD, FFT test, Crest Factor, Damped Sine Wave, SINC signal, Multi-Tone signal.

1. Introduction

Classical methods for testing the dynamic parameters of DACs are stated in a draft [1]. The non-standard methods for DAC testing are using multi-tone, AM and FM signals [2-4]. One of the possibilities how to create an impulse signal is to sum up the several tones which are equidistantly distributed and they have zero phase shifts. The contribution analyzes the possibilities how to use the DSW and SINC signals for DAC testing. These signals are generated by the DUT – a 16-bit-resolution multifunction card. The reference device is a 24 bit digitizer, see Fig. 1. There are two ways how to directly compare the impulse methods using a non-sinusoidal signal with the Sin Wave (SW) FFT test. The first approach is to use a general formula (2) and to recalculate the *SINAD* (from the corrected *ENOB*) using the standard formula for a SW signal [5].

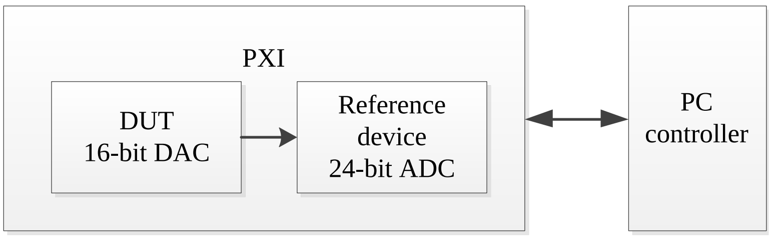

Fig. 1. The measurement setup.

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$$CF = \frac{Amplitude}{RMS}(-), \tag{1}$$

$$ENOB_{\mathrm{GENERAL}} = \frac{SINAD - 4.77 + 20\log CF}{6.02}(bit). \tag{2}$$

Second way is to use the standard formula and correct the *SINAD* and the *ENOB* using the Δ*SINAD* (3) and the Δ*ENOB* correction factors (4).

$$SINAD_{Corrected} = SINAD_{Measured} + \Delta SINAD = SINAD_{Measured} + 20\log CF\ (\mathrm{dB}), \tag{3}$$

$$ENOB_{Corrected} = ENOB_{Measured} + \Delta ENOB = ENOB_{Measured} + \frac{\Delta SINAD}{6.02}(\mathrm{bit}). \tag{4}$$

2. Testing with a Multi-Tone signal

There are two general but important presumptions for testing a DAC: [6-9]

- to test near *Full Scale*(*FS*) - it is necessary to ensure that the *Peak-to-Peak* value covers *FS* of DUT;
- the second condition is that the *Slew Rate SR* of the signal is slower than the *SR* of the DUT (a generator) and the digitizer as well (5). The *SR*'s maximum of the DSW has the same value as the *SR*'s maximum of the sine-wave so it is possible to use the standard formula for the *SR*.

$$SR_{MAX\_SIGNAL} < SR_{\mathrm{DUT}} < SR_{\mathrm{DIGITIZER}}. \tag{5}$$

The multi-tone signal with discrete frequency components (each tone has zero phase shift) is defined by the formula:

$$u_{\mathrm{MT}} = \sum_{i=1}^{m} U_i \sin(2\pi f_i t), \tag{6}$$

where U_i, f_i, $i = 1,2,..m$ are amplitudes and frequencies of m spectral signal components. The *Effective Number of Bits* of the tested ADC and DAC is given by the equation (2). The CF_{MT} of this multi-tone signal is defined by the formula (7).

$$CF_{\mathrm{MT}} = \frac{\sum_{i=1}^{m} U_i}{\sqrt{\sum_{i=1}^{m} \frac{U_i^2}{2}}}. \tag{7}$$

CF is equal to $\sqrt{2m}$ for equal amplitudes of frequency components $U_i = 1/2m$. Table 1 shows how *CF* affects *SINAD* and *ENOB*. Δ*SINAD*, Δ*ENOB* are differences between the SW test and MT test. The signal contains only tones with the same amplitude. Frequencies are chosen such that the maximum (minimum) of the fundamental is supported by the maximum of other tones. For example $f_2=4f_1$. It is possible to say that the maximum of the signal is equal to the sum of the amplitudes.

Table 1. The dependence of *SINAD* and *ENOB* on the number of tones.

Number of Tones	1	2	4
CF_{MT}	√2	2	2√2
Δ*SINAD* (dB)	0	- 3.01	- 6.02
Δ*ENOB* (bit)	0	- 0.5	- 1.0

A disadvantage of this method is that you need to set up the frequency of each tone. It means that it is necessary to pick up 20 parameters for a 20-tone test.

3. Damped Sine Wave Test

The DSW is a natural signal which is easy to generate. For ADC testing we can design a circuit with only one operational amplifier to generate the DSW from a square signal. In case of DAC testing, the signal is generated directly by the DAC.

This signal has one disadvantage. The *CF* differs from that of the sine-wave signal. It is necessary to compute the *CF* to correct the *ENOB* and *SINAD*. Let us do the analysis of the DSW. The solution of the harmonic motion of the under damping system can be expressed [10, 11]

$$u(t) = e^{-2\cdot\pi\cdot f_2\cdot d\cdot t}\sin\left(2\cdot\pi\cdot f_2\cdot t\right), \tag{8}$$

where d is the damping ratio, $f_1 = 1/T_2$ is the natural frequency, see Fig. 2.

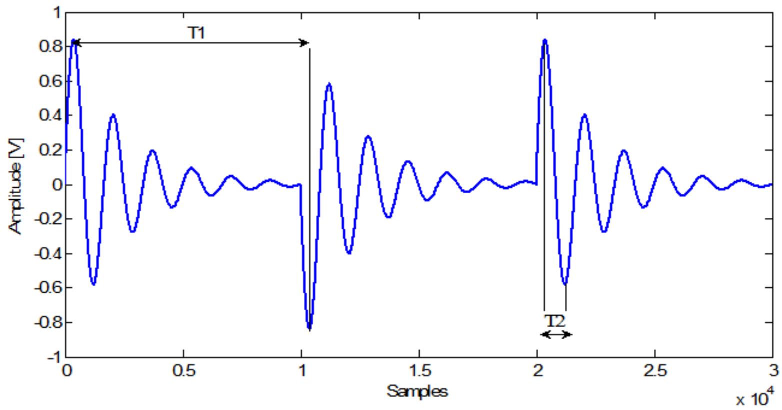

Fig. 2. Time plot of a DSW signal.

The signal's *RMS*, which is periodically generated with the repetitive frequency $f_1 = 1/T_1$, can be computed using (9):

$$RMS = \sqrt{\frac{1}{T_1}\int_0^{T_1} u^2(t)\,\mathrm{d}t}\ , \tag{9}$$

$$RMS = \sqrt{\frac{f_1 e^{\frac{-4\pi d f_2}{f_1}} \left(-1 - d^2 + e^{\frac{4\pi d f_2}{f_1}} + d^2 \cos\left(\frac{4\pi f_2}{f_1}\right) - d \sin\left(\frac{4\pi f_2}{f_1}\right) \right)}{8\pi f_2 d (1 + d^2)}} . \tag{10}$$

This formula can be simplified for the ratio of frequencies f_1 / f_2 from 0.5 to 2.

$$RMS \approx \sqrt{\frac{e^{\frac{-4\cdot\pi\cdot d\cdot f_2}{f_1}} f_1 \left\{ -1 + e^{\frac{4\cdot\pi\cdot d\cdot f_2}{f_1}} \right\}}{8 \cdot \pi \cdot f_2 \cdot d \cdot (1 + d^2)}} . \tag{11}$$

The amplitude of this signal is the first maximum. It can be expressed as the following formula

$$Amplitude = \sqrt{\frac{f_2^2}{d^2 + f_2^2}} e^{-2d \arccos\sqrt{\frac{1}{2} + \frac{d}{2\sqrt{1+d^2}}}} . \tag{12}$$

After some math the *CF* can be formulated as the equation:

$$CF = \frac{2\sqrt{\frac{1}{1+d^2}} e^{-2d \arccos\sqrt{\frac{1}{2} + \frac{d}{2\sqrt{1+d^2}}}}}{\sqrt{\frac{\left(1 - e^{\frac{-4\cdot\pi\cdot d\cdot f_2}{f_1}}\right) f_1}{2 \cdot \pi \cdot f_2 \cdot d \cdot (1 + d^2)}}} . \tag{13}$$

This formula can be reduced if $d << 1$ (i.e. significantly or strongly under-damped systems). Then the *CF* is given by the formula:

$$CF \cong \frac{2e^{-d\frac{\pi}{2}}}{\sqrt{\frac{\left(1 - e^{\frac{-4\cdot\pi\cdot d\cdot f_2}{f_1}}\right) f_1}{2 \cdot \pi \cdot f_2 \cdot d}}} . \tag{14}$$

Just for illustration, there is the *CF* shown in Table 2 for the chosen parameters f_1, f_2 and *d*.

Table 2. The damped sinewave's crest factor for the chosen damping ratio and frequencies.

f_1 (kHz)	f_2 (kHz)	d (-)	*CF* (-)
5	1	0.016	1.74
5	1	0.032	2.05
5	1	0.064	2.59
5	1	0.127	3.33
5	1	0.255	4.04

4. Test with a Sinc signal

The SINC signal seems to be a very perspective signal for testing of dynamic parameters of DACs [12]. This signal is composed of two SINC signals with the same parameters, only the second part is inverted, see Fig. 3.

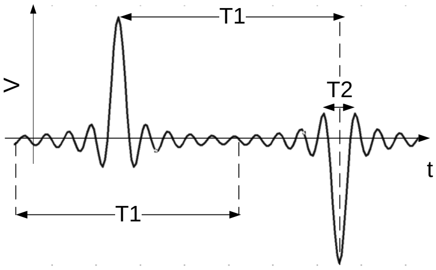

Fig. 3. Time plot of a *SINC* signal.

The first period of the signal is described by the next formula:

$$\mathrm{u}(\mathrm{t}) = \mathrm{H}\left(t+\frac{T_1}{2}\right)\left(\frac{\sin\left(\frac{\omega t}{T_2}\right)}{\frac{\omega t}{T_2}}\right) - \mathrm{H}\left(t-\frac{T_1}{2}\right)\left(\frac{\sin\left(\frac{\omega t}{T_2}\right)}{\frac{\omega t}{T_2}}\right),\ t \in \left\langle -\frac{T_1}{2}, \frac{T_1}{2}\right),\tag{15}$$

where H is the Heaviside function, which chops the SINC function in time, in the range $< -T_1/2, T_1/2)$.

In the frequency domain the signal contains equidistantly distributed components with uniform amplitudes in the ideal case. Because the function is chopped and inverted, the effect of the rectangular window causes leakage. The digital filter limits the frequency band and reduces this effect, but it also reduces the amplitude near the filter's cut-off frequency.

Thus the generated signal symmetrically covers the *FS* of the DAC. *CF* can be computed from one period of our signal (16). When the filter is applied, the amplitude of the signal must be normalized to cover the *FS* of the DAC; the signal *u*, which is stated in formula (16) is then the output signal behind the filter.

$$CF = \frac{Max[u(t)]}{RMS} = \frac{FS_{DAC}}{\sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} u^2(\mathrm{t})dt}}\,.\tag{16}$$

However this integral in equation (16) can be solved only numerically.

5. Test setup and results

A PXI system was used for the practical verification of the test methods with MT, DSW and SINC signals. The first output channel of the DAQ NI PXI 6251 (2 analog outputs: 16-bit, 1.25 MSa/s or 1 analog output: 16-bit 1.8MSa/s, 16 analog inputs: 16-bit ADC, 1.25 MSa/s) was tested. The Digitizer (NI PXI 5922 24 bit, 500 kSa/s, or 16 bits, 15 MSa/s) was chosen as the reference device, see Fig. 1. All impulse methods were compared with the sine wave DFT test. The Hanning 4th order window was used for all measurements. The frequency bandwidth is limited in range from 20 Hz to 20 kHz, for the reduction of the *Noise Floor* in all tests. The test system is programmed in NI LabView. The 2-MSamples were acquired during the test. The sampling frequency is 400 kHz.

Firstly the DUT was testing the internal DAC in DAQ NI PXI 6251 by single-, double-, triple- and quadruple-tone test. The results of this test are shown in Fig. 4 and Fig. 7.

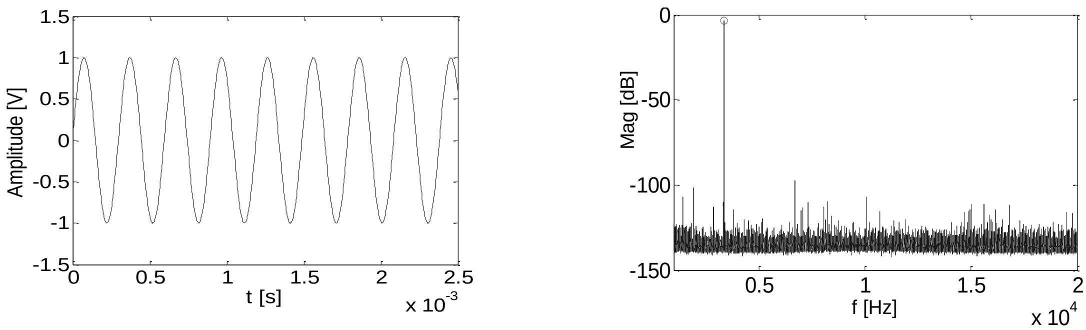

Fig. 4. The time plot and the spectrum of the sinewave signal (3357.87 Hz).

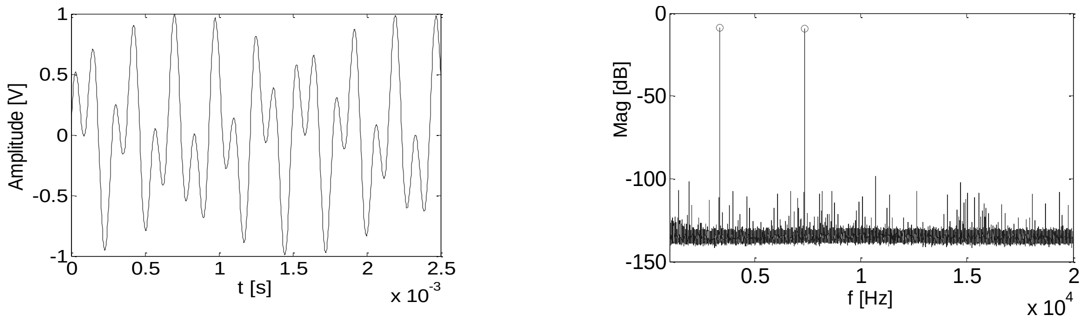

Fig. 5. The time plot and the spectrum of the 2-tone signal (3357.87 Hz, 7359.87 Hz).

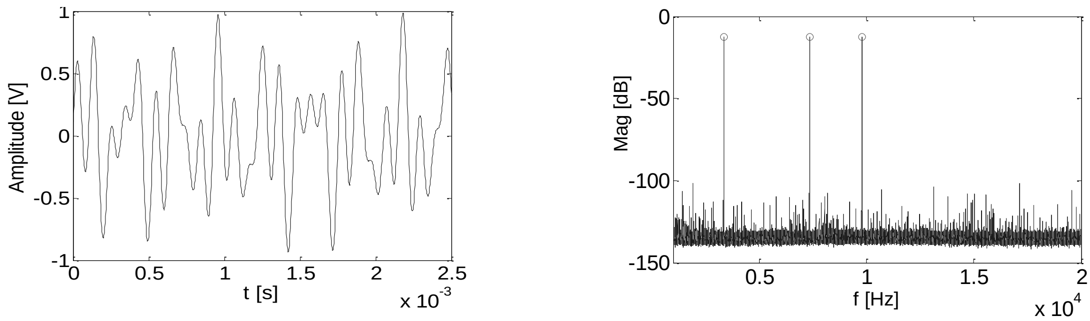

Fig. 6. The time plot and the spectrum of a 3-tone signal (3357.87 Hz, 7359.87 Hz, 9784.52 Hz).

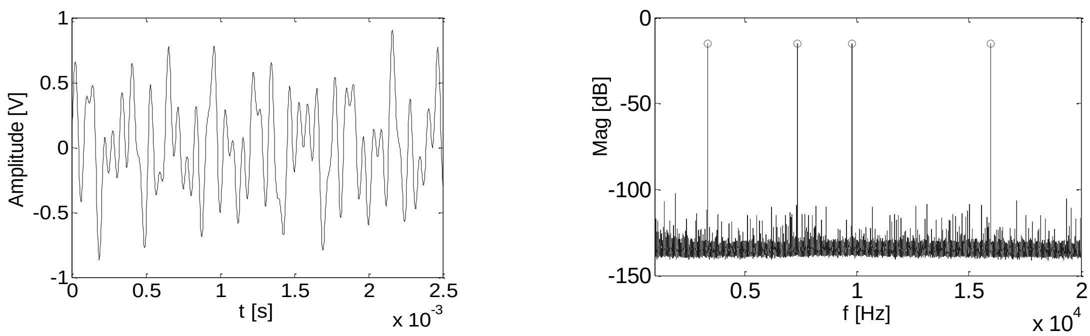

Fig. 7. The time plot and the spectrum of the 4-tone signal, (3357.87 Hz, 7359.87 Hz, 9784.52 Hz, 15987.41 Hz).

The spectral plots of the DSWs with the different damping ratios are shown in Fig. 8 and Fig. 11.

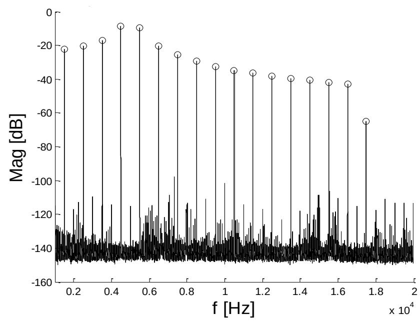

Fig. 8. The spectrum of the DSW, f_1 = 5 kHz, f_2 = 1kHz, *d* = 0.016.

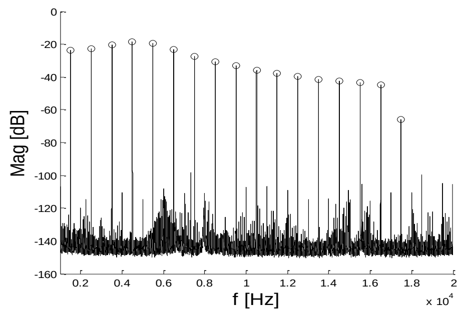

Fig. 9. The spectrum of the DSW , f_1 = 5kHz, f_2 = 1 kHz, *d* = 0.26.

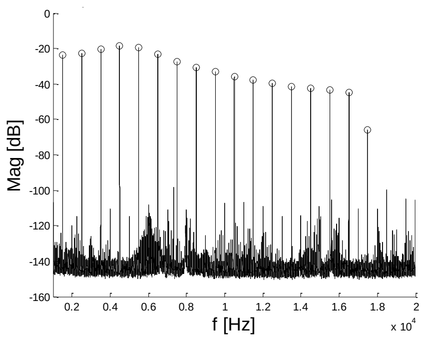

Fig. 10. The spectrum of the DSW *d* = 0.255, f_1 = 5 kHz, f_2 = 1 kHz.

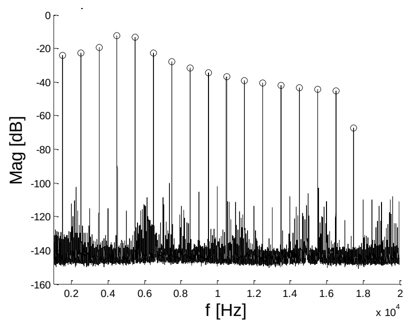

Fig. 11. The spectrum of the DSW *d* = 0.64, f_1 = 5 kHz, f_2 = 1 kHz.

The time and spectral plots of the SINC signals with the different *CF*s and number of tones are shown in Fig. 12 and Fig. 14.

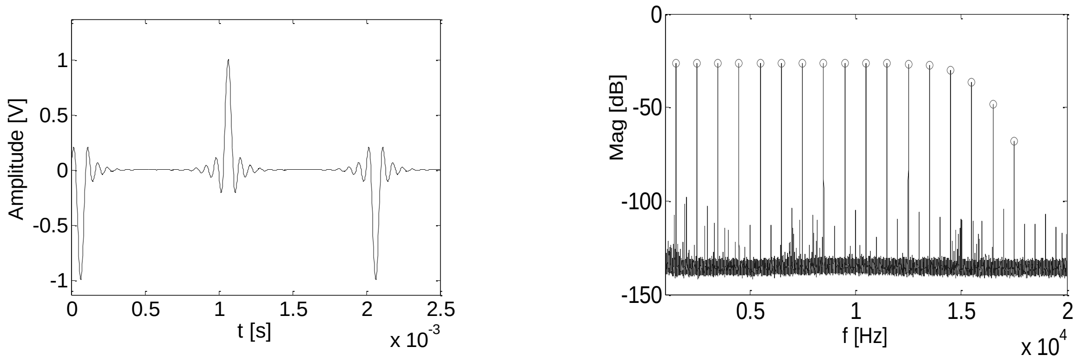

Fig. 12. The time plot of the SINC signal – 17, Tones *CF* = 5.5, *BW* = 15 kHz.

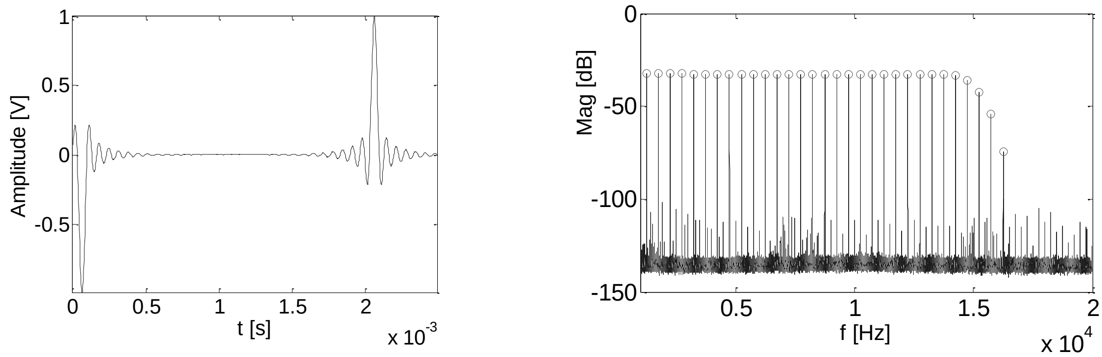

Fig. 13. The time plot of the SINC signal - 31 Tones, *CF* = 7.8, *BW* = 15 kHz.

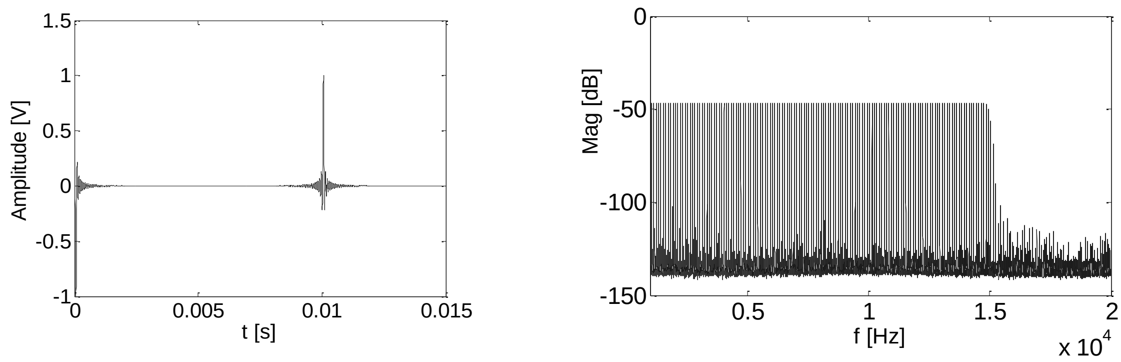

Fig. 14. The time plot of the SINC signal - 143 Tones, *CF* = 17, *BW* =15 kHz.

The internal DAC in the multifunction PXI card from National Instrumests (DAQ NI PXI 6251) was chosen as the DUT. The sampling frequency was set to 400 kSa/s. The bandwidth was limited to the range from 20 Hz to 20 kHz. The results of the tests are summarized in Table 3.

Table 3. Summarized results of DAC testing.

Signal	Frequency	Number of tones	*CF*	*ENOB* (bit)	*SINAD* (dB)
SW	3358.87 Hz	1	1.4	14.3	87.8
SW	7359.87 Hz	1	1.4	14.1	86.5
SW	9784.52 Hz	1	1.4	13.9	85.6
SW	9784.52 Hz	1	1.4	14.3	87.9
Dual Sine	3357.87 Hz, 7359.87 Hz	2	2.0	14.2	87.4
Triple Sine	3357.87 Hz, 7359.87 Hz, 9784.52 Hz	3	2.5	14.2	87.5
Quad Sine	3357.87 Hz, 7359.87 Hz, 9784.52 Hz, 15987.41 Hz	4	2.8	14.3	87.6
DSW	170 Hz – 17 kHz	17	1.7	14.6	89.7
DSW	170 Hz – 17 kHz	17	2.6	14.5	89.1
DSW	170 Hz – 17 kHz	17	4.0	14.4	88.5
SINC	170 Hz – 17 kHz	17	5.6	14.2	86.9
SINC	170 Hz – 17 kHz	31	7.8	14.3	87.7
SINC	170 Hz – 17 kHz	141	17.3	14.3	87.6

6. Conclusions

This work comes from the methods using a multi-tone signal with uniform amplitude distribution in the spectral domain for ADC testing. In addition, this paper shows other ways to test or generate the testing signal with significant numbers of the spectral components. An elegant solution is to use a pulsed SINC signal, which has also uniform amplitude displacement of the spectral components. A benefit of those SINC or DSW methods is that you do not need to choose all parameters for every tone in comparison with the sum of sine-wave signals. For a FFT test it is better to have uniformly distributed amplitudes of the spectral components. Then every tone has the same impact on the results. In this case, the *ENOB* obtained from the test expresses the average performance of the DUT in the chosen frequency range. The results show that it is possible to use MT, DSW and SINC signals for DAC testing. After correction, the results are comparable with the standard *SW* FFT test. When the signal is generated by the DDS system, it is necessary to limit the frequency range by a digital filter. This method should be primarily used in a fast audio-codec test in a wide frequency bandwidth. However, the 2-MSamples were acquired during one test. Therefore, future research will be focused on reducing the number of samples with linear or non-linear fit test methods with economic aspects of short testing in the industry area.

To measure such a performance using a Single Sine Wave signal it is necessary to do as many tests as the number of tones of the band signal. For example, a 17-tone signal is used for testing. The standard procedure, which covers this frequency range, will consume 17 times much time in comparison with a band signal test. Another important thing how to save time is to sample a smaller amount of samples. It was proved that it is sufficient to use only 30 kSamples to reach 0.2 bit deviation of the *ENOB* in comparison with results obtained from 4 MSamples [4]. Typical costs for testing a 16-bit ADC are approximately 100 USD per hour. A typical test takes 80 seconds, a 17-tone test takes 22 minutes. The test using a band signal still takes 80 seconds; it means we save 35USD (during one 17-tone test). An interesting observation during the test is that the characteristic of the *ENOB* is slightly rippled. The ripple maximum is 0.4 bits. The question is how it affects the pulse signal test. From its nature, the pulse signal tests the average performance in a chosen frequency range. The average *ENOB* obtained from a set of single tests is 14.2 bits and the ENOB from a *SINC* test is between 14.2 bits and 14.3 bits. Therefore, it possible to say the results follow the idea. In future, it will be interesting to test a device that has a significant drop in the *ENOB* characteristic, for example a low cost audio codec.

Acknowledgment

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